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The Simplest Scale-Invariant Solutions of the Vlasov-Einstein Equations and Their Cosmological Consequences

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Abstract

The solutions of the Vlasov-Einstein equations for a system of gravitating particles, for which initial conditions with a high degree of symmetry take place, have a form close to the De Sitter metric. The concept of inertial motion of matter under homothetic transformation of space can be applied to explain the formation of large-scale cosmological structures. For this purpose, methods for analyzing the motion of particles in the inertial mode of motion are considered.

Keywords

Vlasov Equation, Covariant Derivative, Cosmological Structures

1. Introduction

The emergence of large-scale low-dimensional structures in modern scientific literature is associated with fluctuation inhomogeneities in matter, which have passed from the inflationary era to the new cosmological era of flat space. At the same time, the dominant paradigm for more than 100 years was the Friedmann model of homogeneous space (in the early stages, it was transformed into the De Sitter model). However, by now an extremely large amount of observational material has been obtained, indicating that the concept of "homogeneity" of the Universe requires a special interpretation. The presence of (quasi)order in the form of ubiquitous large low-dimensional (coherent) structures instead of the expected homogeneous distribution of matter in the region of the Universe accessible to observation suggests the existence of at least background physical mechanisms that effectively compete over long time intervals with the Friedmann expansion (in terms of the spatial distribution of gravitating matter).

The consideration of cosmological models has historically been associated with the analysis of the properties of metrics, which are solutions of Einstein's field equations. The distribution of many masses, which creates a fairly arbitrary, and time-varying gravitational field, together with the field equations can be solved analytically only in exceptional cases, and their numerical solution requires colossal labor costs even with the use of the most modern computing technology. However, there are powerful analytical methods for the qualitative analysis of the properties of solutions to systems of such equations, which can be used to study the properties of astrophysical processes, leaving regions with complex geometry or with fast processes for machine modeling. However, the primary issue for the correct writing of the Vlasov-Einstein system of matter-field equations with a self-consistent field at the stage of cosmological structure formation is the identification of the dependencies of the metric coefficients at the current time stage. At present, the prevailing hypothesis about the initial epoch of the expansion of the Universe is the inflationary one, which uses for its explanation a certain formal "inflaton field", which goes beyond the framework of the classical model ΛCDM. However, based on the concept of a self-consistent field, one can, relying on the ideas of A. A. Vlasov, consider the possibility of realizing the de Sitter stage of expansion as a special form of action of pseudo-forces of metric origin, caused by a special initial distribution of particles on the manifold $\mathcal{M}_8$ of Laptev supporting elements [1] [2] (a generalization of the E. Cartan space), which is a submanifold of the tangent bundle over the configuration set of the system of particles $\mathcal{M}_x$; In this article we will limit ourselves to introducing on it the usual Riemannian metric [3], the operations with which do not require the definition of new concepts.

It can be shown that for long time intervals (including the modern era) the equations with a self-consistent field smoothly transform into the Vlasov-Poisson equations. In this case, their integral Volterra form has, under certain, quite physically obvious and reasonable assumptions, periodic in time solutions [4]. In the region of structure growth, a local change in the metric occurs, caused by this process (as a phase transition of matter in hydrodynamic macroparameters), and it is possible to trace the dynamics of the development of structures from their birth to decay (caused both by the expansion of space and by the increase in internal desynchronization of the constituent parts of macrostructures—which is an essential important part of their genesis: statistical decoherence of multi-time ensembles of particles at large distances).

In this paper, we consider a method for analyzing a system of kinetic Vlasov equations field and equations for the gravitational field, which consistently presents the cosmological aspects of the approaches proposed in the works [5] [6], and allows us to obtain in a very simple way the general evolution of the observed macrostructures in the Universe, as well as to establish general criteria for their development, using only completely transparent analytical approaches.

2. Scale Invariance of the Metric as a Condition for Integrability of Covariant Equations of Motion

We consider the self-consistent system of Vlasov-Einstein equations for a set of massive particles described by the distribution function $F(\boldsymbol{x},\boldsymbol{v})$ ($\boldsymbol{x}\in\boldsymbol{X}\subseteq\mathbb{R}_x^4$, $\boldsymbol{V}\subseteq\mathbb{R}_v^4$):

$$\operatorname{div}_4\left(\boldsymbol{v}F(\boldsymbol{x},\boldsymbol{v})\right)=0,\ R^{\mu\nu}-\frac{1}{2}R^{\dagger}g^{\mu\nu}=\kappa T^{\mu\nu},\ R^{\dagger}\equiv R-2\Lambda,\ \kappa=\frac{8\pi G}{c^4},\tag{1}$$

where: $\operatorname{div}_4(\ldots)$—covariant divergence operator:

$$\operatorname{div}_4(\boldsymbol{v}F)=v^{\mu}\nabla_{\mu}F+F\nabla_{\mu}v^{\mu},\ \nabla_{\mu}F=\frac{\partial F}{\partial x^{\mu}}+\frac{\partial F}{\partial v^{\nu}}\frac{\partial v^{\nu}}{\partial x^{\mu}},\ \mu,\nu,\dots=\overline{0,3}\tag{2}$$

$$\nabla_{\eta}v^{\nu}=\frac{\partial v^{\nu}}{\partial x^{\eta}}+\Gamma^{\nu}_{\eta\zeta}v^{\zeta}\equiv w^{\nu}_{\eta}\left[x^{\eta},v^{\nu},f\right],\ \nabla_{\eta}\left(v^{\nu}F\right)=v^{\nu}\nabla_{\eta}F+F\left(\frac{\partial v^{\nu}}{\partial x^{\eta}}+\Gamma^{\nu}_{\eta\zeta}v^{\zeta}\right).\tag{3}$$

Multiplication by $v^{\eta}\equiv \mathrm{d}x^{\eta}/\mathrm{d}\tau$ and convolution of the first Equation (3) by index η leads to the classical form of the Euler-Lagrange equations given in manuals on general relativity, however, on the right-hand side there will be a quantity with the dimension of acceleration: $v^{\eta}w^{\nu}_{\eta}\equiv\mathfrak{a}^{\nu}$. In this case, the system (3) (with respect to 4-velocities) is overdetermined ($\max\mu\cdot\max\nu>\dim\boldsymbol{v}$) and requires the introduction of additional integrability conditions. As such, we can take the conditions $\hat{\Omega}[w]\equiv\nabla_{\nu}w^{\eta}_{\mu}-\nabla_{\mu}w^{\eta}_{\nu}=0$. Since for (3) we have $\nabla_{\nu}\nabla_{\mu}v^{\eta}-\nabla_{\mu}\nabla_{\nu}v^{\eta}=\nabla_{\nu}w^{\eta}_{\mu}-\nabla_{\mu}w^{\eta}_{\nu}$, $\left(v^{\eta}\right)''_{\nu\mu}=\left(v^{\eta}\right)''_{\mu\nu}$, then $\hat{\Omega}[w]=R^{\eta}_{\alpha\mu\nu}v^{\alpha}=0$ (for a flat space the integrability condition is satisfied automatically, because in this case $R^{\eta}_{\alpha\mu\nu}=0$).

In the case of a space with a non-vanishing Riemann tensor, there is a special form of the tensor $w^{\eta}_{\nu}={}_{(0)}w^{\eta}_{\nu}$, which corresponds to the velocity field emanating from one point in the absence of dynamic forces (accelerations) in the system: ${}_{(0)}w^{\eta}_{\nu}=\mathcal{H}\left(\delta^{\eta}_{\nu}-v^{\eta}v_{\nu}\big|_{v=v\left[{}_{(0)}w^{\eta}_{\nu}\right]}/c^2\right)$, $\mathcal{H}=\text{const}$. The latter, according to is a consequence of the energy conservation condition (in terms of momenta $\boldsymbol{p}=\boldsymbol{p}(\boldsymbol{v})$) on "the mass hyperboloid $\mathcal{M}_v$" [7], and also, according to [5], the fact that the expression for the acceleration there automatically cancels out:

$$\begin{aligned}v^{\eta}\nabla_{\eta}v^{\nu}\Big|_{v=v\left[{}_{(0)}w^{\eta}_{\nu}\right]}&=\left(\frac{\mathrm{d}v^{\nu}}{\mathrm{d}\tau}+\Gamma^{\nu}_{\eta\mu}v^{\eta}v^{\mu}\right)\Bigg|_{v=v\left[{}_{(0)}w^{\eta}_{\nu}\right]}\\&=v^{\eta}\mathcal{H}\left(\delta^{\nu}_{\eta}-v_{\eta}v^{\nu}/c^2\right)\Big|_{v=v\left[{}_{(0)}w^{\eta}_{\nu}\right]}=\mathcal{H}\left(v^{\nu}-v^{\nu}\right)=0.\end{aligned}$$

On the other hand, convolution of the fundamental tensor $g_{\nu\mu}$ with the Equation (3) for ${}_{(0)}w^{\eta}_{\nu}$ gives:

$$\nabla_{\eta}c\equiv0,\ \nabla_{\eta}e_{\mu}=\mathcal{H}c^{-1}\left(g_{\mu\eta}-e_{\mu}e_{\eta}\right),\ e_{\eta}e^{\eta}=1.\tag{4}$$

In the geodetic coordinate system $\mathfrak{K}_4$, defined by a congruence of geodesics, one of which (parameterized by the natural real parameter s) is perpendicular to the 3-dimensional hypersurface on which the coordinates ${}_{(g)}x^k\big|_{k=1,2,3}$ are given (the $\mathfrak{K}_4$ system describes the entire 4-space), we have a general expression for the coordinates $s = \mathfrak{X}^0 \equiv x^0$, ${}_{(g)}x^k = \mathfrak{X}^k\left(x^0,\cdots,x^3\right) = x^k\big|_{k=1,2,3}$ (redesignation of coordinates). The interval in it can be expressed in the form:

$\mathrm{d}s^2 = g_{00}\left(\mathrm{d}x^0\right)^2 - \mathfrak{G}_{ik}\mathrm{d}\mathfrak{X}^i\mathrm{d}\mathfrak{X}^k = g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu$, if $g_{\mu\nu} = \mathfrak{G}_{\mu\nu} + \underbrace{e_\mu e_\nu}_{\text{const}}$, since $e_\nu = e^\nu = (1,0,0,0)$ in the geodetic coordinate system, $g_{\mu 0}\big|_{\mu>0} \equiv 0$ ($g_{00} \equiv 1$).

Thus, from the second equation of the system (4), and the last obtained relation it follows that $\nabla_\eta e_\nu = \mathcal{H}c^{-1}\mathfrak{G}_{\mu\nu}$. Since in geodetic coordinates $g_{00} = 1$, $g_{0k}\big|_{k>0} \equiv 0$, then $\nabla_\eta e_\nu = -\Gamma^\mu_{\eta\nu}e_\mu = \frac{1}{2}\partial g_{\eta\nu}/\partial x^0$.

Consequently, $\partial g_{\eta\nu}/\partial x^0 = \partial\mathfrak{G}/\partial x^0$; finally we obtain

$\nabla_\eta e_\nu = \frac{1}{2}\partial\mathfrak{G}_\eta e_\nu/\partial x^0 = (\mathcal{H}/c)\mathfrak{G}_\eta e_\nu$. Assuming $c, \mathcal{H} = \text{const}$, the last equation can be considered as an ordinary differential equation, the solution of which is expressed through an arbitrary tensor function of three arguments $\Xi_{\eta\nu}$:

$\mathfrak{G}_\eta e_\nu = \exp\left(2\mathcal{H}x^0/c\right)\Xi_{\eta\nu}\left(\mathfrak{X}^1,\mathfrak{X}^2,\mathfrak{X}^3\right)$. Interval for the divergent velocity field of the medium

$$\mathrm{d}s^2 = \left(\mathrm{d}x^0\right)^2 - \exp\left(2\mathcal{H}x^0/c\right)\cdot\Xi_{\eta\nu}\left(\mathfrak{X}^1,\mathfrak{X}^2,\mathfrak{X}^3\right)\mathrm{d}x^\eta\mathrm{d}x^\nu. \tag{5}$$

For Riemannian geometry, the statement about local equivalence in the neighborhood of a chosen point $\mathfrak{O} \in \mathcal{X}_2$ is valid on the hypersurface $\mathcal{X}_2 = \left\{x^0 = \text{const}\right\}$ the above metric and the quasi-Euclidean metric of the local neighborhood $O\left(\tilde{\mathfrak{O}}\right) \subset \mathcal{X}_2$

$$\mathrm{d}s^2 = \left(\mathrm{d}x^0\right)^2 - \sum_{k=1,2,3}\mathrm{d}\left(\tilde{\mathfrak{X}}^k\right)^2,\ \tilde{\mathfrak{X}}^k \equiv \exp\left(\mathcal{H}x^0/c\right)\mathfrak{X}^k \tag{6}$$

(the "tilde" symbol will denote coordinate scaling). Spatial coordinates change on manifolds to which the "time/evolutionary" axis is orthogonal in accordance with the law $\mathfrak{X}^k\left(\mathfrak{X}^0\right) = \exp\left(\mathcal{H}\mathfrak{X}^0/c\right)\cdot\mathfrak{X}^k\left(\mathfrak{X}^0 = 0\right)$ with the expansion rate $\mathrm{d}\mathfrak{X}^k/\mathrm{d}\mathfrak{X}^0 = \left(\mathcal{H}\mathfrak{X}^0/c\right)\mathfrak{X}^k$. Obviously, the similarity to the De Sitter metric cannot be just a coincidence.

The condition $\hat{\Omega}[w] = R_{\alpha\zeta,\mu\nu}v^\alpha = 0$ leads, after applying the covariant derivative to it and using the Bianchi identity, to a covariant differential equation for the Riemann tensor $v^\eta\nabla_\eta R_{\alpha\zeta,\mu\nu} + 2\mathcal{H}R_{\alpha\zeta,\mu\nu} = 0$; for the scalar curvature R, from here we have an ordinary differential equation (along the evolutionary axis of the geodetic coordinate system, which leads to a formal replacement of $\nabla \to \mathrm{d}/\mathrm{d}\mathfrak{X}^0$) for the independent variable $\mathfrak{X}^0$), the solution of which is:

$R\left(\mathfrak{X}^0\right) = R(0)\exp\left(-\mathcal{H}x^0/c\right)$. Thus, the primary curvature of the 4–space should decrease with time (if $\mathcal{H} = \text{const} > 0$).

It should be noted that the assumptions made above do not include the assumption of isotropy and homogeneity of space. The only important factor that complements the classical consideration of the metric of expanding space-time in general relativity is the assumption of an initially present diverging field of matter velocities at a certain initial moment of consideration (that is, in fact, a specialized Cauchy condition). In this case, matter moves "in a non-force regime" (by inertia), without taking into account the influence of the self-consistent field. The duration of the expansion phase of the need for significant consideration of the energy-momentum tensor can be estimated by the value of the parameter $\mathcal{H}$.

Recent observations of the inhomogeneous structure of the Universe [8] can lead us to a model in which at some point there are many regions with matter in a state with the distribution of particle velocities that were described above. It is quite possible that the synchronization of the mentioned regions is absent, and the competition of the inertial motion of matter particles with their motion in a self-consistent field leads (for sufficiently long times) to the establishment of a regime of effective negative temperature of the system, which leads to the appearance of megastructures such as walls and voids. It should be noted that in this case the function $\mathcal{H} \to \{\mathfrak{h}(z)\}$, where $\{\mathfrak{h}(z)\}$ is a some set of local indicators of inertial expansion depending on internal parameters (density, temperature, etc.); its may lead to different growth rates of cosmological systems and demonstrate the so-called "Hubble tension" effect (the reasons for this conclusion are given in the next paragraph).

3. Evolution of Cosmological Structures as a Consequence of Nonequilibrium Processes in Curved Space-Time

The classical diffusion equation for processes (with characteristic time t_0) in 3-dimensional space can be obtained from the continuity equation

$$\partial\rho\left({}_3\boldsymbol{x},t\right)/\partial t+\hat{N}\left[{}_3\boldsymbol{v};f\right]=0,\ \ \hat{N}\left[{}_3\boldsymbol{v};f\right]\equiv\int \operatorname{div}\left({}_3\boldsymbol{v}f\left({}_3\boldsymbol{x},{}_3\boldsymbol{v},t\right)\right)\mathrm{d}_3\boldsymbol{v},$$

when passing to finite differences:

$\hat{N}\left[{}_3\boldsymbol{v};f\right]\to\sum_{j=1}^{3}\int{}_3v_j\left(f\left({}_3\boldsymbol{x}+{}_3\boldsymbol{v}\mathrm{t}_0+\cdots\right)-f\left({}_3\boldsymbol{x}\right)\right)/\left({}_3\boldsymbol{v}\mathrm{t}_0+\cdots\right)\mathrm{d}_3\boldsymbol{v}$. After the transformations we obtain $\partial\rho/\partial t+\operatorname{div}\left({}_3\boldsymbol{v}^{\dagger}+{}_3\boldsymbol{V}_D^{\dagger}\right)\rho=\Psi(\rho)$, where ${}_3\boldsymbol{v}^{\dagger}=\int{}_3\boldsymbol{v}f\,\mathrm{d}_3\boldsymbol{v}/\rho$, ${}_3\boldsymbol{V}_D^{\dagger}\rho=-D\nabla_3\rho$, $D=\left(\mathrm{t}_0/2\right)\int{}_3v_j^2 f\,\mathrm{d}_3\boldsymbol{v}/\rho$. For the region of cosmological macrostructure growth against the background of a homogeneous medium in the inertial regime (if we neglect the presence of force interaction from sources of an external or self-consistent gravitational field, for example, in a channel of matter motion with an ordered average velocity), the covariant diffusion equation has the form $\nabla_4\left(\boldsymbol{v}+\boldsymbol{V}_D\right)\varrho=\Psi(\rho)$, where: $\varrho(\boldsymbol{x})=\mathrm{d}N/\left(|g|^{1/2}\,\mathrm{d}\boldsymbol{x}\right)$—the particle density in the 4-volume, $\nabla_\mu v^\nu=w_\mu^\nu$, $V_D^\nu\varrho=-\mathcal{D}^{\mu\nu}\nabla_\nu\varrho$, $\mathcal{D}^{\mu\nu}$—diffusion tensor, V_D^ν—diffusion flow velocity, function $\Psi(\varrho)=\nu_{ph}\left(\varrho-\varrho_0\right)\left(1-\varrho/\varrho_{\max}\right)^n\Big|_{n=1,2}$ displays the phase transition process "isotropic medium—ordered structure" (ν_{ph} is phenomenologic coefficient, $\varrho_0,\varrho_{\max}$ are concentrations of particles in both phases). For us, however, it is essential that the equation for the velocity vector of the medium by multiplying both of its parts by $\mathrm{d}x^{\mu}/\mathrm{d}\tau$ and folding by index μ, leads, as previously noted, to the classical form of the Euler-Lagrange equation of geodesics, and on the right-hand side there is a quantity with the dimension of acceleration $v^{\beta}w^{\nu}_{\beta}$; this system of 16 equations requires additional restrictions on the Christoffel coefficients, since it is significantly overdetermined ($\dim \boldsymbol{v} = 4 < 16$). We will restrict ourselves to the form of the tensor already considered earlier when imposing the necessary additional conditions: $w^{\nu}_{\mu} = \mathfrak{h}\left(\delta^{\nu}_{\mu} - v^{\nu}v_{\mu}/\boldsymbol{v}^2\right)\big|_{\boldsymbol{v}^2 = v^{\nu}v_{\mu}}$ ($\mathfrak{h} = \mathrm{const}$, the diagonal structure of the tensor $w^{\nu}_{\mu} = \mathrm{diag}(\mathfrak{h},\cdots,\mathfrak{h})$). Obviously, in this case for $\mathfrak{h} \neq 0$ we postulate a situation in which there is no rest state (the growth of the structure, ensured by the condition $\nabla \boldsymbol{v} = w^{\nu}_{\nu} > 0$, continues indefinitely).

Let us introduce a geodetic coordinate system $\mathfrak{K}_4$ (similar to that in item 2). In the congruence of geodesics, we choose a direction with a unit vector $e^{\mu}\left(=\left(1,0^3\right)\right)$ and perform a parametrization on the chosen curve by a real parameter s, on the 3-hypersurface drawn normally to the direction e^{μ} we introduce the coordinates ${}_{(g)}x^k\big|_{k=1,2,3}$. An arbitrary point in 4-space can now be described in new coordinates $s = \mathfrak{X}^0$, ${}_{(g)}x^k = \mathfrak{X}^k\left(x^0,\cdots,x^3\right)$. The interval element in this case has the form $\mathrm{d}s^2 = \mathrm{d}\left(\mathfrak{X}^0\right)^2 - \mathfrak{G}_{ij}\left(\mathfrak{X}^{\mu}\right)\mathrm{d}\mathfrak{X}^i\mathrm{d}\mathfrak{X}^j$ ($\mu = \overline{0,3}$; $i, j = \overline{1,3}$). The metric coefficients can, as before, be written in the form
$g_{\mu\nu} = \mathfrak{G}_{\mu\nu} + e^{\mu}e^{\nu}$.

Thus, the covariant derivative

$$\nabla_{\mu}\left({}_{(e)}v_{\nu}\right) = \mathfrak{h}|\boldsymbol{v}|^{-1}\left(\mathfrak{G}_{\mu\nu} + e_{\mu}e_{\nu} - \left({}_{(e)}v_{\mu}\right)\left({}_{(e)}v_{\nu}\right)\right),\ \boldsymbol{v}_{(e)}v^{\nu} = v^{\nu}.$$

Using the coordinate system introduced earlier, solutions for the velocities can be sought in the form ${}_{(e)}v_{\nu} = {}_{(e)}v^{\nu} = e^{\nu} = \left(1,0^3\right)$, so that the equation with a covariant derivative is reduced to the form $\partial\mathfrak{G}_{\mu\nu}/\partial\mathfrak{X}^0 = \left(2\mathfrak{h}/|v|\right)\mathfrak{G}_{\mu\nu}$. From here we obtain the general form of the metric tensor
$\partial\mathfrak{G}_{\mu\nu} = {}_{(1)}\mathfrak{Q}\left(\mathfrak{X}^0\right)\cdot{}_{(2)}\mathfrak{Q}_{\mu\nu}\left(\mathfrak{X}^1,\mathfrak{X}^2,\mathfrak{X}^3\right)$, where ${}_{(1)}\mathfrak{Q}\left(\mathfrak{X}^0\right) = \exp\left(\mathfrak{h}\mathfrak{X}^0/|v|\right)$,
${}_{(2)}\mathfrak{Q}_{\mu\nu}(\ldots)$ are some admissible functions of their arguments).

Non-zero Christoffel symbols of the second kind for this metric:

$$\Gamma^i_{0j} = \delta^i_j\left({}_{(1)}\mathfrak{Q}\right)'_{\mathfrak{X}^0}\Big/{}_{(1)}\mathfrak{Q},\ \Gamma^0_{ij} = {}_{(2)}\mathfrak{Q}_{ij}\,{}_{(1)}\mathfrak{Q}\left({}_{(1)}\mathfrak{Q}\right)'_{\mathfrak{X}^0},\ \Gamma^k_{ij} = \Gamma^k_{ij}\left({}_{(2)}\mathfrak{Q}_{ij}\right).$$

The features of the motion of particles located in the region of a growing large structure can be determined from the Euler-Lagrange equations:

$$\frac{\mathrm{d}v^i}{\mathrm{d}\mathfrak{X}^0} + \Gamma^i_{kj}v^kv^j + 2\left(\left({}_{(1)}\mathfrak{Q}\right)'_{\mathfrak{X}^0}\Big/{}_{(1)}\mathfrak{Q}\right)v^iv^0 = \mathfrak{a}^i.$$

For small velocities, the 2nd term on the left-hand side can be neglected (compared to the 3rd), and we have an ordinary inhomogeneous differential equation, the solution of the evolution problem (analogically to crystals or plasmoids as in [5]) can be written in quadratures:

$$v^i\left(\mathfrak{X}^0\right) \sim v^i\left(\mathfrak{X}^0=0\right)\exp\left(-\int_0^{\mathfrak{X}^0}\hat{\omega}_1\left({}_{(1)}\mathfrak{Q}\right)\mathrm{d}\mathfrak{X}^0\right)+\int_0^{\mathfrak{X}^0}\hat{\omega}_2\left({}_{(1)}\mathfrak{Q}\left(\mathfrak{X}_*^0\right),\mathfrak{a}^i\left(\mathfrak{X}_*^0\right)\right)\mathrm{d}\mathfrak{X}_*^0,$$

$$\hat{\omega}_1\left({}_{(1)}\mathfrak{Q}\right)=2\left(\left({}_{(1)}\mathfrak{Q}\right)'_{\mathfrak{X}^0}\Big/{}_{(1)}\mathfrak{Q}\right)v^0,$$

$$\hat{\omega}_2\left({}_{(1)}\mathfrak{Q}\left(\mathfrak{X}_*^0\right)\right)=\exp\left(-\int_0^{\mathfrak{X}^0-\mathfrak{X}_*^0}\hat{\omega}_1\left({}_{(1)}\mathfrak{Q}\right)\right)\mathfrak{a}^i\left(\mathfrak{X}_*^0\right)\mathrm{d}\mathfrak{X}_*^0.$$

This gives rise to the effect of a non-force metric impact on particles, leading to their slowing down in the region of growth of the cosmological structure (at $\left({}_{(1)}\mathfrak{Q}\right)'_{\mathfrak{X}^0}\Big/{}_{(1)}\mathfrak{Q}>0$).

In addition to the simplest spherically symmetric conditions on the velocity, one should also consider filamentary structures, to which a significant number of large-scale cosmological objects observed by astronomers belong. In this case, one should consider the interval element

$$\mathrm{d}s^2=\left(\mathrm{d}\mathfrak{X}^0\right)^2-\left(\mathrm{d}\mathfrak{X}^3\right)^2-\left({}_{(I)}\mathfrak{Q}\left(\mathfrak{X}^0\right)\right)^2\sum_{i,j=1}^{2}{}_{(II)}\mathfrak{Q}_{ij}\left(\mathfrak{X}^1,\mathfrak{X}^2\right)\mathrm{d}\mathfrak{X}^i\mathrm{d}\mathfrak{X}^j.$$

In this case, a significant number of Christoffel coefficients are non-zero:

$$\Gamma^i_{j,k}\Big|_{i,j,k=1,2},\ \Gamma^2_{2,4},\ \Gamma^4_{j,k}\Big|_{j,k=1,2}.$$

Their singular points are determined by the conditions: 1) ${}_{(I)}\mathfrak{Q}=0$, 2) ${}_{(II)}\mathfrak{Q}_{11}\cdot{}_{(II)}\mathfrak{Q}_{22}-{}_{(II)}\mathfrak{Q}_{12}=0$. These same conditions determine the singularities for the elements of the Riemann curvature tensor $R_{12,12}$, $R_{14,14}$, $R_{14,24}$, $R_{24,24}$. Thus, at these points, conditions arise for the growth of the cosmological structure—in particular, the first condition describes the axis of symmetry of the mega-object, the second—the degeneration of the first quadratic form ($EG-F^2=0$—the simplest case is plane or "Zel'dovich pancake") of the two-dimensional surface (the stratification of surfaces). Moreover, at a singular point of the 2nd type, the scalar curvature locally changes sign, which indicates the reverse influence of the process of structure emergence on the conditions created there (the transformation of the singularity into the starting point of the process).

We can write out the Euler-Lagrange equations for the new metric to determine the motion of particles in the vicinity of the cosmological structure:

$$\frac{\mathrm{d}v^i}{\mathrm{d}\mathfrak{X}^0}+\Gamma^2_{20}v^2v^0+\Gamma^1_{10}v^1v^0=\mathfrak{a}^i,\ \Gamma^j_{j0}\Big|_{j=1,2}=\frac{1}{2}\left(\ln\left({}_{(I)}\mathfrak{Q}\left(\mathfrak{X}^0\right)\right)\right)_{\mathfrak{X}^0}.$$

Its solution will completely coincide with the previously obtained one in terms of structure.

4. Conclusion

At present, the prevailing concept of describing the early stage of the Universe's evolution is the inflationary theory. But its implementation requires the fulfillment of very unusual conditions. At the same time, A. A. Vlasov, within the framework of his theory, proposed a simple model of biological and crystalline structures, which can be formulated for cosmology and astrophysical objects, such as topological associations of voids and cosmic walls and the cosmic web. In this paper, we consider the mechanism of inertial motion in a system of many gravitationally bound particles under the condition that special requirements are met for the initial distribution of matter. It is shown that in this case the emerging effect of forceless motion can be used to explain the structure of cosmological objects.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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