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The G&G Model: A PDE Approach with Entropy and Irreversibility in Leadership and Policy Rehabilitation in the Democratic Republic of the Congo

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Abstract

This paper introduces the G&G Model with memory (Glody & Gilles Leadership Model), which uses a parabolic-to-pseudo-hyperbolic transition to investigate governance performances in the Democratic Republic of the Congo (DRC). The model inspired from Kurt Lewin's field theory, formalizes how driving and restraining forces interact in social and political environment. The explicit inclusion of Shannon-type entropy as a source of disruption in the dissemination, interpretation, and implementation of public policies is a fundamental component of the model. While the shift to a pseudo-hyperbolic structure indicates the possibility of leadership stabilization, the parabolic model illustrates the irreversible nature of societal imbalances. For in depth accuracy to pinpoint crucial areas of instability or evolution, the model incorporates behavioral reactivity terms, impact gradients, and entropic components. This work provides a rigorous approach and lays the groundwork for mathematical modeling of governance and leadership, paving the way for further reform initiatives based on both quantitative analysis of intricate social systems and qualitative insight.

Keywords

G&G Model, Policy Rehabilitation, Pseudo-Hyperbolic Equation, Irreversibility, Shannon-Type Entropy, Kurt Lewin's Field Theory, GUI (Graphical User Interface), Finite Difference Methods (FDM)

1. Introduction

In our paper entitled "On Inclusive growth: an economic growth time-frequency analysis of the Democratic Republic of the Congo from 1975-2016" [1], we highlighted the first aspect of inclusive growth from a participatory perspective. In other words, to show how the promotion of sectors that catalyze economic growth can be a source of secure and sustainable wealth that can be redistributed in the needy (pro-poor) sectors as well as the equitable distribution of income to each layer of the Congolese population (a marginal productivity remuneration of each economic actor).

On the strength of this, we can detect that this approach denotes a coherent and coordinated mechanism that we call "economic policy", an instrument that allows a state to interfere in the key sectors of a country's economy.

To this end, [2] is the set of government measures intended to influence the production, consumption and distribution of goods in a country. In other words, [3] aim at all the actions put in place by the public authorities to correct economic "imbalances" deemed damaging.

This instrument is subdivided into two channels, namely:

1) **A short-term economic policy**, seeking to control aggregate demand by acting in the short term on one or more of its components. The most likely situation is that of insufficient demand, which must therefore be sustained. Short-term economic policies are said to be Keynesian-inspired in particular because of their time horizon and comprise four objectives (growth, price stability, employment and external equilibrium) and four tools (budgetary policy, wage and income policy, monetary policy and exchange rate policy) which are based on two major functions (the resource allocation function and the income redistribution function).

2) **A structural economic policy**, concerned in the longer term with the environmental and social sustainability of the country's economic development, its potential growth but also the conditions of market functioning (evolution of the market structure and modification of the behaviour of agents in particular).

Using the DRC as an example, since the 1970s, the country has demonstrated an unrelenting desire to support economic recovery through policies that, regrettably, have failed miserably. This mismanagement of public funds led to the DRC falling into a vicious cycle of economic stagnation and excessive debt to the Bretton Woods institutions.

Additionally, the exogenous effects of dollarization combined with an existing floating exchange rate regime, a major consequence of the "hydra of hyperinflation", have made matters worse by causing the exchange rate to fluctuate repeatedly, despite its decision to use monetary seigniorage to combat this crisis.

For this reason, the author [4] claims that "the impotence of public action" is a special affliction that plagues the DRC. The Congolese state is one that declares policies but lacks the capacity to implement them, rescinds or forgets them, then develops plans that are never carried out, retreats into the past while claiming to be a part of a future revolution, and does so without providing the resources necessary to make this leap into a better, more transformative future.

[5] Conversely, it stigmatizes "political instability", which is largely to blame for

the Congolese economy's "shrinking". Therefore, political instability would have hindered investment and the upkeep of installed capacities, particularly since the Congolese economy's lack of diversification deters its growth internationally and renders it uncompetitive in terms of free trade; the DRC is afflicted by "Dutch disease".

In a holistic way, given stylized economic realities, we may state that government intervention, or leadership, is the root cause of the non-inclusiveness of economic growth rather than economic policymaking. [6] appears to agree with this viewpoint by critiquing the "absence of the state", particularly in the 1980s and 1990s.

He claimed that the lack of the State was the cause of Congo's economic suffering over these two decades. However, leadership holds a decisive priority in any kind of organization (in this case, the state) because it allows one to assess the wide range of potential combinations of methods and means that could undermine the goals of structural and conjunctural economic policies while persuading a group of people (the government) to implement the latter for the sake of good governance.

In order to impact the economic policy milestones for potential and inclusive economic growth, we demonstrate throughout this article how important it is to support effective and efficient leadership. This study suggests a novel mathematical method for simulating the performance of Congolese governance by using partial differential equations (PDEs) to reflect social behaviors and leadership dynamics. The goal is to mathematically translate a socio-economic-political system's stability or instability under the influence of several aspects, such as social force dynamics, informational coherence, and management quality.

This work is a component of an interdisciplinary approach that combines ideas from social psychology, economic and political science, and information theory with tools from mathematical physics.

2. Theoretical Basis

2.1. Human Behavior: Kurt Lewin's Model [7]

Collective behavior is modeled according to Lewin's famous equation:

$$B = f(p, e) \tag{1}$$

where:

- B is the behavior,
- p the person (or socio-political profile of a group),
- e the environment.

2.2. Force Field Theory [8]

Despite being qualitatively developed at first, Lewin's force-field theory is thoroughly explored in Social Conflict Resolution and Field Theory in the Social Sciences. A mathematical reformulation of social dynamics in terms of enabling and impeding factors is made possible by this conceptual framework.

This theory, which draws inspiration from group dynamics, asserts that any process of change results from a balance (or imbalance) between:

- Driving forces (*FF*) for change,
- restrictive forces (*RF*) opposing it.

We introduce the total force field:

$$F(t,x) = FF(t,x) - RF(t,x) \tag{2}$$

In this context, leadership becomes a strategic competency for modifying the field of stress in this situation. As a system modulator, the leader can diagnose resistance by using a sophisticated grasp of social, psychological, and structural dynamics; activate or reinforce the forces that propel change; and remove barriers through reorganization, influence, and vision. That's why, throughout this article, we're going to consider $F(t,x)$ as the forces of change (leadership) to be the most important ones. In the next article, we'll take a closer look at local leadership behavior, inserting differential games from game theory and a stochastic process (the Îto process) to better understand the various probabilistic scenarios likely to have a global impact on people's behavior. This will give us a more global and robust representation.

2.3. Role of Information and Entropy [9]

Information is essential to political institutions because it organizes decisions, motivates behavior, and ensures that reforms are coherent. However, significant distortions may result from involuntary or voluntary poor transmission.

To model this uncertainty, we introduce Shannon's entropy:

$$H_{(p)} = -\sum_{i=1}^{m} p_i \log(p_i) \tag{3}$$

where p_i represents the probability of occurrence of a message or interpretation. This entropy will be integrated as a disruptor in our behavioral equation.

3. Modeling

Mathematical modelling of the phenomena of political inefficiency in the DRC is based on the use of partial differential equations (PDEs), in particular the parabolic and hyperbolic systems. We want to depict the dynamics of collective behavior $B(t,x)$, which is impacted by a number of variables, including informational entropy $H(t,x)$, social forces $F(t,x)$.

3.1. Variables

Before detailing the equation of the system, let's define the main variables and functions used in our model:

- $B(t,x)$: collective behavior of the population at a time *t* and at a place *x*, which represents the overall response of the system to public policies and the socio-economic environment.
- $F(t,x)$: social force field, which models the interaction between forces in favour of change ($FF(t,x)$) and forces restricting or opposing ($RF(t,x)$) to

change. These forces can be associated with social, cultural or economic resistance.

- $H(t,x)$: the uncertainty or deterioration of the information flowing through the system is measured by informational entropy. The loss of consistency and clarity in the dissemination of governmental mandates and public policy is measured by entropy.

We deduce the differential forms of the following equations:

a) Equation of Modified Behavior (Kurt Lewin)

$$B(t,x) = f\left(P(t,x), e(t,x)\right) \tag{4}$$

with

- $P(t,x)$: Socio-political profile of the population

Explicit form:

We can posit, for example, that:

$$B(t,x) = \phi\left(P(t,x), e(t,x)\right) \tag{5}$$

where ϕ is a behavioral function. To remain general but mathematically clear, we could linearize ϕ in the form of:

$$B(t,x) = \mu_1\left(P(t,x)\right) + \mu_2\left(e(t,x)\right) \tag{6}$$

with $\mu_1, \mu_2 \in \mathbb{R}$ representing the sensitivity of the behaviour to the person and the environment, respectively.

- Term Update $-\beta B$

By replacing *B*, we obtain:

$$-\beta B(t,x) = -\beta\left[\mu_1\left(P(t,x)\right) + \mu_2\left(e(t,x)\right)\right] \tag{7}$$

b) Shannon-type entropy

$$H(t,x) = -p(t,x)\cdot\log p(t,x) \tag{8}$$

where:

- $H(t,x)$ is the local entropy density at a point *x* and at time *t*.
- $-p(t,x)$ is the probability density of the information (or disinformation) present at that place and time.

3.2. Basic Parabolic System

The parabolic PDE describes the diffusion of collective behavior in space and time. The basic model is given by the following equation:

$$\frac{\partial B}{\partial t} = D\nabla^2 B + \alpha F(t,x) - \beta B + \delta H(t,x) \tag{9}$$

explicit form of the EDP:

$$\frac{\partial B}{\partial t} = D\nabla^2 B + \alpha F(t,x) - \beta\left[\mu_1\left(P(t,x)\right) + \mu_2\left(e(t,x)\right)\right] - \delta p(t,x)\cdot\log p(t,x) \tag{10}$$

where:

- $\frac{\partial B}{\partial t}$ is the rate of change in collective behaviour over time.

- $D\nabla^2 B$ represents the spatial diffusion of behavior, with D being a diffusion coefficient that determines how quickly the effects of leadership and policies spread through the territory.
- $\alpha F(t,x)$ represent the effect of social forces on collective behavior whereas $F(t,x)$ includes dynamics including political movements, civil society mobilizations, and popular pressure. Leadership itself is ingrained in these dynamics, functioning as a catalyst that can guide, intensify, or alter group reactions and paths.
- $\beta\left[\mu_1\left(P(t,x)\right)+\mu_2\left(e(t,x)\right)\right]$: **Behavioural resistance**

- $\beta > 0$ measures the intensity of resistance or social disaffection.
- $\mu_1\left(P(t,x)\right)$: intrinsic influence of personality or collective psychology.
- $\mu_2\left(e(t,x)\right)$: structural effect of the institutional or political environment.
- Together, this term represents the restrictive forces (Lewin) that hinder change: social fatigue, distrust, corruption, bureaucratic inertia, etc.

- $\delta p(t,x)\cdot\log p(t,x)$: **Social Entropy (Shannon)**

- $\delta > 0$ is a factor of sensitivity to entropy.
- $p(t,x)$: behavioral probability distribution, representing the distribution of intentions, opinions, or trends in society.
- The expression $-p\log p$ is informational entropy, measuring disorder, uncertainty, or disinformation in the system
- This term reflects the negative weight of informational instability on collective behaviour: poor communication of public policies, propaganda, social fragmentation, etc.

3.3. G&G Leadership Model (Pseudo-Hyperbolic PDE with Memory)

In order to analyze the conditions under which a system could return to a stable or oscillating state, we consider the transition to a hyperbolic equation, which introduces inertia and allows to capture reversible dynamics. This transition is demonstrated as follows:

a) According to the parabolic equation, we have the following:

$$\frac{\partial B}{\partial t}=D\nabla^2 B+\alpha F(t,x)-\beta\left[\mu_1\left(P(t,x)\right)+\mu_2\left(e(t,x)\right)\right]-\delta p(t,x)\cdot\log p(t,x) \quad (11)$$

b) We derive the following expression with respect to time:

$$\frac{\partial}{\partial t}\left[\frac{\partial B}{\partial t}\right]+\frac{\partial}{\partial t}[\beta B]=\frac{\partial}{\partial t}\left[D\nabla^2 B\right]+\frac{\partial}{\partial t}\left[\alpha F(t,x)\right]+\frac{\partial}{\partial t}\left[\delta H(t,x)\right] \quad (12)$$

c) Let's break down the following terms:

1) $\frac{\partial}{\partial t}\left[\frac{\partial B}{\partial t}\right]=\frac{\partial^2}{\partial t^2}B$

2) $\frac{\partial}{\partial t}[\beta B]=\beta\left(\mu_1\frac{\partial p}{\partial t}+\mu_2\frac{\partial e}{\partial t}\right)$

3) $\frac{\partial}{\partial t}\left[D\nabla^2 B\right]=D\nabla^2\left[\frac{\partial B}{\partial t}\right]$

$$\begin{cases} \frac{\partial}{\partial t}\left[D\nabla^2 B\right] = D\nabla^2\left[\frac{\partial B}{\partial t}\right] \text{ with Young's inequality} \\ \text{with this relation } \frac{\partial}{\partial t}\left[\frac{D\partial^2}{\partial x^2}B\right] = \frac{\partial^2}{\partial x^2}\left[\frac{D\partial}{\partial t}B\right] \end{cases}$$

4) $\frac{\partial}{\partial t}\left[\alpha F(t,x)\right] = \alpha\left[\frac{\partial}{\partial t}FF(t,x) - \frac{\partial}{\partial t}RF(t,x)\right]$

✓ Let's show that

$$\frac{\partial}{\partial t}FF(t,x) = FF(t,x) \tag{13}$$

We must consider that *FF* is a function of time and possibly space, and that it satisfies a particular differential equation. This equality is not always true by default, it defines a differential equation that is satisfied only by certain functions.

Hence (13) is first-order linear ODE (with respect to time), with constant coefficients.

It is well known and its general solution is:

$$FF(t,x) = C(x)\mathrm{e}^t \tag{14}$$

where:

- $C(x)$: is a function that depends on *x*, *i.e.* space (since we only derive with respect to time),
- e^t: comes from solving the homogeneous differential equation.

Thus, let us calculate the derivative of (4) with respect to t

$$\Rightarrow \frac{\partial}{\partial t}FF(t,x) = \frac{\partial}{\partial t}C(x)\mathrm{e}^t$$

$$\Rightarrow C(x)\frac{\partial}{\partial t}\left(\mathrm{e}^t\right) = C(x)\mathrm{e}^t$$

$$\Rightarrow FF(t,x) \tag{15}$$

Therefore (15) is indeed the solution of (13).

✓ Let's show that

$$\frac{\partial}{\partial t}RF(t,x) = RF(t,x) \tag{16}$$

This equation means that the restrictive forces grow exponentially over time at a constant rate, independent of the x-space. It has exactly the same form as that of $FF(t,x)$, and therefore its solution is also of exponential type:

$$RF(t,x) = K(x)\mathrm{e}^t \tag{17}$$

where:

- $K(x)$: is a spatial function (such as a local density or intensity of restrictive forces),
- e^t: comes from solving the homogeneous differential equation.

So let's take (6) and calculate its derivative with respect to t

$$\Rightarrow \frac{\partial}{\partial t}RF(t,x) = \frac{\partial}{\partial t}K(x)\mathrm{e}^t$$

$$\Rightarrow K(x)\frac{\partial}{\partial t}\left(\mathrm{e}^{t}\right)=K(x)\mathrm{e}^{t}$$

$$\Rightarrow RF(t,x) \tag{18}$$

Therefore (18) is indeed the solution of (16).

✓ Summary of the two forces *FF* and *RF*

$$FF(t,x)=C(x)\mathrm{e}^{t}\Rightarrow\frac{\partial}{\partial t}FF(t,x)=FF(t,x)$$

$$RF(t,x)=K(x)\mathrm{e}^{t}\Rightarrow\frac{\partial}{\partial t}RF(t,x)=RF(t,x)$$

5) $$\frac{\partial}{\partial t}\left[\delta H(t,x)\right]=-\delta\frac{\partial}{\partial t}\left[p(t,x)\cdot\log p(t,x)\right] \tag{19}$$

✓ Let us differentiate the function $\delta H(t,x)=-\delta p(t,x)\cdot\log p(t,x)$ with respect to *t*.

It is a compound derivative: we apply the product rule and the logarithm derivative.

$$\Rightarrow\frac{\partial}{\partial t}\left[\delta H(t,x)\right]=-\delta\frac{\partial}{\partial t}\left[p(t,x)\cdot\log p(t,x)\right]$$

$$\Rightarrow-\delta\left[\frac{\partial p}{\partial t}\log p+p\cdot\frac{1}{p}\frac{\partial p}{\partial t}\right]$$

$$\Rightarrow-\delta\left[\frac{\partial p}{\partial t}\log p+\frac{\partial p}{\partial t}\right]$$

$$\Rightarrow-\delta\frac{\partial p}{\partial t}(\log p+1) \tag{20}$$

d) Let's gather all the broken-down terms

This gives us the following pseudo-hyperbolic equation with memory (G&G leadership model):

$$\frac{\partial^2}{\partial t^2}B=D\nabla^2\left[\frac{\partial B}{\partial t}\right]+\left[\alpha F(t,x)\right]-\beta\left(\mu_1\frac{\partial p}{\partial t}+\mu_2\frac{\partial e}{\partial t}\right)-\delta\frac{\partial p}{\partial t}(\log p+1) \tag{21}$$

where:

- $\frac{\partial^2}{\partial t^2}B$ **(Social inertia):** this is a hyperbolic phrase, expressing the acceleration of behavioral change. He demonstrates how behaviors follow a dynamic of temporal inertia rather than changing instantly, much like a civilization that opposes sudden change.
- $\beta\left(\mu_1\frac{\partial p}{\partial t}+\mu_2\frac{\partial e}{\partial t}\right)$: **Behavioral dissipation term**

- β is an overall coefficient of behavioral friction (resistance).
- Two components:
- $P(t,x)$: intrinsic psychological tendency, representing internal culture, prejudices, fear of change, etc.
- $e(t,x)$: external environmental energy, linked to the structure, the political climate, the governance system

- $D\nabla^2\left[\frac{\partial B}{\partial t}\right]$: **(Temporal diffusion with memory):** D is a sociopolitical diffusion coefficient. This concept applies a diffusive spatial effect to the behavior variance. It conveys the notion that societal changes occur in space but with memory or delay; for instance, it can depict the gradual spread of reform or dissatisfaction. This term makes (21) a pseudo PDE because of the mixing of both derivation in space and in time.
- $\alpha F(t,x)$ **(Time Driving Force):** Represents the forces for change (motivations, leadership, political impulses). The derivative $\frac{\partial}{\partial t}\left[\alpha F(t,x)\right]$ indicates the dynamic variations of these forces. α is a factor of sensitivity to positive change, it is the effectiveness of leadership in driving a dynamic.
- $\delta\frac{\partial p}{\partial t}(\log p+1)$ **(Entropic Informational Perturbation):** This term is directly derived from Shannon's entropy $H(t,x)=-p(t,x)\cdot\log p(t,x)$, where $p(t,x)$ is the probability or density of correctly transmitted information. It models the negative effects of poor transmission or manipulation of information (voluntary or not). δ is a coefficient of intensity of the effect of information. If there is informational instability or uncertainty (disinformation, rumors, propaganda), this disrupts the overall social dynamic.

Theorem 1. Stability Theorem for the Pseudo-Hyperbolic PDE with Entropic Perturbation

We consider the following pseudo-hyperbolic partial differential equation:

$$\frac{\partial^2}{\partial t^2}B=D\nabla^2\left[\frac{\partial B}{\partial t}\right]+\left[\alpha F(t,x)\right]-\beta\left(\mu_1\frac{\partial p}{\partial t}+\mu_2\frac{\partial e}{\partial t}\right)-\delta\frac{\partial p}{\partial t}(\log p+1) \tag{22}$$

where $B(t,x)$ is the main evolving field, and F,p,e are smooth source terms with

$$p(t,x)>0.$$

We define the energy of the system as:

$$\varepsilon_{(t)}:=\frac{1}{2}\int_\Omega\left|\frac{\partial B}{\partial t}\right|^2\mathrm{d}x+\frac{D}{2}\int_\Omega|\nabla B|^2\,\mathrm{d}x \tag{23}$$

<u>Assumptions</u>

Assume:

- $F,p,e\in C^2\left([0,T]\times\bar{\Omega}\right)$,
- $p(t,x)>0$ and $\log p\in L^\infty$,
- $B=0$ on $\partial\Omega$.

Initial conditions $B_0(x)\in H_0^1(\Omega)$ and $\partial_t B(0,x)\in L^2(\Omega)$, then there exist constants $\lambda>0$ and $C\geq 0$ such that:

$$\varepsilon(t)\leq\varepsilon(0)\mathrm{e}^{-\lambda t}+C \tag{24}$$

Lemma 1. Control of source term

Let

$$S(t,x) := \alpha\partial_t F - \beta(\mu_1\partial_t p + \mu_2\partial_t e) - \delta\partial_t p(\log p + 1) \tag{25}$$

Then for any $\varepsilon > 0$, there exists $C_\varepsilon > 0$ such that:

$$\left|\int_\Omega \partial_t B \cdot S\mathrm{d}x\right| \le \varepsilon\|\partial_t B\|_{L^2}^2 + C_\varepsilon\left(\|\partial_t F\|_{L^2}^2 + \|\partial_t p\|_{L^2}^2 + \|\partial_t e\|_{L^2}^2\right) \tag{26}$$

Proof of the Theorem

Multiply the equation by $\partial_t B$ and integrate over Ω:

$$\int_\Omega \frac{\partial^2 B}{\partial t^2}\cdot\partial_t B\mathrm{d}x = \int_\Omega D\nabla^2\left(\frac{\partial B}{\partial t}\right)\cdot\partial_t B\mathrm{d}x + \int_\Omega S\cdot\partial_t B\mathrm{d}x$$

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\partial_t B\|_{L^2}^2 = -D\|\nabla\partial_t B\|_{L^2}^2 + \int_\Omega S\cdot\partial_t B\mathrm{d}x \tag{27}$$

Apply the lemma to obtain:

$$\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon(t) \le -D\|\nabla\partial_t B\|_{L^2}^2 + \varepsilon\|\partial_t B\|_{L^2}^2 + C_\varepsilon\left(\|\partial_t F\|_{L^2}^2 + \|\partial_t p\|_{L^2}^2 + \|\partial_t e\|_{L^2}^2\right) \tag{28}$$

Applying Grönwall's inequality concludes the proof.

Corollary 1. Boundedness of Energy

If the source terms *F*, *p*, and *e* are regular and their time derivatives are bounded in L^2, then:

$$\sup \varepsilon(t) < \infty$$

$$t \in [0,T]$$

✓ Statement of the theorem

Under these assumptions, there is a critical threshold $\delta^* > 0$ such that:

- If $\delta < \delta^*$ then the behavior $B(t,x)$ tends towards an asymptotically stable state, *i.e.* the effects of informational perturbations attenuate over time.
- If $\delta \ge \delta^*$ then the system becomes unstable, and small variations in $p(t,x)$ can generate significant behavioral divergence, making policies ineffective in the long run.

✓ Political and social interpretation:

- The threshold δ^* measures the sensitivity of the social system to informational disturbances.
- Too high δ means that the poor flow of information becomes a major factor in social disorganization, regardless of the favorable political forces at play.
- This theorem illustrates the importance of a healthy and transparent information ecosystem to ensure the behavioral stability of social reforms.

3.4. Impact of Entropy on System Dynamics

We can mimic the impact of inadequate information transmission and uncertainty in public policy by introducing Shannon entropy as a disruptor. People find it more difficult to support and carry out reforms when entropy is high because policy decisions and actions become less cohesive.

In our model, the force field $F(t,x)$ represents the set of social, economic and political dynamics that influence collective behaviour. These forces are composed of two main families:

$$F(t,x) = FF(t,x) - RF(t,x) \quad (29)$$

where:

- $FF(t,x)$ are the forces for change,
- $RF(t,x)$ the restrictive or resisting forces.

The informational entropy $H(t,x)$, measuring uncertainty in the flow of information, acts as a negative modulator of these forces:

$$F(t,x)^{*} = \left[\left(FF(t,x) - RF(t,x)\right)\cdot\left(1-\eta H(t,x)\right)\right] \quad (30)$$

where η is a coefficient determining the influence of entropy on the force field. The higher $H(t,x)$ is, the more unbalanced the social forces are and the more likely they are to generate tensions, making the political balance more unstable.

3.4.1. Vector Field Analysis

To understand the internal dynamics of this field, we use two classic operators in vector analysis:

a) The divergence $\nabla\cdot F^{*}$

- Interpretation: Measures the net flow in or out of a given point in the field.
- Political significance: A positive divergence ($\nabla\cdot F^{*} > 0$) indicates an area of expansion of tensions or reforms, a possible social "explosion" (e.g., citizen mobilization).
- Conversely, a negative divergence ($\nabla\cdot F^{*} < 0$) indicates a withdrawal, blocking, or compression of the dynamics of change, often caused by censorship, fear, or misinformation.

b) The rotational $\nabla\times F^{*}$

- Interpretation: Measures the tendency of the field to rotate around a point, *i.e.* to generate cycles, recurrent tensions or instability loops.
- Political significance: A non-zero rotational reflects the presence of persistent internal conflicts, strategic misalignment or a chain of crises: contradictory decisions, ministerial instability, cyclical mobilization/repression, etc.

3.4.2. Influence of Entropy on Divergence and Rotational

The effect of the entropy $H(t,x)$ on the field F^{*} is indirect but structuring:

- It reduces the intensity of social forces, but also modifies their geometry.
- By its heterogeneous nature in space (urban vs. rural areas, elites vs. populations), it distorts the structure of the field.

Thus, we can write:

$$\nabla\times F^{*} = \nabla\times\left[\left(FF-RF\right)\left(1-\eta H\right)\right] \quad (31)$$

These derivatives imply cross-terms: the gradients of $H(t,x)$ interact with those of FF and RF, producing areas of high structural instability.

We can thus say that the force field becomes a real diagnostic tool depending on whether:

- Its divergence makes it possible to identify areas of active culling or deep blockage.
- Its rotational makes it possible to detect political feedback loops, unresolved internal tensions, and cycles of instability.

The integration of entropy in this field makes it possible to mathematically represent the destructive role of the poor circulation of information, a key factor in the failure of many public policies.

4. Qualitative Analysis of the System

We can comprehend how variables interact in space and time, as well as the circumstances in which the collective behavior $B(t,x)$ might tend towards a stable, unstable, or chaotic state, by studying the system qualitatively. Here, we differentiate between a number of scenarios based on the equation's structure (hyperbolic or parabolic), entropy levels, and force imbalance.

4.1. Stability of Collective Behaviour

When the development of collective behavior stabilizes in the face of external disturbances, or when $B(t,x)$ converges to a stationary solution over time, the system is said to be stable.

Stability Conditions:

- Low entropy $H(t,x) \to 0$
- Balanced Social Forces: $FF \approx RF$,
- Low divergence $\nabla \cdot F \approx 0$
- Low rotational $\nabla \times F \approx 0$

In this case, the parabolic PDE acts as a dissipative system, gradually leading to an equilibrium, often slow, but globally convergent

4.2. Instability and Collapse of the System

On the other hand, unstable behaviour occurs when $B(t,x)$ diverges in certain regions of space or exhibits oscillations that are not damped over time, often under the effect of strong or persistent perturbations.

Conditions of instability:

- High entropy $H(t,x) \gg 0$
- Strong imbalance between FF and RF
- Strong positive or negative divergence $|\nabla \cdot F| \gg 0$
- Significant rotational: $\nabla \times F \neq 0$, revealing a persistent conflictual structuring

In this case, the parabolic model becomes insufficient to capture complex dynamics. The transition to a hyperbolic model is necessary to represent the phenomena of crisis, rupture or rapid collapse.

5. Numerical Analysis

This article's theoretical examination provides the groundwork for a thorough numerical investigation of the system. In fact, without thorough numerical simulation, the complexity of the relationships among the variable's leadership, social forces, collective behavior, and informational entropy cannot be completely utilized.

5.1. Simulation of Parabolic and the G&G Leadership Model (Pseudo-Hyperbolic EDP with Memory)

The two mathematical formulations (parabolic and pseudo-hyperbolic) will be implemented using adapted numerical methods:

- Finite difference methods [10] (FDM) for spatial approximation ($\nabla^2 B$)
- Explicit or implicit schemas depending on the stability needed
- Real-time analysis of $B(t,x)$ solutions for different initial configurations

The objective will be to observe:

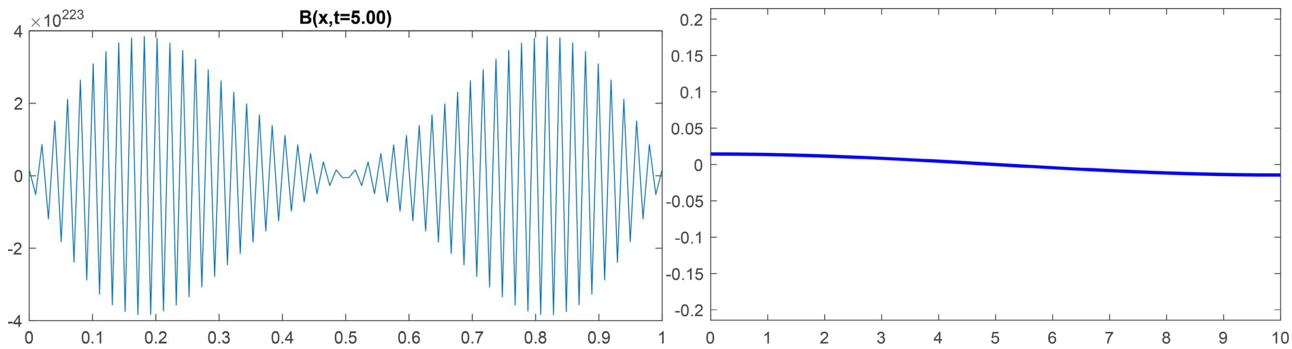
- The evolution of collective B behavior in high or low entropy contexts
- The spatial spread of crises or reforms
- The effect of different leadership profiles on system stabilization

NB: the simulation will be done with a GUI (Graphical User Interface) using MATLAB.

5.2. Results of Simulation & Discussion

In **Figure 1**, the parabolic model (11), $B(t,x)$ displays an hourglass oscillation as we vary the parameter α. The amplitude of this wave reaches $\mathbf{10^{223}}$ and fades between $t_{0.4}$ and $t_{0.6}$, before resuming its course. Conversely, when we do the same in the G&G model (21), we notice a line with a slight monotonic decreasing trend between [0.01, −0.01]. In practice, this means that in a parabolic regime, the forces of change (leadership) don't need diffusion as such to be manifest, but they will be followed by enormous tensions likely to disrupt the socio-politico-economic ecosystem (hence the amplitude), which will stabilize for a period ($t_{0.4}$ and $t_{0.6}$), which can be dubbed a "phase of partial stability", and which amplifies at the end of long-term leadership (or absence of leadership in $t_{0.7}$ and $t_{0.9}$). On the other hand, in a hyperbolic regime, we see a form of continuous and almost decreasing stability, which tells us that we're in the presence of passive leadership, so that we'd start from the assumption that political superstructures have a solid foundation. This may lead us to say that the socio-political-economic ecosystem evolves autonomously, so much so that even in the absence of leadership, all other things remain equal.

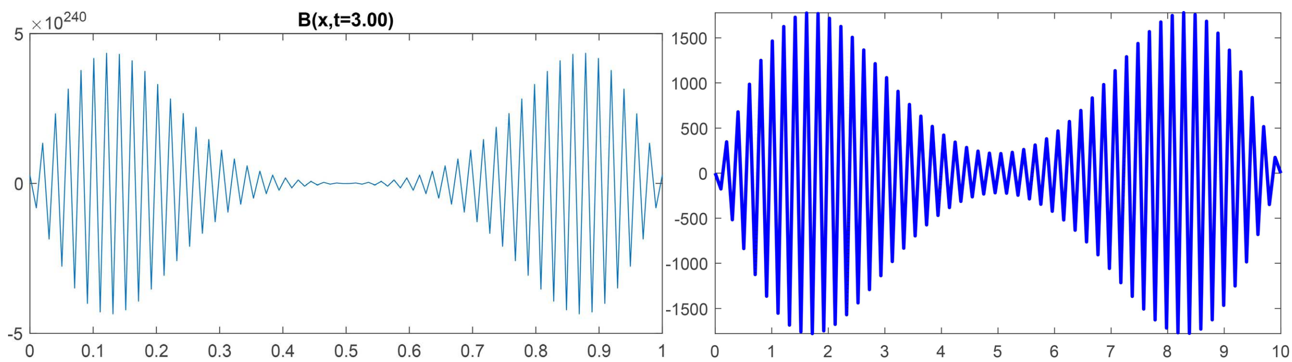
Figure 2 shows in (11) that $B(t,x)$ still exhibits an hourglass oscillation when, in addition to varying α, we vary the diffusion parameter D. The amplitude reaches $\mathbf{10^{240}}$ at the start of the oscillation and tends to stabilize between $t_{0.3}$ and $t_{0.7}$, before resuming its course from $t_{0.8}$. On the other hand, we observe that (21) also exhibits an hourglass oscillation with an amplitude of $\mathbf{(1.5 \times 10^3)}$, which stabilizes between t_4 and t_6, only to resume its course from t_8. Pragmatically speaking, we note that in a parabolic regime, diffusion (the spread of a reform) tends to make the leadership process more unstable in the installation phase (in other



(a) (b)

Figure 1. Parabolic model and G&G model with α variations and without D variations.

words, it will generate strong tension within the population). However, societal behavior will tend to stabilize for a period ($t_{0.3}$ and $t_{0.7}$), returning once again to a state of high tension at the end of leadership in the long term. On the other hand, in a hyperbolic regime, given that the socio-politico-economic foundations are stable, accentuating leadership through excessive diffusion risks distorting the ecosystem, causing slight tensions during the installation phase and ultimately leadership over the long term. This allows us to deduce that, in the case in point, leadership does not necessarily require diffusion to be effective, depending on whether it is passive or not. In a holistic way, we can thus say that for both regimes, policy-makers should carefully manage the propagation of reforms, at the risk of creating distortions within the population.

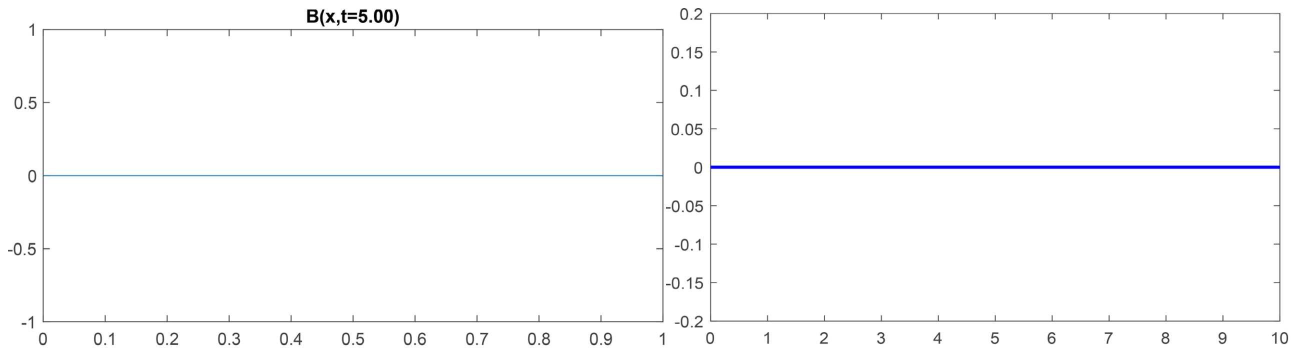


(a) (b)

Figure 2. Parabolic model and G&G model with α variations and without D variations.

In both models shown in **Figure 3**, $B(t,x)$ shows similar graphs with and without variations in D. This may lead us to say that resistance to change can only be felt if its main components are present: people (e.g. political opponents, etc.) and an environment conducive to political upheaval.

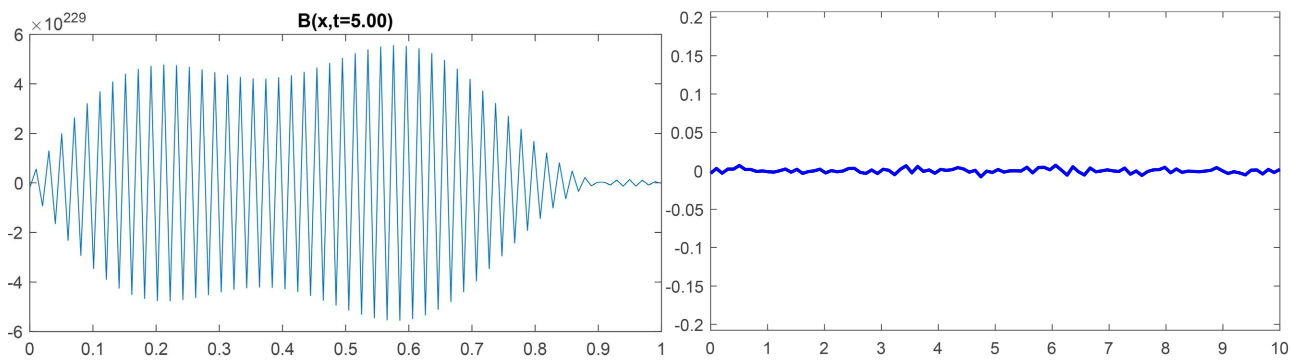
Figure 4 shows in (11), a strong oscillation with an amplitude of $\mathbf{10^{229}}$, tending to fade at the end of the stroke. In the (21), on the other hand, we observe a line moving along axis 0 with very low oscillations. We can see that the parameters μ_1 and μ_2 play a catalytic role in the frequency distribution of $B(t,x)$. In practice,



(a) (b)

Figure 3. Parabolic model and G&G model with β variations and with and without D variations.

this leads us to understand that, in a parabolic regime, any political upheaval can create strong tensions and distortions within the population, such that the notable instability in this regime makes it sensitive to any form of change at the risk of causing its collapse. On the other hand, in a hyperbolic regime, we can say that all forms of resistance to change are absorbed, provided the socio-economic-political ecosystem is stable.



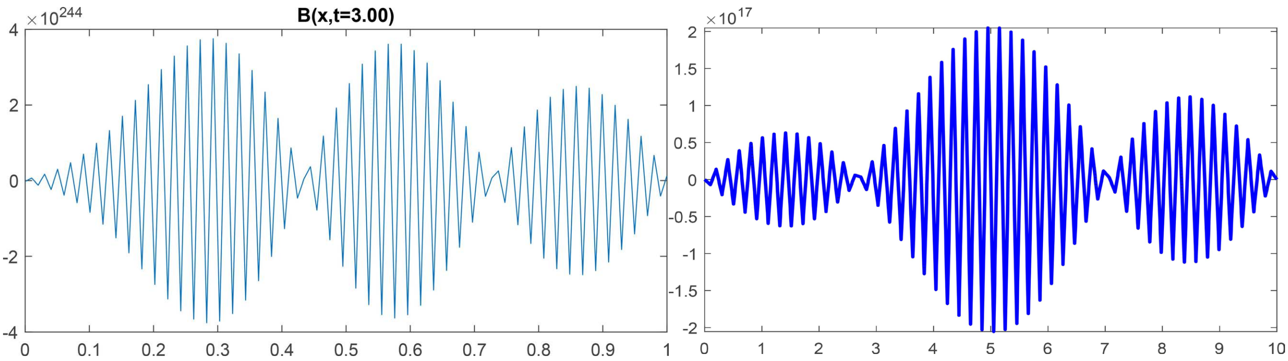
(a) (b)

Figure 4. Parabolic model and G&G model with β, μ_1, μ_2 variations without D variations.

In both models shown in **Figure 5**, we see very strong oscillations in $B(t,x)$, almost translating over time. In (11) the amplitude reaches $\mathbf{10^{244}}$ and $\mathbf{10^{17}}$ in (21). We see that D plays a fundamental role in the variational amplification of the parameters μ_1 and μ_2, giving deep meaning to β.

In practice, in a parabolic regime, we already notice without reform propagation that there is strong disturbance in the socio-economic-political ecosystem. When the latter is present, it causes a collapse of this ecosystem, in turn collapsing any possibility of leadership. Furthermore, in the light of this representation, we can distinguish three sequences that can be translated as periods of experimentation (short term, medium term and long term), which could lead us to say that the phenomena of tension would be more intense in the short and medium term, and could slightly decrease in intensity over the long term.

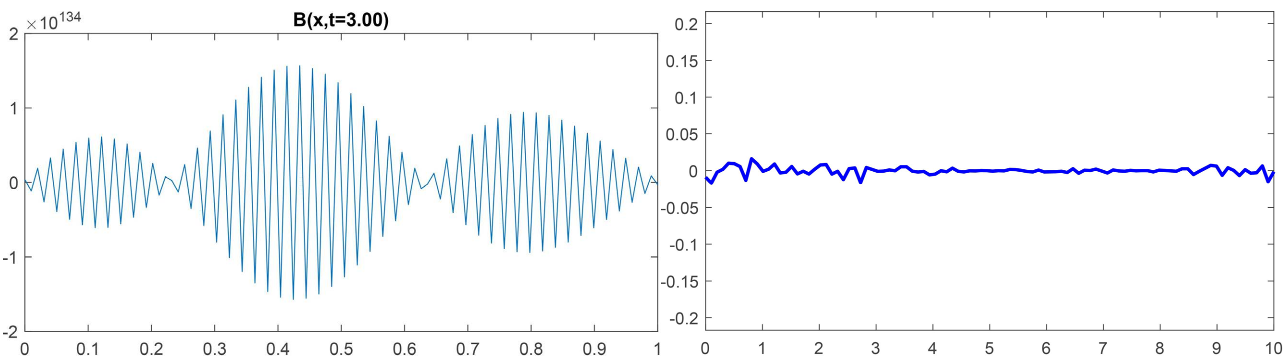
However, this phenomenon can be observed in a hyperbolic regime, with only one difference: the chaotic dimension observed in the parabolic regime is more sensitive and therefore more unstable. What's more, we can also say that strong tensions could manifest themselves over the medium term, just as tensions observed over the short term could reproduce themselves over the long term.



(a) (b)

Figure 5. Parabolic model and G&G model with β, μ_1, μ_2 variations and D variations.

Figure 6 shows in (11), an oscillation that is almost translational and fades at the end of the period. With an amplitude of $\mathbf{10^{134}}$, the entropy δ causes a significant perturbation in $B(t,x)$. Yet in (21), we observe a line with weak oscillations evolving on the 0 axis and smoothing out at the end of the period. Pragmatically, we can say that in a parabolic regime, informational disruption plays a major role in creating high tension within the socio-economic-political ecosystem, leaving the population in a state of total misinformation, a situation that could be remedied in the long term. On the other hand, in a hyperbolic regime, we note that there may be some minor tensions as a result of informational disruption, albeit controllable. We can thus say that state structures have control over the flow of information and transmit it faithfully. Entropic disturbances are therefore under absorption.



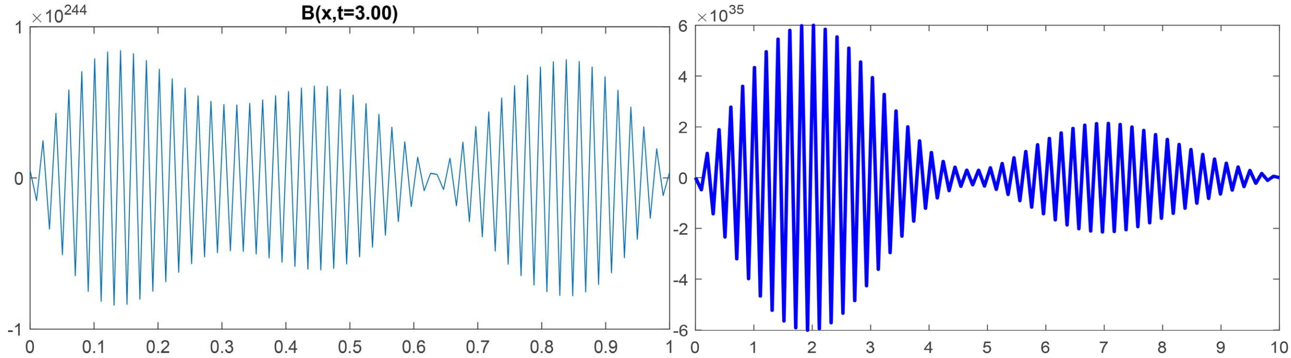
(a) (b)

Figure 6. Parabolic model and G&G model with δ variations without D variations.

In **Figure 7** we observe in (11) that $B(t,x)$ exhibits a strong oscillation and tends to fade at the end of the period with a slight translation ($t_{0.4}$ to $t_{0.6}$ and $t_{0.8}$ to

t_1). With an amplitude of $\mathbf{10^{244}}$, we see that D amplifies δ so that $\mathbf{10^{244}} > \mathbf{10^{134}}$. Furthermore, in (21) we observe a strong oscillation at the beginning of the period, fading at the end. With an amplitude of $\mathbf{10^{35}}$, we see that D amplifies δ so that we move from a line with quasi-weak oscillations to a large frequency movement. From a contextual point of view, in a parabolic regime, the strong propagation of an informational disturbance plays a destructive role in the socio-economic-political ecosystem and its collapse. What's more, the latter may fade over the medium term, only to be translated into the long term.

On the other hand, in a hyperbolic regime, we find that a strong propagation of an informational disturbance can nevertheless cause tensions within the socio-economic-political ecosystem over the short term, but which may fade over the long term.



(a) (b)

Figure 7. Parabolic model and G&G model with δ variations with D variations.

Figure 8 shows in the (11), that $B(t,x)$ exhibits a large frequency sequence of oscillation that amplifies at the end of the period, with an amplitude reaching $\mathbf{10^{134}}$ when α and δ are varied. However, in (21), a monotonically decaying line with slight oscillation is observed and located between [0.01, −0.01]. From a contextual point of view, in a parabolic regime, if the forces of leadership are accompanied by an informational disturbance, this will have serious consequences, even causing total chaos over the long term within the socio-economic-political ecosystem, giving rise to great agitation in societal behavior. On the other hand, in a hyperbolic regime, we may see very slight tensions at the start of the process, but these will be kept under control over the long term, depending on our assumption that state milestones are sufficiently hardened to ensure that leadership is sustained even in the event of an informational disruption.

Figure 9 shows in (11), that $B(t,x)$ exhibits a regular oscillation at the beginning of the period, which amplifies at the end of the period, reaching an amplitude of $\mathbf{10^{244}}$ when α and δ vary under the influence of D. On the other hand, in the (21) we observe a strong amplification of oscillations reaching an amplitude of 10^{65}. In the practical case, within a parabolic regime, when the propagation of

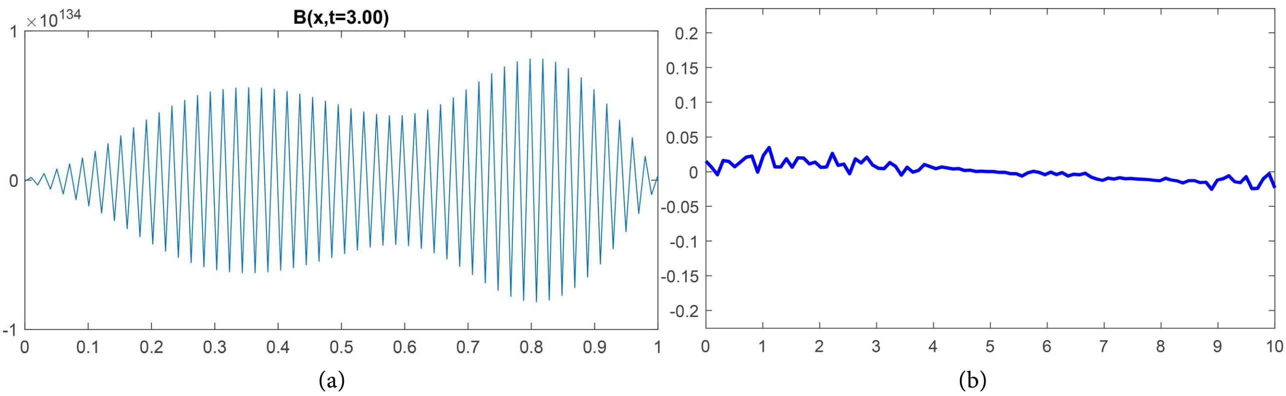


Figure 8. Parabolic model and G&G model with α, δ variations without D variations.

leadership reforms is accelerated with informational disruptions, the devastating effects will not be felt in the short term but rather in the long-term causing irreparable collapse of the leadership and the socio-economic-political ecosystem. However, in a hyperbolic regime, when the propagation of leadership reforms is accelerated with informational disruptions, this already causes at first sight a strong tension at the beginning of the period and an amplification of the latter over the long term, which may also cause a collapse of the leadership and the socio-economic-political ecosystem over the long-term.

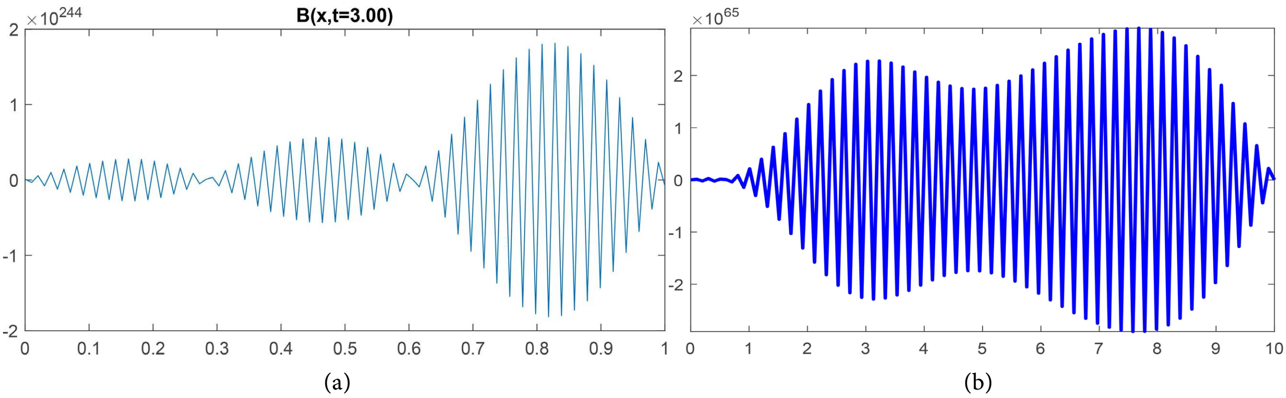


Figure 9. Parabolic model and G&G model with α, δ variations with D variations.

General analysis and contextualization

Parabolic model

As a result of this simulation, we were able to observe that the parabolic model shows a real sensitivity to all variations ($\alpha, \beta, \delta, D, \mu_1$ & μ_2) causing it to reach oscillations approaching 10^{240} (maximum threshold), which denotes numerical instability and this especially, under entropic disturbance. This reality, which runs counter to its theoretical and analytical stability, also enables us to understand that, as a socio-economic-political ecosystem sensitive to any positive or negative change, it is highly susceptible to turning into chaos, which also enables us to highlight an irreversible situation with no possibility of remedy. In this respect, leadership under a parabolic regime will find it difficult to take root without strong tensions within the population, with or without informational entropy. What's more, the acceleration of reforms could make the system chaotic regardless of its presence or absence, since societal instability will always be preyed to arousing strong tensions. The parabolic representation of leadership allows us to understand, in fine, that we are in the presence of a regime where the psychology of the population is the antithesis of the latter, and which is prey to exploding or self-destructing at any moment in an irreversible manner.

G&G leadership model (Pseudo-hyperbolic PDE with memory)

With regard to the simulation carried out, we were able to note that (21) exhibits notable stability with regard to all variations ($\alpha, \beta, \delta, \mu_1$ & μ_2) only causing it to reach oscillations approaching 10^{60} (maximum threshold) following potential variations in parameter D. Its numerical stability also allows us to attest its analytical stability following the behavioral stability theorem under entropic perturbation. To this end, we can say that a hyperbolic regime has a high probability of converging towards effective stability if and only if the reform propagation parameter is carefully regulated to avoid any form of explosion or collapse of the socio-economic-political ecosystem. Moreover, it has a strong capacity to resist resistance to change, as well as any form of misinformation or informational disruption; a situation made possible by the fact that the leadership present in this regime is of the passive type, as the state superstructures coupled with the homogeneous movement of the population evolve autonomously. In addition to informational mastery, this regime will be able to prevent any form of political revolution against the established system, and plan a resurgence in advance.

In conclusion, given the above, in a system where leadership is structured with a temporal memory (21), entropic disturbances and resistance to change do not brutally disorganize the system. However, in a governance system without inertia (parabolic model), any disturbance can tumble into chaos if it is not instantly absorbed by a good diffusion.

6. Conclusions

Article Overview

The results expected after simulations have given us an insight into the potential regimes to be found in most countries, especially in Africa, in this case the Democratic Republic of the Congo, currently in a chaotic socio-economic-political situation. Indeed, the absence of effective leadership is leading the country towards self-destruction of its state structures, making it sensitive to any changes (positive or negative).

As the Congolese population is subject to high levels of tension linked to political instability (corruption, bad governance, civil war), it is faced with significant misinformation, preventing it from being aware of all realities, whether endogenous or exogenous; a situation which is at the root of potential internal revolts, inexorably impacting the Congolese economy.

This is why [11] *asserts that political instability is the form of existence of the Democratic Republic of the Congo; the DRC is a never-ending crisis. It is a long and lasting consequence of the violent creation, through colonial conquest, of its territory; a creation that was sanctioned by the Berlin International Conference on the Congo* (1884-1885). *The Congolese people are the product of* 500 *years of struggle against the system, now globalized, in all its phases. They are the product of victories, but above all defeats, in these struggles against the slave trade and Atlantic and Arabic-speaking slavery. Until almost the* 1920s, *against colonial conquest, expropriation, predation, the plundering of their resources, wars, neo-colonial authoritarianism demanded by the hegemonic antagonism of the Cold War and growing impoverishment.*

In the light of this gloomy reality, we understand why the DRC is in the grip of a political regime that has no means of regaining a state of stability in all its spheres; a reality that faithfully reflects the characteristics of the parabolic partial differential equation model known as the "**parabolic regime**".

Recommendations

The aim of our study was to present a new leadership model, the "**G&G Leadership model**", mathematically constructed via a transition from a parabolic PDE to a pseudo-hyperbolic one, in order to prove that it is possible to recover a state of socio-economic-political stability that we would describe as a "**hyperbolic regime**" under certain conditions:

a) Establishing adaptive transformational leadership

The model indicates that the reactivity of the social system is influenced by the dynamics of change forces ($\alpha \frac{\partial F}{\partial t}$) in response to resistances (β). Transformational leadership, which can anticipate social change and manage tremors, helps to reduce behavioral instabilities. With this in mind, it is essential to train public leaders in the systemic and dynamic approach to change so that they become tension regulators rather than mere managers.

b) Optimize channels for listening to and involving citizens.

Political entropy is influenced by the parameter $P(t,x)$, which represents collective consciousness. When the population is excluded from the decision-making process, social entropy increases, leading to systemic instability. To ensure a solid interconnection between leadership and the citizen base, it is necessary to introduce inclusive procedures such as public consultations, participative digital platforms or polls.

c) Adjust the structural parameters of the environment $e(t,x)$

Structural elements such as economic crises, institutional conflicts or legal instability act as catalysts for resistance to change. Radical reform of public institutions, improved social justice and economic stability are needed to create a structural environment conducive to transformation.

d) Refining the propagation of change (parameter D)

In the G&G model, diffusion D acts as a regulator. It favors a balanced distribution of transformation dynamics within the social structure. It would therefore be necessary to develop strategies for information, civic education and citizen mobilization, in order to guarantee a homogeneous diffusion of change across all strata of the population.

e) Reducing political entropy through uniform strategies

Entropy, symbolized by $-\delta \frac{\partial p}{\partial t}(\log p+1)$, illustrates information ambiguities, policy discordances and strategic divergences. To remedy this, it is imperative to design policies that are explicit, coherent and in line with a collective national vision. It is essential that political communication is logical, intelligible and cohesive.

Thus, in view of all these recommendations, we believe that it will be possible for the DRC to achieve inclusivity in its economic growth, since the economic policies enabling this optimum will be founded on solid, structured and clearly defined foundations such that the functions of resource allocation and distribution will be conveyed by crystal-clear coherence and without the possibility of being diverted to personal ends favoring the security of the public treasury.

Relative limits of our working paper

We encountered a real difficulty in our search for a mathematical model of leadership on which to base the G&G model. Lewin's behavioral model and his theory of force fields helped us to develop our model.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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