Scientific Research Publishing

Journal of Applied Mathematics and Physics, 2025, 13(7), 2354-2362
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Cosmic Neutrino Background (CνB) Temperature Derived from the Stefan Boltzmann Law in $R_{H_t} = ct$ Cosmolgy

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How to cite this paper: Haug, E.G. (2025) Cosmic Neutrino Background (CνB) Temperature Derived from the Stefan Boltzmann Law in $R_{H_t} = ct$ Cosmolgy. *Journal of Applied Mathematics and Physics*, **13**, 2354-2362.
https://doi.org/10.4236/jamp.2025.137134

Received: June 23, 2025
Accepted: July 26, 2025
Published: July 29, 2025

Abstract

We are proposing a temperature formula for the Cosmic Neutrino background (CνB) temperature that can be derived from the Stefan-Boltzman law under certain assumptions, such as $R_{H_t} = ct$ cosmology. Our derived formula gives a prediction of the CνB temperature of 1.9336 K ± 0.0072 K which is in line with the current literature on the topic.

Keywords

CνB Temperature, CMB Temperature, Reissner-Nordstrom Metric, Haug Spavieri Metric

1. CMB Temperature

In contrast to the cosmic microwave background the cosmic neutrino background is very hard to detect directly as neutrinos are only known to have very weak interactions with other particles [1], so before we get to how to predict it we will sumamrize some important development in relation to the CMB temperature. The CMB temperature is approximately 2.725 K [2]-[6]. Tatum, Seshavatharam, and SLakshminarayana [7] suggested based on heuristics that the CMB temperature for $R_H = ct$ cosmology could be given by:

$$T_{cmb,t} = \frac{\hbar c^3}{k_b 8\pi G\sqrt{M_{cr,t} m_p}} = \frac{\hbar c}{k_b 4\pi\sqrt{R_t 2 l_p}} \tag{1}$$

where $R_t = \frac{c}{H_t} = ct$, $l_p = \sqrt{\frac{G\hbar}{c^3}}$ represents the Planck [8] [9] length, and k_b

denotes the Boltzmann constant. Furthermore $M_{cr,t} = \frac{c^2 R_{H_t}}{2G}$ is the time dependent critical Friedmann [10] mass.

Haug and Wojnow [11] [12] are the first to demonstrate that the following formula can be derived from the Stefan-Boltzmann law:

$$T_{cmb,t} = \frac{T_p}{8\pi}\sqrt{\frac{2l_p}{R_{H_t}}} \tag{2}$$

where $T_p = \sqrt{\frac{\hbar c^5}{G}} = \frac{m_p c^2}{k_b}$ is the Planck temperature. This Haug and Wojnow also demonstrated can be re-written into the Tatum et al formula.

Haug and Tatum [13] have indicated the CMB temperature also can be derived from geometric principles in a $R_h = ct$ type universe. See, for example, [14]. Haug [15] has demonstrated that the CMB temperature simply can be predicted as:

$$T_{cmb,t} = \sqrt{T_{max} T_{min,t}} \tag{3}$$

where $T_{max} = \frac{\hbar c^3}{k_b 8\pi G m_p} = \frac{\hbar c}{k_b 8\pi l_p}$ is the Hawking [16] temperature of a Planck mass black hole and $T_{min,t} = \frac{\hbar c^3}{k_b 8\pi G M_{bh}} = \frac{\hbar c}{k_b 4\pi R_{H_t}}$ is the Hawking temperature of the Hubble sphere, when $M_{bh} = M_{cr} = \frac{c^2 R_{H_t}}{2G}$ is the black hole mass equal to the critical Friedmann mass.

Equation (1), (2), and (3) are different ways to express the same concept. The most intuitive way is: $T_{cmb,t} = \sqrt{T_{max} T_{min,t}}$, as it shows that the CMB temperature is simply the geometric mean of the lowest and highest possible temperatures in the universe—something that is consistent with the universe being an ideal heat engine (Carnot engine); see [17]. The idea that the universe can operate as a heat engine is not new, nor is the idea that a black hole can possibly operate as a heat engine; see [18]-[24]. What is new is that Haug has recently demonstrated this leads to a CMB temperature that is the geometric mean of the maximum and minimum Hawking temperatures in a black hole universe.

2. Derivation of CνB Temperature from the Stefan-Boltzmann Law

The Cosmic Neutrino Background temperature is assumed to be approximately 1.95 K (see [25]). The cosmic microwave background (CMB) radiation spectrum is known to be that of a nearly perfect black body. This has been pointed out by for example Muller *et al.* [26]:

> *"Observations with the COBE satellite have demonstrated that the CMB corresponds to a nearly perfect black body, characterized by a temperature* T_0 *at* $z = 0$*, which is measured with very high accuracy,* $T_0 = 2.72548 \pm 0.00057\ \text{K}$*."*

It is natural to assume also the Neutrino background fits well the assumption of almost a perfect black body and thereby one should be able to use the Stefan-Boltzmann law for deriving the CνB temperature. Relic neutrinos, while fermions, are often approximated as an almost perfect blackbody spectrum due to their thermal decoupling in the early universe while still relativistic. Their distribution follows a Fermi-Dirac function with zero chemical potential, contributing an energy density suppressed relative to photons by a factor of 7/8, characteristic of relativistic fermions. After electron-positron annihilation, entropy conservation yields the canonical temperature ratio $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_{\text{CMB}}$. While bosonic treatments neglect the 7/8 factor, they can still capture the qualitative thermodynamic behavior of neutrino backgrounds, with the induced error in energy density being about 12.5%, a known and easily correctable factor when needed for precision cosmology.

From the Stefan-Boltzmann law, the Neutrino "luminosity" of the Hubble sphere must should be given by:

$$L_H = 4\pi R_H^2 \sigma T_\nu^4 \tag{4}$$

where T_ν is the Neutrino background temperature of the Hubble sphere. This should likely at least be a good first approximation as Neutrinos moves with almost indistinguishable speed to the speed of light so they behave like heat radiation.

Further the Hawking temperature of a Planck mass black hole must be as shown by Haug and Wojnow [11]:

$$T_{Hw,p} = \frac{\hbar g}{2\pi c k_b} = \frac{\hbar \frac{G m_p}{r_s^2}}{2\pi c k_b} = \frac{\hbar c}{8\pi l_p k_b} = \frac{T_p}{8\pi}. \tag{5}$$

We will assume Planck mass black holes are building blocks are linked to a hypothetical Planck mass particle that plays a central role in nature and cosmos, something discussed by Motz and Eppstein [27] as well as Haug [28]. The energy passing through a sphere with a radius equal to the Reissner-Nordström [29] [30] extremal solution or the Haug Spavieri [31] minimal solution of a Planck mass black hole must be:

$$E = \frac{L_H}{4\pi r_h^2}. \tag{6}$$

where $r_h = \frac{GM}{c^2} = \frac{1}{2} r_s$ is the extremal black hole event horizon. The radiant flux absorbed by the Planck mass black hole cross-section $\pi r_h^2 = \pi l_p^2$ is thus expressed as:

$$4\pi l_p^2 \sigma T_{Hw,p}^4 = \pi \left(2 l_p\right)^2 E = 4\pi l_p^2 \frac{4\pi R_H^2 \sigma T_\nu^4}{4\pi l_p^2} = 4\pi R_H^2 \sigma T_\nu^4 \tag{7}$$

Solved for T_ν this gives:

$$
\begin{aligned}
T_v &= T_{Hw,p}\sqrt{\frac{l_p}{R_H}} \\
T_v &= \frac{T_p}{8\pi}\sqrt{\frac{l_p}{R_H}} = \frac{\hbar c}{8\pi k_b\sqrt{R_H l_p}} \approx 1.9336\ \text{K} \pm 0.0072\ \text{K}.
\end{aligned} \tag{8}
$$

which is very close to what is also estimated by the standard cosmological model, as seen in [25] [32] [33], of about 1.95 K. We have used a $H_0 = 67.4 \pm 0.5$ km/s/Mpc as given by the Particle Data Group (PDG)[1] in their Astrophysical Constants and Parameters. We have naturally also taken into account the uncertanity in the Planck length and used the NIST CODATA value of $l_p = (1.616255 \pm 0.000018) \times 10^{-35}\ \text{m}$. However, unlike the CMB temperature [2] [3] [6], the CνB temperature has not been measured, as neutrinos interact so weakly with other particles that it is hard to measure, however it could be that we in the future are able to measure the CνB background temperature also.

3. Further Discussion

Equation (8) above is likely related to metrics such the Haug Spavieri metric and the extremal solutions off Kerr [34], and Kerr-Newman [35] [36], as seen in [37]). The Haug-Spavieri metric as well as the extremal Reissner-Nordstrom metric leads to a total mass-energy in the observable universe given by:

$$
M_u = \frac{c^2 R_H}{2G} + \frac{c^3 \Lambda R_H^2}{6G} = \frac{c^2 R_H}{G} \tag{9}
$$

The first part, $\frac{c^2 R_H}{2G}$, is identical to the critical Friedmann mass. The second part is linked to relativistic gravitational energy and the cosmological constant ("Dark energy"), which has an exact value of $\Lambda = 3\left(\frac{H_t}{c}\right)^2 = \frac{3}{R_H^2}$. This latter part of the total mass represents relativistic gravitational energy, which can also be considered dark energy since only gravitational interactions from it are likely observable. We can also write the CνB temperature as:

$$
\begin{aligned}
T_v &= \frac{\hbar c^3}{k_b 8\pi G\sqrt{M_u m_p}} = \hbar \frac{c}{\sqrt{R_H l_p}} \frac{1}{k_b 8\pi} = \frac{\hbar}{k_b 8\pi\sqrt{t_h t_p}} \\
&= \frac{\hbar\sqrt{H_0}}{k_b 8\pi\sqrt{t_p}} \approx 1.9336\ \text{K} \pm 0.0072\ \text{K}.
\end{aligned} \tag{10}
$$

where M_u represents the Haug-Spavieri universe mass as well as the Reissner-Nordström extremal universe mass [37] given by $M_u = 2M_{cr} = \frac{c^2 R_t}{G}$, where $R_t = ct$. This is equivalent to the critical Friedmann mass plus 50% dark energy, which essentially constitutes relativistic gravitational energy interacting with matter. The Haug-Spavieri universe adheres to $R_H = ct$ and also to the Haug ex-

[1]https://pdg.lbl.gov/2023/reviews/rpp2023-rev-astrophysical-constants.pdf

tremal universe. This naturally implies that the formula is applicable to any cosmic epoch. However, further investigation is warranted, both mathematically and experimentally, as each metric and argument used must be carefully investigated before one hastily draws conclusions.

4. Neutrino Density and Neutrino Density Parameter

We further predict that the density of the CνB radiation must be based on Planck's radiation law which should result in:

$$\rho_{\nu,t} = \frac{1}{c^2}\int_0^\infty h\nu n(\nu)\mathrm{d}\nu = \frac{a_b T_{\nu,t}^4}{c^2} \tag{11}$$

where $a_b = \frac{4}{c}\sigma = \frac{\pi^2 k_b^4}{15c^3\hbar^3}$ is the radiation constant.

Haug and Tatum [38] has demonstrated that the critical Friedmann density can be written as:

$$\rho_{cr,t} = \frac{3H_t^2}{8\pi G} = T_{cmb,t}^4 \frac{23040\pi}{c^3}\sigma \tag{12}$$

where $\sigma = a_b \frac{c}{4} = \frac{\pi^2 k_b^4}{60c^2\hbar^3}$ is the Stefan-Boltzmann constant and $a_b = \frac{4}{c}\sigma$ is the radiation constant and $T_{cmb,t}$ is the CMB temperature. The neutrino density parameter is defined as $\Omega_\nu = \frac{\rho_\nu}{\rho_{cr}}$ so we get:

$$\Omega_\nu = \frac{\rho_\nu}{\rho_{cr}} = \frac{\frac{\frac{4}{c}\sigma T_{\nu,t}^4}{c^2}}{T_{cmb,t}^4 \frac{23040\pi}{c^3}\sigma}$$

$$\Omega_\nu = \frac{T_{\nu,t}^4}{T_{cmb,t}^4 5760\pi} \tag{13}$$

$$\Omega_\nu = \frac{1}{23040\pi} \approx 1.38\times 10^{-5}$$

This means the Neutrino temperature after the neutrino decoupling can be written as (see also [39])

$$T_{\nu,t} = \hbar \frac{c}{\sqrt{R_{H_t} l_p}} \frac{1}{k_b 8\pi} = \sqrt[4]{\frac{3c^2\Omega_\nu}{8\pi G a_b}} \frac{1}{t^{1/2}} \tag{14}$$

If we input $t = \frac{1}{H_0}$ and are using a $H_0 = 66.9$ km/s/Mpc (see Haug and Tatum [40] for estimation of such a H_0) we get

$$T_{\nu,0} = \sqrt[4]{\frac{3c^2\Omega_\nu}{8\pi G a_b}} \frac{1}{\left(\frac{1}{H_0}\right)^{1/2}} \approx 1.93\ \mathrm{K} \tag{15}$$

Which is very close to the assumed CνB temperature in standard theory also [25], despite calculated from $R_{H_t} = ct$.

Equation (14) is not restricted only to a specific $R_{H_t} = ct$ model, but can be used to predict the CνB temperature also in other models that uses different assumptions and therefore have a different function or value for Ω_ν.

It is also interesting to note that we have:

$$\frac{\Omega_\nu}{\Omega_\gamma} = \frac{\rho_\nu}{\rho_\gamma} = \frac{5760\pi}{23040\pi} = \frac{1}{4} \tag{16}$$

An interesting question is why the CνB density would be exactly one fourth of the CMB density. If this is correct also in the real universe, what would be the physical explanation for this? We do not yet have the answer, but we encourage others to research this as well.

5. Further Discussion

The derivations above are based on certain assumptions, such as the metric solutions of Reissner-Nordström or the Haug-Spavieri metric. While the CMB temperature has been measured with high precision and aligns well with recent developments in CMB predictions, the CνB temperature remains highly uncertain from an observational standpoint. Therefore, theoretical models regarding it should remain open to revision. The uncertainty primarily lies in Ω_ν; however, there is no such uncertainty in the model derived above, which yields an exact value of $\Omega_\nu = \frac{1}{23040\pi}$. Still, this result is based on assumptions that, with today's technology, are difficult to test due to the challenges involved in measuring the CνB temperature.

An interesting question is why the predicted neutrino and CMB energy densities are so close in value. Specifically, why do we find that the CνB density is exactly a quarter of the CMB density: $\frac{\rho_\nu}{\rho_\gamma} = \frac{1}{4}$. Could the C$\nu$B be more closely related to the CMB than is commonly assumed? This and many other questions warrant further investigation. We have to be aware that we are working in a $R_{H_t} = ct$ model here. Also, the ratio between the CMB density and the critical density remains constant from CMB decoupling until today; it is $\Omega_\gamma = \frac{\rho_\gamma}{\rho_{cr}} = \frac{1}{5760\pi}$, see [41]. The physical explanation for this is likely either that the universe is a growing black hole, where the critical mass also increases linearly with R_{H_t}, or that the universe is an extremal black hole, where at least the apparent density changes as we observe matter farther away, see [42].

In the standard model, the neutrino temperature is given by $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_{cmb}$, while in our model it is $T_\nu = \frac{1}{\sqrt{2}} T_{cmb}$. The difference in model predictions of the CνB temperature is about 1%. Unfortunately, to this day, the neutrino temperature has not been measured accurately enough to distinguish between the two models. However in the future one can hope this is possible.

6. Conclusion

We have suggested a formula that links the CνB temperature and the Hubble constant (Hubble radius and Hubble time), as well as actually also linking the CνB to the CMB temperature, as we, based on the derivations above, must have the following relation: $\frac{T_\nu}{T_{cmb}} = \frac{1}{\sqrt{2}} \approx 0.70711$ and further that $\frac{\rho_\nu}{\rho_\gamma} = \frac{1}{4}$, through the cosmic epochs of $R_{H_t} = ct$ cosmology. Furthermore, we have derived a CνB density parameter equal to $\Omega_\nu = \frac{1}{23040\pi} = \frac{1}{4}\Omega_\gamma$, valid for any time after neutrino decoupling inside $R_{H_t} = ct$ cosmology.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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