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Finding the Planck Length from the Union2 Supernova Database in a Way That Appears to Resolve the Hubble Tension

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Abstract

Haug and Tatum have developed a cosmological model which links the CMB temperature, the Hubble parameter, cosmological redshift, and the Planck length in a manner fully consistent with general relativity and the Stefan-Boltzmann law. This means that one can easily extract the Planck length from observed cosmological redshifts. We demonstrate this by extracting the current NIST CODATA Planck length from the Union2 supernova database using the observed redshifts from all 580 type Ia supernovae. Our new mathematical approach places tight constraints on a Hubble constant extracted from supernovae in a manner that appears to resolve the Hubble tension. Importantly, our linear expansion model offers a near-perfect match using these astronomical observations without necessitating the expansion of space beyond the speed of light c or the introduction of an accelerating dark energy. We believe that this approach strongly favors growing black hole $R_H = ct$ cosmological models over the Λ-CDM model.

Keywords

Hubble Tension, Planck Length, Hubble Constant, CMB, Cosmological Redshift, Upsilon Constant, General Relativity, Cosmological Models

1. Introduction and Background

Haug and Tatum [1] have recently presented a cosmological model that is consistent with both general relativity theory and a newly-quantized version of general relativity theory, which we will discuss shortly. Haug and Tatum's cosmological model represents the culmination of many years of work by several researchers, wherein each piece of the puzzle has gradually fallen into place, leading to the sudden convergence of various elements into a single model that appears to be a very simple and powerful cosmological framework. Here, we will outline some of these components. In 2015, Tatum *et al.* [2] arrived at the following predictive formula for the CMB temperature:

$$T_{cmb} = \frac{\hbar c^3}{k_b 8\pi G\sqrt{M_c m_p}} = \frac{\hbar c}{k_b 4\pi\sqrt{R_H 2l_p}} = \frac{\hbar c}{k_b 4\pi\sqrt{\frac{c}{H_0}2l_p}} \tag{1}$$

where k_b is the Boltzmann constant, M_c is the mass equivalent of the critical Friedmann [3] universe $M_c = \frac{c^2 R_H}{2G}$, R_H is the Hubble radius, H_0 is the Hubble parameter, l_p is the Planck length, and m_p is the Planck mass. It should be noted that the left side of this equation is very similar to the Hawking black hole radiation temperature formula, except that the M in the Hawking denominator is replaced by $\sqrt{M_c m_p}$.

Recently, Haug and Wojnow [4] [5] have derived the same formula from the Stefan-Boltzmann law. Furthermore, since Haug and Tatum [6] have also derived the same formula from general geometrical principles of the Hubble sphere, this suggests that this formula could be valid within multiple black hole cosmological models. In addition, Haug [7] has also shown how one can derive this formula if one assumes light bending (space-time bending) is quantized and linked to the Planck scale.

Haug and Tatum [8] have recently also presented a thermodynamic Friedmann-type framework consistent with the CMB formula above.

Equation (1) essentially directly binds together the CMB temperature and the Hubble constant, something that has not been done in other models. It also links the Planck length with the CMB temperature and the Hubble constant, as elaborated further in great detail by Haug [4]. We will soon revisit how this is also consistent with a Planck-scale quantized version of general relativity theory, which quantizes general relativity theory without altering any output predictions. Insight into general relativity from a deeper perspective indeed seems to link gravity with the Planck scale, something that was predicted by Eddington [9] as early as 1918 and has only recently come to pass.

Haug and Tatum [1] recently have also provided mathematical proof that Equation (1) is consistent with a cosmological redshift of:

$$z = \frac{R_H}{R_t} - 1 \tag{2}$$

which Haug and Tatum demonstrate can only be consistent with $T_t = T_0(1+z)^{\frac{1}{2}}$. Alternatively, Equation (1) is also consistent with a cosmological redshift of the form:

$$z = \sqrt{\frac{R_H}{R_t}} - 1 \tag{3}$$

which they demonstrate is consistent with the well-known CMB temperature and z relation $T_t = T_0(1+z)$. Observations seem to favor the latter CMB and z relation, as seen in [10]. From the Haug and Tatum model we also must have:

$$t_h = \frac{D(1+z)^2}{2cz + cz^2} \tag{4}$$

and

$$R_H = \frac{D(1+z)^2}{2z + z^2} \tag{5}$$

and

$$H_0 = \frac{2cz + cz^2}{D(1+z)^2} \tag{6}$$

applying the first term of the Taylor series expansion, which is valid for $z \ll 1$, gives:

$$H_0 \approx \frac{2cz}{D} \tag{7}$$

Furthermore, for any z we have:

$$D = \frac{2cz + cz^2}{H_0(1+z)^2} \tag{8}$$

This represents a proper distance to an object at redshift z. Notably, none of the three distances in the Λ-CDM model corresponds to the distance in our model. At first glance, one might think that our model must already be ruled out, as the distances it predicts would not match any of those derived from a given observed redshift within the Λ-CDM framework. However, this assumption would be incorrect. All cosmological distances in the Λ-CDM model, or any other model for that matter, are theoretical, model-dependent predictions, and none of them is directly observed.

At long and short cosmological distances, this result is different than the distance given by the Λ-CDM model, since the first term of its Taylor series expansion alone is given by:

$$D \approx \frac{2cz}{H_0} \tag{9}$$

This is twice the distance predicted by the Λ-CDM model: $d \approx \frac{cz}{H_0}$, when $z \ll 1$.

The only truly independent method for measuring distances is parallax, which relies on pure geometric principles. However, parallax is practically limited to very short cosmological distances. Hypothetically, parallax could be applied to distant galaxies. Yet, for such distances, standard parallax methods do not retain their robustness or accuracy. It is well known that applying standard parallax to distant galaxies would require redshift adjustments. Thus, even parallax-based distance predictions for such distant objects would be influenced by the underlying cosmological model. See, for example, Hogg [11].

The fact that our linear expansion model can accurately match all observed supernova redshifts, as we will soon demonstrate, without requiring adjustments for phenomena such as an accelerating dark energy is, in our view, a very strong indication that we are on the right track with our new cosmological redshift and corresponding distance formulae.

It is important to note here that we will be referring to a particular subclass of $R_H = ct$ linear expansion cosmology models. $R_H = ct$ cosmology is an actively-explored cosmology to this day. See, for example, [12]-[15]. There are multiple sub-classes of $R_H = ct$ cosmology models. Herein, we will be working under the assumption that the Hubble sphere acts as a growing black hole. The idea that the Hubble sphere could be treated as a black hole was pointed out as early as 1972 by Pathria [16]. Even if black hole cosmology models are much less well-known than the Λ-CDM model, they are seriously discussed to this day. See, for example, [17]-[26]. Recent observations by JWST have also brought back the discussions of whether our observable universe (the Hubble sphere) could be a black hole, see Shamir [27].

2. Planck-Quantized General Relativity Theory

Max Planck [28] [29] assumed in 1899 that there were three universal constants: the speed of light c, the gravitational constant G, and the Planck constant $\hbar$. Combining these with dimensional analysis, he arrived at a unique length: $l_p = \sqrt{\frac{G\hbar}{c^3}}$, time: $t_p = \sqrt{\frac{G\hbar}{c^5}}$, mass: $m_p = \sqrt{\frac{\hbar c}{G}}$, and temperature: $T_p = \frac{1}{k_b}\sqrt{\frac{\hbar c^5}{G}}$. These are known today as the Planck units or natural units. However, in Planck's day, it was far from clear whether these Planck units had a real physical significance, as they emerged solely from dimensional analysis. For example, Bridgman [30], who received the 1946 Nobel Prize in physics, considered them to be pure mathematical artifacts unrelated to physical reality.

Quantum gravity has been an unsolved challenge for more than 100 years. Already in 1916, Einstein [31] pointed out that the next step in gravity was quantum gravity, or in his own words:

"*While, according to the intra-atomic electron movement, atoms might emit not only electromagnetic but also gravitational energy, albeit in a minuscule amount. Since this should not be true in nature, it seems that the quantum theory must modify not only Maxwell's electrodynamics but also the new theory of gravitation.*"—Albert Einstein, 1916.

Einstein devoted much of the remainder of his life to this endeavor, but with little success. However, Eddington [9] had already provided an important hint in 1918, suggesting that quantum gravity likely had to be dependent in some way on the Planck length. Today, most researchers, particularly those working on developing quantum gravity theory, acknowledge the significance of Planck units [32]-

[34]. The two best-known attempts to develop a reasonable quantum gravity theory have been string theory and loop quantum gravity theory (LQG). However, despite the initial enthusiasm, these theories have not achieved any consensus among experts in the field. Despite the considerable talents of researchers involved in the development of string theory and LQG, development of a useful quantum gravity theory has eluded them.

Since there has been no lack of talent in this pursuit, and yet quantum gravity theory remains unsolved even after 100 years, it could be that, at some point in the history of physics, a wrong turn was taken. If such was the case, we must first backtrack to the point where the wrong turn was possibly taken and proceed from there. This is what Haug [35] has recently tried to do, leading to a new and simple way to Planck-quantize general relativity theory. While this is not the main focus of our paper, a brief historical context will provide the reader with an interesting background for what we will soon demonstrate: a close connection between the quantum scale and the cosmic scale, and how the quantum scale, in the form of the Planck length value, constrains the H_0 value when considering cosmological redshift. We propose the result will be our newly-quantized version of general relativity theory.

This brief history begins with Newton [36] in 1686. Newton's original gravity force formula was simply $F = \frac{M_n m_n}{r^2}$. There was no gravitational constant in Newton's formula, which he only expressed in words in his *Principia*. Despite this, Newton provided a series of accurate gravitational effect predictions. See Cohen [37]. For hundreds of years, Newton's gravity force formula was used to find planetary orbital velocities, the masses of planets, as well as their gravitational accelerations. However, Newton's mass definition was quite different from today's. Maxwell [38] used Newton's original framework as late as 1873, describing gravitational acceleration simply as $g = \frac{M_n}{r^2}$ (in contrast to today's formula of $g = G\frac{M}{r^2}$), meaning that the Newtonian mass of the Earth was $M_n = gr^2$. Since gravitational acceleration has dimensions of $L \cdot T^{-2}$, this implies that Newtonian mass dimensions were $L^3 \cdot T^{-2}$. So, obviously, this Newtonian unit was very different from today's kilogram mass unit. Maxwell actually took note of this history. It was known as "astronomical mass" and, for many years, had been understood in astrophysics in relation to Newton's theory. However, for earthly macroscopic objects and even microscopic ones, the kilogram had become the standard in France and the pound in Great Britain, as Maxwell also mentioned.

There had been discussions for some years on whether it would be favorable to use the same mass standard for astronomical objects as for everyday macroscopic and microscopic ones, preferably across countries. The kilogram was ultimately implemented as the standard in all areas of physics. This meant that the kilogram mass had to be introduced into Newton's formula. However, the original Newton formula no longer worked if one simply replaced M_n and m_n with their kilogram counterparts M and m. Something was now missing from the formula, which could be fixed by introducing a constant that soon would be known as Newton's gravitational constant, despite Newton never attempting to invent it or even search for it. Furthermore, Cavendish [39] in 1798 also did not attempt to measure the gravitational constant, nor did he mention a gravitational constant, as incorrectly claimed in multiple papers and books, including by Feynman. See [40] [41]. What is known today as Newton's gravitational constant was actually first introduced in 1873 by Cornu and Baille [42] to make the Newtonian formula still work after replacing the Newtonian mass with the kilogram mass definition. Thüring [43] pointed out that this gravitational constant was introduced without a deep understanding of its physical significance. We realize that there is nothing inherently wrong with the gravitational constant; it is indeed a constant and it is clearly needed when working with the kilogram definition of mass. The key question, however, is: "What exactly does the gravitational constant represent, from a deeper perspective?"

Hossenfelder [44], in her otherwise excellent book, claims: "*Newton's constant* (*G*) *quantifies the strength of gravity.*" However, this does not seem to be the case, since one could clearly predict the same Newtonian gravity phenomena with Newton's original formula, which had no explicit gravitational constant. As early as 1984, Cahill [45] [46] solved the Planck mass formula for G and obtained $G = \frac{\hbar c}{m_p^2}$. He suggested that the Planck mass might be more fundamental than G and that the gravitational constant could be expressed in this way as a composite constant. However, in 1987, Cohen correctly pointed out that, since no one had demonstrated how to derive the Planck units without first knowing G, Cahill's approach led to a circular problem in reasoning. Such views persisted at least until 2016, as mentioned in a paper by McCulloch [47], wherein the circular reasoning problem is also addressed.

Nevertheless, in 2017, Haug [48] demonstrated that one could find the Planck length from even small macroscopic objects using a Cavendish apparatus without knowledge of G, and later he showed that one can find the Planck length without relying on G or $\hbar$ at all [49] [50].

Furthermore, in 2021, Haug demonstrated how one could find the Planck length from cosmological redshift without knowing G, but by assuming $z \approx \frac{H_0 c}{d}$, although this formula is only an approximation valid for low z. Herein, we extend this history by developing a method that can be used to find the Planck length even from higher z cosmological measurements. The main focus of our paper, however, is to demonstrate that the Planck length imposes constraints on the acceptable values of the Hubble constant, even when extracting it from observed cosmological redshifts.

It is also highly significant that we can find the Planck length using two entirely different methods. The first method follows Max Planck's approach based on dimensional analysis, yielding:

$$l_p = \sqrt{\frac{G\hbar}{c^3}} \tag{10}$$

Since the Planck constant and the speed of light today are defined as exact (by NIST CODATA 2018) with no uncertainty, the only uncertainty in the Planck length, using this definition, must come from uncertainty in the gravitational constant. The gravitational constant is one of the least precise physical constants, and enormous work has been done to measure it more precisely, see for example [51]-[54]. Today, the NIST CODATA 2018 standard gives it a value of $G = 6.67430\times10^{-11} \pm 0.00015\times10^{-11}\ \mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$. This means we must have:

$$\begin{aligned} l_p &= \sqrt{\frac{G\hbar}{c^3}} = \sqrt{\frac{6.67430\times10^{-11} \pm 0.00015\times10^{-11} \times 1.054571817\times10^{-34}}{299792458^3}} \\ &= 1.616255\times10^{-35} \pm 0.000018\times10^{-35}\ \mathrm{m} \end{aligned}$$

Here, one might well ask how the above historical context may guide us in quantizing general relativity theory. We propose that if one can find the Planck length without knowledge of G, it is indeed possible to express G in terms of Planck units. If so, then we can use $G = \frac{l_p^2 c^3}{\hbar}$ and now substitute this identity into Einstein's field equation, yielding (see [4] [55]):

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi l_p^2}{c\hbar}T_{\mu\nu} \tag{11}$$

The Planck length now becomes a part of Einstein's field equation. However, the benefit of doing this is not entirely clear until we solve the equation for certain boundary conditions and examine the metric solution, such as the Schwarzschild solution. Before doing this, however, we will first take advantage of one more very simple but, in our view, very important way to consider a kilogram mass. In 1923, Compton [56] described the Compton wavelength of an electron as $\bar{\lambda} = \frac{\hbar}{mc}$ and measured it through Compton scattering. If we solve the Compton wavelength formula for the mass, we obtain:

$$m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c} \tag{12}$$

In the spirit of complementarity, we will assert that any kilogram mass can be represented in this way, not only the mass of an electron. The idea that protons could also have a Compton wavelength has been discussed by multiple authors [57] [58]. In actuality, it is likely that only fundamental particles have a "physical" Compton wavelength. The Compton wavelength of a composite mass can be seen as the aggregate of the Compton wavelengths of all of the constituent elementary particles, including even photons, as the rest mass energy of the photon can be expressed through the Compton wavelength, see [59]. To make a long story short, this means, for example, that the Schwarzschild metric can be expressed as (by replacing G with $G=\frac{l_p^2c^3}{\hbar}$ and the kilogram mass M with $M=\frac{\hbar}{\bar{\lambda}}\frac{1}{c}$):

$$\mathrm{d}s^2=\left(1-\frac{2GM}{c^2r}\right)c^2\mathrm{d}t^2-\left(1-\frac{2GM}{rc^2}\right)^{-1}\mathrm{d}r^2-r^2\mathrm{d}\Omega^2$$

$$\mathrm{d}s^2=\left(1-\frac{2l_p}{r}\frac{l_p}{\bar{\lambda}_M}\right)c^2\mathrm{d}t^2-\left(1-\frac{2l_p}{r}\frac{l_p}{\bar{\lambda}_M}\right)^{-1}\mathrm{d}r^2-r^2\mathrm{d}\Omega^2 \tag{13}$$

The term $\frac{l_p}{\bar{\lambda}_M}$ represents the reduced Compton frequency in the gravitational mass of interest. This is natural, since we have the reduced Compton frequency per second as $f=\frac{c}{\bar{\lambda}}$ and the reduced Compton frequency per Planck time is then $f=\frac{c}{\bar{\lambda}}t_p=\frac{l_p}{\bar{\lambda}_M}$, which, in our view, achieves the quantization of matter and gravity. Interestingly, multiple recent research studies do indeed indicate that matter ticks at the Compton frequency, see [60] [61].

Similarly, this approach can be applied to other metric solutions, such as the Kerr [62] or Kerr-Newman [63] [64] solutions, which are often used to describe black holes. The extent to which this quantized general relativity can be unified with quantum mechanics is beyond the scope of this paper, but will be addressed in the near future.

For our current purpose, we now have a straightforward formulation of general relativity that includes the Planck length, yet does not alter any predictions from general relativity. The potential benefit of Equations (11) and (13) is that they likely allow for a new and deeper insight into the phenomenon of quantum gravity. Modern cosmology theory has its origin in general relativity theory; and $R_H=ct$ cosmology clearly has a general relativistic framework. As one will see from our particular sub-class of $R_H=ct$ cosmology models, it is now possible to extract the Planck length directly from cosmological redshift, by using the entire Union2 supernova redshift database. In view of our new approach to general relativity theory, this result is fully consistent. What is most important in this paper is that the Planck length and its mathematical relationship to the Hubble constant appears to impose a constraint on the Hubble constant that one can extract from the supernova database. We maintain that this discovery appears to solve the Hubble tension. Nevertheless, extraordinary claims require extraordinary proof, so we will attempt to demonstrate this carefully in the next sections.

3. How Finding the Planck Length from the Union2 Supernova Redshift Database Appears to Resolve the Hubble Tension by Putting Constraints on H_0

Here, we will demonstrate that one can find the Planck length from the Union2 supernova database without first relying on a knowledge of the value of G. Let us first revisit the CMB prediction redshift formula of Haug and Tatum [1]. In its most general form, it is given by:

$$T_{cmb,t} = \frac{\hbar c}{k_b 4\pi\sqrt{R_t 2l_p}} = T_0\left(1+z\right) \tag{14}$$

Solving for R_t we get:

$$R_t = \left(\frac{\hbar c}{T_0\left(1+z_{obs}\right)k_b 4\pi}\right)^2 \frac{1}{2l_p} \tag{15}$$

Next, we input this for R_t in our cosmological redshift formula:

$$z_{pre} = \sqrt{\frac{R_H}{R_t}} - 1 = \sqrt{\frac{\frac{c}{H_0}}{\left(\frac{\hbar c}{T_0\left(1+z_{obs}\right)k_b 4\pi}\right)^2 \frac{1}{2l_p}}} - 1 \tag{16}$$

To use this redshift prediction formula to predict or, more precisely, attempt to match, the redshift from the Union2 supernova database, we need to know the value of the Planck constant. It is defined exactly today based on the NIST CODATA 2018 value of $\hbar = 1.054571817\times10^{-34}\ \text{J}\cdot\text{s}$. The speed of light is also exactly defined as $c = 299792458\ \text{m/s}$, and the Boltzmann constant is exactly defined as $k_b = 1.380649\times10^{-23}\ \text{J}\cdot\text{K}^{-1}$. Therefore, in c, $\hbar$, and k_b, there is no uncertainty. In addition, we need the CMB temperature, the Hubble constant, and the Planck length to predict the cosmological redshift. For T_0, we require the CMB temperature at present. The CMB temperature in the current cosmic epoch is measured very accurately in a series of recent studies, as seen in [65]-[68]. We will use the CMB temperature published by Dhal *et al.* [68] of 2.725007 ± 0.000024 K.

For the Hubble constant, we will initially use the value from one of the most recent H_0 studies involving supernovae by one of the leading research teams. It is important to note here that there is little or no disagreement regarding the CMB temperature at present. For example, the 2009 Fixen [67] study of the current CMB temperature reports the value 2.72548 ± 0.00057 K. However, for the Hubble constant, the standard uncertainties are much larger. Additionally, measurements of the Hubble constant from supernovae have yielded considerably different values compared to those obtained from the CMB. This phenomenon is known as the Hubble tension.

In our model, remarkably, we also need the Planck length. It's important to note that, in the standard view of Newtonian physics and general relativity theory, there hasn't been a successful attempt to connect gravity theory with the Planck length. As pointed out in the last section, Haug has recently claimed to have developed a quantized version of general relativity theory where, from a deeper perspective, we see the Planck length and the Compton wavelength as playing an important role. Theory is one thing, but herein we will use real observations in comparison to theoretical predictions, in much of the remainder of this paper.

Here, we will assume that we do not know the exact value of the Planck length except for very rough estimates and qualified guesses. We will start by guessing the Planck length as $l_p = 5\times10^{-35}$ meters. Then, we input this value together with the CMB temperature from the Dhal study and the Hubble constant from Murakami *et al.* [69] of 73.01 ± 0.85 km/s/Mpc into our redshift prediction formula for each type Ia supernova, and plot our findings relative to the real observations. This is illustrated in **Figure 1**. The predicted redshifts based on our Planck length guess of $l_p = 5\times10^{-35}$ are represented by the red line, which is far above the observed redshifts represented by the blue line. This indicates that our guess for the Planck length is too high. Therefore, we make another qualified, but still wild, guess of the Planck length being $l_p = 0.5\times10^{-35}$. Based on this Planck length guess, we obtain predicted redshifts represented by the green line. As we can see, the green line gives way too low predicted redshifts compared to the observed ones. Hence, we deduce that the Planck length must lie between 0.5 × 10^{-35} m and 5 × 10^{-35} m. We can continue with trial-and-error like this to minimize the error between the observed and predicted redshifts: $\min\sum_{i=1}^{580} z_{obs}(i) - z_{pred}(i)$. The number 580 is used in our calculation because we take into account every single supernova in the Union2 database; there are 580 observed type Ia supernova redshifts in the database. A simple "manual" trial-and-error method will work, or we can use a more efficient and "intelligent" trial-and-error method like the bisection method or the Newton-Raphson method. Both algorithms are much faster. One can even use the nearly instantaneous Goal Seek function in Excel, which likely employs a bisection method.

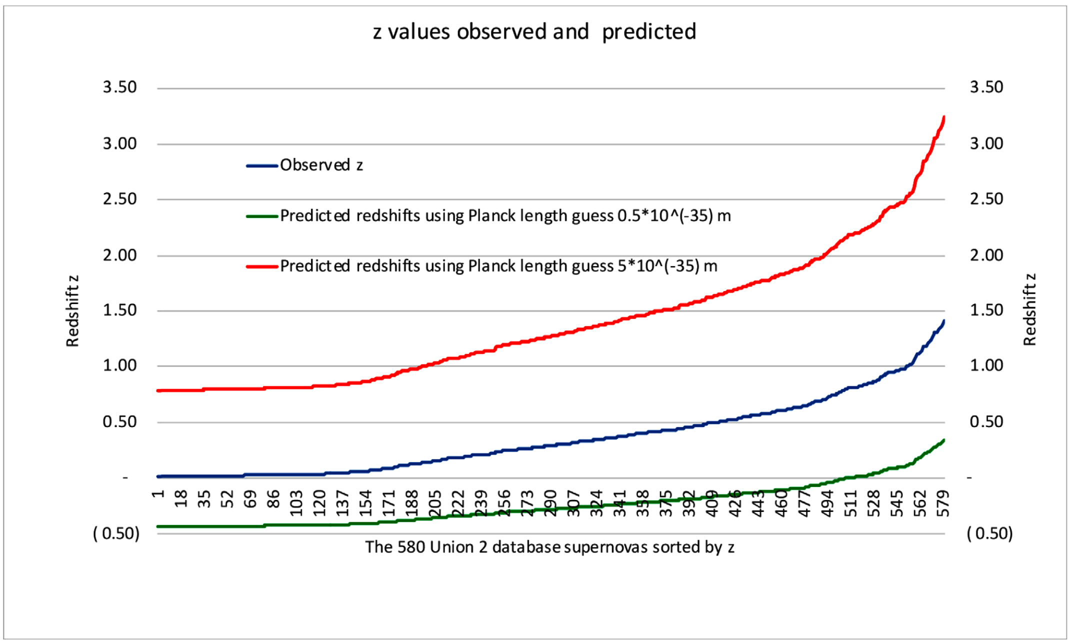

Figure 1. This figure shows the predicted supernova redshifts with an assumed $H_0 = 73.01$ km/s/Mpc and wild guesses of the Planck length of 5 × 10^{-35} m and 0.5 × 10^{-35} m, as well as the observed redshifts.

Our trial-and-error method that minimizes the difference between the predicted and observed redshifts yields an estimated Planck length of $l_p \approx 1.7646\times10^{-35}$ m, as shown in **Figure 2**. It is important to note that this estimated Planck length carries additional uncertainty due to the uncertainties in the Cosmic Microwave Background (CMB) temperature and the Hubble constant (H_0) that we used. The uncertainty in CMB observations is significantly smaller compared to that for the Hubble constant. Specifically, considering the Hubble tension, the uncertainty in H_0 becomes considerably large. Not surprisingly, a Planck length of 1.7646 × 10^{-35} m significantly deviates from the Planck length estimated through dimensional analysis. Returning to Max Planck's formula, the Planck length is defined as $l_p = \sqrt{\frac{G\hbar}{c^3}}$, where the uncertainty in the Planck length estimate then primarily arises from the uncertainty in the best estimates of G. According to NIST CODATA 2018, the reported value of l_p is 1.616255 × 10^{-35} m, with a standard uncertainty of 0.000018 × 10^{-35} m. The Planck length estimate derived from the supernova database, using the Hubble constant from Murakami *et al.* of 73.01 ± 0.85 km/s/Mpc, is 8241σ outside the NIST CODATA estimate (based solely on dimensional analysis). We can slightly adjust the CMB temperature in our input based on its standard deviation, but we still remain far from the recognized Planck length based on dimensional analysis.

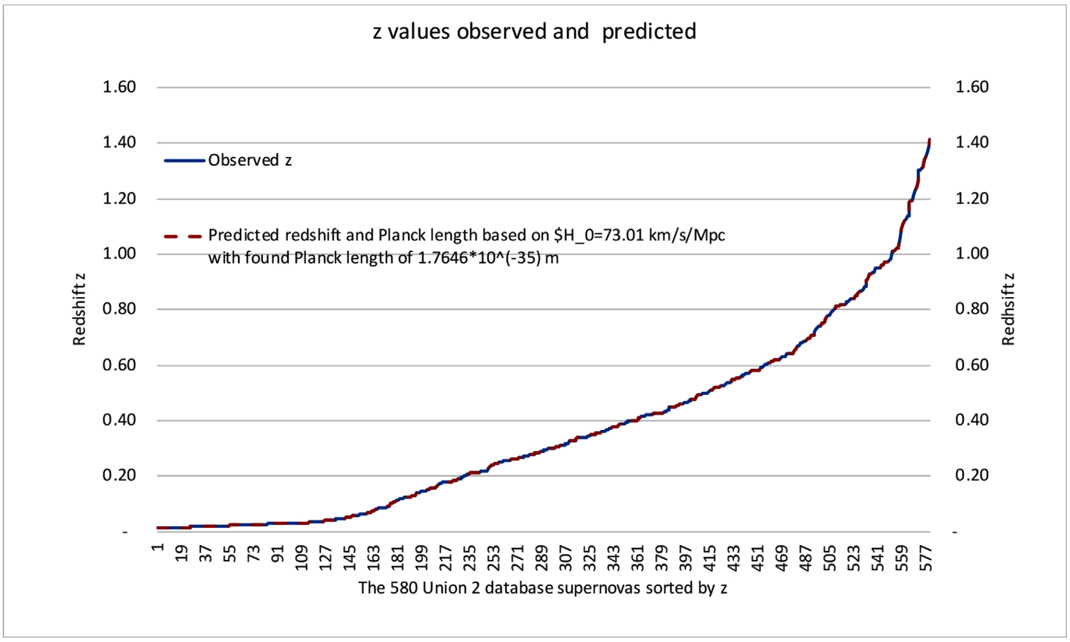

Figure 2. This figure shows that if we assume a Hubble constant value of $H_0 = 73.01 \pm 0.81$ km/s/Mpc, the matching found Planck length must be 1.7646 ± 0.0020 × 10^{-35} m.

In **Figure 3**, we propose that the Planck length must fall within its standard deviation (STD) as defined by the NIST CODATA 2018. This assumption leads to an estimated Hubble constant from the Union2 redshift database of $66.8711^{+0.0019}_{-0.0019}$, km/s/Mpc. This estimation potentially resolves the Hubble tension, because it utilizes the entire supernova database, with observed redshifts ranging from $z = 0.015$ to $z = 1.414$, to find the matching Hubble constant. In other words, to maintain the Planck length within its uncertainty range, especially when considering the small standard deviation in the Cosmic Microwave Background (CMB) temperature, this is the matching value necessary for the Hubble constant.

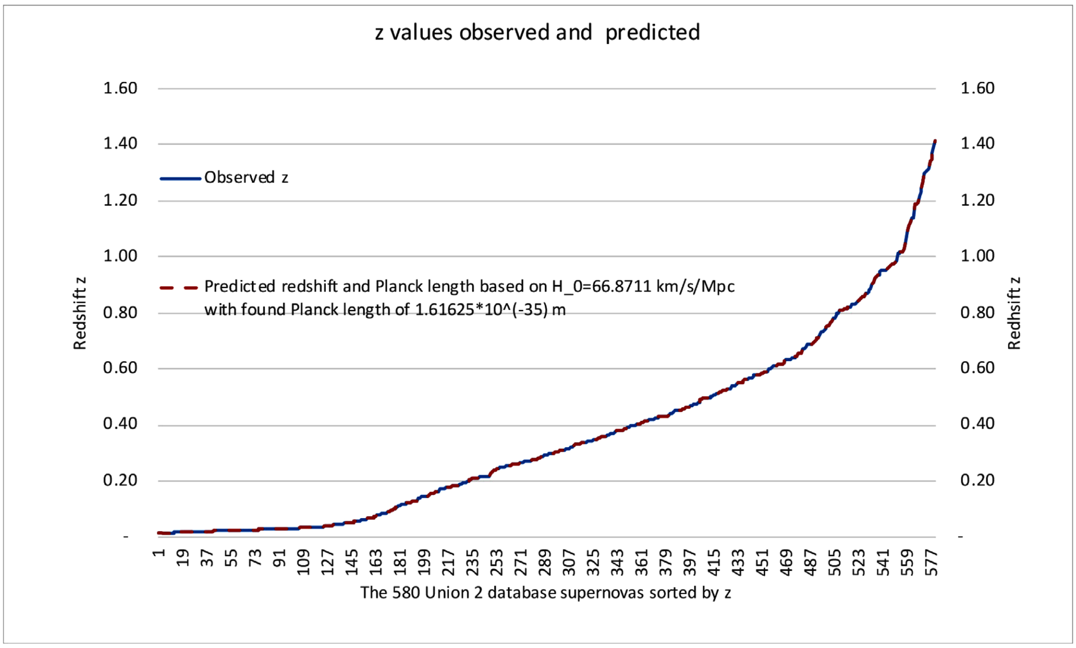

Figure 3. This figure shows the predicted redshifts and found Planck length when using a Hubble constant of $H_0 = 66.8711$ km/s/Mpc. We take into account the uncertainty in the current CMB temperature from the Dhal et.al study and find that there is a match with the NIST CODATA 2018 value of the Planck length as a constraint on the Planck length. We find that, to be inside the acceptable Planck length uncertainty, we must match the Union2 supernova redshift database with a Hubble constant value of $H_0 = 66.8711 \pm 0.0019$ km/s/Mpc. To put it another way, if we want the Hubble constant value to be outside this value in relation to the observed supernova redshifts, then we must accept a Planck length outside of the one STD uncertainty given by NIST CODATA for the Planck length. As we have seen from the previous figure, a Hubble constant value of around 72 to 73 km/s/Mpc is totally unacceptable in our model, as it leads to unacceptable Planck length tension. We conclude that neither Hubble tension nor Planck length tension is necessary, if one uses our model.

4. Supernova Team H_0 Determinations Point to Incorrect CMB Temperature Predictions from Union2 Supernova Redshifts

We can also find the CMB temperature from cosmological redshift based on Equation (16) that we repeat here for convinence:

$$z_{pre} = \sqrt{\frac{R_H}{R_t}} - 1 = \sqrt{\frac{\frac{c}{H_0}}{\left(\frac{\hbar c}{T_0\left(1+z_{obs}\right)k_b 4\pi}\right)^2 \frac{1}{2l_p}}} - 1 \tag{17}$$

This time, we will assume that the Planck length is known, and we will used the NIST CODATA 2018 value for it. In addition, we will use the H_0 value estimated by Riess *et al.* [70] at $H_0 = 73.04 \pm 1.04$ km/s/Mpc. Based on these inputs, we can now minimize the difference between the predicted and observed redshifts by adjusting the T_0 until we have minimized the error, according to $\min \sum_I^{580} z_{obs,i} - z_{pre,i}$. This is illustrated in **Figure 4**.

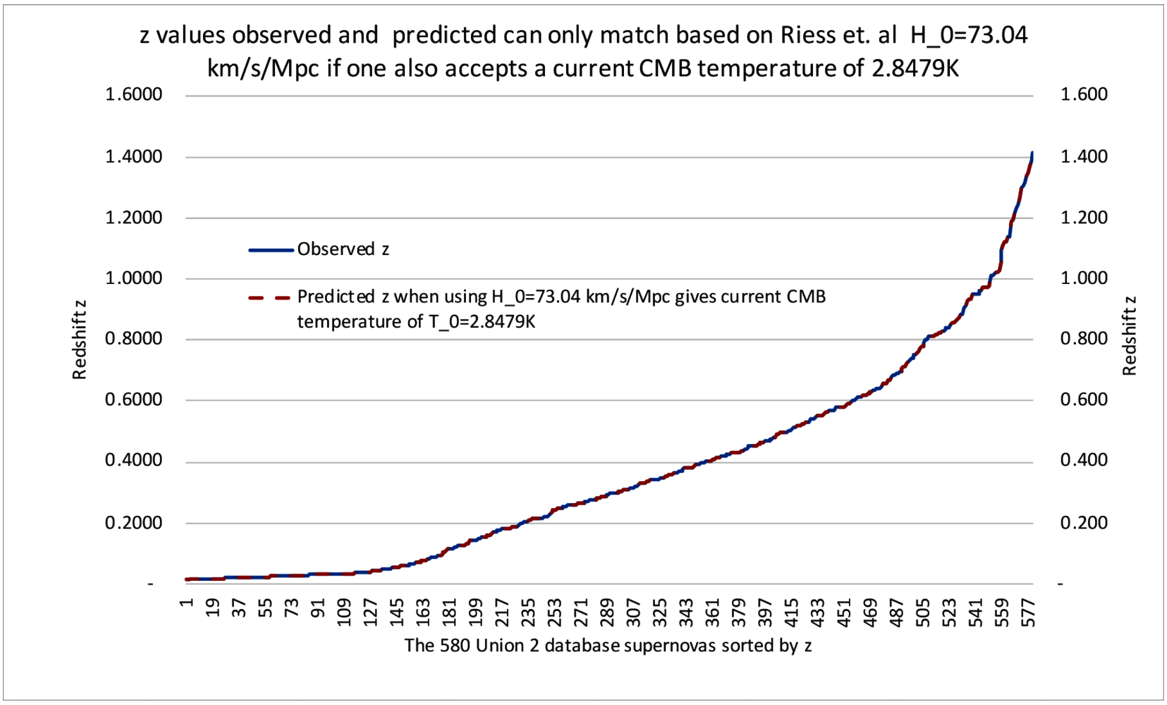

Figure 4. This figure shows that, based on the Riess et.al Hubble constant determination of $H_0 = 73.04 \pm 1.04$ km/s/Mpc, one can match the observed Union2 supernova redshifts in our model only if one accepts a current CMB best-fit temperature of $T_0 = 2.8479^{+0.0203}_{-0.0203}$ K. This is far outside the measured current CMB temperature of $T_0 = 2.725007 \pm 0.000024$ K by Dhal *et al.* and indicates that the Riess *et al.* H_0 value is way too high, and not consistent with our new model.

5. Additional Arguments in Support of a Hubble Tension Solution

In addition to the above arguments in support of a Hubble tension solution, one can also employ a different approach which reaches the same basic conclusion and is complementary to the one above. This approach makes use of our newly-derived "Upsilon equation" which couples H_0 with the current CMB temperature T_0 (see [71]-[73]) by the simple and exact formula:

$$H_0 = \mho T_0^2 \tag{18}$$

The Latin Capital Upsilon symbol $\mho$ is a compound coupling constant with the following value: $\mho = 2.91845601 \times 10^{-19} \pm 0.00003279 \times 10^{-19}\ \mathrm{s^{-1} \cdot K^{-2}}$. This is the value for this composite constant based on the NIST CODATA values of its constituent constants. This composite constant was derived in relation to Equation (18) and given first by Tatum *et al.* [71]:

$$\mho = \frac{k_b^2 32\pi^2 G^{1/2}}{c^{5/2}\hbar^{3/2}} = 2.91845601\times10^{-19} \pm 0.00003279\times10^{-19}\ \mathrm{s^{-1}\cdot K^{-2}} \tag{19}$$

This is a composite constant composed entirely of already very well-known physical constants. There is no uncertainty in the Boltzmann constant k_b, the reduced Planck constant $\hbar$, or the speed of light c, as these are all exactly defined in today's most updated S.I. unit system, the NIST CODATA 2018 standard: $k_b = 1.380649\times10^{-23}\ \mathrm{J\cdot K^{-1}}$; $c = 299792458\ \mathrm{m/s}$; $\hbar = 1.054571817\times10^{-34}\ \mathrm{J\cdot s}$. Only the gravitational constant G has a residual small uncertainty; its NIST CODATA value is given as $G = 6.67430\times10^{-11} \pm 0.00015\times10^{-11}\ \mathrm{m^3\cdot kg^{-1}\cdot s^{-2}}$.

Rearrangement of Equation (18) gives:

$$T_0 = \left(\frac{H_0}{\mho}\right)^{1/2} \tag{20}$$

which can be used to calculate a current T_0 value for any given H_0 or vice versa. We can then compare the coupled H_0 and T_0 values from four recent CMB temperature studies (see **Table 1**) with the coupled H_0 and T_0 values of the most recent high precision SH0ES Team study reported in 2022 by Riess *et al.* [70].

Table 1. This table presents Hubble constant estimates derived from Equation (18) across several notable CMB studies. The uncertainties in the predicted H_0 account for both the uncertainty in the measured T_0 and the uncertainty in the Upsilon constant.

CMB study:	Temperature measurement:	$H_0 = \mho T_0^2$:
2004: Fixsen *et al.* [65]:	2.721 ± 0.010 K	$H_0 = 66.68 \pm 0.49$ km/s/Mpc
2009: Fixsen *et al.* [67]:	2.72548 ± 0.00057 K	$H_0 = 66.8944 \pm 0.0287$ km/s/Mpc
2011: Noterdaeme *et al.* [66]:	2.725 ± 0.002 K	$H_0 = 66.8708 \pm 0.0989$ km/s/Mpc
2023: Dhal *et al.* [68]:	2.725007 ± 0.000024 K	$H_0 = 66.8712 \pm 0.0019$ km/s/Mpc

In **Table 1**, the values of H_0 in units of km/s/Mpc (after conversion from their S.I. unit values) are coupled to a tight T_0 range of 2.721 ± 0.010 K to 2.72548 ± 0.00057 K. As a result of these high precision T_0 measurements, the calculated H_0 values using Equation (18) show a tight range of 66.68 ± 0.49 km/s/Mpc to 66.8944 ± 0.0287 km/s/Mpc. This is much higher H_0 precision than given by any other method and is fully consistent with our findings from the last section, wherein we incorporated all 580 type Ia supernova redshifts in the Union2 database, and found a best-fitting $H_0 = 66.8711 \pm 0.0019$ km/s/Mpc.

We can then compare what T_0 value would, according to Equation (20), be coupled to the SH0ES study H_0 value of 73.04 ± 1.04 km/s/Mpc. Given this H_0 range of 72.0 to 74.08 km/s/Mpc (once converted to S.I. units), Equation (20) indicates that the coupled T_0 value should be 2.8479 ± 0.0203 K. This surprisingly high T_0 value, greater than 0.1 K higher than the measured T_0 value, is clearly an outlier when analyzing it using our "Upsilon equation".

Using our CMB redshift prediction formula and method of reference [1], we can also show how the Union2 database of 580 type Ia supernova redshifts, in combination with the Riess *et al.* H_0 value of 73.04 ± 1.04 km/s/Mpc, is a best match for a T_0 value of 2.8479 ± 0.0203 K. See **Figure 4**. This is simply yet another way to show the same outlier appearance of local H_0 determination, in comparison to H_0 determinations made from CMB studies. So, we conclude that the equivalent found Planck length and Upsilon equation approaches demonstrated herein add further support to the impression that the Hubble tension is now solved in favor of the Planck Collaboration result, particularly since, in every case, we have used the local universe supernova redshift data to do so.

Furthermore, if we solve the Planck length formula $l_p = \sqrt{\frac{G\hbar}{c^3}}$ for G, we get $G = \frac{l_p^2 c^3}{\hbar}$ (see [55]), so the Upsilon constant used above can also be expressed as:

$$\mho = \frac{k_b^2 32\pi^2 l_p}{\hbar^2 c} = 2.91845601\times10^{-19} \pm 0.00003279\times10^{-19}\ \mathrm{s}^{-1}\cdot\mathrm{K}^{-2} \tag{21}$$

or from the Planck time:

$$\mho = \frac{k_b^2 32\pi^2 t_p}{\hbar^2} = 2.91845601\times10^{-19} \pm 0.00003279\times10^{-19}\ \mathrm{s}^{-1}\cdot\mathrm{K}^{-2} \tag{22}$$

Since the Planck energy is given by $E_p = \frac{\hbar}{l_p}c$, we can also re-write the Upsilon constant as:

$$\mho = \frac{k_b^2 32\pi^2}{\hbar E_p} = 2.91845601\times10^{-19} \pm 0.00003279\times10^{-19}\ \mathrm{s}^{-1}\cdot\mathrm{K}^{-2} \tag{23}$$

Again, the only uncertainty in the Upsilon constant comes from G or alterntively l_p, as we have $E_p = \sqrt{\frac{\hbar c^5}{G}} = \frac{\hbar}{l_p}c$. The uncertainty now comes from the Planck length NIST CODATA $l_p = 1.616255\times10^{-35} \pm 0.000018\times10^{-35}\ \mathrm{m}$. The relative standard uncertainty in the Planck length: $\frac{0.000018\times10^{-35}\ \mathrm{m}}{l_p} = 1.1\times10^{-5}$ is exactly half that of the relative standard uncertainty in G: $\frac{0.00015\times10^{-11}\ \mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}}{G} \approx 2.2\times10^{-5}$. However, the uncertainty in Upsilon is the same no matter if calculated from G or l_p. Because the formula relying on G uses $\sqrt{G}$ and the Planck length formula uses $G = \frac{l_p^2 c^3}{\hbar}$, the uncertainty affecting Upsilon must be the same. The relative standard uncertainty in Upsilon based on inputs from NIST CODATA 2018 must be:

$$\frac{0.00003279\times10^{-19}\ \mathrm{s}^{-1}\cdot\mathrm{K}^{-2}}{\mho} \approx 1.1\times10^{-5} \tag{24}$$

In other words, exactly the same as for the Planck length, as we would expect. So, from Equation (18) we can most readily see that there are key inter-relationships between the CMB temperature, H_0 and the Planck length, fully consistent with the previous sections of this paper. This also means that we naturally have:

$$H_0 = \mho T_0^2$$

$$H_0 = \frac{k_b^2 32\pi^2 l_p}{\hbar^2 c} T_0^2$$

$$l_p = \frac{H_0}{T_0^2} \frac{\hbar^2 c}{k_b^2 32\pi^2} \tag{25}$$

and since we have $T_t = T_0(1+z)$ we must also have:

$$l_p = \frac{H_0 (1+z)^2}{T_t^2} \frac{\hbar^2 c}{k_b^2 32\pi^2} \tag{26}$$

Equation (25) was recently presented and discussed by Haug [4], who derived it by simply rearranging the formula of Tatum *et al.* [2]. One can readily see that this equation yields the Planck length from T_0 and H_0. One can do this derivation either by using the best current high precision measurements of the CMB temperature and the Hubble constant, or even more precisely by incorporating all 580 type Ia supernova redshifts in the Union2 database, as demonstrated herein. This confirms the consistency of our framework and also supports the contention that the Planck length is constant through time, as expected by many physicists and quantum gravity theorists. Furthermore, since $\frac{\hbar^2 c}{k_b^2 32\pi^2}$ has an exact value due to $\hbar$, and c and k_b being exact constants, it implies also that, if l_p is constant over cosmic time, there are compelling reasons to believe that, by Equation (25), l_p imposes a constraint on the ratio relation between H_0 and T_0, as clearly seen in our Upsilon equation and other work in this paper. Given that T_0 is extremely accurately measured (with very low standard deviation), it is the uncertainty in H_0 that we have greatly reduced by understanding this deeper relation.

We also can easily incorporate cosmological redshift into our Upsilon equation. We can start with:

$$T_t = T_0(1+z) \tag{27}$$

and then naturally we must have:

$$T_0 = \frac{T_t}{1+z} \tag{28}$$

We can now replace T_0 with this in the Upsilon equation and we get:

$$H_0 = \frac{\mho T_t^2}{(1+z)^2} \tag{29}$$

and

$$T_t = \left(\frac{H_0}{\mho}\right)^{1/2} (1+z) \tag{30}$$

Furthermore, we can also have:

$$D = \frac{cz + cz^2}{\mho T_0^2 (1+z)^2} \tag{31}$$

Table 2 summarizes additional key inter-relationships between the CMB temperature, H_0, and the Planck units. It is important to note here that the only uncertainty in the Planck units arises from difficulties in measuring l_p with precision. The uncertainty is the same in every Planck unit since we have $t_p = \frac{l_p}{c}$, $m_p = \frac{\hbar}{l_p}\frac{1}{c}$, $E_p = m_p c^2 = \hbar \frac{c}{l_p}$, and $a_p = \frac{c^2}{l_p}$. Since c and $\hbar$ are exact constants, the uncertainty in all of these arises only from l_p. The considerable uncertainty in l_p and t_p should not be surprising, as it is likely that they respectively represent the smallest length and time interval possible. They cannot be measured directly, but indirectly, we can measure them by finding the best fit with high precision Hubble constant and CMB temperature values, and now we can even incorporate a large database of cosmological redshifts. See also [49]. Despite the high uncertainty in l_p, its uncertainty is still very small compared to that found in H_0 by traditional studies. We believe that our Upsilon Equation (18) is the key to minimizing uncertainty in H_0, thus representing an important development in quantum cosmology.

Table 2. This table illustrates how to determine the Hubble constant from the current CMB temperature and various Planck units as well as the CMB temperature from H_0 and various Planck units.

	Hubble constant H_0:	CMB temperature T_0:
From Upsilon formula:	$H_0 = \mho T_0^2$	$T_0 = \left(\frac{H_0}{\mho}\right)^{1/2}$
From Upsilon formula:	$H_0 = \mho \frac{T_t^2}{(1+z)^2}$	$T_t = \left(\frac{H_0}{\mho}\right)^{1/2}(1+z)$
where	$\mho = \frac{k_b^2 32\pi^2 G^{1/2}}{c^{5/2}\hbar^{3/2}}$	is a composite constant
or	$\mho = \frac{k_b^2 32\pi^2 l_p}{\hbar^2 c}$	the same as above, but re-written.
or	$\mho = \frac{k_b^2 32\pi^2 t_p}{\hbar^2}$	the same as above, but re-written.
or	$\mho = \frac{k_b^2 32\pi^2}{\hbar E_p}$	the same as above, but re-written.
Planck unit:		
From Planck length:	$H_0 = l_p T_0^2 \frac{k_b^2 32\pi^2}{\hbar^2 c}$	$T_0 = \sqrt{\frac{H_0 c}{l_p}}\frac{\hbar}{k_b \pi\sqrt{32}}$
From Planck time:	$H_0 = t_p T_0^2 \frac{k_b^2 32\pi^2}{\hbar^2}$	$T_0 = \sqrt{\frac{H_0}{t_p}}\frac{\hbar}{k_b \pi\sqrt{32}}$

Continued

From Planck mass:	$H_0 = \frac{T_0^2}{m_p}\frac{k_b^2 32\pi^2}{\hbar c^2}$	$T_0 = c\frac{\sqrt{H_0 m_p \hbar}}{k_b \pi\sqrt{32}}$
From Planck energy:	$H_0 = \frac{T_0^2}{E_p}\frac{k_b^2 32\pi^2}{\hbar}$	$T_0 = \frac{\sqrt{H_0 E_p \hbar}}{k_b \pi\sqrt{32}}$
From Planck acceleration:	$H_0 = \frac{T_0^2}{a_p}\frac{c k_b^2 32\pi^2}{\hbar^2}$	$T_0 = \sqrt{\frac{H_0 a_p}{c}\frac{\hbar}{k_b \pi\sqrt{32}}}$
From Planck force:	$H_0 = \frac{T_0^2 l_p}{F_p}\frac{k_b^2 32\pi^2}{\hbar}$	$T_0 = \frac{\sqrt{H_0 F_p \hbar}}{k_b \pi\sqrt{32 l_p}}$

Table 3 gives additional relationships between cosmic parameters, expressed in terms of the Upsilon constant.

Table 3. This table summarizes a series of relations between cosmic parameters, expressed in terms of the Upsilon constant.

Entity:	Equation:
Upsilon constant	$\mho = \frac{k_b^2 32\pi^2 G^{1/2}}{c^{5/2}\hbar^{3/2}}$
or	$\mho = \frac{k_b^2 32\pi^2 t_p}{\hbar^2}$
value (NIST CODATA 2018)	$2.91845601\times10^{-19} \pm 0.00003279\times10^{-19}\ \text{s}^{-1}\cdot\text{K}^{-2}$
Hubble constant:	$H_0 = \mho T_0^2$
CMB temperature	$T_0 = \sqrt{\frac{H_0}{\mho}}$
Hubble time:	$t_H = \frac{1}{\mho T_0^2}$
CMB temperature	$T_0 = \sqrt{\frac{1}{\mho t_H}}$
Hubble radius:	$R_H = \frac{c}{\mho T_0^2}$
CMB temperature	$T_0 = \sqrt{\frac{c}{\mho R_H}}$
Cosmological redshift	$z = T_t\sqrt{\frac{\mho}{H_0}} - 1 = \frac{T_t}{T_0} - 1 = \sqrt{\frac{R_H}{R_t}} - 1$
Cosmological redshift	$z \approx \frac{D\mho T_0^2}{2c} = \frac{d\mho T_0^2}{c} = \frac{dH_0}{c}$
Redshift proper distance	$D = \frac{2cz + cz^2}{\mho T_0^2 (1+z)^2}$
Redshift proper distance	$D = \frac{2cz + cz^2}{\mho T_t^2}$

Continued

Redshift proper distance	$D \approx \frac{2cz}{\mho T_0^2} = \frac{2cz}{H_0}$, when $z \ll 1$
Critical mass (Friedmann)	$M_c = \frac{c^3}{2GT_0^2\mho}$
Critical density (Friedmann)	$\rho_c = \frac{3\mho^2 T_0^4}{8\pi G}$
CMB temperature	$T_0 = \sqrt{\frac{c^3}{2GM_c\mho}}$

Table 4 summarizes a series of relations between cosmic parameters in past cosmic epochs, in terms of the Upsilon constant.

Table 4. This table summarizes a series of relations between cosmic parameters in past cosmic epochs, in terms of the Upsilon constant.

Entity:	Equation:
Upsilon constant	$\mho = \frac{k_b^2 32\pi^2 G^{1/2}}{c^{5/2}\hbar^{3/2}}$
or	$\mho = \frac{k_b^2 32\pi^2 t_p}{\hbar^2}$
value (NIST CODATA 2018)	$2.91845601\times10^{-19} \pm 0.00003279\times10^{-19}\ \text{s}^{-1}\cdot\text{K}^{-2}$
Hubble constant in the past:	$H_t = \mho T_0^2 (1+z)^2$
Hubble constant in the past:	$H_t = \mho T_t^2$
Hubble (CMB) temperature in the past:	$T_t = \sqrt{\frac{H_0}{\mho}}(1+z)$
Hubble time in the past:	$t_t = \frac{1}{\mho} T_0^2 (1+z)^2$
Hubble radius in the past:	$R_t = \frac{c}{\mho} T_0^2 (1+z)^2$
Critical mass (Friedmann)	$M_t = \frac{c^3}{2GT_0^2(1+z)^2\mho}$
Critical density (Friedmann)	$\rho_{c,t} = \frac{3\mho^2 T_0^4 (1+z)^4}{8\pi G} = p_c (1+z)^4$
CMB temperature in the past	$T_t = \sqrt{\frac{H_t}{\mho}}$
CMB temperature in the past	$T_t = (1+z)\sqrt{\frac{c^3}{2GM_c\mho}}$
CMB temperature in the past	$T_t = (1+z)\sqrt{\frac{H_0}{\mho}}$
CMB temperature in the past	$T_t = \frac{(\rho_{c,t} 8\pi G)^{\frac{1}{4}}}{3^{\frac{1}{4}}\sqrt{\mho}}$

6. Possible Underlying Reasons Why We Appear to Have Resolved the Hubble Tension and Dramatically Increased Precision in Estimates

We have claimed to have resolved the Hubble tension and, in addition, to have dramatically reduced the uncertainty (standard deviation, STD) in H_0 estimates. This almost seems too good to be true. However, we have carefully reviewed our logic and derivations, and find no obvious errors. The reason our method appears to be much more powerful than the existing Λ-CDM model is that we have established exact relations between H_0, T_0, z, and the Planck length, as we illustrate in Figure 5. These appear to be exact relations between the smallest and largest scales of the universe. Thus, our model appears to have a solid framework. If we know any three of these parameters, we can find the remaining one with high precision. It makes sense to take what is measured most accurately, namely, the CMB temperature, the Planck length, and the redshifts, and then use these high-precision measurements to determine the Hubble constant with high precision. In the Λ-CDM model, such exact relations have not yet been established between these parameters; and for this reason, the Λ-CDM model does not appear to be as good at describing certain aspects of the universe as the model we have presented; nor can it predict the Hubble constant with the precision that we can achieve. That said, the Λ-CDM model has evolved from work over time; it has been adjusted over time. We do not exclude the possibility that our findings can be incorporated into that model as well. Both models should be carefully investigated and compared by multiple researchers over time.

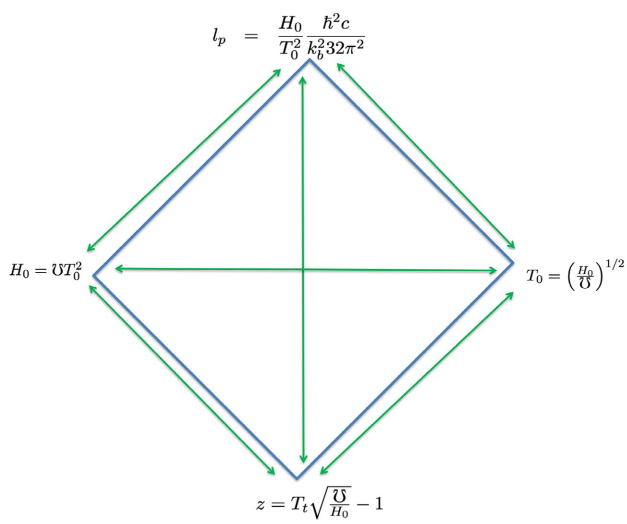

Figure 5. This figure illustrates that we have established exact relations between the Hubble constant H_0, the current CMB temperature T_0, and CMB temperatures from the past T_t, the cosmological redshift z, and even the Planck length l_p. It is these newly-established exact inter-relationships that appear to have allowed us to resolve the Hubble tension. In addition, they allow for dramatically-improved Hubble constant prediction. Here, there appear to be foundational relationships between the microcosmos and the macrocosmos.

Figure 6 shows key equations for the current universal parameters which incorporate the Upsilon constant.

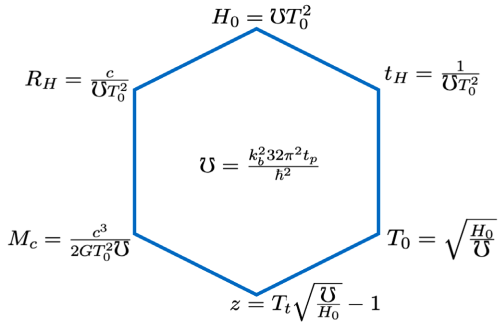

Figure 6. This figure shows how the Upsilon constant can be used to calculate current universal parameters. Note that the cosmological redshift equation at the bottom gives the link between past and present.

Figure 7 illustrates the different global parameters of the universe in past cosmic epochs, and shows how they are inter-related in terms of the Upsilon constant; this scenario is based on expansion of the universe in so-called $R_H = ct$ growing black hole cosmology.

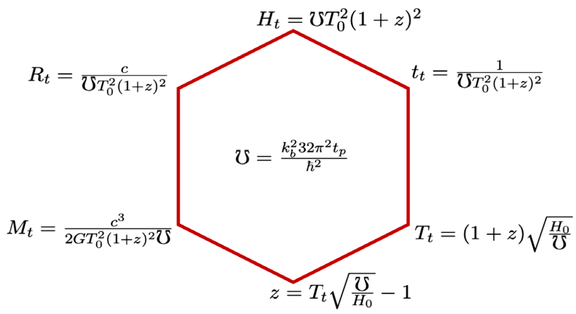

Figure 7. This figure illustrates the different global parameters of the universe in past cosmic epochs in terms of the Upsilon constant and cosmological redshift.

Figure 6 and **Figure 7** bring us back to Isaac Newton. In his *Principia*, he actually mentioned an absolute minimum time interval, and further stated that all of his philosophy was based on minimum units (see [74]). Herein, we have demonstrated that the Planck scale plays an important role in cosmology. The Planck length and the Planck time impose constraints on the values acceptable for such parameters as the Hubble constant. They even appear to resolve the Hubble tension in favor of the Planck Collaboration Hubble constant value. Of course, our claims should not be taken for granted. Our work should be studied and scrutinized over time by multiple researchers. After all, time is the best referee.

7. CMB Power Spectrum

An important aspect of the CMB is the CMB power spectrum and the baryonic acoustic oscillations (BAO). **Figure 8** shows the observed power spectrum as well as the theoretical fit to the ΛCDM model. The ΛCDM model uses a series of degrees of freedom parameters to match the CMB power spectrum, such as the amount of dark matter, dark energy, baryons, space curvature k, and more. An important outstanding question is, therefore, whether the $R_H = ct$ cosmology can also explain and match the CMB power spectrum.

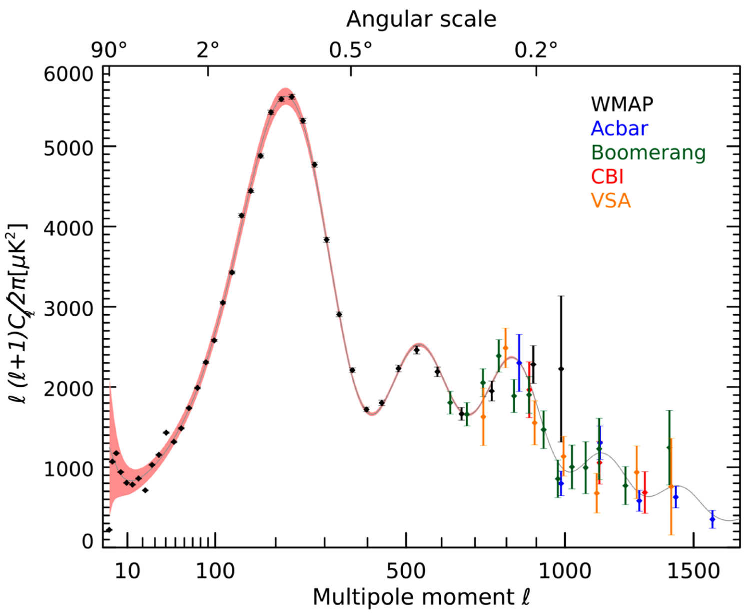

Figure 8. This figure illustrates the CMB power spectrum. The placement of the first peak is consistent with flat space. Source of the figure: https://en.wikipedia.org/wiki/Cosmic_microwave_background.

First of all, studies have shown that the placement of the first major peak is consistent with a nearly flat space, $k = 0$, which is an assumption in our model (see [8]]). Flat-space predicts that the first peak is at $l \sim 220$ which it is. Whether our model can match and explain all the observed peaks in the power spectrum is too early to say. Melia [75] [76], with his $R_H = ct$ model, has done some interesting work in this area, claiming that the $R_H = ct$ cosmology outperforms the ΛCDM model. However, his model differs considerably from ours in many aspects, so it is too early to determine how well our model can describe baryonic acoustic oscillations. We also encourage other researchers to investigate this further.

8. Conclusion

We have demonstrated that it is possible to extract the Planck length from the 580 type Ia supernova redshifts in the Union2 database by using the current temperature of the Cosmic Microwave Background (CMB) and the Hubble constant. However, this Planck length extraction approach imposes significant constraints on the Hubble constant value, which must be $66.8711^{+0.0019}_{-0.0019}$, km/s/Mpc to match the observed redshifts in the Union2 database, so long as we accept the NIST CODATA value for the Planck length. Alternatively, we would have to introduce the idea that the Planck length has changed since the beginning of the universe, something that seems much less likely. For the local universe Hubble constant determinations by Riess and others to be compatible with the Union2 supernova redshift database, the best-fitting Planck length would have to deviate by more than 8241σ from the NIST CODATA value. This deviation appears to be unacceptable, since the Planck length almost surely must remain constant. Although the uncertainty in the Planck length may be higher than in most constants, it nevertheless imposes significant constraints on the Hubble constant value that best-fits the redshift database. To our knowledge, ours is likely the first cosmological model to establish a clear connection between the CMB temperature, the Hubble constant (H_0), cosmological redshift (z), and the Planck length. We believe that the Λ-CDM model does not provide for a method for estimating the Planck length by imposing constraints on the Hubble constant. Our method of extracting a Hubble constant tightly constrained by the uncertainty in the Planck length, suggests a solution to the Hubble tension in favor of the Planck Collaboration CMB Hubble constant determination. We invite others to study our model and to evaluate its potential usefulness in the context of Planck-scale quantum cosmology.

Data Availability Statements

The supernova Union-2 database that we have used can be found here:
https://supernova.lbl.gov/Union/figures/SCPUnion2.1_mu_vs_z.txt.

Conflicts of Interest

The authors declare no conflicts of interest.

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