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Vector Flux through a Closed Surface Compared to Eye Drops That Dissolve Protein Clumps That Cause Cataracts

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Abstract

Researchers are investigating the possibility of developing Eye drops that will dissolve protein clumps that cause cataracts, but this study is in its early stages. Our aim in this article is to think about geometric-mathematical tools, that compare such Eye drops to a vector flux through a closed curved surface as an Eye lens, with the help of Gauss's Theorem.

Keywords

Vector Flux through a Closed Surface, Gauss's Theorem, Curved Surface

1. Introduction

In article [1], we developed vector area and volume elements in curved coordinate for flux vector fields into open and closed curved spaces compared to Eye retinas, by proving Gauss's Theorem that we use in this article by **Appendix A** and **Appendix B**.

In article [2], we developed flux vector fields with stem cells into open and curved spaces in the retinas of recently blind people and (AMD) patients to enable the growth of visual cells in their retinas. We also used curved coordinates that we use in this article by **Appendix C**.

In articles [3] [4] and [5], we presented mirror eye lens and telescopic eye lens consisting of 3 lenses with a variable point radius eye lens for the aid of age macular degeneration (AMD) but we did not present the structure of the lens.

In article [6], researchers are investigating the possibility of developing eye drops with special vitamins that will dissolve protein clumps that cause cataracts, but this study is in its early stages. Also, early research showed that lanstrol and

N-acetylcarnosine eye drops could reduce cataract-related cloudiness in animal models, but more extensive human trials are still needed.

2. Eye Drops Compared to a Vector Flux through an Eye Lens with Cataracts, Compared to a Closed Ellipsoid Surface

Eye lens as an ellipsoid lens or a spherical lens which is a closed surface, let's say: $x^2+y^2+a^2z^2=a^2$, $a>1$, or: $x^2+y^2+z^2=a^2$, depends on the patients Eye lens, is shown in **Figure 1**.

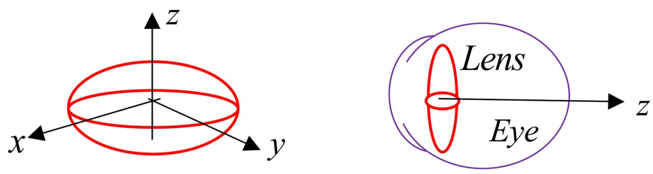

Figure 1. The Eye lens as an ellipsoid or a spherical closed surface.

3. Eye Drops as a Flux of a Vector Field through a Closed Surface That Is the Lens of the Eye, According to Gauss's Theorem

Gauss's theorem: The flux of a vector field $\vec{F}=Pi+Qj+Rk$ through a closed surface is equal to the scalar operation of the Hamilton operator $\vec{\nabla}=\frac{\partial}{\partial x}i+\frac{\partial}{\partial y}j+\frac{\partial}{\partial z}k$ on the vector field into the entire volume that the surface closes: $\oiint_S \vec{F}\bullet \mathrm{d}\vec{S}=\iiint_V \vec{\nabla}\bullet\vec{F}\cdot \mathrm{d}V=\iiint_V div\left(\vec{F}\right)\cdot \mathrm{d}V$, which is proofed in **Appendix A** and **Appendix B**.

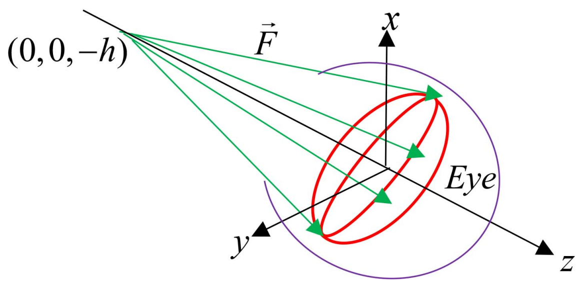

Figure 2. Eye drops, as a flux of a vector field through the Eye lens which is a closed surface.

Using **Figure 2**, and if we assume that the equation of an Eye lens is: $x^2+y^2+a^2z^2=a^2$ or $x^2+y^2+z^2=a^2$, and the general vector field flux is: $\vec{F}=m\left(xi+yj+\left(z-h\right)k\right)$, where: m is Eyedrop's density, and where m,x,y,z depend on drop viscosity, lens permeability, and protein clump distribution, limiting its predictive power, that depends on Eye lens which are now in research.

If drop viscosity, lens permeability, and protein clump distribution are not functions of m,x,y,z then according to Gauss's theorem:

$$\oiint_S \vec{F} \bullet \mathrm{d}\vec{S} = \iiint_V \vec{\nabla} \bullet \vec{F} \cdot \mathrm{d}V$$
$$= \iiint_V \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \right) \bullet m\left(xi + yj + (z-h)k \right) \cdot \mathrm{d}V$$
$$= 3m \iiint_V \mathrm{d}V$$

Continued calculation in curve coordinates ρ, φ, z, with the help of **Appendix C**.

1) For an ellipsoid Eye lens:

$$x^2 + y^2 + a^2 z^2 = a^2 \Rightarrow z = \pm \frac{1}{a} \sqrt{a^2 - \left(x^2 + y^2 \right)} = \pm \sqrt{1 - \left(\frac{\rho}{a} \right)^2}$$

$$\oiint_S \vec{F} \bullet \mathrm{d}\vec{S} = \iiint_V \vec{\nabla} \bullet \vec{F} \cdot \mathrm{d}V = 3m \iiint_V \mathrm{d}V = 3m \iiint_V \rho \mathrm{d}\varphi \mathrm{d}\rho \mathrm{d}z$$

$$= 3m \int_0^{2\pi} \mathrm{d}\varphi \int_0^a \rho \mathrm{d}\rho \int_{-\sqrt{1-\left(\frac{\rho}{a}\right)^2}}^{\sqrt{1-\left(\frac{\rho}{a}\right)^2}} \mathrm{d}z = 3m \cdot 2\pi \int_0^a 2\rho \sqrt{1 - \left(\frac{\rho}{a} \right)^2} \mathrm{d}\rho$$

$$= 12\pi m \int_0^a \sqrt{1 - \left(\frac{\rho}{a} \right)^2} \rho \mathrm{d}\rho = 12\pi m \int_0^a \mathrm{d} \left(-\frac{a^2}{3} \sqrt{\left(1 - \left(\frac{\rho}{a} \right)^2 \right)^3} \right)$$

$$= 12\pi m \left[-\frac{a^2}{3} \sqrt{\left(1 - \left(\frac{\rho}{a} \right)^2 \right)^3} \right]_0^a = 4\pi m a^2,$$

$$\oiint_S \vec{F} \bullet \mathrm{d}\vec{S} = \iiint_V \vec{\nabla} \bullet \vec{F} \cdot \mathrm{d}V = 4\pi m a^2$$

2) For a spherical Eye lens:

$$x^2 + y^2 + z^2 = a^2 \Rightarrow z = \pm \sqrt{a^2 - \left(x^2 + y^2 \right)} = \pm \sqrt{a^2 - \rho^2}$$

$$\oiint_S \vec{F} \bullet \mathrm{d}\vec{S} = \iiint_V \vec{\nabla} \bullet \vec{F} \cdot \mathrm{d}V = 3m \iiint_V \mathrm{d}V = 3m \iiint_V \rho \mathrm{d}\varphi \mathrm{d}\rho \mathrm{d}z$$

$$= 3m \int_0^{2\pi} \mathrm{d}\varphi \int_0^a \rho \mathrm{d}\rho \int_{-\sqrt{a^2-\rho^2}}^{\sqrt{a^2-\rho^2}} \mathrm{d}z = 3m \cdot 2\pi \int_0^a 2\rho \sqrt{a^2 - \rho^2} \mathrm{d}\rho$$

$$= 12\pi m \int_0^a \sqrt{a^2 - \rho^2} \rho \mathrm{d}\rho = 12\pi m \int_0^a \mathrm{d} \left(-\frac{1}{3} \sqrt{\left(a^2 - \rho^2 \right)^3} \right)$$

$$= 12\pi m \left[-\frac{1}{3} \sqrt{\left(a^2 - \rho^2 \right)^3} \right]_0^a = 4\pi m a^3,$$

$$\oiint_S \vec{F} \bullet \mathrm{d}\vec{S} = \iiint_V \vec{\nabla} \bullet \vec{F} \cdot \mathrm{d}V = 4\pi m a^3$$

The flux of a vector field through a closed spherical Eye lens by spherical coordinates: θ, φ, r with the help of **Appendix D**.

$$\oint\!\!\oint_S \vec{F} \bullet \mathrm{d}\vec{S} = \iiint_V \vec{\nabla} \bullet \vec{F} \cdot \mathrm{d}V = 3m \iiint_V \mathrm{d}V = 3m \iiint_V r^2 \sin(\theta) \mathrm{d}\varphi \mathrm{d}\theta \mathrm{d}r$$

$$= 3m \int_0^{2\pi} d\varphi \int_0^{\pi} \sin(\theta) \mathrm{d}\theta \int_0^{a} r^2 \mathrm{d}r = 3m \cdot 2\pi \cdot \left[\frac{r^3}{3}\right]_0^a \cdot \int_0^{\pi} \sin(\theta) \mathrm{d}\theta$$

$$= 6\pi m \frac{a^3}{3} \left[-\cos(\theta)\right]_0^{\pi} = 4\pi m a^3$$

$$\oint\!\!\oint_S \vec{F} \bullet \mathrm{d}\vec{S} = \iiint_V \vec{\nabla} \bullet \vec{F} \cdot \mathrm{d}V = 4\pi m a^3$$

4. Conclusions

We hope that by geometric-mathematical tools, by the flux of Spatial drops into the closed curved space of the Eye lens, Eye drops that dissolve protein clumps causing cataracts, all over the closed Eye lens, we will help to destroy cataracts in the Eye lens by Eye drops, instead of surgeries.

Advantages of the article:

1) The model proposed in the article is based on an innovative method for using eye drops against cataracts instead of surgery. The model proposed in the paper is a mathematical model based on vector flux through a closed surface, using Gauss's theorem.

2) The advantage of using special eye drops against cataracts is a non-invasive treatment, given the risks associated with any surgical intervention.

3) The model does not take into account the parameters: lens permeability, and droplet viscosity but can encourage advances in medical and pharmaceutical research on non-surgical treatments for cataracts and provide the possibility of additional treatment for the dreamer.

Medical and pharmacological research is in its infancy, so that there are no results that could be mentioned about drop viscosity, lens permeability, and protein clump distribution.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A: Introduction to the Proof of Gauss's Theorem, According to Reference [1]

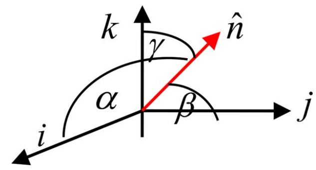


Figure A1. A unit vector: $\hat{n} = \cos(\alpha)i + \cos(\beta)j + \cos(\gamma)k$.

The vector flux through an open surface, according to **Figure A1**:

$$\iint_S \vec{F} \bullet \mathrm{d}\vec{S} = \iint_S \vec{F} \bullet \hat{n}\mathrm{d}S = \iint_S \vec{F} \bullet \left(\mathrm{d}S\cos(\alpha)i + \mathrm{d}S\cos(\beta)j + \mathrm{d}S\cos(\gamma)k\right)$$
$$= \iint_S (Pi + Qj + Rk) \bullet (\mathrm{dydz}i + \mathrm{dxdz}j + \mathrm{dxdy}k)$$
$$= \iint_S (P\mathrm{dydz} + Q\mathrm{dxdz} + R\mathrm{dxdy})$$

When: $\mathrm{d}S\cos(\alpha) = \mathrm{dydz} \perp i$, $\mathrm{d}S\cos(\beta) = \mathrm{dxdz} \perp j$, $\mathrm{d}S\cos(\gamma) = \mathrm{dxdy} \perp k$ (**Figure A2**).

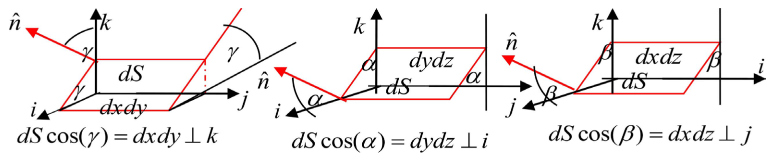


Figure A2. Projections of the area element $\mathrm{d}S$ by the angles α, β, γ on the axis planes.

Appendix B: Development of the Proof of Gauss's Theorem about Vector Flux through a Closed Surface, According to Reference [1]

$$\oiint_S \vec{F} \bullet \mathrm{d}\vec{S} = \oiint_S (Pi + Qj + Rk) \bullet \mathrm{d}S\hat{n}$$
$$= \oiint_S (Pi + Qj + Rk) \bullet (\mathrm{dydz}i + \mathrm{dxdz}j + \mathrm{dxdy}k)$$
$$= \oiint_S (P\mathrm{dydz} + Q\mathrm{dxdz} + R\mathrm{dxdy})$$
$$= \oiint_S Pi \bullet \mathrm{dydz}i + \oiint_S Qj \bullet \mathrm{dxdz}\,j + \oiint_S Rk \bullet \mathrm{dxdy}k$$
$$= 3 + 2 + 1$$

1. $\oiint_S R(x,y,z)k \bullet \mathrm{dxdy}k$

$$= \iint_{S, z=f_2(x,y)} R\left(x,y,f_2(x,y)\right)k \bullet \mathrm{dxdy}k$$
$$+ \iint_{S, z=f_1(x,y)} R\left(x,y,f_1(x,y)\right)k \bullet \mathrm{dxdy}(-k)$$
$$= \iint_S \mathrm{dxdy}\left(R\left(x,y,f_2(x,y)\right) - R\left(x,y,f_1(x,y)\right)\right)$$

$$= \iint_S \mathrm{dxdy}\left[R(x,y,z)\right]_{z=f_1(x,y)}^{z=f_2(x,y)}$$

$$= \iint_S \mathrm{dxdy} \int_{f_1(x,y)}^{f_2(x,y)} \frac{\partial R}{\partial z}\mathrm{dz} = \iiint_V \frac{\partial R}{\partial z}\mathrm{dxdydz} = \iiint_V \frac{\partial R}{\partial z}\mathrm{d}V$$

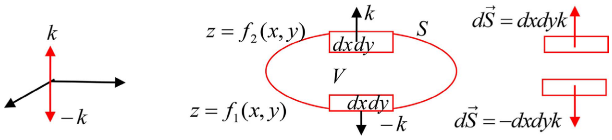


Figure A3. Vector field flux throe z axis.

2. $\oiint_S Q(x,y,z)\, j \bullet \mathrm{dxdz}j$

$$= \iint_{S,y=g_2(x,z)} Q(x,g_2(x,z),z)\, j \bullet \mathrm{dxdz}j$$

$$+ \iint_{S,y=g_1(x,z)} Q(x,g_1(x,z),z)\, j \bullet \mathrm{dxdz}(-j)$$

$$= \iint_S \mathrm{dxdz}\left(Q(x,g_2(x,z),z) - Q(x,g_1(x,z),z)\right)$$

$$= \iint_S \mathrm{dxdz}\left[Q(x,y,z)\right]_{y=g_1(x,z)}^{y=g_2(x,z)}$$

$$= \iint_S \mathrm{dxdz} \int_{g_1(x,z)}^{g_2(x,z)} \frac{\partial Q}{\partial y}\mathrm{dy} = \iiint_V \frac{\partial Q}{\partial y}\mathrm{dxdydz} = \iiint_V \frac{\partial Q}{\partial y}\mathrm{d}V$$

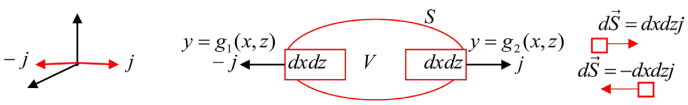


Figure A4. Vector field flux throe y axis.

3. $\oiint_S P(x,y,z)\, i \bullet \mathrm{dydz}i$

$$= \iint_{S,x=h_2(y,z)} P(h_2(y,z),y,z)\, i \bullet \mathrm{dydz}i$$

$$+ \iint_{S,x=h_1(y,z)} P(h_1(y,z),y,z)\, i \bullet \mathrm{dydz}(-i)$$

$$= \iint_S \mathrm{dydz}\left(P(h_2(y,z),y,z) - P(h_1(y,z),y,z)\right)$$

$$= \iint_S \mathrm{dydz}\left[P(x,y,z)\right]_{x=h_1(y,z)}^{x=h_2(y,z)}$$

$$= \iint_S \mathrm{dydz} \int_{h_1(y,z)}^{h_2(y,z)} \frac{\partial P}{\partial x}\mathrm{dx} = \iiint_V \frac{\partial P}{\partial x}\mathrm{dxdydz} = \iiint_V \frac{\partial P}{\partial x}\mathrm{d}V$$

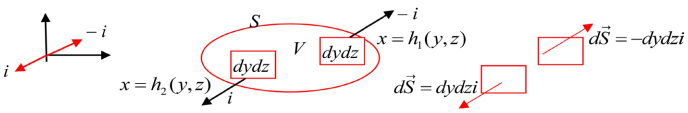


Figure A5. Vector field flux throe x axis.

By **Figures A3-A5**, we prove Gauss's Theorem.

$$3+2+1=\oint\!\!\!\oint_S \vec{F}\bullet \mathrm{d}\vec{S}=\iiint_V \frac{\partial P}{\partial x}\mathrm{d}V+\iiint_V \frac{\partial Q}{\partial y}\mathrm{d}V+\iiint_V \frac{\partial R}{\partial z}\mathrm{d}V$$

$$=\iiint_V\left(\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}\right)\cdot \mathrm{d}V$$

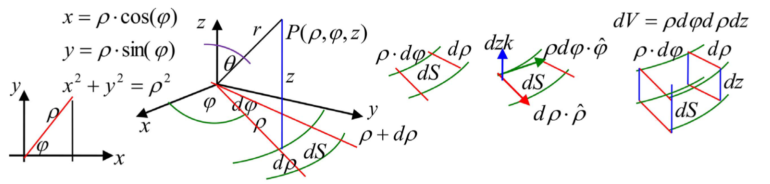
$$=\iiint_V\left(\frac{\partial}{\partial x}i+\frac{\partial}{\partial y}j+\frac{\partial}{\partial z}k\right)\bullet\left(Pi+Qj+Rk\right)\cdot \mathrm{d}V$$

$$=\iiint_V \vec{\nabla}\bullet\vec{F}\cdot \mathrm{d}V=\iiint_V div\left(\vec{F}\right)\cdot \mathrm{d}V$$

$$\oint\!\!\!\oint_S \vec{F}\bullet \mathrm{d}\vec{S}=\iiint_V \vec{\nabla}\bullet\vec{F}\cdot \mathrm{d}V=\iiint_V div\left(\vec{F}\right)\cdot \mathrm{d}V$$

Appendix C

Development of the volume element in curved coordinates ρ,φ,z, by **Figure A6**, according to Reference [2].



$$dS=\left|d\vec{S}\right|=\left|d\rho\cdot\hat{\rho}\times\rho d\varphi\cdot\hat{\varphi}\right|=\rho\cdot d\varphi\cdot d\rho\left|k\right|=\rho\cdot d\varphi\cdot d\rho$$

$$dV=(d\rho\cdot\hat{\rho}\times\rho\cdot d\varphi\cdot\hat{\varphi})\bullet dz\cdot k=\rho\cdot d\varphi\cdot d\rho\cdot dz\cdot(\hat{\rho}\times\hat{\varphi})\bullet k=\rho\cdot d\varphi\cdot d\rho\cdot dz\cdot k\bullet k=\rho\cdot d\varphi\cdot d\rho\cdot dz$$

Figure A6. The volume element $\mathrm{d}V=\rho\mathrm{d}\varphi\mathrm{d}\rho\mathrm{d}z$ in curved coordinates: ρ,φ,z system.

Appendix D

Development of the volume element in spherical coordinates: θ,φ,r, by **Figure A7**.

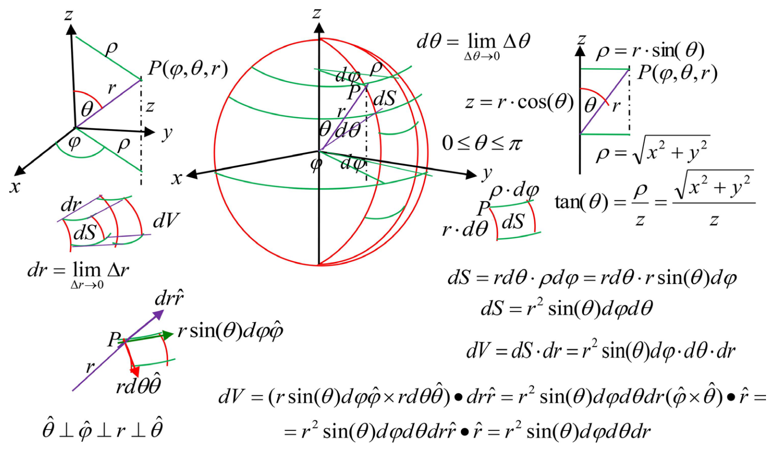


Figure A7. The volume element $\mathrm{d}V=r^2\sin\left(\theta\right)\mathrm{d}\varphi\cdot\mathrm{d}\theta\cdot\mathrm{d}r$ in spherical coordinates: θ,φ,r system.