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Length, Simultaneity, Time, Mass, Kinetic Energy, Quantization, and Subspace-Times Diagrams According to the Inverse Relativity Model

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Abstract

Third paper on the inverse relativity model, the first paper [1] included the splitting of total spacetime into positive and negative subspace-times, the second paper [2] included a description of relativistic mechanics in each subspace-time. In the third paper here, we study the properties, quantization and diagrams of each subspace-time and as a result we get the inverse length relativity where the length expands from the value zero to the original length in the negative subspace, inverse relativity of simultaneity where simultaneity is absolute and time is real and not an illusion in positive subspace, inverse relativity of time where time contracts to become supertime in negative subspace, inverse relativity of mass and kinetic energy where both are reduced in positive subspace. The paper also included new concepts of mass, work, and motion, An explanation of Heisenberg's principle and discovering the existence of the inverse of the principle, and the quantization of transformation equations in each subspace. According to this quantization, we do not need infinite energy for a material particle to reach the speed of light in the special case, and there is no infinite energy density for a dust field in negative subspace in the general case. The structure of the negative subspace-time does not collapse when the speed of the reference frame reaches the speed of light, but the negative subspace-time turns into reverse subspace-time. We also propose in the paper a possibility for testing the model.

Keywords

Inverse Length Relativity, Supertime, Inverse Mass Relativity, Inverse Spacetime, Inverse Heisenberg Principle, Inverse Theory of Relativity

1. Introduction

In the first and second papers, we presented the idea of the inverse relativity model and the approach followed with that model. According to this presentation, we can say that the inverse relativity model, whether in the special or general case, is a model for studying the structure of spacetime itself as a complex structure of subspace-times. And not to study a physical problem in itself, as in special relativity, the purpose of which was to find a solution to the problem of the stability of the speed of light in the experiment of Michelson and Morley [3] and the results resulting from that solution, or as in general relativity, where Einstein sought to provide an explanation of gravity as a curvature of the fabric of space-time [4] and the results resulting from that as well after Newton's failure to provide an explanation of the nature of gravity. However, the model paves the way for solving some problems of modern physics such as the problem of relativistic thermodynamics [5]. In the third paper we complete the model by studying the following properties: the measurement of length in the direction of motion or the proper distance, the simultaneity between two events, the proper time, the mass and kinetic energy in each subspace-time, and also the shape of the diagrams of positive and negative subspace-times and the structure of causality associated with these diagrams. We will also try to understand the relation between mass, energy, work and motion more deeply according to the new energy and momentum transformations for each subspace-time described in the second paper. As we know, the measurements of length, distance, time, simultaneity and mass in classical Galilean relativity [6] are physical constants when transferring from one inertial frame of reference to another. However, with the emergence of special relativity [7] our view of these physical constants changed, as these constants appeared as relativistic variables when transferring from one inertial frame of reference to another when the relative speed between the frames of reference is close to the speed of light. For example, the length in the direction of motion contracts and the time taken for an event expands with the motion of the reference frame, events that are simultaneous with respect to one observer are not necessarily simultaneous with respect to the other observer, mass increases with the motion of the reference frame [8]. The results of special relativity were revolutionary for its time, as it changed many concepts of physics at that time. However, on the other hand, we find that relativity went too far in describing the nature of time, as it considered time an illusion and presented the concept of a block universe [9], thus contradicting the thermodynamic concept of time. We also find that the structure of space-time in the special case collapses when the speed of the reference frame reaches the speed of light theoretically, where length contracts to infinity and time expands to infinity as well. In the general case, when the gravitational particle reaches the Schwarzschild radius, we have a spacetime singularity and an infinite energy density. Both special and general relativity are classical theories [10] because they ignore quantum properties such as the Heisenberg principle [11], which results in great difficulty in quantizing spacetime, or in other words, showing the quantum properties of particles on flat spacetime in the special case or on curved spacetime in the general case. Considering inverse relativity a modern model for the structure of space-time in the special and general cases, does it reveal to us new properties of time and space that none of the previous theories have reached? In other words, will the new model reveal to us another behavior for the relativistic variables of length, time, simultaneity, mass, and energy? Can the new model theoretically solve the problems rooted in special and general relativity? This is what we will try to answer in this paper.

2. Methods

2.1. Inverse Relativity of Length

Assuming that we have two inertial reference frames [12] S and S' from orthogonal coordinate systems, each frame of reference has an observer at the origin O and O', and that the frame S' is moving at a constant speed V_s with respect to the frame S in the positive direction of the x-axis. We also assume that we have in frame S' a stationary rod whose length is parallel to the x-axis, see **Figure 1**.

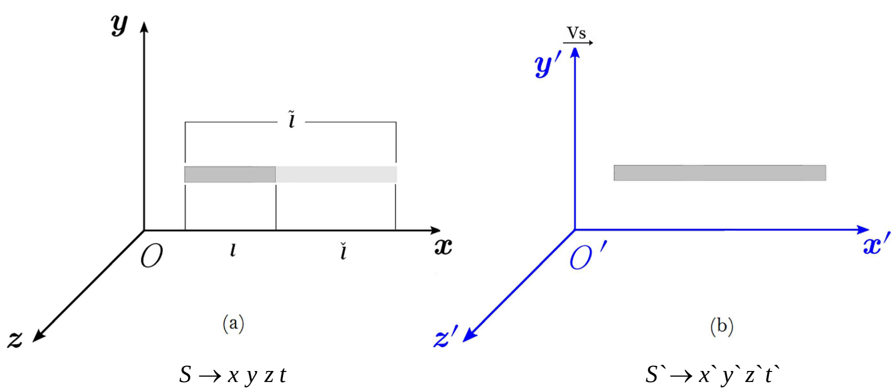

Figure 1. (a) shows the length of the positive and negative rod relative to the reference frame S, (b) shows the length of the rod relative to the reference frame S'.

Because the rod is fixed with respect to the frame of reference S' and its length is parallel to the $x^{`}$-axis, the ends of the rod lie on the $x^{`}$-axis. The observer O' measures the first end of the rod at the point $x^{`}_1$ and the time $t^{`}_1$, and the second end at the point $x^{`}_2$ and the time $t^{`}_2$, the length of the rod represents the difference between these two points $l^{`}_0 = \Delta x^{`}$. As for the observer O, the rod will appear to be moving. Therefore, the two ends of the rod must be measured at the same instant in time. If we want to observe the length of the rod in the positive subspace, the first end of the rod will be at the point $\tilde{x}_1$ and the time $\tilde{t}_1$, and the second end will be at the point $\tilde{x}_2$ and the same time $\tilde{t}_1$, and the positive length of the rod represents the difference between these two points $\tilde{l} = \Delta\tilde{x}$. From the inverse positive modified Lorentz transformations shown in the first paper [1], Equation (11.1), we obtain the transformation of the coordinates of each point on the x-axis in the positive subspace.

$$\tilde{x}_1 = x\grave{}_1 \qquad \tilde{x}_2 = x\grave{}_2 \tag{1.3}$$

By subtracting the two equations, we obtain the transformation of the spatial period into the positive subspace, which also represents the positive length of the rod

$$\Delta\tilde{x}_{21} = \Delta x\grave{}_{21} \quad \Rightarrow \quad \tilde{l} = l\grave{}_0 \tag{2.3}$$

Equation (2.3) shows us that the length of the rod in the positive subspace is constant for both observers. This is due to the symmetry of the positive subspace structure for both observers, as shown in the first paper [1], Section 2.3.

$$\breve{x}_1 = \gamma\left(x\grave{}_1\left(1-\gamma^{-1}\right)+V_s t\grave{}_1\right) \qquad \breve{x}_2 = \gamma\left(x\grave{}_2\left(1-\gamma^{-1}\right)+V_s t\grave{}_2\right) \tag{3.3}$$

By subtracting the two equations

$$\Delta\breve{x}_{21} = \gamma\left(\Delta x\grave{}_{21}\left(1-\gamma^{-1}\right)+V_s \Delta t\grave{}_{21}\right) \tag{4.3}$$

But Equation (4.3) shows the time of the observer O' and not the time of the observer O, so we use the fourth equation of the inverse negative modified Lorentz transformation, Equation (45.1), shown in the first paper, to transfer time from $t\grave{}$ to $\breve{t}$ at each point in the observer's location O.

$$\breve{t}_1 = \gamma\left(t\grave{}_1 + \frac{V_s x\grave{}_1}{c^2}\right) \qquad \breve{t}_2 = \gamma\left(t\grave{}_2 + \frac{V_s x\grave{}_2}{c^2}\right) \tag{5.3}$$

By also subtracting the two equations, we get

$$\Delta\breve{t}_{21} = \gamma\left(\Delta t\grave{}_{21} + \frac{V_s \Delta x\grave{}_{21}}{c^2}\right) \tag{6.3}$$

$$\Delta\breve{t}_{21} = \Delta t\grave{}_{21}\gamma + \gamma\frac{V_s \Delta x\grave{}_{21}}{c^2} \tag{7.3}$$

$$\Delta t\grave{}_{21}\gamma = \Delta\breve{t}_{21} - \gamma\frac{V_s \Delta x\grave{}_{21}}{c^2} \tag{8.3}$$

$$\Delta t\grave{}_{21} = \Delta\breve{t}_{21}\gamma^{-1} - \frac{V_s \Delta x\grave{}_{21}}{c^2} \tag{9.3}$$

Substitute (9.3) in (4.3)

$$\Delta\breve{x}_{21} = \gamma\left(\Delta x\grave{}_{21}\left(1-\gamma^{-1}\right)+V_s \Delta\breve{t}_{21}\gamma^{-1} - \frac{V_S^2 \Delta x\grave{}_{21}}{c^2}\right) \tag{10.3}$$

The last equation represents the transformation of the spatial period in the negative subspace in terms of the observer's time O. As we assumed above, the observer O measures both ends of the rod at the same time because the rod is moving relative to him, *i.e.* $\Delta\breve{t}_{21} = 0$, by substituting this in Equation (10.3).

$$\Delta\breve{x}_{21} = \gamma\left(\Delta x\grave{}_{21}\left(1-\gamma^{-1}\right) - \frac{V_S^2 \Delta x\grave{}_{21}}{c^2}\right) \tag{11.3}$$

By taking $\Delta x\grave{}_{21}$ as a common factor

$$\Delta\breve{x}_{21} = \gamma\Delta x\grave{}_{21}\left(1-\gamma^{-1} - \frac{V_S^2}{c^2}\right) \tag{12.3}$$

But, $\left(1-\frac{V_S^2}{c^2}\right)=\gamma^{-2}$

$$\Delta\breve{x}_{21}=\gamma\Delta x`_{21}\left(\gamma^{-2}-\gamma^{-1}\right) \quad (13.3)$$

$$\Delta\breve{x}_{21}=\Delta x`_{21}\left(\gamma^{-1}-1\right) \quad (14.3)$$

We finally obtain the transformation of the spatial period in the negative subspace with the condition of simultaneity in the length measurement process with respect to the observer O, by substituting the value of each of $\Delta\breve{x}_{21}$ and $\Delta x`_{21}$

$$\breve{l}=l`_0\left(\gamma^{-1}-1\right) \quad (15.3)$$

Equation (15.3) shows us that the length of the rod in the negative subspace expands with respect to the observer O with the increase in the speed of the reference frame (the speed of rod) from the value zero to the value $l`_0$. When the rod is at rest, or $V_s \ll c$, the reciprocal of the Lorentz factor is $\gamma^{-1}\approx 1$, and therefore $\breve{l}=0$, but when the speed of the rod approaches the speed of light, the reciprocal of the Lorentz factor in this case $\gamma^{-1}\approx 0$ and therefore $\breve{l}\approx -l`_0$,This result is the opposite of the result of special relativity ([13], p. 23), so the length relativity here is called inverse length relativity. However, it does not represent a contradiction with special relativity, because the expansion here is negative as shown above. Therefore, the expansion here represents the amount of decrease in the length of the rod resulting from the real contraction of length in special relativity. When the rod in rest or $V_s \ll c$, $\breve{l}=0$, which means that the amount of contraction in the length of the rod according to special relativity is equal to zero, and when the speed of the rod reaches the speed of light, theoretically, the expansion increases by a negative value until it reaches the same amount as the length of the rod, $\breve{l}\approx -l`_0$, *i.e.* the rod contracts by an amount equal to its length, therefore the real length according to special relativity will be equal to zero. So by adding the positive and negative lengths of the rod $l=\tilde{l}+\breve{l}$, we get the length contraction of the rod according to special relativity.

$$l=l`_0+l`_0\left(\gamma^{-1}-1\right) \quad (16.3)$$

$$l=l`_0\gamma^{-1} \quad (17.3)$$

The negative subspace contains positive distances because it expresses real motion, which is the motion of the rod with the frame of reference according to the first and second papers, where the speed in the negative subspace is always equal to the speed of the frame of reference, but positive distances are related to time. The negative distance appears only when we assume that there is no time in the negative subspace or that $\Delta\breve{t}_{21}=0$ as we assumed above. According to special relativity, we explained above that when the speed of the rod reaches the speed of light, theoretically only the length of the rod becomes zero, but this result represents a collapse in the structure of space [14] in the length transformation equation and a loss of information as well for the observer O, while the structure of space and the information about the length of the rod remain present for the observer O', which constitutes a paradox or contradiction in the special relativity. In the inverse relativity model, there are no collapses in the length transformation equation at the speed of light, as both observers have the same rod length, but one of them has a negative sign, which represents the preservation of the structure of the negative subspace in addition to the structure of the positive space.

2.2. Inverse Relativity of Simultaneity

If we have two events A, B (for example, two lightning bolts in the sky) separated by a distance on the $x`$-axis, and we want to observe the two events in the positive subspace-time where the moment the two events occurred, the coordinates were $\tilde{x}_0 = x`_0 = 0$ and $\tilde{t}_0 = t`_0 = 0$, look at Figure 2. The observer O' observes the coordinates of the first event $(x`_A, y`_A, z`_A, t_A`)$ and the coordinates of the second event $(x`_B, y`_B, z`_B, t_B`)$ with respect to the reference frame S'. As for observer O, he observes the coordinates of the first event $(\tilde{x}_A, \tilde{y}_A, \tilde{z}_A, \tilde{t}_A)$ and the coordinates of the second event $(\tilde{x}_B, \tilde{y}_B, \tilde{z}_B, \tilde{t}_B)$ with respect to the reference frame S but in the positive subspace.

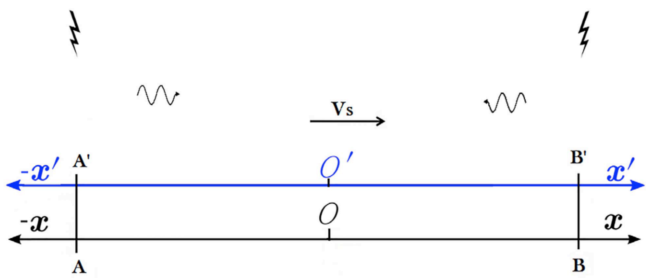

Figure 2. Shows two lightning bolts occur at the moment when the reference coordinates of frames S and S' coincide.

From the inverse positive modified Lorentz transformations shown in Equation (15.1), we obtain the time transformation for each event in the positive subspace-time.

$$\tilde{t}_A = t_A`\gamma \qquad \tilde{t}_B = t_B`\gamma \tag{18.3}$$

By subtracting the two equations, we obtain the transformations of the time period between the two events in the positive subspace-time.

$$\Delta\tilde{t}_{AB} = \Delta t`_{AB}\gamma \tag{19.3}$$

If the two events are simultaneous with respect to the observer O', that is, the two lightning bolts occurred at the same time, then $\Delta t`_{AB} = 0$. By substituting this into the previous equation, we get

$$\Delta\tilde{t}_{AB} = 0 \qquad \tilde{t}_A - \tilde{t}_B = 0 \qquad \tilde{t}_A = \tilde{t}_B \tag{20.3}$$

This means that the two events are also simultaneous with respect to the observer O in the positive subspace-time. Thus, simultaneous events separated by a distance on the x-axis with respect to one observer in the positive subspace-time are also simultaneous with respect to the other observer. This result is the opposite of the relativity of simultaneity in special relativity ([13], p. 16), As we know, simultaneous events that are separated by a distance in the total spacetime (Minkowski spacetime) with respect to one observer are not simultaneous with respect to the other observer, so we call the relativity of simultaneity here the inverse relativity of simultaneity, which simply means that the simultaneity between events is absolute with respect to both observers in the positive subspace-time. This result reveals more about the nature of time. For example, if we have three consecutive events in time D F G with respect to the observer O'. If event F represents the present or now with respect to the observer O', then event D is past and event G is future and there can be a causality between them such that event G is the result of event D. According to special relativity, the two causally separate asynchronous events D F with respect to observer O' can be simultaneous with respect to one observer in the universe. We also find that the two causally separate asynchronous events F G can be simultaneous with respect to another observer in the universe, *i.e.* event F is simultaneous with event D and event G. This means that the moment of now is synchronous with the moment of the past and the moment of the future. This conclusion led Einstein to believe that time is merely an illusion and that the past, present and future of all cosmic events exist at the same moment on the total fabric of spacetime for different observers, which is known as the block universe [15]. But such a conception of the nature of time results in many philosophical and logical problems, especially the thermodynamic conception of time. Either in the inverse relativity model or in positive subspace-time, we find that non-synchronous events remain non-synchronous for all observers in the universe. This means that observer O or any other observer in the universe will observe events D F G in the same temporal order, *i.e.* the moment of the past precedes the moment of now and the moment of now precedes the moment of the future. Thus, time appears here as an arrow passing from the past to the future, which is consistent with the concept of thermodynamic time. This represents a fundamental difference between the nature of time in special relativity as an illusion and the nature of time in inverse relativity as a reality.

If we want to describe the same two events in negative subspace-time, the observer O observes the coordinates of the first event $\left(\breve{x}_A, \breve{y}_A, \breve{z}_A, \breve{t}_A\right)$ and the coordinates of the second event $\left(\breve{x}_B, \breve{y}_B, \breve{z}_B, \breve{t}_B\right)$ with respect to the reference frame S where at the moment the two events began to occur the coordinates were also $\breve{x}_0 = x`_0 = 0$ and $\breve{t}_0 = t`_0 = 0$, Through the inverse negative modified Lorentz transformations shown in the first paper [1], Equation (45.1), we obtain a time coordinate transformation for every event in negative subspace-time.

$$\breve{t}_A = \gamma\left(t`_A + \frac{V_s x`_A}{c^2}\right) \qquad \breve{t}_B = \gamma\left(t`_B + \frac{V_s x`_B}{c^2}\right) \tag{21.3}$$

By also subtracting the two equations, we get the transformation of the time period between the two events in the negative subspace-time.

$$\Delta\breve{t}_{AB} = \gamma\left(\Delta t`_{AB} + \frac{V_s \Delta x`_{AB}}{c^2}\right) \tag{22.3}$$

Also by substituting $\Delta t`_{AB} = 0$ in the previous equation and because the two events are simultaneous with respect to the observer O' as we assumed above

$$\Delta \breve{t}_{AB} > 0 \qquad \breve{t}_A - \breve{t}_B > 0 \qquad \breve{t}_A > \breve{t}_B \tag{23.3}$$

This means that the two events are not simultaneous with respect to the observer O in the negative subspace-time, *i.e.* events that are simultaneous with respect to one observer are not simultaneous with respect to the other observer in the negative subspace-time, so the simultaneity of events here is relative and not absolute. This result is consistent with special relativity. But as we explained in the first paper, Section 2.4, negative subspace-time is spacetime devoid of causality, *i.e.* time here does not express the time of causality. Accordingly, the events D G shown in the previous example above, which exist between them causality with respect to the observer O', are not causally connected with respect to the observer O in negative subspace-time. Thus, the simultaneity of these events in negative subspace-time does not express the illusion of causal time.

2.3. Inverse Relativity of Time

But if we want to observe here the time of the event A independently (It is the emission of a light pulse resulting from the first lightning bolt) in the positive subspace-time. For the observer O' the event is observed in the time period $\Delta t` = t`_A$, while for the observer O it is in the time period $\Delta \tilde{t} = \tilde{t}_A$ where $\tilde{t}_0 = t`_0 = 0$ as we assumed before. Through the inverse positive modified Lorentz transformations we obtain the transformation of the time period of the event A.

$$\Delta \tilde{t} = \Delta t` \gamma \tag{24.3}$$

Equation (24.3) shows us that the time period dilates with respect to the observer O in the positive subspace-time as the velocity of the reference frame V_s increases. This result agrees with the results of special relativity ([13], p. 17) with a slight difference in that here we find that the dilation of the time period of the event does not depend on the distance traveled by the pulse on the $x`$-axis. That is, time dilates here regardless of the direction or axis in which the light pulse moves or in which the event is located in general. But if we want to describe the same event in negative subspace-time, we obtain the transformation of the time period of the event directly through the inverse negative modified Lorentz transformations according to the following equation:

$$\Delta \breve{t} = \gamma \left(\Delta t` + \frac{V_s \Delta x`}{c^2} \right) \tag{25.3}$$

The last equation shows that the time period in general expands with respect to the observer O in the negative subspace-time as the velocity of the reference frame V_s increases. But what if we want to transform time in negative subspace-time when the speed is symmetric between the reference frames, as we find in special relativity or in Minkowski's total spacetime, the speed of light is same or constant for both observers? Here we can also set the speed stability condition in the negative subspace-time with respect to both observers. As we explained in the first and second papers, the velocity of light or any particle or event in general with respect to the observer O in the negative subspace-time is equal to the velocity of the reference frame V_s, by analyzing the velocity of the pulse in the reference frame S' for either event A or B on the x`-axis, We get the pulse velocity $V_x^`$ equal to the frame velocity V_s and it has a negative value $V_x^` = -V_s$ because the reference frame S moves in the negative direction on the $x$`-axis, so $\Delta x` = -V_s \Delta t`$. By substituting this in the equation we get

$$\Delta \breve{t} = \gamma\left(\Delta t` - \frac{V_s V_s \Delta t`}{c^2}\right) \tag{26.3}$$

By taking $\Delta t`$ as a common factor

$$\Delta \breve{t} = \Delta t` \gamma\left(1 - \frac{V_S^2}{c^2}\right) \tag{27.3}$$

But $\left(1 - \frac{V_S^2}{c^2}\right) = \gamma^{-2}$

$$\Delta \breve{t} = \Delta t` \gamma \gamma^{-2} \tag{28.3}$$

$$\Delta \breve{t} = \Delta t` \gamma^{-1} \tag{29.3}$$

$$\Delta \breve{t} = \Delta t` \sqrt{1 - \frac{V_S^2}{c^2}} \tag{30.3}$$

Equation (30.3) is a special case of Equation (25.3). It shows us that time contracts for the observer O in the negative subspace-time as the velocity of the reference frame V_s increases if the velocity of the event is the same for both observers in the negative subspace-time. It is the opposite result of the dilation of time in special relativity, so the relativity of time here is called inverse time relativity. If the reference frame S' moves at speeds less than the speed of light, the amount of contraction in time is very slight and can be neglected. But when the reference frame S' moves at speeds close to the speed of light, the amount of contraction in time is very large, and in this case we call it supertime. The purpose of this result is to understand the nature of time and to reveal new properties of time. Just as time expands over the total space-time in special relativity or over the positive subspace-time in inverse relativity, time can also contract in the negative subspace-time. However, as we mentioned, negative space is a space without causality, so the contraction of time here does not represent an acceleration of causally linked events, but rather it is specific to the energy of that space. We will explain the importance of that in Section 2-7.

2.4. Inverse Relativity of Mass

In the second paper, we obtained the transformation of the relativistic energy of a particle with rest mass from the reference frame S' to S in each subspace, where we showed in the second paper, Equation (32.2) [2], that the transformation of the positive relativistic energy $\tilde{E}$ of the particle decreases with respect to the observer O in the positive subspace with increasing velocity of the reference frame

V_s, According to the following equation.

$$\tilde{E} = E`\gamma^{-1} \quad \Rightarrow \quad \tilde{E} = E`\sqrt{1-\frac{V_S^2}{c^2}} \tag{32.2}$$

where $E` = m`c^2$ and $\tilde{E} = m\tilde{V}^2$ is the positive relativistic energy, $\tilde{V}$ is the speed of light in the positive subspace-time. m is the total relativistic mass with respect to the reference frame S, so the mass m is the mass equivalent to the total relativistic energy and not to the positive relativistic energy, in other words it is not a mass characteristic of the positive subspace energy. Because relativistic energy is a function of relativistic mass according to special relativity ([16], pp. 48-49), Therefore, we can write positive relativistic energy in terms of positive relativistic mass $\tilde{m}$, which represents the mass equivalent to positive energy, where the change in positive relativistic energy depends only on the change in positive mass, In this case, the relation between positive mass and positive energy is written in the same formula used in special relativity: $\tilde{E} = \tilde{m}c^2$, where c here represents a constant in the formula of the law.

$$\tilde{E} = m\tilde{V}^2 = \tilde{m}c^2 \tag{31.3}$$

By substituting the value of each energy into Equation (32.2)

$$\tilde{m}c^2 = m`c^2\gamma^{-1} \tag{32.3}$$

$$\tilde{m} = m`\gamma^{-1} \quad \Rightarrow \quad \tilde{m} = m`\sqrt{1-\frac{V_S^2}{c^2}} \tag{33.3}$$

From Equation (33.3) we find that the relativistic mass of the particle with respect to the observer O in the positive subspace decreases as the velocity of the reference frame increases. This result is also the opposite of the relativity of mass in special relativity ([16], pp. 43-44), so the relativity of mass here is called inverse mass relativity. As for the mass transformation in the negative subspace is through the relativistic total energy transformation equation, Equation (70.2), also shown in the second paper [2]

$$\breve{E} = \frac{V_s}{c}\gamma\left[p`_x V_s + E`\left(1-\gamma^{-1}\right)\right] \tag{70.2}$$

where $\breve{E}$ represents the negative relativistic energy and is equal to $\breve{E} = m\breve{V}^2$, $\breve{V}$ represents the speed of light in the negative subspace-time, and here we also find that the negative relativistic energy is written in terms of the total relativistic mass m equivalent to the total relativistic energy, and not in terms of the mass equivalent to the negative energy or a distinct mass of the energy of the negative subspace. Thus, here we can also write the negative relativistic energy in terms of the negative relativistic mass $\breve{m}$ which represents the mass equivalent to the negative energy in the same formula used in special relativity $\breve{E} = \breve{m}c^2$, where c is, as we mentioned before, a constant in the formula of the law

$$\breve{E} = m\breve{V}^2 = \breve{m}c^2 \tag{34.3}$$

By substituting the value of each energy in the equation

$$\breve{m}c^2 = \frac{V_s}{c}\gamma\left[p\grave{}_x V_s + m\grave{}c^2\left(1-\gamma^{-1}\right)\right] \quad (35.3)$$

In the case that the particle does not move along the $x\grave{}$-axis, that is, $p\grave{}_x = 0$, the equation is reduced to the following formula:

$$\breve{m}c^2 = m\grave{}c^2\frac{V_s}{c}(\gamma-1) \quad (36.3)$$

$$\breve{m} = m\grave{}\frac{V_s}{c}(\gamma-1) \quad (37.3)$$

Equation (37.3) shows us that the relativistic mass increases with respect to the observer O in the negative subspace with the increase in the speed of the reference frame. This result agrees with the relativity of mass in special relativity, with a different mathematical formulation. Although the positive relativistic mass is less than the mass $m\grave{}$ (which contains the rest mass m_0) until it approaches zero when the speed of the reference frame approaches the speed of light, this theoretically. However, this does not represent a loss of the actual rest mass of the particle, but rather a transfer of mass from the positive to the negative subspace, as the rest mass remains conserved in total space. We conclude from the above that the analysis of the total relativistic energy into positive and negative energy, as explained in the second paper, Section 2.4 and Section 2.8, necessarily also leads to the analysis of the total mass into positive mass and negative mass equivalent to each type of energy. It must be noted here that the concept of negative mass is specific to the inverse relativity model and is completely different from the negative mass hypothesis in general relativity, which is incompatible with masses similar to it in sign [17].

2.5. Inverse Relativistic Kinetic Energy

Although the second paper included a description of relativistic mechanics in both positive and negative subspaces. It did not include the transformation of relativistic kinetic energy from the reference frame S' to S in each subspace. From the equation for the transformation of the total relativistic energy from the reference frame S' to S in the positive subspace shown above in Equation (32.2), we obtain the equation for the transformation of the change in positive relativistic energy.

$$\Delta\tilde{E} = \Delta E\grave{}\gamma^{-1} \quad (38.3)$$

According to special relativity, the change in relativistic total energy is the difference between the relativistic total energy and the energy of the rest mass ([16], pp. 47-48), which is equal to the relativistic work done on the particle or the relativistic kinetic energy, *i.e.* $\Delta E\grave{} = kE\grave{}$ and $\Delta\tilde{E} = k\tilde{E}$. Substituting this into Equation (38.3) we get

$$k\tilde{E} = kE\grave{}\gamma^{-1} \Rightarrow k\tilde{E} = kE\grave{}\sqrt{1-\frac{V_S^2}{c^2}} \quad (39.3)$$

Equation (39.3) represents the transformation of relativistic kinetic energy from the reference frame S' to S in the positive subspace. It shows us that the relativistic kinetic energy decreases with respect to the observer O in the positive subspace as the speed of the reference frame increases. This result is also the opposite of the transformation of relativistic kinetic energy in special relativity, so positive relativistic kinetic energy is called inverse relativistic kinetic energy. From the equation for the transformation of the total relativistic energy from the reference frame S' to S in the negative subspace given above in Equation (70.2), we also obtain the equation for the transformation of the negative relativistic energy change.

$$\Delta \breve{E} = \frac{V_s}{c} \gamma \left[\Delta p\grave{}_x V_s + \Delta E\grave{} \left(1 - \gamma^{-1}\right) \right] \tag{40.3}$$

As we mentioned above, the change in total relativistic energy represents relativistic kinetic energy, so the change in negative relativistic energy represents negative relativistic kinetic energy, *i.e.* $\Delta \breve{E} = k\breve{E}$. Substituting this into the previous equation, we get

$$k\breve{E} = \frac{V_s}{c} \gamma \left[\Delta p\grave{}_x V_s + kE\grave{} \left(1 - \gamma^{-1}\right) \right] \tag{41.3}$$

In the case that the particle does not move along the $x\grave{}$-axis, that is, $p\grave{}_x = 0$, the equation is reduced to the following formula:

$$k\breve{E} = kE\grave{} \frac{V_s}{c} (\gamma - 1) \tag{42.3}$$

Equation (42.3) shows us that the relativistic kinetic energy increases with respect to the observer O in the negative subspace as the velocity of the reference frame increases. This result also agrees with the relativistic kinetic energy in special relativity. Although it differs in mathematical formulation, we conclude from the above that the analysis of the total relativistic energy into positive and negative energy, as explained in the second paper, necessarily also leads to the analysis of the relativistic kinetic energy into positive and negative relativistic kinetic energy. It should also be noted here that the concept of negative energy is specific to the inverse relativity model and is completely different from the negative energy in quantum mechanics or general relativity, which generates antigravity [18].

2.6. The Concept of Mass, Work and Motion According to the Inverse Relativity

Special relativity provided us with the principle of mass-energy equivalence ([16], p. 48) [19], which explained that mass is a form of energy. Einstein also defined the relations between mass and energy in the famous equation $E = mc^2$, but despite that, that equation explains the relationship between matter and energy is only normative, that is, how much matter is equivalent to how much energy, or vice versa. The nature of this equivalence remains ambiguous as physicists use expressions for rest mass as rest energy or concentrated energy. Here we will try through the new model "Inverse Relativity" to provide a deeper understanding of the relationship between matter and energy. This includes new concepts for each of mass, work and motion using the reference frame system shown above in **Fig-**

ure 1, but assuming the existence of a box at rest with respect to the reference frame S' containing a light source at point P that emits a photon in the positive direction of the $x`$-axis from point P to point Q. See Figure 3.

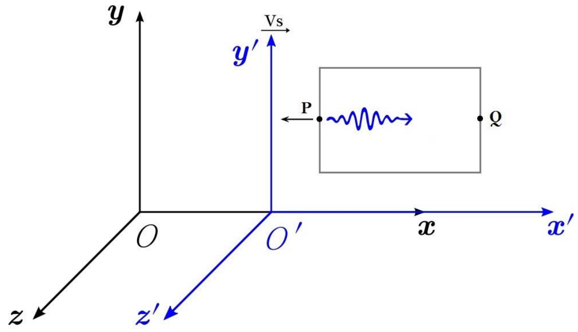

Figure 3. Shows a box with a light source whose frame of reference is S', moving at constant velocity relative to the frame of reference S.

According to quantum mechanics, photons have momentum [20] and therefore also have force. Here we can determine the momentum of the photon with high accuracy by considering the known energy source versus the uncertainty in the photon's position according to Heisenberg's principle. When a photon is propelled with a force $F`_{\gamma}$ from the light source at point P in the positive direction of the $x`$-axis, the box is propelled backwards in the negative direction of the $x`$-axis with a force of $-F`_{box}$ as a reaction resulting from the photon propelling forward according to Newton's third law. When the photon reaches point Q, it collides with the box and loses energy and momentum. As a result, the box is propelled in the positive direction of the $x`$-axis to return to its original position again.

$$F`_{\gamma} = -F`_{box} \quad \Rightarrow \quad F`_{\gamma} + F`_{box} = 0 \tag{43.3}$$

We conclude from the previous equation that the sum of the forces on the $x`$-axis affecting the system (*i.e.* the box and the photon) is equal to zero $\sum F`_x = 0$, which means that the system is stationary with respect to the observer O'. We can compensate for every force acting on the system in terms of the change in momentum, where $\Delta m`_{\gamma} c / \Delta t`$ represents the change in the photon's momentum, and $M`_{box} \Delta u`_x / \Delta t`$ represents the change in momentum of the box.

$$\frac{\Delta m`_{\gamma} c}{\Delta t`} = -\frac{M`_{box} \Delta u`_x}{\Delta t`} \tag{44.3}$$

But at time zero the initial momentum of both the photon and the box was zero. Substituting this into the previous equation and rearranging the equation we get.

$$m`_{\gamma} = -M`_{box} \frac{u`_x}{c} \tag{45.3}$$

According to the principle of inverse relativity, which represents an application of the principle of special relativity [21] but in the positive subspace shown in the first paper, item 2.3, we can apply Newton's third law in the positive subspace with the same mathematical formula as Equation (44.3), and thus we obtain the same relationship shown in Equation (45.3) but in the positive subspace.

$$m_\gamma = -M_{box} \frac{\tilde{u}_x}{\tilde{V}} \tag{46.3}$$

By substituting from the transformation equation for the speed of light in the positive subspace Equation (23.1) shown in the first paper, by also substituting from the transformation equation for the speed of the particle on the $x`$-axis in the positive subspace from the first equation of set No. 15.2 shown in the second paper, and also by substituting from the transformation equation the box mass according to special relativity in the previous equation we get.

$$m_\gamma = m`_\gamma \gamma \tag{47.3}$$

We find here that the total mass of the photon with respect to the observer O increases with the increase in the speed of the reference frame (the speed of the box), and we conclude from this two important points. The first point is that the mass equivalent to the energy of the photon appears here as the rest mass $m`_\gamma \equiv m`_0$ where we can consider the energy of the photon as part of the energy of the rest system, so it is rest energy and is equivalent to rest mas. The second point, part of the mechanical work done on the box is transferred to the photon and turns into an increase in the total energy of the photon, and thus an increase in the mass equivalent to that energy. But if the photon is moving at the maximum possible speed, which is the speed of light, why is work done on the photon? To answer this question, we apply the relativistic energy transformation equations for a particle with rest mass in both positive and negative subspace to the photon stuck inside the moving box. According to the relativistic energy transformation equation in positive subspace shown in the second paper, Equation (32.2), we find that

$$\tilde{E}_\gamma = E`_\gamma \gamma^{-1} \tag{48.3}$$

Substituting for the value of $\tilde{E}_\gamma, E`_\gamma$ according to what is shown above and in the second paper.

$$m_\gamma \tilde{V}_x^2 = m`_\gamma c^2 \gamma^{-1} \tag{49.3}$$

Substitute (47.3) in (49.3)

$$\tilde{V}_x = c\gamma^{-1} \tag{50.3}$$

Here we find that the energy of the photon decreases with respect to observer O in the positive subspace as the speed of the reference frame increases, although the total mass m_γ equivalent to the energy of the photon increases as a result of the work done, but the decrease in the speed of light in the subspace has a greater effect than the increase in mass, and by also applying the relativistic energy transformation equation for a particle with rest mass in the negative subspace Equation (70.2) shown above to the photon, we find that

$$\breve{E}_\gamma = \frac{V_s}{c} \gamma \left[p`_x V_s + E`_\gamma \left(1 - \gamma^{-1}\right) \right] \tag{51.3}$$

where $p\grave{}_x$ here represents the momentum of the photon that moves with the system and not inside the system, and because the system is stationary with respect to the reference frame S', thus $p\grave{}_x = 0$, so the previous equation is reduced to the following formula.

$$\breve{E}_\gamma = E\grave{}_\gamma \frac{V_s}{c}(\gamma - 1) \;\Rightarrow\; \breve{E}_\gamma < W_\gamma \tag{52.3}$$

Equation (52.3) shows us that the relativistic energy of the photon increases with respect to the observer O in the negative subspace as the speed of the reference frame increases. However, this energy, which represents the sum of the energy of the rest mass and the relativistic kinetic energy which the photon moves with the box, it is less than the relativistic work done on the photon $W_\gamma = E\grave{}_\gamma(\gamma - 1)$. This is an unexpected result and simply means that the work done on the photon is not converted into kinetic energy. But it also makes sense. If the photon is moving at its maximum possible speed, which is the speed of light c on the x-axis with respect to the reference frame S. Therefore, the work done here cannot increase the speed of the photon according to the second postulate in special relativity. So the work is only converted into an increase in the total energy of the photon and an increase in the mass of the photon equivalent to that energy, by substituting the value of each of $\breve{E}_\gamma, E\grave{}_\gamma$ according to what is explained above and in the second paper as well.

$$m_\gamma \breve{V}_x^2 = m\grave{}_\gamma c^2 \frac{V_s}{c}(\gamma - 1) \tag{53.3}$$

Substitute (47.3) in (53.3)

$$m_\gamma \breve{V}_x^2 = m_\gamma \gamma^{-1} c^2 \frac{V_s}{c}(\gamma - 1) \tag{54.3}$$

By dividing both sides of the equation by V_s, taking into account that $V_s = \breve{V}$

$$\breve{V}_x = c\left(1 - \gamma^{-1}\right) \tag{55.3}$$

By substituting (50.3) into (55.3)

$$\breve{V}_x = c - \tilde{V}_x \tag{56.3}$$

We conclude from Equations (56.3) and (50.3) that when the photon velocity $\breve{V}_x$ increases in the negative subspace, the photon velocity $\tilde{V}_x$ decreases in the positive subspace so that the resultant speed of the photon remains equal to the speed of light c in total space. The work done in the two equations also appears as an increase in the relativistic mass equivalent to the photon's energy, not the kinetic energy. This means that the relativistic work is not to add a new speed or motion to the photon, but to transfer the velocity from the dimensions of positive subspace (the space in which the structure of the system appears, *i.e.* the photon and the box) to the dimensions of negative subspace (the space in which the motion of the system appears). In other words, it works to analyze the speed of light to move the photon with the reference frame (the box), and thus the speed analysis appears here as a real analysis made by the mechanical work done on the particle.

It is the basis on which the inverse relativity model is based. Through the previous example, we can obtain a new concept for each of mass, work, and motion.

In classical physics [22], the inertial mass of a particle is defined as the particle's resistance to motion, and in special relativity it is a form of energy, but in the new model we can conclude that the inertial mass of a particle it is "the resistance of relativistic energy to transition from the dimensions of positive subspace to the dimensions of negative subspace. The work done is not to produce motion but it is "the work required to transfer motion from the dimensions of positive subspace to the dimensions of negative subspace". We also conclude that the motion of a particle is not a property acquired by the particle when work is done on it but it is "a property possessed by the particle in positive space and transferred to negative space by doing work on the particle." Through the new concepts we can understand the mechanism of motion of subatomic particles.

For example, the motion of the proton, as we know that the proton consists of quarks with an infinitesimal rest mass that move at a speed very close to the speed of light and gluons without rest mass that move at the speed of light, so the rest mass of the proton [23] is considered the mass equivalent to the relativistic energy of the quarks and gluons. When mechanical work is done on the proton to make it move, this work cannot give more speed to the proton particles according to the second postulate of special relativity [24]. Therefore, the work done works to transfer the speed of the quarks and gluons from inside the proton (*i.e.* from the positive subspace that represents the causal space and in which the strong nuclear forces appear) to the negative subspace that represents the motion of the system as a whole (the proton). It is also interesting to apply this mechanism to elementary particles. This leads us to conclude that particles with rest mass must have an internal wave structure because it is the wave that has the property of motion. It may be a standing wave in a field or a closed wave in a string. This is not the subject of our research, but what is important is that it moves at the speed of light and is subject to the previous transformation equations.

2.7. Hypothesis of Energy Fluctuation between Subspaces and Quantization of Transformation Equations

Using the reference frames system shown in the previous example, but replacing the source of photons with a source of electrons, if the observer O' accurately observes the energy of the quantum particle with respect to the reference frame S', he will have uncertainty in the time period that the particle takes to possess this energy and vice versa according to the Heisenberg energy-time principle [25], assuming that the uncertain quantities of energy and time are subject to the energy-time transformation equations of the positive subspace-time.

$$\Delta\tilde{E}_{un} = \Delta E^{`}_{un}\gamma^{-1} \qquad \Delta\tilde{t}_{un} = \Delta t^{`}_{un}\gamma \tag{57.3}$$

where $\Delta E^{`}_{un}, \Delta t^{`}_{un}$ represent the amount of uncertainty in energy and time for the quantum particle with respect to the reference frame S', while $\Delta\tilde{E}_{un}, \Delta\tilde{t}_{un}$ represent the amount of uncertainty in energy and time for the quantum particle with respect to the reference frame S but in the positive subspace-time. By multiplying both sides of the first equation by both sides of the second equation, we get.

$$\Delta\tilde{E}_{un}\Delta\tilde{t}_{un} = \Delta\grave{E}_{un}\Delta\grave{t}_{un} \tag{58.3}$$

The previous equation shows us that the formula of Heisenberg's uncertainty principle is constant for both observers under positive subspace transformations. This result agrees with the principle of inverse relativity, as we mentioned above. According to Heisenberg's principle, $\Delta\grave{E}_{un}\Delta\grave{t}_{un} = \hbar/2$, by substituting this in the previous equation, we get.

$$\Delta\tilde{E}_{un}\Delta\tilde{t}_{un} \geq \frac{1}{2}\hbar \tag{59.3}$$

Substituting the value of $\Delta\tilde{t}_{un}$ from Equation (57.3) into Equation (59.3), assuming that $\Delta\grave{t}_{un} = 1$, we have here two values of the uncertainty in the positive energy $\Delta\tilde{E}_{un}$, the first value when $V_s = 0$ where $\gamma^{-1} = 1$, the second value when $V_s = c$, which is theoretically where $\gamma^{-1} = 0$ according to the following equation.

$$\Delta\tilde{E}_{un} \geq \frac{\hbar}{2\Delta\grave{t}_{un}}\gamma^{-1} \Rightarrow \left(\Delta\tilde{E}_{un}\right)_{min} \geq \frac{1}{2}\hbar \text{ or } \Delta\tilde{E}_{un} = 0 \tag{60.3}$$

Equation (60.3) shows us that we cannot confirm the presence or absence of the infinitesimal part $\frac{1}{2}\hbar$ of the positive energy of the quantum particle (which here represents part of the internal energy of the system) in positive space when the speed of the reference frame is much less than the speed of light. The equation also shows us that the uncertainty in positive energy decreases until it reaches zero when the speed of the reference frame reaches the speed of light theoretically, as a result of the dilation of time to infinity in the positive subspace-time. But this result represents a contradiction with the Heisenberg principle in positive subspace-time where the minimum limit must not equal zero. By following the same previous steps we can also obtain the transformation of uncertain quantities of energy and time in negative subspace-time according to the transformations of negative subspace-time, *i.e.* according to the following equations.

$$\Delta\breve{E}_{un} = \Delta\grave{E}_{un}\frac{V_s}{c}(\gamma - 1) \qquad \Delta\breve{t}_{un} = \Delta\grave{t}_{un}\gamma^{-1} \tag{61.3}$$

where each $\Delta\breve{E}_{un}, \Delta\breve{t}_{un}$ represents the amount of uncertainty in energy and time for the quantum particle with respect to the reference frame S but in the negative subspace-time, by multiplying both sides of the first equation by both sides of the second equation we get.

$$\Delta\breve{E}_{un}\Delta\breve{t}_{un} = \Delta\grave{E}_{un}\Delta\grave{t}_{un}\frac{V_s}{c}\left(1 - \gamma^{-1}\right) \tag{62.3}$$

But

$$\frac{V_s}{c}\left(1 - \gamma^{-1}\right) \leq 1 \tag{63.3}$$

Substitute (63.3) in (62.3)

$$\Delta\breve{E}_{un}\Delta\breve{t}_{un} \leq \Delta E^{`}_{un}\Delta t^{`}_{un} \tag{64.3}$$

Also, by substituting the value of $\Delta E^{`}_{un}\Delta t^{`}_{un}$ according to the Heisenberg principle in the previous equation.

$$\Delta\breve{E}_{un}\Delta\breve{t}_{un} \leq \frac{1}{2}\hbar \tag{65.3}$$

Equation (65.3) is the inverse of Heisenberg's principle, where $\frac{1}{2}\hbar$ in negative subspace represents the maximum uncertainty in the negative energy of the quantum particle. Substituting the value of $\Delta\breve{t}_{un}$ from Equation (61.3) into Equation (65.3), assuming $\Delta t^{`}_{un} = 1$, we also have two values of $\Delta\breve{E}_{un}$, the first value when $V_s = 0$ where $\gamma = 1$, the second value when $V_s = c$ from the theoretical side where $\gamma = \infty$

$$\Delta\breve{E}_{un} \leq \frac{\hbar}{2\Delta t^{`}_{un}}\gamma \Rightarrow \left(\Delta\breve{E}_{un}\right)_{max} \leq \frac{1}{2}\hbar \text{ or } \Delta\breve{E}_{un} \approx \breve{E}_{max} \tag{66.3}$$

Equation (66.3) shows us that we cannot confirm the presence or absence of the infinitesimal part $\frac{1}{2}\hbar$ of negative energy in negative subspace when the speed of the reference frame is much less than the speed of light, The equation also shows us that the uncertainty in negative energy increases until it reaches infinity when the speed of the reference frame reaches the speed of light, which is equivalent to all the negative energy as a result of the contraction of time to zero in negative subspace-time. This result also represents a contradiction with Heisenberg's inverse principle in negative subspace-time, where the maximum limit of energy should not be more than $\frac{1}{2}\hbar$, but here we find that the uncertain quantity is equal to all the negative energy in negative subspace when $V_S = c$. Why do we have uncertainty about the presence of all the negative energy in negative subspace? and also uncertainty about the presence of an infinitesimal part of positive energy in positive subspace? According to classical physics we have no obvious reason to doubt these quantities. To explain this and also to resolve the contradiction between time dilation and the Heisenberg principle in positive subspace and the contradiction between time contraction and the inverse Heisenberg principle in negative subspace, we put forward a non-classical hypothesis, which is the fluctuation of energy between these subspaces. According to this hypothesis, we can replace the positive time in the positive space shown in Equation (59.3) with the negative time $\Delta\breve{t}_{un}$ and rewrite the equation as follows.

$$\Delta\tilde{E}_{un} \geq \frac{\hbar}{2\Delta\breve{t}_{un}} \tag{67.3}$$

Substituting the value of $\Delta\breve{t}_{un}$ from Equation (61.3) into Equation (67.3), assuming that $\Delta t^{`}_{un} = 1$, we have here new values of $\Delta\tilde{E}_{un}$, the first value when $\gamma = 1$, and the second value when $\gamma = \infty$

$$\Delta\tilde{E}_{un} \geq \frac{\hbar}{2\Delta t^{`}_{un}}\gamma \Rightarrow \left(\Delta\tilde{E}_{un}\right)_{min} \geq \frac{1}{2}\hbar \text{ or } \left(\Delta\tilde{E}_{un}\right)_{max} = \breve{E}_{max} \tag{68.3}$$

Equation (68.3) shows us that we cannot confirm the presence or absence of most or all of the negative energy in positive space. This result is consistent with the hypothesis of energy fluctuation between positive and negative space. But the important result here is that the minimum limit of the uncertainty in the energy does not decrease but increases until it reaches $\breve{E}_{max}$ when the speed of the reference frame reaches the speed of light. This preserves the Heisenberg principle in positive space, even when the uncertainty of time is infinite and $V_S = c$. We have a minimum uncertainty in positive energy equal to $\frac{1}{2}\hbar$. According to the energy fluctuation hypothesis we can also replace the negative time in negative space shown in Equation (65.3) with the positive time $\Delta\tilde{t}_{un}$ we get the following equation

$$\Delta\breve{E}_{un} \leq \frac{\hbar}{2\Delta\tilde{t}_{un}} \tag{69.3}$$

By following the same steps where substituting the value of $\Delta\tilde{t}_{un}$ from Equation (57.3) in Equation (69.3) and assuming that $\Delta t^{`}_{un} = 1$, we also have here new values $\Delta\breve{E}_{un}$, the first value when $\gamma^{-1} = 1$, and the second value when $\gamma^{-1} = 0$ from the theoretical side as well.

$$\Delta\breve{E}_{un} \leq \frac{\hbar}{2\Delta t^{`}_{un}}\gamma^{-1} \Rightarrow \left(\Delta\breve{E}_{un}\right)_{max} \geq \frac{1}{2}\hbar \text{ or } \left(\Delta\breve{E}_{un}\right)_{min} = 0 \tag{70.3}$$

Equation (70.3) shows us that we cannot confirm the presence or absence of the infinitesimal part $\frac{1}{2}\hbar$ of positive energy in negative space. This result also agrees with the hypothesis of energy fluctuation between subspaces. But the important result here is also that the maximum limit of the uncertainty in the energy does not increase when the speed of the reference frame reaches the speed of light, but decreases until it reaches zero, so when the uncertainty in time is infinitely small we have a maximum limit of uncertainty in negative energy equal to $\frac{1}{2}\hbar$. This preserves the inverse Heisenberg principle in negative subspace. We conclude from all of the above that, thanks to super-time, all negative energy can fluctuate between positive and negative space, while positive energy, as a result of time dilation, cannot fluctuate completely, but rather a smaller quantity of it fluctuates, which is the quantity $\frac{1}{2}\hbar$. Therefore, we have doubt or uncertainty about the existence of energy always linked to time in every subspace-time. This represents an explanation of the Heisenberg principle and the Heisenberg inverse principle, but what is the importance of each of them?

According to the Heisenberg principle, the minimum limit of uncertainty in energy $\frac{1}{2}\hbar$ is constant under transformation for all quantum particles belonging to the same frame of reference in the positive subspace, *i.e.* it is not subject to the Lorentz factor in the transformation equation. Therefore, when this quantity is added to the transformation equation, it becomes a transformation inequality. For example, but not limited to, the transformation equations of energy and energy density in the special and general case shown in the second paper [2], Equation (32.2) and (104.2), change to the following inequalities:

$$\tilde{E}n \geq E^{`}n\gamma^{-1} + \frac{1}{2}n\hbar \quad \Rightarrow \quad \tilde{E}n \geq \frac{1}{2}n\hbar \qquad V_S = c$$

$$\tilde{\rho}\tilde{V}^2 \geq \rho_0 c^2 \gamma^{-1} + \frac{n\hbar}{2V_{ol}^{`}} \quad \Rightarrow \quad \tilde{\rho}\tilde{V}^2 \geq \frac{n\hbar}{2V_{ol}^{`}} \qquad V_S = c,\ r = r_S \tag{71.3}$$

where $\tilde{\rho}\tilde{V}^2$ is the positive energy density of the dust field, $\rho_0 c^2$ is the proper density, N is the number of particles, $V_{ol}^{`}$ is the proper volume, r_S is the Schwarzschild radius, and γ in the general case is equal to $\gamma = 1\big/\sqrt{1 - r_S/r}$.

The transformation inequalities in the positive subspace show us that when $V_S = c$ in the special case or when $r = r_S$ in the general case, we have doubts about the existence of the minimum quantity of energy and energy density. In the same way we can also change the transformation equations for velocity, acceleration, forces and mass in the special case into transformation inequalities, so we also have doubts about the existence of the minimum of the previous physical quantity in both the special and general cases. This means that we cannot confirm the disappearance of information for any physical system from the positive subspace, whether in the special or general case. Therefore, the importance of Heisenberg's principle is to preserve the existence of information or causality and the laws of physics that represent the properties of the positive subspace.

As for the Heisenberg inverse principle, the maximum uncertainty in energy is constant under the transformation for all quantum particles belonging to the same frame of reference in the negative subspace, *i.e.* it is also not subject to the Lorentz factor in the transformation equation. Therefore, when this quantity is added to the transformation equation, it also becomes a transformation inequality. Through the previous examples, the energy transformation equations and energy density in negative subspace in the special and general case shown in the second paper [2], Equation (71.2) and (120.2), change to the following inequalities.

$$\breve{E}n \leq E^{`}n\frac{V_s}{c}(\gamma - 1) + \frac{1}{2}n\hbar \quad \Rightarrow \quad \breve{E}n \leq \frac{1}{2}n\hbar \qquad V_S = 0$$

$$\breve{\rho}\breve{V}^2 \leq \frac{nm_0c^2\left(\gamma^2 - 1\right) + \frac{1}{2}n\hbar}{\left(V_{ol}^{`}\right)(1-\gamma) + \left(V_{ol}\right)_{\hbar}} \Rightarrow \quad \breve{\rho}\breve{V}^2 \leq \frac{n\hbar}{2\left(V_{ol}\right)_{\hbar}} \qquad V_S = 0,\ r = \infty \tag{72.3}$$

The transformation inequalities in negative subspace show us that when $V_S = 0$ in the special case or when $r = \infty$ in the general case we have doubts about the existence of an infinitesimal part of energy and energy density in negative subspace despite the fact that the frame of reference is at rest or the particle is at rest because it is far from the influence of gravity. This result is of great importance, as we have doubts about the existence of self-acceleration within limits $\Delta\breve{V}$ for each quantum particle resulting from the quantity $\frac{1}{2}\hbar$, and because $\Delta\breve{V} = \Delta V_S$ in the special case, therefore the reference frame has a self-acceleration of ΔV_S, although this does not represent a violation of the principle of special relativity because this acceleration is infinitesimal and its effect on the macroscopic level can be ignored, by substituting $\Delta\breve{E}_{un} = m_0 \Delta\breve{V}^2$ in the first equation, group 72.3, we get

$$nm_0 \Delta\breve{V}^2 = \frac{1}{2} n\hbar \quad \Rightarrow \quad \Delta\breve{V} = \sqrt{\frac{\hbar}{2m_0}} \tag{73.3}$$

Based on the previous result, it is not necessary to do infinite mechanical work for any material particle (particle with rest mass) to reach the speed of light. It is enough to do only mechanical work for the particle to reach the critical speed V_C, which is less than the speed of light by an amount of ΔV_S, Where each quantum particle in the reference frame can use an infinitesimal part of positive energy (which represents the internal energy of the particle) to reach the speed of the reference frame to the speed of light.

$$\breve{V}_C = c - \Delta V_S \quad \Rightarrow \quad \breve{V}_C = c - \sqrt{\frac{\hbar}{2m_0}} \tag{74.3}$$

This result represents a fundamental difference between inverse relativity and special relativity, which places an increase in mass as a barrier to prevent any material particle from reaching the speed of light. In the general case, we also find that the velocity of a particle in a curved negative subspace-time under the influence of the gravitational mass M equals the escape velocity, as we mentioned in the first paper, *i.e.* $\Delta\breve{V} = \Delta V_{esc}$. Accordingly, it is not necessary for a dust field to reach the Schwarzschild radius for its velocity in a negative subspace-time to equal the speed of light c. It is enough for the particles to reach the critical velocity $\breve{V}_c$, which is less than the speed of light by an amount ΔV_{esc}

$$\breve{V}_c = c - \Delta V_{esc} \tag{75.3}$$

As a result of the existence of a critical escape velocity here also, we have a critical radius $\breve{r}_c$ according to the escape velocity equation.

$$\breve{r}_c = \frac{2MG}{\breve{V}_c^2} \qquad \breve{r}_c > r_s \tag{76.3}$$

That is, when the dust field reaches the critical radius, it can self-accelerate to reach the Schwarzschild radius, but this is temporary due to the energy fluctuation according to the inverse Heisenberg principle. Therefore, the difference $\Delta\breve{r}_{un} = \breve{r}_c - r_s$ represents the uncertainty in the black hole radius. By substituting the value of the radius and taking $2MG$ as a common factor, we get.

$$\Delta\breve{r}_{un} = 2MG\left(\frac{1}{\breve{V}_c^2} - \frac{1}{c^2}\right) \tag{77.3}$$

This result, in the general case, also represents a fundamental difference between inverse relativity and general relativity, as we do not have infinite energy density in negative subspace-time in the general case, but we have a critical energy density. In general, we can say that the inverse Heisenberg principle in negative subspace-time, in both the special and general cases, prevents the existence of infinite energy and energy density. In short, we find that the inverse relativity model can provide us with an explanation for the Heisenberg principle, one of the most important pillars of quantum mechanics. It also explains to us the importance of this principle in positive subspace-time for the preservation of information, and the importance of the principle's existence in inverse in negative subspace-time for material particles to reach the speed of light without the need for infinite energy.

2.8. Positive Subspace-Time Diagram

In the first paper [1], we used to write the line element (Metric) in the positive subspace-time in the form shown in Equation (60.1). It is a constant quantity under the positive modified Lorentz transformation or the inverse positive modified Lorentz transformation from one frame of reference to another, as we explained in the first paper, Section 2.3. Where the square magnitude of line element remains constant under transformation, it is therefore represented in a subspace-time split from the total spacetime, although it is not algebraically clear that the split spacetime is a subspace-time. By substituting Equation (23.1) into Equation (60.1) shown in the first paper, we can rewrite the positive subspace-time metric formula as follows.

$$\mathrm{d}\tilde{s}^2 = \mathrm{d}\tilde{x}^2 + \mathrm{d}\tilde{y}^2 + \mathrm{d}\tilde{z}^2 - c^2\mathrm{d}\tilde{t}^2\gamma^{-2} \tag{78.3}$$

The new formulation of the metric is distinguished by its linear independence from the Minkowski spacetime metric [26], and it also contains the zero vector, and its vectors are closed under addition and multiplication. Therefore, its representation must be in a subspace-time. According to the previous formula, we also have a new and distinctive formula for the metric tensor of positive subspace-time, which differs from the metric tensor of Minkowski spacetime, where in the first paper both spacetimes had the same formula without discrimination.

$$\tilde{\eta}^{\epsilon\tau} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\gamma^2 \end{bmatrix}, \quad \tilde{\eta}_{\epsilon\tau} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\gamma^{-2} \end{bmatrix} \tag{79.3}$$

From the previous equations we can also obtain the characteristic basis vectors of the positive subspace-time, whose number represents the number of dimensions of this subspace.

$$\tilde{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \tilde{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \tilde{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \tilde{e}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ i\gamma^{-1} \end{bmatrix} \quad i = \sqrt{-1} \tag{80.3}$$

So the spacetime diagram here will also be four-dimensional, three spatial dimensions $\tilde{x}\,\tilde{y}\,\tilde{z}$ and the time dimension $\tilde{V}\tilde{t}$ or $c\tilde{t}\gamma^{-1}$ according to the formula shown above, and because the speed in this dimension is variable, *i.e.* it differs from one observer to another depending on the difference in the speed of the reference frame, therefore we cannot represent it on the time axis directly, So we use the time axis in the form $c\tilde{t}$ where the units of measurement used are agreed upon by all observers, where the light second which is equal to 3×10^{8} meters or the light meter which is equal to 3×10^{-8} parts of a second, then we calculate the speed $\tilde{V}$ on the diagram and then we project the speed onto the time axis $c\tilde{t}$ to get the value of $c\tilde{t}\gamma^{-1}$. Because it is difficult to draw or visualize a four-dimensional diagram, we use the simplified spacetime diagram that is known [27]. It is a diagram of two positive spatial dimensions, which are the axes $\tilde{x}, \tilde{y}$ and the positive time dimension, which is the $c\tilde{t}$ -axis, and the origin of this diagram is $\tilde{t}_0 = \tilde{x}_0 = \tilde{y}_0 = 0$ as shown in **Figure 4**.

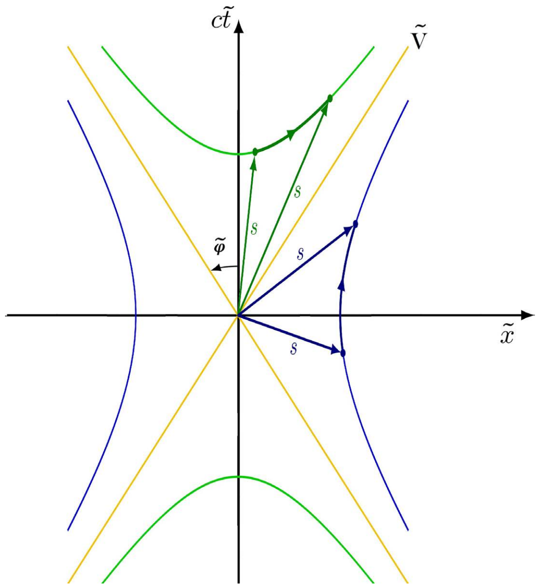

Figure 4. Shows the structure of causality on the positive subspace-time diagram, the 2D drawing is for simplification.

As we know, each point on this diagram represents an event, and the path followed by the particle or event on this diagram is called the positive world line. From the drawing, we can find that the slope of the positive straight world line is equal to

$$\tanh\tilde{\varphi} = \frac{d\tilde{x}}{cd\tilde{t}} \Rightarrow \tanh\tilde{\varphi} = \frac{\tilde{u}}{c} \Rightarrow \tanh\tilde{\varphi} = \tilde{u} \quad c = 1 \tag{81.3}$$

where $\tilde{u}$ represents the velocity of the particle in the positive subspace-time, *i.e.* the slope of the positive world line is equal to the velocity of the particle on the positive diagram. Replacing the velocity of the particle $\tilde{u}$ with the speed of light in the positive subspace-time $\tilde{V}$, which is called here the speed of positive light, and substituting the value of $\tilde{V}$ from Equation (23.1) shown in the first paper [1], we obtain the slope of the positive world line of light.

$$\tanh\tilde{\varphi} = c\gamma^{-1} \tag{82.3}$$

The previous equation represents the slope of the positive world line of light passing through the origin point with respect to the observer O, and thus all positive world lines of light passing through the origin point that have the same slope represent the light cone [28] in the positive subspace-time with respect to the observer O. The cone here is called the positive light cone, and it is a flat cone in the 3D diagram or linear in the 2D diagram shown in **Figure 4** in yellow. The previous equation also shows us that the angle of the positive cone varies from one observer to another according to the speed of each observer's reference frame. The spacetime interval between any two points or events on the positive diagram is called the positive spacetime interval, $\Delta\tilde{s}^2 = \Delta\tilde{\alpha}^2 - c^2\Delta\tilde{t}^2\gamma^{-2}$ where, $\Delta\tilde{\alpha}^2 = \Delta\tilde{x}^2 + \Delta\tilde{y}^2 + \Delta\tilde{z}^2$, and on the sub-diagram it represents the world line of a particle or signal that can move between two points or events on the positive sub-diagram, it has three values, the first value is $\Delta\tilde{s}^2 = 0$

$$\Delta\tilde{\alpha}^2 - c^2\Delta\tilde{t}^2\gamma^{-2} = 0 \;\;\Rightarrow\;\; \Delta\tilde{\alpha}^2 = \tilde{V}^2\Delta\tilde{t}^2 \tag{83.3}$$

By dividing both sides of the equation by $\Delta\tilde{t}^2$, we find that $\tilde{u} = \tilde{V}$, meaning that the speed of the signal or the causal speed between the two events is equal to the value of the speed of light in the positive subspace-time with respect to the observer O. Therefore, the two events here occur on the surface of the positive light cone, and the spacetime interval here is called the positive lightlike, as shown in **Figure 4** in yellow. Because the positive speed of light represents the maximum speed physically allowed in this subspace-time, the two events can be causally linked through a light signal only. As for the second value of the positive interval $\Delta\tilde{s}^2 < 0$

$$\Delta\tilde{\alpha}^2 - c^2\Delta\tilde{t}^2\gamma^{-2} < 0 \;\;\Rightarrow\;\; \Delta\tilde{\alpha}^2 < \tilde{V}^2\Delta\tilde{t}^2 \tag{84.3}$$

By dividing both sides of the inequality by $\Delta\tilde{t}^2$, we find that $\tilde{u} < \tilde{V}$. This means that the speed of the signal or the causal speed between the two events is less than the value of the speed of positive light. Therefore, the two events here occur within the positive light cone, and the positive interval here is called the positive timelike. All points that have the same square as this interval or the same distance from the origin in the 3D diagram are represented by a hyperbolic surface or hyperbola in the 2D diagram, as shown in **Figure 4** in green, and because the causal speed is less than the maximum speed physically allowed in this subspace-time, therefore, two events can be causally linked through a light signal or particle with rest mass. As for the third value of the positive interval, $\Delta\tilde{s}^2 > 0$

$$\Delta\tilde{\alpha}^2 - c^2\Delta\tilde{t}^2\gamma^{-2} > 0 \;\;\Rightarrow\;\; \Delta\tilde{\alpha}^2 > \tilde{V}^2\Delta\tilde{t}^2 \tag{85.3}$$

By dividing both sides of the inequality by $\Delta\tilde{t}^2$, we find that $\tilde{u} > \tilde{V}$. This means that the speed of the signal or the causal speed between the two events is greater than the value of the speed of positive light. Therefore, the two events here occur outside the positive light cone, and the positive interval here is called the positive spacelike. All points that have the same square as this interval or the same distance from the origin for the 3D diagram are represented by a hyperbolic surface or a hyperbola for the 2D diagram, as shown in **Figure 4** in blue. Because the causal velocity is greater than the maximum velocity physically allowed in this subspace-time with respect to the observer O, the two events cannot be causally related.

We conclude from the above that events outside the positive light cone are causally separate, while events inside the positive cone can be causally connected. If we have an event at the origin point in the previous diagram which represents now, for example, event F, so all events that occur in the upper part of the positive light cone are in the future of event F, and event F can causally affect one of them. As for the events that occur in the lower part of the positive light cone, they are in the past of event F, and one of them can causally affect on event F. We can generalize these results to any observer other than observer O, where only the inclination angle of the positive cone varies, but the geometry of the diagram remains constant relative to any other observer. The above description shows that the structure of causality on a positive subspace-time diagram is exactly the same as the structure of causality on a Minkowski spacetime diagram [28], but with only the difference in the value of the speed of light, which makes the positive subspace-time a causal spacetime. We also demonstrated this result in the first paper, but at the level of 3D positive subspace, and we also gave a precise description of this.

As for the rotation of the observer's coordinates O' with respect to the observer's coordinates O on the positive subspace-time diagram, it differs from the rotation of the observer's coordinates followed on the Minkowski diagram, because the transformation matrix for the positive subspace-time is an identity matrix. See the first paper [1], Equation (18.1), By equating the rotation matrix in a hyperbolic 4D space [29] with the identity matrix of the positive subspace-time, we obtain the values of $\cosh\tilde{\varnothing}$ and $\sinh\tilde{\varnothing}$ for the rotation of the coordinates of the observer O' in this subspace-time in the $\tilde{x}c\tilde{t}$ -plane.

$$\tilde{\Lambda}^{\epsilon}_{\mu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cosh\tilde{\varnothing} & 0 & 0 & \sinh\tilde{\varnothing} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sinh\tilde{\varnothing} & 0 & 0 & \cosh\tilde{\varnothing} \end{bmatrix} \tag{86.3}$$

Here we find $\cosh\tilde{\varnothing} = 1$ and $\sinh\tilde{\varnothing} = 0$, and we conclude from this that the only angle allowed here is $\tilde{\varnothing} = 0$. This means that the axes of the space and time coordinates of the observer O' are identical to the axes of the space and time coordinates of the observer O, and thus there is no rotation of the coordinates of an observer with respect to another. However, despite the fact that the coordinate axes of both observers are identical, we find that the slope of the world line of light differs from one observer to another because the speed of light differs from one observer to another. We find that the slope of the observer's world line O' is equal to $\tanh\varphi` = \mathrm{d}x`/c\mathrm{d}t`$, by following the same previous steps in Equations (81.3), (82.3), we get the slope of the world light line equal to $\tanh\varphi` = c$ And because $c = 1$, the angle of the light cone for this observer is always equal to 45 degree. See **Figure 5**. By substituting in Equation (82.3), we obtain an equation that transforms the slope of the world line of light from observer O' to observer O on the diagram, as shown in the following figure.

$$\tanh \tilde{\varphi} = \tanh \varphi` \gamma^{-1} \quad (87.3)$$

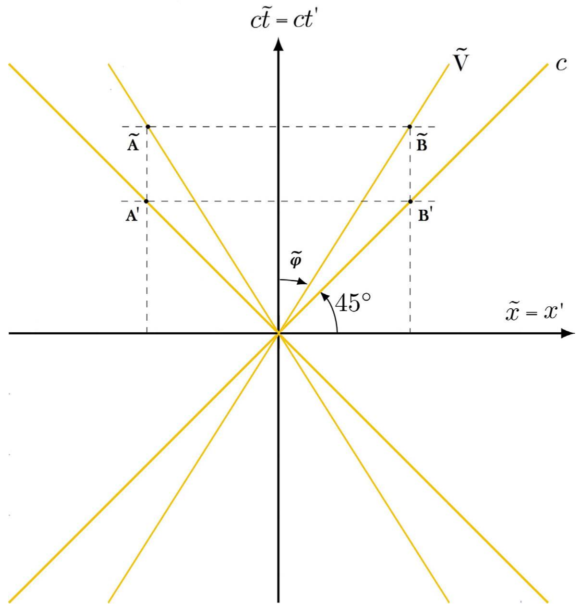

Figure 5. Shows the coincidence of observers' coordinates and the difference in the angles of the light cone as an alternative to rotation in the positive subspace-time.

Through the previous diagram, we can explain both the symmetry of the proper distance, time dilation, and the inverse relativity of simultaneity with respect to the observer O in the positive subspace-time directly.

Distance Symmetry: To show the transformation of positive length or the symmetry of the proper distance with respect to both observers on the diagram directly. We represent the length of the rod or the proper distance shown above in item 2-1 as a spatial period $\Delta x`_{12}$ on the $x`$-axis in the previous diagram with respect to the observer O'. The positive length or positive distance is also represented as a spatial period $\Delta \tilde{x}_{12}$ on the $\tilde{x}$-axis with respect to the observer O. because the spatial coordinates with respect to both observers O' and O on the positive diagram are identical, therefore the spatial period on the $x`\tilde{x}$-axis is equal.

$$\Delta \tilde{x}_{12} = \Delta x`_{12} \quad \Rightarrow \quad \tilde{l} = l` \quad (88.3)$$

This means that the length or the proper distance here is constant or symmetrical for both observers. Not only the spatial period on the $x`\tilde{x}$-axis, but any spatial period in positive subspace. As a result of the matching of all spatial coordinates for both observers. See **Figure 6**, where the structure of subspace appears symmetric between observers, in exchange for reducing the speed of light and dilating time with respect to observer O, we have also explained this in the first paper, Section 2-3.

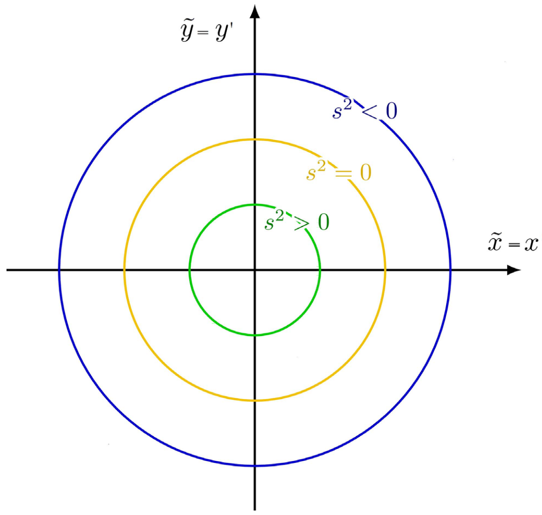

Figure 6. Shows the coincidence of spatial coordinates for both observers in the positive subspace.

Inverse simultaneity: To show the inverse simultaneity relativity on the positive subspace-time diagram, we can represent the two simultaneous events $A`, B`$ shown in Section 2.2 above, as two points on the surface of the light cone with respect to the observer O'. Because the two events are simultaneous, the line passing through the two points or events is a straight line parallel to the $x`\tilde{x}$-axis, as shown in **Figure 5**. The representation of the same two events on the surface of the positive light cone with respect to observer O is through the extension of the spatial projection of each event, where the spatial coordinates of both observers are identical, as shown in **Figure 5**. We also find that the line passing between the two points of intersection on the surface of the observer's cone O is also a straight line parallel to the $x`\tilde{x}$-axis, which means that the two events also remain simultaneous with respect to the observer O. By substituting the spatial and time periods between these events in the previous slope transformation equation, we obtain the transformation of the time period from observer O' to observer O, and by substituting the value of $\Delta t`_{AB} = 0$ in the resulting equation because the two events are simultaneous with respect to observer O', we also obtain simultaneity with respect to observer O.

$$\frac{\Delta \tilde{x}_{AB}}{c\Delta \tilde{t}_{AB}} = \frac{\Delta x`_{AB}}{c\Delta t`_{AB}}\gamma^{-1} \Rightarrow \Delta \tilde{t}_{AB} = \Delta t`_{AB}\gamma \Rightarrow \tilde{t}_A = \tilde{t}_B \quad \Delta t`_{AB} = 0 \tag{89.3}$$

We conclude from the previous example that the line passing between two events on the light cone with respect to observer O is always parallel to the line passing between the same two events with respect to observer O'. If the two events $A`, B`$ are not simultaneous, *i.e.* the line passing between them is oblique with respect to observer O', observer O will also obtain an oblique line between the two points $\tilde{A}, \tilde{B}$ parallel to the previous line, this means that the two events are not simultaneous. Generalizing this result for any other observer, we conclude that non-simultaneous events remain non-simultaneous for all observers in the universe in the positive subspace-time.

Time dilation: As for time dilation or the transformation of the proper time on the positive subspace-time diagram, by representing the previous event A on the light cones with respect to the two observers O' and O, We find from **Figure 5** that, as a result of the difference in the angle of inclination of the light cones of each of them, the projection of event $\tilde{A}$ onto the time dimension $c\tilde{t}$ is longer than the projection of the same event onto the time dimension $ct\grave{}$, which represents the dilation of time with respect to observer O. By substituting the spatial and time period of this event into the previous slope transformation equation, we obtain the transformation of the time period of the event from observer O' to observer O.

$$\frac{\Delta\tilde{x}_A}{c\Delta\tilde{t}_A} = \frac{\Delta x\grave{}_A}{c\Delta t\grave{}_A}\gamma^{-1} \Rightarrow \Delta\tilde{t}_A = \Delta t\grave{}_A\gamma \tag{90.3}$$

If the previous events occur on the hyperbolic surfaces in the 3D diagram or the hyperbola in the 2D diagram, *i.e.* inside or outside the light cone with respect to O', the extension of the spatial projection of any of them also represents its location on the hyperbolic surface or hyperbola inside or outside the positive light cone with respect to the observer O. In this case, the slope transformation equation expresses the transformation of the slope of the tangent line to a point on the hyperbolic surface or hyperbola of each observer.

2.9. Negative Subspace-Time Diagram

In the first paper [1], we also used to write the negative subspace-time metric in the form shown in Equation (72.1) which is a constant quantity under the negative modified Lorentz transformation or the inverse negative modified Lorentz transformation from one frame of reference to another, as we previously explained in the first paper, item 2.4. Where the square magnitude of line element remains constant under transformation, so it is also represented in a subspace-time split from the total spacetime, Although here too it is not algebraically clear that the split spacetime is a subspace-time. By substituting the value of the velocity $\breve{V}$ in terms of both $c,\tilde{V}$ in (72.1) according to the parallelogram law of vector addition shown in **Figure 1** in the first and second papers [2], where $\breve{V}^2 = c^2 + \tilde{V}^2 - c\tilde{V}\cos\theta$ and also substituting the value of $\tilde{V}$ from the Equation in (23.1) in the first paper, the equation becomes $\breve{V}^2 = c^2\left(1+\gamma^{-2} - 2\gamma^{-1}\cos\theta\right)$, we can rewrite the negative spacetime metric formula as follows:

$$\mathrm{d}\breve{s}^2 = \mathrm{d}\breve{x}^2 + \mathrm{d}\breve{y}^2 + \mathrm{d}\breve{z}^2 - c^2\mathrm{d}\breve{t}^2\left(1+\gamma^{-2} - 2\gamma^{-1}\cos\theta\right) \tag{91.3}$$

can be shortened $-\left(1+\gamma^{-2} - 2\gamma^{-1}\cos\theta\right) = -\Omega^2$

$$\mathrm{d}\breve{s}^2 = \mathrm{d}\breve{x}^2 + \mathrm{d}\breve{y}^2 + \mathrm{d}\breve{z}^2 - c^2\mathrm{d}\breve{t}^2\Omega^2 \tag{92.3}$$

The new formulation of the metric here is also distinguished by its linear independence from the Minkowski spacetime metric, and it also contains the zero vector and its vectors are closed under addition and multiplication, thus its representation must be in a subspace-time. We also have a new and distinctive formula for the metric tensor of negative subspace-time, which differs from the metric tensor of Minkowski spacetime, where in the first paper both spacetimes had the same formula.

$$\breve{\eta}^{\sigma\rho} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\Omega^{-2} \end{bmatrix}, \ \breve{\eta}_{\sigma\rho} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\Omega^{2} \end{bmatrix} \tag{93.3}$$

From the previous equations we can also obtain the characteristic basis vectors of the negative subspace, whose number represents the number of dimensions of this subspace.

$$\breve{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \breve{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \ \breve{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \ \breve{e}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ i\Omega \end{bmatrix} \tag{94.3}$$

Therefore, the diagram of this space-time will also be four-dimensional, with three spatial dimensions $\breve{x}\,\breve{y}\,\breve{z}$ and one time dimension $\breve{V}\breve{t}$ or $c\breve{t}\Omega$, according to the formula shown above. Because the speed on this dimension is also variable and varies from one observer to another depending on the speed of the reference frame, we cannot represent it directly on the time axis. Therefore, we use the time axis in the form $c\breve{t}$, where the units of measurement used are agreed upon for all observers, as explained in the previous section. Then we calculate the velocity $\breve{V}$ on the diagram, and then we project the velocity onto the time axis $c\breve{t}$ to obtain the value of $c\breve{t}\Omega$. According to the inverse negative modified Lorentz transformations, we find that the following dimensions are $\breve{y} = \breve{z} = 0$. Therefore, the diagram will be drawn in two dimensions, one spatial dimension, which is the $\breve{x}$-axis, and one time dimension, which is the $c\breve{t}$-axis. The origin of this diagram is $\breve{t}_0 = \breve{x}_0 = 0$, as shown in **Figure 7**.

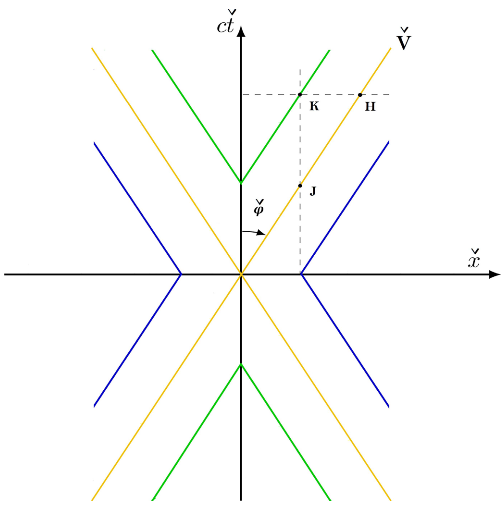

Figure 7. Shows a non-causal structure on the negative subspace-time diagram.

The negative subspace-time diagram differs from the positive one in some respects, which we will explain gradually. However, here we also find the path followed by the particle or event on this diagram, which is called the negative world line. From the drawing we can also find the slope of the negative straight line, which is equal to

$$\tanh\breve{\varphi} = \frac{\mathrm{d}\breve{x}}{c\mathrm{d}\breve{t}} \quad \Rightarrow \quad \tanh\breve{\varphi} = \frac{\breve{u}}{c} \quad \Rightarrow \quad \tanh\breve{\varphi} = \breve{u} \quad c = 1 \tag{95.3}$$

where $\breve{u}$ represents the velocity of the particle in the negative subspace-time, and thus the slope of the negative world line here is also equal to the velocity of the particle on the negative diagram as in the positive subspace-time. When we replace the velocity of the particle $\breve{u}$ with the speed of light in the negative subspace-time $\breve{V}$, which is called here the negative light speed, and by substituting the value of $\breve{V}$ from Equations (54.1) and (55.1) shown in the first paper [1], we obtain the slope of the negative world line of light.

$$\tanh\breve{\varphi} = V_S \tag{96.3}$$

The previous equation shows the slope of the negative world line of light passing through the origin point with respect to an observer O, which is equal to the velocity of the reference frame V_s and has a positive or negative value according to the direction of motion of the reference frame S' on the x-axis with respect to the reference frame S, *i.e.* here the observer O has only one world line of negative light. But for the sake of geometric symmetry, all negative world lines of light passing through the origin point have the same slope, representing the light cone in the negative subspace-time with respect to the observer O, shown in **Figure 7** in yellow. The light cone here is called the negative cone, and it is a linear cone because the diagram here is 2D, as we explained above. The previous equation also shows us that the angle of the negative cone varies from one observer to another according to the speed of the reference frame of each observer. Because there is no acceleration in negative subspace-time, we explained this in the second paper [2], section 2.7. Also, the velocity of a particle in space is equal to the velocity of the particle in the time dimension in negative subspace-time, we also explained this in the second paper, Section 2.6. Therefore, the velocity $\breve{V}$ here represents the minimum and maximum limit of permissible velocity in negative subspace-time. The spacetime interval between any two points or events on this diagram is called the negative spacetime interval, $\Delta\breve{s}^2 = \Delta\breve{\alpha}^2 - \breve{V}^2\Delta\breve{t}^2$, where $\Delta\breve{\alpha}^2 = \Delta\breve{x}^2 + \Delta\breve{y}^2 + \Delta\breve{z}^2$, and on the diagram it represents the world line of a particle or signal that travels between the two events, it also has three values, the first value $\Delta\breve{s}^2 = 0$.

$$\Delta\breve{\alpha}^2 - c^2\Delta\breve{t}^2\Omega^2 = 0 \quad \Rightarrow \quad \Delta\breve{\alpha}^2 = \breve{V}^2\Delta\breve{t}^2 \tag{97.3}$$

By dividing both sides of the equation by $\Delta\breve{t}^2$, we find that $\breve{u} = \breve{V}$ meaning that the speed of the signal or the causal speed between the two events is equal to the value of the speed of light in the negative subspace-time. Therefore, the two events here fall on the surface of the negative light cone with respect to an observer

O, and the negative interval here is called the negative lightlike, as shown in **Figure 7** in yellow. Because the speed $\breve{V}$ represents the minimum and maximum limit, *i.e.* the same speed for all events on the surface of the negative cone, therefore, just as a signal light moves from the first event with speed $\breve{V}$, the second event also moves with the same speed on the same world line. This means that the signal does not reach between the two events, therefore the two events cannot be causally linked through a signal light. As for the second value of the positive interval $\Delta\breve{s}^2 < 0$

$$\Delta\breve{\alpha}^2 - c^2\Delta\breve{t}^2\Omega^2 < 0 \quad \Rightarrow \quad \Delta\breve{\alpha}^2 < \breve{V}^2\Delta\breve{t}^2 \tag{98.3}$$

By dividing both sides of the inequality by $\Delta\tilde{t}^2$, we find that $\tilde{u} < \breve{V}$ meaning that the causal velocity between the two events is less than the speed of light in negative subspace-time with respect to the observer O. Therefore, the two events here occur within the negative light cone, and because the causal velocity here is less than the minimum velocity physically permitted in this subspace-time, therefore the two events cannot be causally related. In other words, the physically permissible negative interval inside the negative cone, it is also a negative lightlike type, where the world lines inside the negative cone are parallel to the lines of the negative cone, as shown in **Figure 7** in green. As for the third value of the positive interval, $\Delta\breve{s}^2 > 0$

$$\Delta\breve{\alpha}^2 - c^2\Delta\breve{t}^2\Omega^2 > 0 \quad \Rightarrow \quad \Delta\breve{\alpha}^2 > \breve{V}^2\Delta\breve{t}^2 \tag{99.3}$$

By dividing both sides of the inequality by $\Delta\tilde{t}^2$, we find that $\tilde{u} > \breve{V}$, meaning that the causal velocity between the two events is greater than the value of the speed of light in negative subspace-time with respect to the observer O. the two events here occur outside the negative light cone, because the causal velocity is greater than the maximum speed physically allowed in this subspace-time. So the two events cannot be causally related either, and the negative interval physically allowed outside the negative cone is also of the negative lightlike type, as shown in **Figure 7** in blue.

We conclude from the above that, as a result of the fact that the minimum is equal to the maximum of the causal velocity on the negative diagram, all events here, including possible signals between events, have a world line of the negative lightlike type, *i.e.* they have world lines parallel to the lines of the negative cone with respect to the observer O. Where we find that any two events on this diagram have the same position on the $\breve{x}$-axis, such as the events J and K. See **Figure 7**—they are asynchronous on the $c\breve{t}$-axis—that is, they are always separated by a period of time and they cannot meet at any moment. Therefore, there cannot be a causal connection between these two events. As we also find that any two simultaneous events on the $c\breve{t}$-axis such as the two events K and H, see **Figure 7**, have different positions on the $\breve{x}$-axis, *i.e.* they are always separated by a spatial period and cannot meet in a place, and thus there is no causal connection between these two events as well. This means that all possible events on the negative subspace-time diagram are causally separate, even if there is a causal relationship between them in the positive subspace-time; they disappear in the negative subspace-time. We demonstrated this result in the first paper as well, but at the level of 3D negative subspace, where particles have the same speed and direction, and therefore there are no intersections or collisions between their paths in the negative subspace and we find the same thing here, but at the level of negative 4D subspace, where there is also no intersection between the worldlines of event. We can also generalize these results to any observer other than O, where only the inclination angle of the negative cone varies from one observer to another, but the geometry of the diagram remains constant for any observer. The above description shows that the causal structure on a negative subspace-time diagram is completely different from the causal structure of a Minkowski spacetime diagram. It is, in fact, a non-causal structure.

As for the rotation of the observer's coordinates O' with respect to O on the negative subspace-time diagram, it is exactly the same as the rotation of the observer's coordinates on the Minkowski diagram, because the transformation matrix for the negative space-time is the result of subtracting the Minkowski spacetime matrix from the positive subspace-time matrix. See the first paper, Equations (30.1) and (31.1).

$$\breve{\Lambda}_{\mu}^{\sigma} = \bar{\Lambda}_{\mu}^{\nu} - \tilde{\Lambda}_{\mu}^{\epsilon} \tag{100.3}$$

Thus, by equating the rotation matrix in a hyperbolic 4D space with the matrix of each spacetime, and by substituting in the previous equation, we obtain the rotation matrix of the negative subspace-time, and from it we obtain the values of both $\cosh\breve{\varnothing}$ and $\sinh\breve{\varnothing}$ of the rotation of the observer's coordinates O' with respect to O. We previously explained above that the matrix of rotational coordinates in the positive hyperbolic subspace-time $\tilde{\Lambda}_{\mu}^{\epsilon}$ remains the same as an identity matrix where the permissible angle of rotation is equal to zero. As for the matrix of rotational coordinates in Minkowski spacetime according to special relativity with inverse Lorentz transformations, where the rotation is in the hyperbolic plane x, ct, it is written in the following formula.

$$\bar{\Lambda}_{\mu}^{\nu} = \begin{bmatrix} \cosh\varnothing & 0 & 0 & \sinh\varnothing \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sinh\varnothing & 0 & 0 & \cosh\varnothing \end{bmatrix} \quad \cosh\varnothing = \gamma, \ \sinh\varnothing = \beta\gamma \tag{101.3}$$

Substitute (86.3) and (101.3) into (100.3)

$$\breve{\Lambda}_{\mu}^{\sigma} = \begin{bmatrix} \cosh\varnothing & 0 & 0 & \sinh\varnothing \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sinh\varnothing & 0 & 0 & \cosh\varnothing \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{102.3}$$

$$\breve{\Lambda}_{\mu}^{\sigma} = \begin{bmatrix} \cosh\varnothing - 1 & 0 & 0 & \sinh\varnothing \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \sinh\varnothing & 0 & 0 & \cosh\varnothing - 1 \end{bmatrix} \tag{103.3}$$

The last matrix represents the rotation matrix of the observer's coordinates O' in the negative hyperbolic subspace-time, where we find that the values of both $\cosh\varnothing = \gamma$ and also $\sinh\varnothing = \beta\gamma$, which means that $\breve{\varnothing} = \varnothing$ or in other words the rotation of the coordinates in the negative hyperbolic subspace-time is the same as the rotation of the coordinates in the Minkowski spacetime. This is also evident in the time transformation equation in the inverse negative modified Lorentz equations, which represents the same time transformation equation in the inverse Lorentz equations. Therefore, the graphic representation of both relativity of simultaneity and time dilation on a negative subspace-time diagram is the same as that shown on a Minkowski diagram. There is nothing new to mention, so we will not represent these results graphically here. Rather, we will represent new results graphically, such as negative length expansion and time contraction on a negative subspace-time diagram.

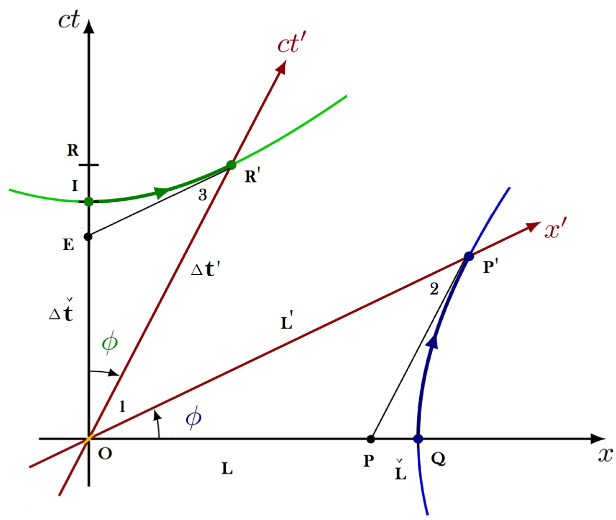

Figure 8. Shows length expansion with a negative value and time contraction in negative subspace-time.

From the previous figure we find that $\text{OP}` = \Delta x`$ represents the length of the stationary rod or the proper distance with respect to the observer O' on the $x`$-axis. We also find that $\text{OP} = \Delta x$ represents the relativistic length of the rod with respect to the observer O on the *x*-axis according to special relativity, but from the geometry of the figure we find that

$$\text{PQ} = \text{OP} - \text{OQ} \tag{104.3}$$

From **Figure 8** we find that angle 2 is a right angle because it is symmetrically equal to angle 1 between the perpendicular coordinates $x`, ct`$, and accordingly $\cosh\varnothing = \text{OP}`/\text{OP}$, and we also find from the figure that the two points $\text{Q}, \text{P}`$ are located on the same hyperbola, and thus $\text{OQ} = \text{OP}`$, by substituting that in Equation (104.3) we get

$$\text{PQ} = \frac{\text{OP}`}{\cosh\varnothing} - \text{OP}` \Rightarrow \text{PQ} = \text{OP}`\left(\frac{1}{\cosh\varnothing} - 1\right) \tag{105.3}$$

But, $OP` = l_0$ and $\cosh \varnothing = \gamma$

$$PQ = l_0 \left(\gamma^{-1} - 1 \right) \tag{106.3}$$

Thus, $PQ = \breve{l}$, which represents the negative length of the rod on the diagram. As shown in the previous diagram, the distance PQ expands in the negative direction on the *x*-axis with respect to the observer O with the increase in the angle of rotation $\varnothing$ or the speed of the reference frame S'. As for the contraction of time from the previous figure, we find that $OR` = c\Delta t`$, which represents the time period with respect to the observer O' on the $ct`$ axis. We also find that, $OR = c\Delta t$, which represents the dilation of the time period with respect to the observer O on the ct-axis according to special relativity. From **Figure 8**, we find that angle 3 is a right angle because it is symmetrically equal to angle 1 between the perpendicular coordinates $x`ct`$. Accordingly

$$\cosh \varnothing = \frac{OR`}{OE} \Rightarrow OE = \frac{OR`}{\cosh \varnothing} \tag{107.3}$$

where $OR` = \Delta ct`$ and $\cosh \varnothing = \gamma$

$$OE = \Delta t` \gamma^{-1} \qquad c = 1 \tag{108.3}$$

Thus, $OE = \Delta \breve{t}$ represents the time contraction in the negative subspace-time on the diagram. As shown in the previous diagram, the Time period OE contracts with respect to the observer O with increasing rotation angle $\varnothing$ or the speed of the reference frame S'.

2.10. Reflection of Negative Subspace-Time

The negative modified Lorentz equations revealed new properties of space and time in negative subspace-time, such as length expansion and time contraction. However, we obtained length dilation through the condition of simultaneous distance measurement (see 2-1 above), and time dilation through the condition of symmetry of the event velocity between reference frames (see 2-3 above). Therefore, these results are independent of each other. We will explain this in detail in the following example. Let us assume that we have an observer O who belongs to the reference frame S. He performs an experiment which is to emit an electron as a signal from the source at the origin O to a lamp at point K to light this lamp, where the electron belonging to the reference frame S' is located at the origin point O' and therefore the speed of the electron here represents the relative speed between the reference frames, as shown in **Figure 9**.

Suppose that at the moment of electron emission the coordinates were $\breve{x}_0 = x`_0 = 0$ and $\breve{t}_0 = t`_0 = 0$. Here we can describe the event with respect to the reference frame S' where the lamp moves along the $x`$-axis in the negative direction from point k to the origin O' a distance $kO' = -\Delta x`$ in a time period $\Delta t`$. As for the reference frame S, the observer O observes the particle moving along the *x*-axis in the positive direction from the origin O to point k a distance $Ok = \Delta \breve{x}$ in a time period $\Delta \breve{t}$.

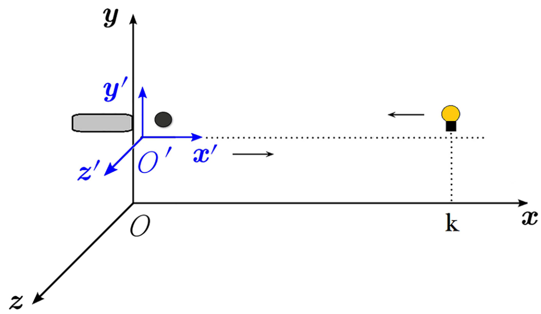

Figure 9. Shows an experiment that fulfills the condition of symmetry of the velocity in negative subspace-time.

This example satisfies the condition of velocity symmetry between reference frames, as we find that the velocity of the lamp $-V_x^{`}$ with respect to S' is equal to the velocity of the electron $\breve{V}_x$ with respect to S, without the need to analyze $V_x^{`}$ as in Section 2.3 above.

$$\breve{V}_x = -V_x^{`} \tag{109.3}$$

By substituting the value of $\breve{V}_x, V_x^{`}$ in the previous equation

$$\frac{\Delta\breve{x}}{\Delta\breve{t}} = -\frac{\Delta x^{`}}{\Delta t^{`}} \tag{110.3}$$

Because the symmetry condition is available here, we can substitute here the value of $\Delta\breve{t}$ according to the time contraction equation that we proved.

$$\frac{\Delta\breve{x}}{\Delta t^{`}\gamma^{-1}} = -\frac{\Delta x^{`}}{\Delta t^{`}} \tag{111.3}$$

$$\Delta\breve{x} = -\Delta x^{`}\gamma^{-1} \tag{112.3}$$

Equation (112.3) shows us that the proper distance decreases with respect to the observer O in the negative subspace-time as the velocity of the reference frame S' (the electron) increases. It should be noted here that the proper distance is not measured simultaneously by the observer O. Therefore, it does not represent a proper distance transformation according to inverse relativity. As shown on the space-time diagram Figure 8, the contraction of the distance OP is related to the contraction of the time period OE with the increase of the rotation angle or the speed of the reference frame, and when it reaches $V_S = c$ (because the particle at the critical speed can at any moment accelerate itself to the speed of light), By substituting this in the equations for the contraction of distance and time, we obtain a collapse in the transformation equations or in the structure of negative subspace-time, and we find the same thing on the diagram Figure 8 when the angle of rotation reaches 45 degrees.

But on the other hand, we find that the distance $\Delta x^{`}$ traveled by the lamp or any other value is a proper distance with respect to the reference frame S', and thus it is subject to the distance expansion equation, If the observer O observes this distance in the reference frame S simultaneously, we have achieved the simultaneity condition here.

$$\Delta\breve{x} = \Delta x`\left(\gamma^{-1} - 1\right) \tag{113.3}$$

By substituting $V_S = c$ into the previous equation, we get

$$\Delta\breve{x} = -\Delta x` \tag{114.3}$$

We mentioned in the first section that the expansion of length or distance by a negative value expresses the amount of decrease in distance resulting from the real contraction of distance due to special relativity. But we can consider that the decrease in positive distance is a negative distance expanding in the positive direction. See **Figure 10**, and vice versa in the negative direction, and when the speed of the reference frame S' reaches the speed of light and the positive distance disappears or the structure of space collapses in the positive direction according to special relativity, we find that the negative distance replaces it according to inverse relativity, and thus the positive direction becomes negative and the same thing in the negative direction. In other words, when the speed of the reference frame S' reaches the speed of light, the observer O observes a reflection of the coordinate $x`$ in the reference frame S' with respect to his own frame S (see **Figure 10**). The structure of the spatial space becomes symmetrical but reversed, and as a result, a reflection of the displacement vectors also occurs. After multiplying Equation (113.3) by −1 and By substituting $V_S = c$, we obtain a reflection of the displacement directions with respect to each reference frame.

$$-\Delta\breve{x} = \Delta x` \tag{115.3}$$

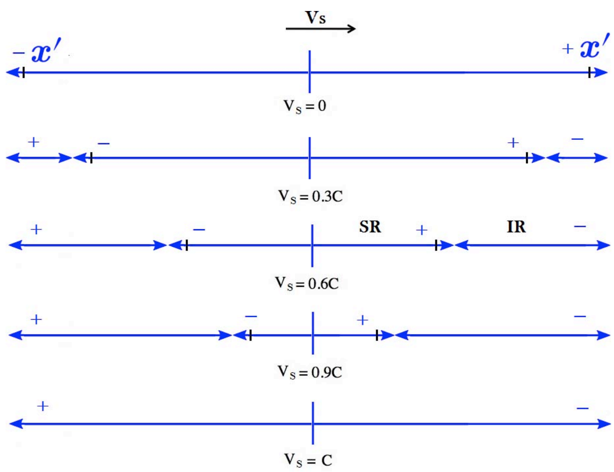

Figure 10. Shows the reflection of negative subspace when the reference frame reaches the speed of light.

But the reflection of the displacement direction necessarily leads to a reflection in the motion and thus time. Just as we know that any spatial period on the negative subspace-time diagram is equivalent to the time period, so we can substitute the value of $\Delta x` = c\Delta t`$ and also $\Delta\breve{x} = c\Delta\breve{t}$ in Equation (113.3)

$$c\Delta\breve{t} = c\Delta t`\left(\gamma^{-1} - 1\right) \tag{116.3}$$

$$\Delta\breve{t} = \Delta t`\left(\gamma^{-1} - 1\right) \tag{117.3}$$

Equation (117.3) shows that time dilates with a negative value as the speed of the reference frame S' (the electron) increases. This equation does not represent a contradiction with the equation for time contraction in negative subspace-time. Because time dilation has a negative value, it only expresses the amount of time decrease resulting from time contraction, as shown on the negative subspace-time diagram, **Figure 8**, where we find that the time period IE dilates negatively with the time period OE contraction as the rotation angle (the velocity of the reference frame) increases. It only shows us the nature of time in an reversed space. When $V_S = c$ at any instant and the time contraction equation breaks down, we obtain from the negative time dilation equation the symmetry of time between the reference frames, but in reverse.

$$\Delta\breve{t} = -\Delta t` \tag{118.3}$$

where the observer's time O passes into the future while the electron's time passes into the past, in other words, the observer O observes in the future the electron's past or the electron's return to its position in the past. As for the electron, it will seem to him that the lamp goes back in time and takes its position in the past. That is, the time reversal here expresses the reversal of the motion of the reference frames relative to each other, which is a logical result after the reversal of the displacement vectors shown in Equation (115.3). We can say then that the negative subspace-time at the speed of light becomes a subspace-time with a symmetric and inverse structure for both observers. But this reflection is temporary and occurs within a very short period of time, due to the energy fluctuations that cause the electron to reach the speed of light, according to the Heisenberg's inverse principle. Because the speed here is equal to the speed of light, *i.e.* it is precisely defined, and thus $\Delta\breve{p}_{un} = 0$, so the reflection in place is within the range of uncertainty in position $\Delta\breve{x}_{un}$ (See comparison **Table 2** below).

Although the results we have arrived at here are in subspace-time and not real, these results do not violate the second postulate of special relativity because the reflection occurs at the speed of light, but does not exceed it. Also, the reflection of time does not violate the principle of causality, because the reflection of time is linked to the reflection of space. In other words, the electron cannot go back in time when it goes to the lamp and reaches the lamp in the past, that is, before its emission, where the result precedes the cause. Just as it returns to the past in time, it returns in space, and therefore the result cannot precede the cause according to this reflection. There is also no conflict with thermodynamic entropy, which creates a direction for time, because the reversal of time here is for an individual quantum particle. However, if we have a number of particles moving at a critical speed in the same direction but not in the same frame, such as a beam of particles, whether electrons or even photons, then their spacetime reflections will be unsynchronous and even random. So we assume the existence of entropy for spacetime fluctuations or spacetime reflections, where the spacetime entropy dominates the beam and creates a direction of time towards the future, and thus it is not possible to observe a reflection in the entire beam even for a very small period of time, So the speed of a ray of light in a vacuum is always equal to the value c, and its direction on the spacetime diagram is always towards the future.

3. Results

From the positive and negative modified Lorentz transformation equations shown in the first paper, we obtain the relativity of length, where the proper length or distance remains constant under the transformation in positive subspace due to the symmetry of the positive space structure, while the length expands from zero to the original length in negative subspace with the motion of the reference frame (which is a result opposite to special relativity). See Equation (2.3, 15.3). The relativity of simultaneity, where we find that the simultaneity between two events is absolute, and the time of causality is real time and not an illusion in the positive subspace-time (which is a result opposite to special relativity). See Equation 19.3. The relativity of time, where time expands in positive subspace-time, while time in negative subspace-time contracts with the motion of the frame of reference until we have super-time (which is the opposite result of special relativity). See Equation (24.3, 30.3). Through the transformation equations of relativistic energy in each subspace-time in the special case shown in the second paper, we also obtained, Relativistic mass: where mass decreases in positive subspace (which is the opposite result of special relativity), while mass increases in negative subspace with the motion of the reference frame (see Equations 33.3, 37.3). Relativistic kinetic energy: where kinetic energy decreases in positive subspace (which is the opposite result of special relativity), while kinetic energy increases in negative subspace with the motion of the reference frame see Equation (39.3, 42.3), See also the comparison **Table 1** below. Through the experiment of the photon stuck inside a moving box, we gained a new concepts of both mass, work, motion, and understanding the mechanism of motion of elementary particles. By hypothesising the fluctuation of energy between subspaces, we were able to explain the Heisenberg principle of energy and time in positive subspace-time, and to discover the existence of the principle inversely in negative subspace-time, and to quantize the transformation equations in each subspace in the form of transformation inequalities (see set of Equation, 71.3, 72.3). As a result of quantization, we obtain, in the special case, the possibility of a quantum particle with a rest mass reaching the speed of light without infinite energy, and in the general case, the possibility of a dust field reaching the Schwartzschild radius in a negative subspace-time without infinite energy density. Through subspace-time diagrams, we obtained a causal structure in the positive subspace-time and a non-causal structure in the negative subspace-time. Through an experiment that fulfills the condition of symmetry of event velocity between reference frames in the negative subspace-time, we have shown that this subspace-time can be inverted without violating causality or exceeding the speed of light.

Table 1. Comparison table between relativistic variables in Special Relativity and Inverse Relativity.

Relativistic Variables	Special Relativity	Inverse Relativity
Relativity of length	$l = l\grave{}_0 \gamma^{-1}$	$\breve{l} = l\grave{}_0 \left(\gamma^{-1} - 1\right)$
Relativity of simultaneity	$\Delta t\grave{}_{AB} = 0, \Delta t_{AB} \neq 0$	$\Delta t\grave{}_{AB} = 0, \Delta \tilde{t}_{AB} = 0$
Relativity of time	$\Delta t = \Delta t\grave{}_0 \gamma$	$\Delta \breve{t} = \Delta t\grave{}_0 \gamma^{-1}$
Relativity of mass	$m = m\grave{}_0 \gamma$	$\tilde{m} = m\grave{}_0 \gamma^{-1}$
Relativistic kinetic energy	$kE = kE\grave{}\gamma$	$k\tilde{E} = kE\grave{}\gamma^{-1}$

4. Discussions

The third paper on the inverse relativity model demonstrates its ability to solve some of the problems inherent in both special and general relativity. The first problem is the collapse of spacetime in special relativity at $V_S = c$ theoretically, where length contracts to infinity, meaning one of the dimensions of space disappears. Time also expands to infinity, or becomes zero, or disappears, we find a similar situation in general relativity when $r = r_S$, where the structure of spacetime also collapses, which is known as spacetime singularity [30]. However, in the inverse relativity model, as we explained above in the Section 2-10, the structure of spacetime does not collapse as happens in special relativity, but rather spacetime becomes inverted. Thus, the inverse relativity model is characterised by the absence of collapse in its equations or in the structure of spacetime.

The second problem: It is the problem of the illusion of time in special relativity as a result of the relativity of simultaneity, as it appears on the Minkowski diagram of space-time that events that are not synchronous with respect to one observer can be simultaneous with respect to another observer. This resulted in the concept of the block universe, which contains all the events of the universe in the past, present, and future at the same moment, and thus time becomes an illusion, as Einstein believed. However, in the inverse relativity model, two types of time are distinguished, causal time associated with events and non-causal time, or time not associated with events. In the section of inverse relativity of simultaneity, the simultaneity of causal time is absolute, and thus causal time appears as real time. This result is consistent with the thermodynamic conception of time (entropy).

The third problem is infinity. In the special relativity, due to the increasing mass of the particle, no material particle can reach the speed of light because it requires infinite energy. In general relativity, we also find a similar situation: when a dust field reaches the Schwarzschild radius, we have infinite energy density. However, in inverse relativity, according to the hypothesis of energy fluctuations between subspaces, a quantum particle can use an infinitely small part of its internal energy to produce self-acceleration. If the particle's speed is a critical speed, *i.e.*, less than the speed of light by the amount of self-acceleration, the quantum particle can reach the speed of light in the special case without infinite energy. We find the same situation in the general case: if the dust field is at the critical radius, *i.e.*, greater than the Schwarzschild radius by the amount of uncertainty in the radius created by self-acceleration, the dust field can reach the Schwarzschild radius without infinite energy and thus infinite energy density.

In this paper, we also tried to bridge the gap between the concept of wave and particle nature, as we explained in the section on the concept of mass, work, and motion the ability to do mechanical work on a packet of waves (photons) like any material particle. Also, particles with rest mass, in order to carry the property of motion, must have a light wave structure, *i.e.*, standing or closed waves that move at the speed of light. We also explained the model's ability to combine seamlessly with quantum mechanics, or quantization. We find that quantum mechanics quantizes the model for us by changing the transformation equations into transformation inequalities. The model also explains quantum mechanics, or specifically the Heisenberg principle of energy and time, as a fluctuation of energy between subspaces, meaning that each of them completes the other. So the third paper is paving the way for understanding quantum mechanics. We will try to find a comprehensive interpretation of quantum mechanics in another paper. By substituting the values of $\Delta\tilde{E}_{un} = m\Delta\tilde{V}_{un}\Delta\tilde{V}_{un}$ and $\Delta\breve{E}_{un} = m\Delta\breve{V}_{un}\Delta\breve{V}_{un}$ into both the Heisenberg and the Inverse-Heisenberg formulas for energy and time, we obtain the Heisenberg and Inverse-Heisenberg formulas for position and momentum, as shown in the following **Table 2**.

Table 2. Comparison table of the Heisenberg Principle in each Subspace-time in Inverse Relativity.

Subspace-times	Positive Subspace-Time	Negative Subspace-Time
Energy and time	$\Delta\tilde{E}_{un}\Delta\tilde{t}_{un} \ge \frac{1}{2}\hbar$	$\Delta\breve{E}_{un}\Delta\breve{t}_{un} \le \frac{1}{2}\hbar$
Momentum and position	$\Delta\tilde{p}_{un}\Delta\tilde{x}_{un} \ge \frac{1}{2}\hbar$	$\Delta\breve{p}_{un}\Delta\breve{x}_{un} \le \frac{1}{2}\hbar$

We conclude the paper by proposing an experiment to test one of the previous results, which is doing mechanical work on the photon according to the following diagram: The diagram represents a laser beam of uniform wavelength coming from the source to a semi-transparent mirror. The beam splits into two simultaneous beams, one of which heads to mirror M_1 and the other to mirror M_2. The mirrors are equidistant from the semi-transparent mirror. The two beams then reflect back to the semi-transparent mirror and combine into a single beam that heads to the detector. Assuming that the diagram is located in the xy-plane and that mirror M_1, M_2 is rotatable, its rotation axes are x and y, respectively, as shown in **Figure 11**. Because the reflected rays come from the same source and travel the same distance at a constant speed, the waves heading to the detector are synchronous and form a specific interference pattern that appears on the detector.

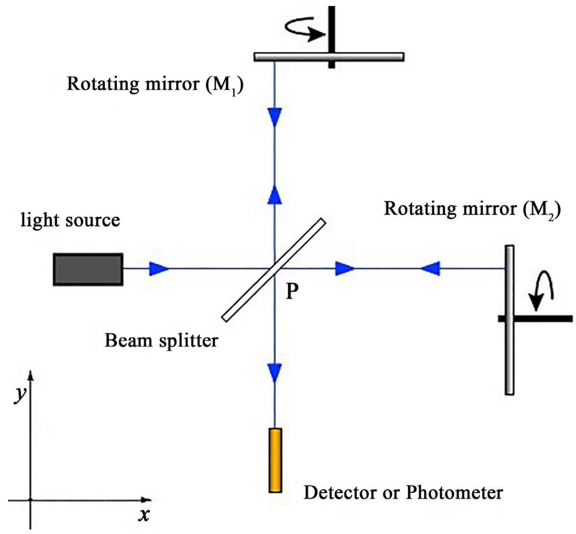

Figure 11. Shows the experiment of doing mechanical work on photons through rotating mirrors.

The first step of the experiment is before rotating the mirror, where the observer adjusts the angles of the rotating mirrors to make them perpendicular to the rays falling on them by obtaining an interference pattern on the detector. Because the photons are perpendicular to the surface of the mirrors, all components of their tangential velocity to the surface of the mirror are equal to zero, and therefore, they are at rest relative to the mirror at the moment of collision. The second step of the experiment is for the observer to rotate the mirror at a relatively high angular velocity and while rotating, observe the interference pattern on the detector. If we get a new interference pattern, this is the result of changing the frequency of the photons because the speed of light is constant in all cases. This is evidence that the rotating mirrors did mechanical work on the photons. But if we get more than one interference pattern or interference patterns that change over time, this is a result of the mirror's vibration during rotation and also the mirror's non-perfectly level surface. In this case, the observer replaces the detector with a photometer because even with the mirror's vibration and imperfect surface, we expect mechanical work to be done on some of the photons and we get a noticeable change in the radiation intensity. The observer measures the radiation intensity before rotating the mirror in the first step, then re-measures the radiation intensity again while the mirror is rotating. If the observer obtains higher radiation intensity, this is evidence that the rotating mirrors have done mechanical work on the photons.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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