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First Principles in Fundamental Physics

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Abstract

"All is Number"—the universe follows a few profound mathematical rules, and pure thought can grasp reality. This paper explores the first principles of fundamental physics, focusing on the principle of relativity, the principle of least action, and the principle of regularity. By illustrating the principle of relativity with an example of coordinate transformation, the paper clarifies the nature of spacetime: spacetime exists objectively, whereas coordinate systems are merely mathematical constructs. It also discusses the uniqueness of natural coordinate systems and their roles in both quantum and classical mechanics. Clifford geometric algebra is introduced as a mathematical framework for physical theories, and the principle of least action is analyzed, emphasizing the Lagrangian as an intrinsic characteristic of physical systems, which can be expressed as a linear combination of the system's energy terms. Through examples from complicated systems, the paper demonstrates the structural characteristics, applications, and limitations of this principle. Furthermore, it examines the fundamental distinction between the finite and the infinite, noting that infinity is merely an analytical variable rather than a number. If a physical equation yields a solution with infinite energy density, the theory must be revised to maintain consistency. These first principles not only constitute the foundation of physics but also unveil profound symmetries and universal patterns in mathematical structures.

Keywords

First Principle, Clifford Algebras, Hypercomplex Numbers, Nonlinear Spinors, Unified Field Theory

1. Introduction

In 350 BCE, Aristotle introduced the concept of first principles in *Physics*: unless we understand the fundamental conditions and principles of a thing and analyze its simplest elements, we cannot truly comprehend it. In *Metaphysics*, he further stated that every inquiry within a system involves fundamental propositions and assumptions that cannot be ignored, omitted, or overturned. In quantum chemistry, first principles specifically refer to the method of computing a system's energy eigenstates from scratch, starting from the Schrödinger equation for each particle. The concept of first principles is also widely applied in engineering and technological innovation. Tesla founder Elon Musk used this approach when designing batteries. In a 2012 interview with Kevin Rose, Musk remarked that people's thinking is often constrained by conventions and past experiences. Few people think from first principles. One must start reasoning from the most fundamental principles and then assess whether the conclusions are feasible and how they differ from traditional methods.

The essence of first principles lies in solving problems through logical reasoning, starting from the most fundamental principles. It emphasizes exploring the nature and origins of things, gradually dissecting them to identify their fundamental components, and then reconstructing the whole based on these elements. Fundamental physics, as the core discipline studying the structure of matter and the laws of motion, has undergone a continuous process of refinement, evolving from experimental facts to theoretical models. In a lecture, Chen-Ning Yang noted that research in fundamental physics generally progresses through four levels: from experimental facts to phenomenological theories, then to theoretical frameworks, and finally to mathematical structures. A discipline with a well-defined mathematical structure is considered a mature field.

So, does fundamental physics have first principles? This is precisely the question explored in this paper. As research deepens, we find that nature indeed operates according to a few simple rules, and pure thought can grasp reality. These universal principles form the core of the Theory of Everything: The Principle of Relativity, The Principle of Least Action and The Principle of Regularity. These three principles are self-evident. While they cannot be fully verified through experiments, rejecting them would be equivalent to denying the existence of objective laws. The following sections will elaborate on these first principles and their applications in specific physical problems.

2. The Principle of Relativity

The principle of relativity was first proposed by Galileo based on a thought experiment [1]: inside a closed cabin, one cannot distinguish whether the ship is stationary or moving at a constant speed. Poincaré regarded it as a universal law of nature and laid the theoretical foundation of special relativity before 1905 [2]. In the development of relativity, Einstein established the covariance of physical equations as a universal principle. However, since the proposal of special relativity, there have been ongoing debates regarding the explanations of time dilation, length contraction, and the relativity of simultaneity. These debates indicate that conceptual misunderstandings of the principle of relativity still exist. Through a simple example—the coordinate representation of a unit sphere—we can reveal the subtle relationship between spacetime and coordinates and further explore the physical significance of the principle of relativity.

2.1. Spacetime and Coordinates

Typically, we use spherical coordinates to describe a sphere, similar to the Earth's latitude and longitude, except for a different choice of the zero point. As shown in Figure 1, the coordinates of point p on the sphere are (θ,φ), with the line element or distance formula given by

$$\mathrm{d}s^2=\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2. \tag{2.1}$$

The coordinate values (θ,φ) range over the rectangular region $(\theta,\varphi)\in(0,\pi]\times[0,2\pi)$.

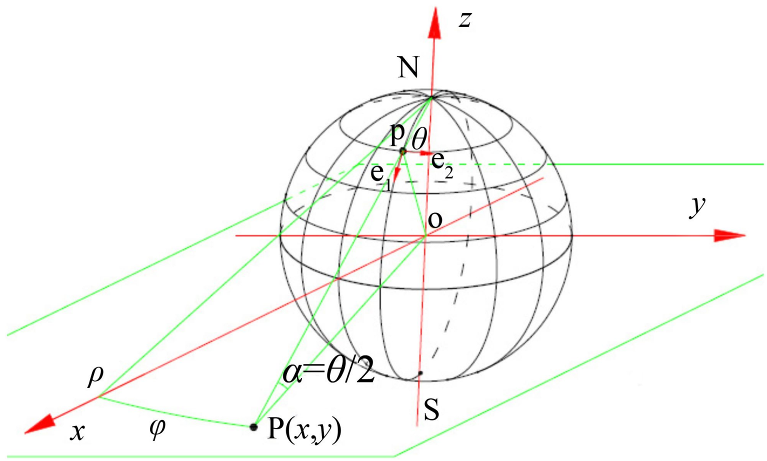

Figure 1. The geometric meaning of plane coordinates (x,y) and vectors $\boldsymbol{e}_1,\boldsymbol{e}_2$ on the unit sphere.

We can also represent point p using Cartesian coordinates. For instance, drawing a ray through the North Pole N and point p, which intersects the equatorial plane at point P, the Cartesian coordinates of P are (x,y). Clearly, this mapping transforms the South Pole S to the origin, maps different latitude circles on the unit sphere to different concentric circles in the equatorial plane, and sends the North Pole to infinity. Except for the North Pole N, there exists a one-to-one and bijective correspondence between points on the sphere and points on the plane. Therefore, both (θ,φ) and (x,y) are coordinate systems describing the unit sphere, and the transformation $(\theta,\varphi)\leftrightarrow(x,y)$ is a reversible and differentiable coordinate transformation satisfying

$$x=\rho\cos\varphi,\ \ y=\rho\sin\varphi,\ \ \rho\equiv\sqrt{x^2+y^2}=\cot\left(\frac{\theta}{2}\right). \tag{2.2}$$

Its inverse transformation is given by

$$\theta=2\arctan\left(\frac{1}{\rho}\right),\ \ \varphi=\arctan\left(\frac{y}{x}\right), \tag{2.3}$$

In the Cartesian coordinate system, the distance formula (2.1) transforms into

$$\mathrm{d}s^2 = \frac{4}{\left(1+x^2+y^2\right)^2}\left(\mathrm{d}x^2+\mathrm{d}y^2\right). \tag{2.4}$$

The relationship between a globe and a world map provides another example of using Cartesian coordinates to describe a sphere. The globe corresponds to a unit sphere, while the world map represents a coordinate system (u,v) describing the unit sphere. Since a sphere cannot be directly flattened onto a plane, maps often employ an equal-area transformation $(\theta,\varphi)\leftrightarrow(u,v)$, mapping any two regions of equal area on the sphere to two regions of equal area on the world map.

From the above description of the unit sphere's coordinates, we can summarize the following conclusions:

1) The unit sphere is an objective existence that needs precise description.

2) A coordinate system is a subjective labeling system that can vary from person to person.

3) A coordinate system must satisfy certain mathematical requirements, such as being in one-to-one correspondence with points on the sphere, maintaining continuity and smoothness, and ensuring that a small circle on the sphere is also a smooth closed curve in the coordinate system.

4) Different coordinate systems are related by reversible and differentiable transformations, meaning different coordinate values correspond to the same original point on the sphere:

$$\mathrm{p}\leftrightarrow(\theta,\varphi)\leftrightarrow(x,y)\leftrightarrow(u,v). \tag{2.5}$$

5) The invariant $\mathrm{d}s$ represents the infinitesimal arc length on the sphere, making $\mathrm{d}s$ an objective quantity rather than the segment length in a particular coordinate system. Similarly, the area element $\mathrm{d}A$ is also an objective quantity independent of the coordinate system.

6) Coordinate differentials $(\mathrm{d}x,\mathrm{d}y)$, $(\mathrm{d}\theta,\mathrm{d}\phi)$, and $(\mathrm{d}u,\mathrm{d}v)$ are not vectors; they are independent variables. Only the basis vectors $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ in the tangent plane at point p are true vectors.

Many ambiguities and paradoxes in special relativity often arise from misunderstandings about the relationship between objective spacetime and subjective coordinate systems [3]. In flat spacetime, transformations between Cartesian coordinate systems must be linear. For example, taking the ground as the reference frame, the spacing between railway sleepers is uniform Δx. Measuring these intervals from a moving train should also yield uniform intervals $\Delta x'$. Similarly, if fireworks are launched at one-minute intervals $\Delta t = 1$ on the ground, the observed launch intervals on the train $\Delta t'$ should also be equal. These "measurements" refer only to coordinate assignments and do not involve the time delay of light propagation. Light propagation is a physical process subject to the Doppler effect: moving toward the fireworks increases the frequency, while moving away decreases it. Standard textbooks discussing special relativity effects often fail to distinguish between coordinate assignment and physical measurement, leading to confusion and misinterpretation. For discussions on light propagation time calculations and interpretations, see [4]. In conclusion, coordinate systems are mere mathematical tools for describing objective spacetime, and transformations between different Cartesian coordinate systems must be linear.

Like (2.1), once the spacetime distance formula is established, many problems reduce to straightforward mathematical calculations. So, what should the spacetime invariant $\mathrm{d}s$ be? Newton's absolute spacetime view holds that spatial and temporal intervals remain the same whether on the ground or on the train. Relativity, however, treats spacetime as an integrated whole, where the invariant is given by $\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}r^2$. Which framework better corresponds to reality? This is an empirical question, not a theoretical one. Experimental results confirm that the speed of light in a vacuum is independent of the coordinate system, which is equivalent to verifying that the spacetime distance formula should be $\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}r^2$. Thus, time and space form an interrelated whole, with coordinate transformations following Lorentz transformations. It is crucial to emphasize, as in the unit sphere example, that spacetime is an objective reality, while coordinate systems are artificial mathematical labeling systems. Special relativity describes the geometry of spacetime, and coordinate systems merely provide mathematical descriptions, distinct from the "light propagation" process observed by an observer. "Light propagation" can be precisely described using coordinate systems.

From the above analysis, we see that the principle of relativity reflects the relationship between objective physical processes and subjective mathematical settings. Violating the principle of relativity would mean denying the objectivity of physical laws, making this principle an unfalsifiable first principle. However, the structure of spacetime itself must be determined and verified through experiments. For instance, string theory proposes that spacetime may not be a 1 + 3-dimensional Lorentzian manifold. This can be tested by examining Lorentz anomalies since extra dimensions would inevitably leave traces in Lorentz transformations. The objectivity of spacetime implies the existence of a special class of reference frames in which time is objectively synchronized, while simultaneity in other coordinate systems is merely a theoretical construct, not a physical reality [3] [5]. In this special reference frame, the proper time of a stationary particle is maximized, whereas a moving particle's proper time is shorter. For any particle, its proper time satisfies $\int_0^T \sqrt{1-v^2}\,\mathrm{d}t \le T$, regardless of its motion direction and acceleration. For simplicity, we set the speed of light $c=1$ if there is no ambiguity. In the real universe, this special reference frame corresponds to the natural coordinate system at rest with respect to the cosmic microwave background. Time in the natural coordinate system corresponds to the unified cosmic time, analogous to Newton's absolute time. In quantum mechanics, energy eigenstates can only be defined in the natural coordinate system, and Noether charges in classical mechanics have objective significance only when integrated in this system. Misconceptions about simultaneity have been addressed in detail in [4] [5]. This issue is subtle, understood only by those with rigorous logic and sharp insight.

2.2. Clifford Geometric Algebra

We continue to explore the mathematical representation of infinitesimal geometry on the unit sphere. The line element vector $\mathrm{d}\boldsymbol{s}$ and the directed area element $\mathrm{d}\boldsymbol{A}$ are given by

$$\mathrm{d}\boldsymbol{s} = \boldsymbol{e}_1 \mathrm{d}\theta + \boldsymbol{e}_2 \mathrm{d}\varphi = \sigma_1 \mathrm{d}\theta + \sigma_2 \sin\theta \mathrm{d}\varphi,$$
$$\mathrm{d}\boldsymbol{A} = \boldsymbol{e}_1 \mathrm{d}\theta \wedge \boldsymbol{e}_2 \mathrm{d}\varphi = \sigma_1 \wedge \sigma_2 \sin\theta \mathrm{d}\theta \mathrm{d}\varphi,$$

where $\boldsymbol{e}_1$ represents the covariant tangent vector along the θ grid line (longitude) at the point p on the sphere, and $\boldsymbol{e}_2$ represents the covariant tangent vector along the φ grid line (latitude) at the point p. The standard orthogonal basis σ_k has operator properties and can be represented using Pauli matrices, satisfying the Clifford geometric algebra. $\sigma_{12} = \sigma_1 \wedge \sigma_2$ represents the direction and unit of the area of the sphere, and the total area of the unit sphere is

$$\boldsymbol{A} = \oint \mathrm{d}\boldsymbol{A} = \sigma_{12} \oint \sin\theta \mathrm{d}\theta \mathrm{d}\varphi = 4\pi\sigma_{12},$$

where $|\sigma_{12}| = 1$ and 4π is the value of the surface area of the sphere.

Geometric algebra is a unified language for describing spacetime and physical theories [6]-[9], and below we briefly introduce the Clifford algebra representation of Riemann geometry. For an n-dimensional pseudo-Riemannian manifold with an indefinite metric $g_{\mu\nu}$, suppose its metric satisfies

$$\left(g_{\mu\nu}\right) \simeq \left(\eta_{ab}\right) = \mathrm{diag}\left(I_p, -I_q\right), \ \left(n = p + q\right). \tag{2.6}$$

At any given point $\boldsymbol{x}$, we have a set of covariant basis vectors $\{\gamma_\mu\}$ and their transformations

$$\gamma_\mu = f_\mu^a \gamma_a, \ \gamma^\mu = f_a^\mu \gamma^a. \tag{2.7}$$

In this paper, Greek letters (μ, ν) are used as indices for curvilinear coordinates, Latin letters (a, b) are used as indices for local orthogonal coordinates, and repeated indices imply Einstein summation. Boldface denotes important concepts or definitions. The set $\{\gamma_\mu\}$ is also called the **frame**, and $f_\mu^a, f_a^\mu \in \mathbb{R}$ are the **frame coefficients**. The vector γ_a is the standard orthogonal basis at the given point, and $\left(\gamma^a = \eta^{ab}\gamma_b, \gamma^\mu = g^{\mu\nu}\gamma_\nu\right)$ is the **co-frame**. The line element vector of the tangent space at the point is

$$\mathrm{d}\boldsymbol{x} \equiv \gamma_\mu \mathrm{d}x^\mu = \left(\gamma_a f_\mu^a\right)\left(f_b^\mu \delta X^b\right) = \gamma_a \left(f_\mu^a f_b^\mu\right) \delta X^b = \gamma_a \delta X^a, \tag{2.8}$$

where δX^a is the orthogonal coordinate differential in the tangent space at the given point, which can only be determined up to a Lorentz transformation. The Lorentz transformation here is not limited to the 1 + 3-dimensional spacetime.

The vector space with the defined Clifford product forms a **Clifford algebra** $C\ell\left(\mathbb{R}^{p+q}\right)$. The frame or basis vectors satisfy the following Clifford relations

$$\frac{1}{2}\left(\gamma_a\gamma_b + \gamma_b\gamma_a\right) = \gamma_a \cdot \gamma_b I = \eta_{ab} I, \ \frac{1}{2}\left(\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu\right) = \gamma_\mu \cdot \gamma_\nu I = g_{\mu\nu} I, \tag{2.9}$$

where $\gamma_a\gamma_b$ and $\gamma_\mu\gamma_\nu$ are the **Clifford product or geometric product** of vectors, and I is the identity element of the Clifford algebra. In cases where no confusion arises, we may replace I with 1. From (2.7) and (2.9), we obtain the relationship between $\left(f_a^\mu, f_\mu^a\right)$ and the metric as

$$f_\mu^a f_b^\mu = \delta_b^a,\ f_\mu^a f_a^\nu = \delta_\mu^\nu,\ f_a^\mu f_b^\nu \eta^{ab} = g^{\mu\nu},\ f_\mu^a f_\nu^b \eta_{ab} = g_{\mu\nu}. \tag{2.10}$$

Clifford algebra has a close relationship with Euclidean geometry, which is why it is also called geometric algebra. Its complete set of basis elements consists of 2^n elements,

$$I, \gamma_a, \gamma_a\gamma_b, \gamma_a\gamma_b\gamma_c, \cdots, \gamma_1\gamma_2\cdots\gamma_n,\ \left(a<b<c<\cdots\right). \tag{2.11}$$

Thus, γ_a is also called a **generator** of the Clifford algebra. According to the representation theorem of Clifford algebra, we know that the basis vectors $\{\gamma_a\}$ are isomorphic to a special set of γ-matrices constructed from Pauli matrices [10]. Therefore, when no confusion arises, we may omit the distinction between the basis vectors γ_a and their matrix representations.

From (2.8) we have

$$\mathrm{d}x^2 = \frac{1}{2}\left(\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu\right)\mathrm{d}x^\mu \mathrm{d}x^\nu = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \eta_{ab}\delta X^a \delta X^b, \tag{2.12}$$

$$\mathrm{d}V_k = \mathrm{d}x_1 \wedge \mathrm{d}x_2 \wedge \cdots \wedge \mathrm{d}x_k = \gamma_{\mu\nu\cdots\omega}\mathrm{d}x_1^\mu \mathrm{d}x_2^\nu \cdots \mathrm{d}x_k^\omega, \tag{2.13}$$

where $1 \le k \le n$, $\mathrm{d}x$ and $\mathrm{d}x_j$ are line element vectors, and $\mathrm{d}V_k$ is the directed volume element of a parallelepiped formed by k vectors $\{\mathrm{d}x_1, \mathrm{d}x_2, \cdots, \mathrm{d}x_k\}$, $\gamma_{\mu\nu\cdots\omega} = \gamma_\mu \wedge \gamma_\nu \wedge \cdots \wedge \gamma_\omega \in \Lambda^k\left(\mathbb{R}^{p,q}\right)$ is the unit of directed volume, and $\wedge$ is the **Grassmann's exterior product**, defined as

$$\gamma_{a_1} \wedge \gamma_{a_2} \cdots \wedge \gamma_{a_k} \equiv \frac{1}{k!}\sum_\sigma \sigma_{a_1 a_2 \cdots a_k}^{b_1 b_2 \cdots b_k} \gamma_{b_1}\gamma_{b_2}\cdots\gamma_{b_k},\ \left(1 \le k \le n\right),$$

where $a_j \ne a_l\ \left(j \ne l\right)$, $\sigma_{a_1 a_2 \cdots a_k}^{b_1 b_2 \cdots b_k}$ is the permutation tensor: if $b_1 b_2 \cdots b_k$ is an even permutation of $a_1 a_2 \cdots a_k$, it equals 1; for an odd permutation, it equals -1; otherwise, it equals 0. The above formula is a sum over all permutations; in other words, it is antisymmetric with respect to all indices.

To relate geometric concepts, we need to transform the Clifford product into the Grassmann product. Thus, the following form of **Clifford-Grassmann number** is used:

$$C = C_0 I + C_a \gamma^a + C_{ab}\gamma^{ab} + \cdots + C_{12\cdots n}\gamma^{12\cdots n} \tag{2.14}$$

which forms a 2^n-dimensional hypercomplex system over the field of real numbers [11] [12], where $C_0, C_a, \cdots, C_{12,\cdots n} \in \mathbb{R}$. The norm of the Clifford algebra is defined as the **González norm** $\|C\| = \sqrt[m]{\left|\det\left(C\right)\right|}$, where m is the order of the matrix representation of C. The González norm is invariant under similarity transformations, and transformations such as rotations, reflections, and translations in coordinate systems are invariant [13]. In fact, for any given unitary matrix, a similarity transformation transforms one set of standard orthogonal bases into another set of standard orthogonal bases. For any Clifford-Grassmann numbers A

and B, the norm satisfies the modulus product rule: $\|AB\|=\|A\|\cdot\|B\|$.

For the real 1 + 3 dimensional spacetime, the generators of the Clifford algebra $C\ell\left(\mathbb{R}^{1,3}\right)$ are represented by the Dirac γ matrices in the lowest-order complex matrix form:

$$\gamma^0=\gamma_0=\begin{pmatrix}0 & I_2\\ I_2 & 0\end{pmatrix},\ \gamma^a=-\gamma_a=\begin{pmatrix}0 & -\sigma_a\\ \sigma_a & 0\end{pmatrix}, \tag{2.15}$$

where σ_a are the Pauli matrices:

$$\vec{\sigma}=\left(\sigma^j\right)=\left\{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right\}. \tag{2.16}$$

The Dirac γ matrices (2.15) generate the following Grassmann basis for $C\ell\left(\mathbb{R}^{1,3}\right)$:

$$I_4,\gamma^a,\gamma^{ab}=\gamma^a\wedge\gamma^b,\gamma^{abc}=-\epsilon^{abcd}\gamma_d\gamma^{0123},\gamma^{0123}=-i\gamma^5, \tag{2.17}$$

where $\gamma^5=\operatorname{diag}\left(I_2,-I_2\right)$, and the Levi-Civita symbol ϵ^{abcd} is defined with the convention $\epsilon^{0123}=-\epsilon_{0123}=1$. We have the Clifford-Grassmann number as

$$A=sI_4+A_a\gamma^a+H_{ab}\gamma^{ab}+Q_a\gamma^a\gamma^{0123}+p\gamma^{0123}, \tag{2.18}$$

where $\left(s,p,A_a,\cdots\in\mathbb{R}\right)$. $sI_4\in\Lambda^0$ is a scalar, $A_a\gamma^a\in\Lambda^1$ is a true vector, $H_{ab}\gamma^{ab}\leftrightarrow\left(\vec{E},\vec{B}\right)\in\Lambda^2$ is a 2-vector (bivector), $Q_a\gamma^a\gamma^{0123}\in\Lambda^3$ is a 3-vector (trivector), and $p\gamma^{0123}\in\Lambda^4$ is a pseudocalar. In general, any Clifford algebra $C\ell\left(\mathbb{R}^{p,q}\right)$ is a **hypercomplex system**.

Clearly, the infinitesimal vector $\mathrm{d}\boldsymbol{x}=\gamma_\mu\mathrm{d}x^\mu$ is more fundamental than the distance formula $\mathrm{d}s^2=g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu$. Moreover, the spinor equation directly involves the matrix representation of the co-frame γ^μ. When calculating the energy-momentum tensor of spinor fields in curved spacetime, we need the definite relation between the variation of the frame $\delta\gamma_\alpha$ and the variation of the metric $\delta g_{\mu\nu}$ [14]. If the frame γ_μ is known, the metric tensor can immediately be obtained from (2.10) or $g_{\mu\nu}=\gamma_\mu\cdot\gamma_\nu$. However, determining the frame from a known metric is more complicated, because the mapping $g_{\mu\nu}\mapsto\gamma_\alpha$ is multivalued and involves an arbitrary Lorentz transformation in the orthogonal coordinate system of the tangent space $\delta X'=\Lambda\delta X$. When a Lorentz transformation is fixed, this mapping can establish a continuous, differentiable one-to-one correspondence $g_{\mu\nu}\leftrightarrow\gamma_\alpha$ in certain connected regions D. If we further introduce the differential operator for the basis vectors, i.e., the connection operator $\eth_\mu\gamma^\alpha=K^\alpha_{\mu\nu}\gamma^\nu$, many differential geometric concepts and operations are greatly simplified [15].

The framework provided above offers a rigorous mathematical formulation of the principle of relativity, ensuring the covariance of physical quantities and equations under coordinate transformations. Clifford geometric algebra has many practical applications in physics. For example, in [12], we derived the complete set of Maxwell equations. In [14], we used the third-order coupling between spinor field spin and rotational gravitational fields to explain the origin of cosmic magnetic fields. Therefore, "relativity is essentially the geometry of spacetime".

Due to the symmetry constraints of geometric algebra, the basic physical fields only include scalars, spinors, vectors, metric tensors, and their derived bivectors, trivectors, and so on. Combining with the principles of least action and regularity discussed below, we can derive the fundamental contents of basic physics. By further incorporating specific hypotheses and experimental conditions, these basic principles can derive nearly all known physical laws, providing a clear and unified theoretical framework for the world. This is the power of first principles [16].

3. The Principle of Least Action

The principle of least action states that the evolution of fundamental physical systems follows a variational principle, ensuring that the integral of a certain state function of the system attains a critical value. This state function is the system's Lagrangian or Lagrangian density $\mathcal{L}$. First, we need to clarify the concept of a "system". A **physical system** refers to an objectively existing entity with stable properties, such as an electron, a star, spacetime, or an electromagnetic field, similar to structured sets in mathematics. Obviously, we cannot study "half an electron" or an isolated "changing electric field" because they lack a relatively complete structure or dynamical significance.

For conservative systems, the Lagrangian density $\mathcal{L}$ is an inherent characteristic that comprehensively encapsulates the system's dynamics. $\mathcal{L}$ possesses the following key properties:

1) **Systematicity**: $\mathcal{L}$ must be defined over a definite physical system, where each term corresponds to an objective physical quantity.

2) **Real-valued**: $\mathcal{L}$ is a real-valued scalar, allowing numerical comparisons.

3) **Covariance**: $\mathcal{L}$ maintains its form under different coordinate systems, reflecting the system's objective properties.

4) **Consistency of Dimensions**: Each term in $\mathcal{L}$ has the dimension of energy or energy density, representing energy components of the system.

5) **Additivity**: $\mathcal{L}$ is a linear combination of the system's energy terms, and the total $\mathcal{L}$ of the system is the sum of the Lagrangians of its subsystems.

Similar to how an analytic function can be approximated by a power series expansion, these properties allow us to approximate the total $\mathcal{L}$ using various energy characteristic terms of the system, thereby fully describing the physical system. The Lagrangians of fundamental fields in nature, such as the spinor field $\mathcal{L}_\psi$, spacetime $\mathcal{L}_g$, and electromagnetic potential $\mathcal{L}_A$ in (5.2), have been extensively discussed [16], and will not be reiterated here. Next, we will examine examples of Lagrangians for complicated systems to illustrate the structural characteristics, application potential, and limitations of the principle of least action as a first principle.

3.1. The Dynamic Equation of Compound Pendulum

This is a lumped-parameter mechanical system, as shown in **Figure 2**. The system consists of two pendulum lengths a, c and a spring with rest length b, with a total of three degrees of freedom (z, θ, φ). It is most convenient to express planar motion using complex numbers. The coordinates of the three mass points (m, m_1, m_2) can be written as

$$\vec{r}_m = (b+z)\mathrm{e}^{\mathrm{i}\theta},\ \vec{r}_1 = a\mathrm{e}^{\mathrm{i}\theta},\ \vec{r}_2 = a\mathrm{e}^{\mathrm{i}\theta} + c\mathrm{e}^{\mathrm{i}\varphi}. \tag{3.1}$$

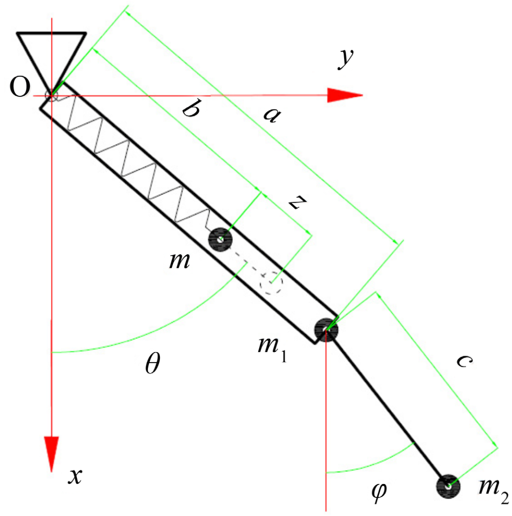

Figure 2. Schematic diagram of a compound pendulum with three degrees of freedom.

The velocities are

$$\vec{v}_m = \left(\dot{z} + \mathrm{i}(b+z)\dot{\theta}\right)\mathrm{e}^{\mathrm{i}\theta},\ \vec{v}_1 = \mathrm{i}a\dot{\theta}\mathrm{e}^{\mathrm{i}\theta},\ \vec{v}_2 = \mathrm{i}\left(a\dot{\theta}\mathrm{e}^{\mathrm{i}\theta} + c\dot{\varphi}\mathrm{e}^{\mathrm{i}\varphi}\right). \tag{3.2}$$

The squared velocity magnitudes are

$$v_m^2 = \dot{z}^2 + (b+z)^2\dot{\theta}^2,\ v_1^2 = \left(a\dot{\theta}\right)^2,\ v_2^2 = \left(a\dot{\theta}\right)^2 + (c\dot{\varphi})^2 + 2ac\cos(\theta-\varphi)\dot{\theta}\dot{\varphi}. \tag{3.3}$$

Substituting the above expressions into $T = \frac{1}{2}\left(mv_m^2 + m_1v_1^2 + m_2v_2^2\right)$ gives the total kinetic energy of the system,

$$T = \frac{1}{2}m\left(\dot{z}^2 + (b+z)^2\dot{\theta}^2\right) + \frac{1}{2}m_1\left(a\dot{\theta}\right)^2 + \frac{1}{2}m_2\left(\left(a\dot{\theta}\right)^2 + (c\dot{\varphi})^2 + 2ac\cos(\theta-\varphi)\dot{\theta}\dot{\varphi}\right). \tag{3.4}$$

Assuming that the spring has a stiffness coefficient k, and taking the system's zero gravitational potential energy at point O, the total potential energy of the system is given by

$$V = \frac{1}{2}kz^2 - mg(b+z)\cos\theta - m_1ga\cos\theta - m_2g(a\cos\theta + c\cos\varphi), \tag{3.5}$$

$$\to \frac{1}{2}\left(kz^2 + \left(m(b+z) + m_1a + m_2a\right)g\theta^2 + m_2gc\varphi^2\right) + C,\ (\theta, \varphi \to 0). \tag{3.6}$$

Substituting (3.4) and (3.5) into $\mathcal{L} = T - V$ gives the system's Lagrangian. By varying the action $I = \int \mathcal{L}\mathrm{d}t$ with respect to the variables (z, θ, φ), we obtain the system's equations of motion. Since $\mathcal{L}$ does not explicitly depend on time t, i.e., $\partial_t\mathcal{L} = 0$, Noether's theorem implies that the system's total energy

$$E=\sum_{\forall k}\frac{\partial\mathcal{L}}{\partial\dot{x}_k}\cdot\dot{x}_k-\mathcal{L}=T+V \tag{3.7}$$

is a conserved quantity.

3.2. Galactic Dynamics

Dark matter is a dissipationless superfluid that satisfies a specific equation of state. Its equations of motion differ significantly from those of ordinary matter, playing a decisive role in galaxy formation and cosmic evolution. For the spinor model of dark matter, refer to §5.1. Observations indicate that, in vast regions of galaxies (excluding the central bulge), the density of ordinary matter is much lower than that of dark matter. Let the kinetic energy density and gravitational potential energy of ordinary luminous matter be K_m and P_m, respectively. Then, we have:

$$K_m=\frac{1}{2}\rho\vec{V}^2=\frac{1}{2}\sum_k m_k\dot{\vec{X}}_k^2\delta^3\left(\vec{x}-\vec{X}_k\right), \tag{3.8}$$

$$P_m=\rho\Phi=\sum_k m_k\Phi_k\delta^3\left(\vec{x}-\vec{X}_k\right),\ \ \Phi_k=\Phi\left(t,\vec{X}_k\right), \tag{3.9}$$

where $\rho=\sum_k m_k\delta\left(\vec{x}-\vec{X}_k\right)$ is the density of ordinary luminous matter. Thus, the Lagrangian density of ordinary matter is given by:

$$\mathcal{L}_m=K_m-P_m=\sum_k m_k\left(\frac{1}{2}\dot{\vec{X}}_k^2-\Phi_k\right)\delta^3\left(\vec{x}-\vec{X}_k\right). \tag{3.10}$$

On large scales within a galaxy, the weak-field, low-velocity approximation of Einstein's field equations must include the retarded potential of the gravitational field. Therefore, we have:

$$\mathcal{L}_\Phi=\frac{1}{2\kappa}\partial_\mu\Phi\partial^\mu\Phi-\rho_d\Phi,\ \ \kappa=4\pi G, \tag{3.11}$$

where ρ_d is the mass density of dark matter. The total Lagrangian of the galaxy is the sum of the Lagrangians of its subsystems [17] [18]:

$$\begin{aligned}\mathcal{L}&=\mathcal{L}_\Phi+\mathcal{L}_m+\mathcal{L}_d\\&=\frac{1}{2\kappa}\partial_\mu\Phi\partial^\mu\Phi-\rho_d\Phi+\sum_k m_k\left(\frac{1}{2}\vec{V}_k^2-\Phi\right)\delta\left(\vec{x}-\vec{X}_k\right)+\mathcal{L}_d,\end{aligned} \tag{3.12}$$

where $\mathcal{L}_d\left(\rho_d,U_d^\mu,P_d\right)$ is the Lagrangian of the dark halo, and $\vec{V}\left(t,\vec{x}\right)=\dot{\vec{X}}$ represents the velocity of ordinary matter (such as stars and dust).

Since the exact form of $\mathcal{L}_d$ is still unclear, ρ_d can only be inferred through observational data. The primary component of dark matter consists of non-baryonic dark particles, which do not participate in electromagnetic or strong interactions. Although multiple theoretical candidates exist, the author believes that dark matter is most likely composed of massive nonlinear spinor fields [14] [17]. Nonlinear spinor fields are a type of superfluid without electromagnetic interactions, and their trajectories differ significantly from those of ordinary matter. In cases where the influence of stellar motion on the dark halo is negligible, stars can be regarded as moving within a steady-state gravitational potential field. If $\Phi_k = \Phi(\vec{r}_k)$ is time-independent, fundamental conclusions in Newtonian gravity, such as the conservation of mechanical energy and the virial theorem, remain valid. This significantly simplifies the analysis and computation of galaxy structures.

By varying the action derived from (3.12) with respect to each parameter, we obtain the equations of galactic dynamics:

$$\partial_\alpha \partial^\alpha \Phi = -\kappa(\rho_d + \rho), \tag{3.13}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{V} \equiv (\partial_t + \vec{V} \cdot \nabla)\vec{V} = -\nabla\Phi, \tag{3.14}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho \equiv (\partial_t + \vec{V} \cdot \nabla)\rho = -\rho \nabla \cdot \vec{V}, \tag{3.15}$$

When $\rho \ll \rho_d$, the term ρ in (3.13) can be neglected, similar to the treatment of planetary motion in the Solar System. The gravitational field Equation (3.13) takes the form of a wave equation, not because the speed of light is constant, but because the vast scale of a galaxy means that changes in the source term take a long time to affect the motion of distant celestial bodies. Since gravitational field variations are slow, Newton's law of universal gravitation remains sufficiently accurate over short distances. As a reasonable approximation, the galactic dynamics Equations (3.13) - (3.15) decouple the primary physical quantities from each other, simplifying analysis and computation. Depending on the specific context, Equation (3.14) can be interpreted either as the dynamics of an inviscid, pressureless fluid or as the equation of motion for particles. This simplified model of galactic dynamics successfully explains numerous observational results, such as the Faber-Jackson relation and the Tully-Fisher relation [18].

3.3. Elastic Mechanics

For infinitesimal vibrations $\vec{u}$ in a uniform elastic medium, the system satisfies the orthogonal invariance of three-dimensional space. In a Cartesian coordinate system, the kinetic energy density T and the strain potential energy density V are given as follows ([19], p. 281),

$$T = \frac{1}{2}\rho \vec{v}^2, \; V = \frac{1}{2}\mu \partial_j u_k \partial_j u_k + \frac{1}{2}(\mu + \lambda)(\partial_k u_k)^2, \tag{3.16}$$

where the medium density ρ is constant, the vibration velocity $\vec{v} = \partial_t \vec{u}$, and μ and λ are the elastic coefficients satisfying the strong ellipticity condition $\mu > 0, \lambda + 2\mu > 0$. Here, repeated indices indicate summation over the three spatial directions. Thus, the Lagrangian density of the system is obtained as

$$\mathcal{L} = T - V + \rho \vec{f} \cdot \vec{u} = \frac{1}{2}\rho \vec{v}^2 - \frac{1}{2}\mu \partial_j u_k \partial_j u_k - \frac{1}{2}(\mu + \lambda)(\nabla \cdot \vec{u})^2 + \rho \vec{f} \cdot \vec{u}, \tag{3.17}$$

where $\vec{f}$ is the external force density. By varying $\vec{u}$, the equations of motion are given by

$$\rho \partial_t^2 \vec{u} = \mu \Delta \vec{u} + (\mu + \lambda)\nabla(\nabla \cdot \vec{u}) + \rho \vec{f}. \tag{3.18}$$

If the vibration displacement $\vec{u}$ is decomposed into longitudinal waves $\vec{z}$

and transverse waves $\vec{w}$,

$$\vec{u}=\vec{z}+\vec{w},\ \nabla\times\vec{z}=0,\ \nabla\cdot\vec{w}=0, \tag{3.19}$$

then we have $\vec{z}=\nabla\phi$. Substituting this into the equations of motion, two standard wave equations are obtained:

$$\partial_t^2\vec{z}=(\lambda+2\mu)\Delta\vec{z},\ \partial_t^2\vec{w}=\mu\Delta\vec{w}. \tag{3.20}$$

It is evident that longitudinal and transverse waves propagate at different speeds.

The above examples illustrate that the principle of least action is well-suited for describing the motion laws of fundamental objects and conservative systems. It is not only a highly general fundamental principle but also a concrete computational method. In practice, as a solving method, boundary values can also be included in the total action and solved using finite element methods. However, this principle is not applicable to dissipative systems, such as Newtonian fluids or mechanical systems with frictional forces. Additionally, for certain mesoscopic systems that neglect many factors (such as the two-dimensional Rashba model in condensed matter physics), the principle of least action may fail to fully describe their dynamical behavior. Such systems either lack a suitable Lagrangian function or their Lagrangian loses its scalar nature due to the absence of certain symmetries and structural details, requiring additional parameters to restore their intrinsic symmetry. For example, the dynamics of turbulence can be improved by introducing specific turbulence energy terms into the dynamical equations.

Based on the above derivation, we can supplement several important conclusions regarding the principle of least action:

1) **Source and external force terms**: These can be introduced as known conditions, such as ρ_d in (3.11) and $\vec{f}$ in (3.17), as well as certain simple forms of friction. When the Lagrangian terms of these components are included in the system, they become interaction terms between different subsystems.

2) **Interaction terms**: These appear in the Lagrangian density $\mathcal{L}$ as scalar products of physical quantities from different subsystems, and each type of interaction should only be accounted for once. For instance, the gravitational potential $\rho\Phi$ has already been included in (3.9) as the potential energy term of ordinary matter, so it should not be counted again as the potential energy of the gravitational field.

3) **Interactions in unified field theory**: In the total Lagrangian (5.2) - (5.5), which includes all physical systems, interactions between matter components are realized through the momentum operator of the spinor field,

$$\hat{p}_{\mu,k}=\mathrm{i}\left(\hbar\partial_\mu+\Upsilon_\mu\right)+wQ_\mu-e_kA_\mu-s_k\Phi_\mu, \tag{3.21}$$

while the Lagrangian (5.4) of the interaction potential only contains derivative and nonlinear terms of the potential field itself [16] [20].

4) **The role of gravity**: Gravity affects all matter through the metric. If there are no cross-product terms between the parameters of two material systems, they remain independent and each satisfies the energy-momentum conservation law

[14].

5) **Symmetry breaking**: Due to approximations, the fluid dynamic Equations (3.14) in galactic dynamics satisfy Galilean invariance, whereas the gravitational field Equation (3.13) satisfies Lorentz invariance. Overall, the galactic dynamics Equations (3.12) only satisfy translation and rotational invariance relative to the galactic center coordinate system. Elastic mechanics is based on the assumption of a homogeneous medium, so its Lagrangian $\mathcal{L}$ also only satisfies translation and rotational invariance, but does not possess mobility invariance.

4. The Principle of Regularity: Finite and Infinite

Although significant progress has been made in big bang theory and black hole research, most physicists still believe that a truly correct physical theory should avoid the occurrence of singularities. If the equations of general relativity predict singularities, this implies that the theory breaks down under extreme conditions. For example, Einstein once pointed out [21]: One cannot assume that these equations remain valid for very high field densities and matter densities, nor can one conclude that the "beginning of expansion" necessarily implies a mathematical singularity. Weinberg also argued [22]: The universe never actually reached a state of infinite density. The current expansion of the universe may have started at the end of a previous contraction, when the density reached a very high but still finite value. A singularity implies that spacetime curvature tends to infinity and mass density $\rho \to \infty$. Even Hawking and Penrose attempted to avoid the actual existence of singularities. This is because the real world, as a Hamiltonian conservative system, cannot exhibit infinite energy density. The singularity theorems are not universal and therefore are not strictly valid [23].

Mathematically, there is a fundamental distinction between finite and infinite. Infinity and infinitesimals cannot be treated as ordinary numbers; they are variables or processes used in analysis. Otherwise, logical contradictions arise, such as $\infty + N = N\infty = \infty$. The concept of infinity must be understood through the language of Cauchy limits. The numerical value 0 also cannot be treated as an infinitesimal; for example, as $x \to 0$, $\sin x/x \to 1$, but $\sin x/0$ is meaningless. Similarly, finite sets and infinite sets exhibit different properties. For example, the set of positive integers $\{n\}$ can be put into one-to-one correspondence with the set of even numbers $\{2n\}$, i.e., $n \leftrightarrow 2n$. Gödel's incompleteness theorem further demonstrates that any axiomatic system containing elementary number theory inevitably has propositions that cannot be proven or disproven, indicating that the properties of infinite sets cannot be fully characterized by a finite number of axioms.

A popular viewpoint holds that physical theories break down near singularities, particularly at the Planck scale, where all theories require quantum corrections. However, infinitesimals are not specific numerical values but rather analytical variables. For example, the Earth is infinitesimal relative to the solar system, yet its actual scale remains enormous. Although $a \to 0$ is often used in studies of cosmic expansion, the minimum size of the universe remains far greater than the Planck length. If differential operations cannot be used at the Planck scale, then all dynamical equations, including Einstein's equations and the Dirac equation, would break down, as their formulation relies on the limit $\Delta x \to 0$. Thus, the notion of "a theory breakdown" is essentially a circular argument. To achieve a self-consistent physical theory, we should adhere to the principle of regularity: there are no intrinsic singularities in the universe, and all physical equations must avoid solutions with infinite energy density.

Quantum field theory attempts to handle infinities through renormalization techniques. For example, in string theory, the following renormalization relations hold:

$$1+2+3+\cdots=-\frac{1}{12}, \tag{4.1}$$

$$1+3+5+\cdots=\frac{1}{12}. \tag{4.2}$$

These formulas are used to compute spacetime dimensions. However, if both sides of (4.2) are multiplied by 2 and combined with (4.1), we obtain:

$$\begin{aligned}\frac{1}{6}&=1+(1+3)+(3+5)+(5+7)+\cdots \\ &=1+4(1+2+3+\cdots)=1+4\left(-\frac{1}{12}\right)=\frac{2}{3}.\end{aligned} \tag{4.3}$$

Simplifying (4.3) yields $1=0$, exposing the inherent contradiction of extracting finite quantities from infinities. This suggests that quantum field theory is fundamentally a mathematical computational technique rather than a strictly physical theory. Of course, generalized functions (such as $\delta(x)$) are reasonable within concentrated parameter models, but real physical systems remain finite. This is particularly important for nonlinear equations, where finite models must be considered.

In summary, the principle of regularity is a self-evident first principle in fundamental physics. Reasonable physical equations should automatically satisfy this principle. Infinity and infinitesimals are merely variables for analysis and should not be treated or manipulated as finite numbers. If a physical equation yields a solution with infinite energy density, this indicates a problem with the equation itself or its initial conditions, necessitating modifications. However, the physics of the realistic world may exhibit discontinuities, such as shock waves generated by supersonic flight. Thus, the principle of regularity requires that reasonable solutions to physical equations must be functions with bounded and measurable energy or energy density, ensuring the consistency and applicability of the theory.

5. Some Difficult Problems in Physics

By understanding the above three first principles, we can provide new solutions to some common difficult problems in physics. The following issues are widely regarded as major challenges in fundamental physics, but some of them may merely stem from subjective conjectures rather than being genuine scientific problems.

5.1. Dark Matter and Dark Energy

Dark matter is a type of matter that does not participate in electromagnetic interactions; it neither emits nor absorbs light. Although it cannot be directly observed, its existence can be inferred indirectly through astronomical observations such as galaxy rotation curves. Additionally, dark matter may also affect the expansion rate of the universe. Dark energy, on the other hand, is a form of energy that permeates the entire universe and causes its accelerated expansion. Observations from the Hubble Space Telescope and other projects have confirmed that the expansion of the universe is accelerating, supporting the existence of dark energy. Although dark matter and dark energy differ in nature, they both play important roles in cosmic evolution: dark matter's gravitational pull maintains the stability of galaxies and galaxy clusters, while the negative pressure of dark energy drives the accelerated expansion of the universe [24]-[26].

Currently, the primary dark matter models include particle dark matter models, which encompass light neutrinos, axions, and Weakly Interacting Massive Particles (WIMPs). The mass of WIMPs ranges from 10^{-6} eV to 10^{9} eV, with long lifetimes and extremely weak interactions with ordinary matter. There are also non-particle dark matter models, which propose that dark matter is not composed of fundamental particles but rather new forms of matter or energy, such as solitons in string theory, superfluid states, or superconducting states. Despite extensive global efforts in experimental research, dark matter has still not been directly detected.

The Perspective of Spinor Fields: The fundamental field quantities of physical systems are constrained by the symmetries of Clifford algebra $C\ell\left(\mathbb{R}^{1,3}\right)$, primarily including spinor fields, vector fields, and metric fields, as well as the field strengths derived from them, such as bivectors and trivectors [6] [7] [23]. From this perspective, dark matter and dark energy should be sought among these field quantities. We propose that dark matter may be nonlinear spinor fields with mass, while dark energy arises from the nonlinear effects of spinors. Consider the Lagrangian for a nonlinear spinor field:

$$\mathcal{L}_s = \sum_n \left(\psi_n^+ \left(\alpha^\mu i\partial_\mu - \mu\gamma\right)\psi_n + V_n\left(\check{\gamma}_n\right)\right),\ \check{\gamma}_n = \psi_n^+\gamma\psi_n \tag{5.1}$$

where ψ_n is the n-th dark spinor, $\mu > 0$ is a mass constant, and $V_n\left(\check{\gamma}_n\right)$ is a nonlinear potential. In a cosmological background, the normalization condition gives:

$$\left|\psi_n\right|^2 \sim \check{\gamma}_n \sim \delta^3\left(\vec{x} - \vec{X}_n\right) \propto a^{-3},$$

where $a(t)$ is the cosmic scale factor. For the simplest self-interaction potential $V_n = \frac{1}{2}\omega\check{\gamma}_n^2$, where $\omega > 0$ is a constant, we obtain

$$V_n \sim \sum_n \left(\delta^3\left(\vec{x}-\vec{X}_n\right)\right)^2 \propto a^{-6}.$$

According to the principle of least action, the Lagrangian of a composite physical system is an additive real scalar, consisting of the sum of the Lagrangians of its subsystems and interaction terms. Therefore, the total Lagrangian in cosmology must have the following form [27] [28]:

$$\mathcal{L}=\mathcal{L}_g+\mathcal{L}_m,\ \mathcal{L}_m=\mathcal{L}_\phi+\mathcal{L}_p+\mathcal{L}_s+\mathcal{L}_A+\mathcal{L}_\Phi+\cdots, \tag{5.2}$$

where $\mathcal{L}_g=\frac{1}{2\kappa}(R-2\Lambda)$ is the Lagrangian for spacetime (gravitational field), with $\kappa=8\pi c^{-4}G$, R as the scalar curvature, and $\mathcal{L}_m$ as the total Lagrangian for all matter. Here, ϕ is a slow-roll scalar field, $\mathcal{L}_p$ corresponds to classical dust or a perfect fluid, $\mathcal{L}_s$ represents the nonlinear spinor field, and in a spacetime where the metric is diagonalizable, we have:

$$\mathcal{L}_s=\sum_n\left(\Re\left\langle\psi_n^+\alpha^\mu\hat{p}_{n|\mu}\psi_n\right\rangle-m_n\psi_n^+\gamma^0\psi_n+V_n\right), \tag{5.3}$$

$$\mathcal{L}_A=-\frac{1}{2}\nabla_\mu A_\nu\nabla^\mu A^\nu,\ \mathcal{L}_\Phi=\frac{1}{2}\left(\nabla_\mu\Phi_\nu\nabla^\mu\Phi^\nu-b^2\Phi_\mu\Phi^\mu\right),\ \cdots \tag{5.4}$$

where A^μ represents the electromagnetic potential, and Φ^μ represents a short-range potential.

$$\hat{p}_k^\mu=i\left(\hbar\partial^\mu+\Upsilon^\mu\right)-e_kA^\mu-s_k\Phi^\mu,\ e_k=0,\pm e,\ s_k=0,s, \tag{5.5}$$

where α^μ is the flow vector, and Υ^μ represents the Keller connection associated with gravity. Since other field quantities have been discussed in detail in ([16], Ch9.2), this paper focuses on the nonlinear effects of spinor fields.

Varying the Lagrangian (5.2) with respect to the metric $g_{\mu\nu}$ gives Einstein's field equation:

$$G^{\mu\nu}+\Lambda g^{\mu\nu}+\kappa T^{\mu\nu}=0,\ G^{\mu\nu}=-\frac{\delta\left(R\sqrt{g}\right)}{\sqrt{g}\delta g_{\mu\nu}}=R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R, \tag{5.6}$$

where $\frac{\delta}{\delta g_{\mu\nu}}$ is the Euler derivative, and $T^{\mu\nu}$ is the energy-momentum tensor of the matter fields:

$$T^{\mu\nu}=-2\frac{\delta\left(\mathcal{L}_m\sqrt{g}\right)}{\sqrt{g}\delta g_{\mu\nu}}=-2\left(\frac{\partial\mathcal{L}_m}{\partial g_{\mu\nu}}-\left(\partial_\alpha+\Gamma_{\alpha\gamma}^\gamma\right)\frac{\partial\mathcal{L}_m}{\partial\left(\partial_\alpha g_{\mu\nu}\right)}\right)-g^{\mu\nu}\mathcal{L}_m. \tag{5.7}$$

Detailed calculations yield [14]:

$$T^{\mu\nu}=T_\phi^{\mu\nu}+T_p^{\mu\nu}+T_s^{\mu\nu}+T_A^{\mu\nu}+T_\Phi^{\mu\nu}+\cdots, \tag{5.8}$$

$$T_p^{\mu\nu}=\sum_n m_nu_n^\mu u_n^\nu\sqrt{1-v_n^2}\delta^3\left(\vec{x}-\vec{X}_n\right),\ u_n^\mu=\frac{\mathrm{d}}{\mathrm{d}s}X_n^\mu, \tag{5.9}$$

$$T_s^{\mu\nu}=\sum_n\left(\frac{1}{2}\Re\left\langle\psi_n^+\left(\alpha^\mu\hat{p}_n^\nu+\alpha^\nu\hat{p}_n^\mu\right)\psi_n\right\rangle+g^{\mu\nu}V_n\right). \tag{5.10}$$

For Equation (5.9), define

$$\bar{m} \equiv \frac{1}{N}\sum_{n=1}^{N} m_n, \quad \bar{\rho} \equiv \frac{N\bar{m}}{V} = \frac{\varrho}{a^3}, \quad J = \frac{kT}{\bar{m}c^2}, \tag{5.11}$$

where ϱ represents the conformal density of intrinsic mass, and J is the dimensionless temperature of the universe. Taking into account the driving effect of gravitational potential energy, the following theorem can be proved [27].

Conclusion 1. The equation of state for a perfect fluid compatible with relativity should be

$$P_p = \frac{\bar{\rho}kT}{\bar{m}c^2}\left(1 - \frac{kT}{2\left(\sigma\bar{m}c^2 + kT\right)}\right). \tag{5.12}$$

The state functions in an adiabatic process satisfy

$$\bar{\rho} = \rho_0\left[J\left(J+2\sigma\right)\right]^{\frac{3}{2}}, \quad \rho_p = \bar{\rho}\left(1 + \frac{3}{2}J\right), \quad P_p = \bar{\rho}\frac{J\left(J+2\sigma\right)}{2\left(J+\sigma\right)}, \tag{5.13}$$

where $J \to a^{-1}, (a \to 0)$ serves as a parameter, and $\rho_0 = \varrho(\sigma b)^{-3}$ is a constant with the dimension of density in cosmology.

It is evident that the pressure of a perfect fluid is always positive, and as $a \to +0$, $P_p \propto a^{-4}$. Vector fields such as electromagnetic fields include two different types of energy-momentum tensors. One type consists of moving photons, which can be incorporated into classical dust particles, whose energy-momentum tensor also takes the form of (5.9). The static electromagnetic field of charged particles belongs to another type and can be incorporated into the spinor energy-momentum tensor ([16], Ch9.2). From the classical approximation of (5.10), we obtain

$$\psi_n^+\alpha^\nu\psi_n \to u_n^\nu\sqrt{1-v_n^2}\,\delta^3\left(\vec{x}-\vec{X}_n\right), \quad \hat{p}_n^\mu\psi_n \to m_n u_n^\mu\psi_n, \quad V_n \to w_n\sqrt{1-v_n^2}\,\delta^3\left(\vec{x}-\vec{X}_n\right). \tag{5.14}$$

Substituting (5.14) into (5.10), the classical approximation of the total energy-momentum tensor for the spinor fields and interaction fields is obtained as

$$T_s^{\mu\nu} \to \sum_n\left(m_n u_n^\mu u_n^\nu + w_n g^{\mu\nu}\right)\sqrt{1-v_n^2}\,\delta^3\left(\vec{x}-\vec{X}_n\right), \tag{5.15}$$

where $w_n \propto a^{-3}$ corresponds to negative pressure. The difference between the above equation and (5.9) is the additional negative pressure term w_n, which plays the same role as the cosmological constant Λ in Einstein's field Equation (5.6). Similar to a perfect fluid, taking the statistical average of (5.15) gives

Conclusion 2. The energy-momentum tensor of a nonlinear spinor gas with interaction potential is

$$T_s^{\mu\nu} \to \left(\rho_s + P_s\right)U^\mu U^\nu + \left(W_s - P_s\right)g^{\mu\nu}. \tag{5.16}$$

The properties of ρ_s and P_s are similar to those of a perfect fluid. The state function W_s has a negative pressure effect and appears in the position of the cosmological "constant" Λ in Einstein's field equation, which may explain the physical origin of the cosmological constant.

For spinor fields with nonlinear potential, the equivalence principle approximately holds, so the state functions (ρ_s, P_s) are approximately equal to the perfect fluid's (ρ_p, P_p). From (5.15), the microscopic definition of W_s is [14] [27]

$$W_s \equiv \frac{1}{V}\int_V \sum_n w_n \delta^3\left(\vec{x}-\vec{X}_n\right)\sqrt{1-v_n^2}\,\mathrm{d}V = \frac{1}{V}\sum_{X_n\in V} w_n\sqrt{1-v_n^2}, \tag{5.17}$$

where the average parameters are

$$\overline{w} = \frac{1}{N}\sum_{n=1}^{N} w_n \to \frac{C_1}{a^3},\ \ \mu = \frac{1}{N}\sum_{n=1}^{N}\frac{w_n}{m_n} \to \frac{C_2}{a^3},\ \ (a \to +0). \tag{5.18}$$

Using the classical approximation of (5.13) and (5.17), we obtain

$$W_s = \overline{\rho}\left(\frac{\overline{w}}{\overline{m}} - \frac{3\mu\sigma J}{2(J+\sigma)}\right) = \frac{\varrho}{a^3}\left(\frac{\overline{w}}{\overline{m}} - \frac{3\mu\sigma\left(\sqrt{a^2+b^2}-a\right)}{2\sqrt{a^2+b^2}}\right) \to \frac{C}{a^6}. \tag{5.19}$$

Thus, we obtain

Conclusion 3. For nonlinear spinors, from (5.16), we derive the diagonal elements of energy-momentum tensor as

$$T^{\tau}_{\tau|s} = \rho_s + W_s,\ \ T^{r}_{r|s} = T^{\theta}_{\theta|s} = T^{\varphi}_{\varphi|s} = W_s - P_s. \tag{5.20}$$

Therefore, the equation of state in cosmology is given by

$$w_s(a) = \frac{T^{r}_{r|s} + T^{\theta}_{\theta|s} + T^{\varphi}_{\varphi|s}}{3T^{\tau}_{\tau|s}} = \frac{P_s - W_s}{\rho_s + W_s} \to -1,\ \ (a \to +0). \tag{5.21}$$

W_s corresponds to negative pressure, and when $a \to +0$, $W_s \propto a^{-6}$ dominates the total pressure P_m. The nonlinear dark spinor determines the initial state and large-scale structure of the universe. The effect of W_s resembles both the mass-energy density of dark matter and the negative pressure of dark energy.

According to the energy-momentum conservation law (5.16), $T^{\mu\nu}_{;\nu} = 0$, we obtain the equation of motion for the dark spinors

$$(\rho_s + P_s)U^{\nu}U^{\mu}_{;\nu} = \left(g^{\mu\nu} - U^{\mu}U^{\nu}\right)\partial_{\nu}\left(P_s - W_s\right). \tag{5.22}$$

Equation (5.22) differs significantly from the geodesic equation $U^{\nu}U^{\mu}_{;\nu} = 0$, which implies that dark halos in galaxies automatically separate from ordinary matter during galaxy formation [17].

In cosmology, although P_m is referred to as the total pressure, it is essentially a state function that incorporates all interaction potentials, making the existence of negative pressure reasonable. Except for the global scalar field ϕ, all other matter fields are small local structures, whose classical approximations of the energy-momentum tensor have the standard form (5.16), with the equation of state being algebraic equations. In summary, we have:

Conclusion 4. Both nonlinear spinor fields and nonlinear scalar fields can provide initial negative pressure. When $a \to +0$,

$$P_{\phi} \propto -a^{-n}\,(0 < n < 3),\ \ P_p \propto a^{-4},\ \ P_s \propto -a^{-6}, \tag{5.23}$$

where P_{ϕ} is the scalar field pressure, P_p is the ideal gas pressure, and P_s is the dark spinor pressure. The total pressure is dominated by P_s, indicating that the nonlinear spinor field can drive cosmic acceleration.

Feasibility of observational tests: The nonlinear dark spinor field resembles a superfluid, exhibiting no dissipative effects; otherwise, it would erase galactic structures. This means that dark spinors interact with ordinary matter only through gravity, making direct detection difficult. However, at cosmological scales, it may leave observable physical imprints, such as:

1) Variation in cosmic expansion rate: For a universe dominated by nonlinear spinor fields, the scale factor satisfies [27]

$$\dot{a}^2 = \frac{1}{a^2}(a-\alpha)(\omega-a)\left(a^2+2\delta a+\varepsilon^2\right),$$

where $0<\alpha,\delta,\varepsilon \ll \omega$. The solution to this equation can be described by elliptic functions, producing observable effects on the Hubble parameter, which may explain the Hubble tension problem.

2) Impact on galactic structures: In galaxies dominated by dark matter, the gravitational field satisfies (3.13)

$$\partial_\mu \partial^\mu \Phi = -\kappa\left(\rho_d+\rho\right) \approx -\kappa\rho_d .$$

Since the equation of motion for dark spinors deviates significantly from the geodesic equation, it is possible to infer the distribution details of the dark halo ρ_d based on galaxy structures, testing whether its motion follows the dark spinor model.

In conclusion, nonlinear dark spinors may be the true origin of dark matter and dark energy. Their nonlinear potential manifests as both positive energy and negative pressure in $T^{\mu\nu}$, naturally explaining cosmic acceleration in the cosmological context and possibly relating to the cosmological constant Λ. Although spinor equations are more complex than scalar fields, due to the normalization condition, the spinor state function depends only on the scale factor a, making calculations relatively manageable. This model was first published in arXiv:0806.4649, with a solid theoretical foundation and results consistent with observational data. In the future, further observations of cosmic expansion rates and galactic dark halos may verify the validity of the nonlinear dark spinor field model, advancing the study of dark matter and dark energy.

5.2. Misconceptions about Quantum Concepts

Quantum is a core concept in modern physics, first introduced by the German physicist Max Planck in 1900 to explain blackbody radiation. He hypothesized that radiation energy is discrete, meaning it can only take integer multiples of specific energy units, thereby successfully explaining experimental results. It is generally believed that a quantum is the smallest unit of energy change in microscopic particles. If any physical quantity has a minimum indivisible unit, it is called "quantization". In quantum mechanics, many physical quantities (such as energy, angular momentum, spin, charge, etc.) are quantized, meaning they can only take certain discrete values. This discontinuity is the fundamental difference between quantum mechanics and classical physics, mainly manifested in the microscopic world.

However, the concept of quantum is not as mysterious and complex as some descriptions suggest. The following analysis is based on actual physical phenomena:

1) Quantization is not universal: The notion that "microscopic particle energy changes discontinuously" is not always true. For example, the neutrino energy spectrum in nuclear reactions is continuous, the energy changes of charged particles in accelerators are also continuous, and the energy spectrum of synchrotron radiation is continuously distributed. Energy discretization usually occurs when charged particles form energy level structures in stable electromagnetic fields, where their energy states become discrete. The fundamental reason is that the spinor field of a charged particle remains metastable only in energy eigenstates, preventing energy exchange with the environment. Thus, externally observed particle behavior exhibits transitions between specific energy levels. The fact that matter has characteristic spectra is a brilliant design that allows humans to identify objects and appreciate a colorful world. For weak interactions, no energy level structures are formed, so the energy spectrum remains continuous.

2) Quantum does not mean the smallest unit: Quantization does not imply the existence of a minimum unit. For example, the hydrogen atom spectrum does not have a smallest unit but rather a maximum ionization energy level (13.6 eV). Only in a one-dimensional harmonic oscillator potential does a charged particle have a minimum energy unit $\hbar\omega$, and even this frequency ω is an adjustable parameter.

3) Photons are not necessarily indivisible: In Compton scattering, high-energy photons collide with charged particles and transform into lower-energy photons and the kinetic energy of the particles. The quantization of light is not an inherent property of the electromagnetic field itself but a result of the energy level structures of luminous matter and the fact that charged spinor fields can only remain stable in energy eigenstates-essentially a passive effect.

4) Charge quantization is not a typical example of quantum concepts: The quantization of angular momentum and spin is closely related to energy level structures, whereas charge is a coupling constant of fundamental interactions, similar to the coupling coefficients of strong and weak interactions. Thus, it is not a typical example of "quantization".

5) Discreteness is not unique to microscopic particles: For instance, the musical scales of a violin string are also quantized-its frequency can only take specific values; otherwise, harmonious music cannot be produced. Thus, discontinuity is not exclusive to microscopic particles and quantum mechanics.

The uncertainty principle was proposed by Heisenberg in 1927, stating that in microscopic systems, certain conjugate physical quantities cannot be precisely measured simultaneously. For example, the uncertainty relation between a particle's position and momentum is given by $\Delta x\Delta p \ge \frac{1}{2}\hbar$. It is commonly believed that this formula reveals a key fact: in the quantum world, all physical quantities are not absolutely deterministic. No matter how precise our measurement methods are, the particle state itself lacks absolute determinacy. The uncertainty principle is not only a fundamental principle of physics but has also profoundly influenced our worldview. In this worldview, physicists no longer pursue determinism but focus on probabilities.

However, the Schrödinger equation and Dirac equation are fundamentally deterministic equations, where coordinate variables merely serve as spacetime labels rather than actual coordinates of particles. Since fundamental particles are not mathematical point particles, discussing measurement errors without clearly defining particle coordinates may lead to logical errors. Moreover, when solving the hydrogen-like atomic spectrum, we assume the atomic nucleus is precisely located at the coordinate origin ($r=0$) and has an electrostatic field ($p=0$), based on which accurate spectra and wavefunctions are solved. If we introduce uncertainties in the nucleus's position r and momentum p, causing it to "oscillate" randomly, solving for the energy eigenstates of the atom would become impossible—contradicting known experimental and theoretical predictions. Thus, the common interpretation of the uncertainty principle is unreasonable. Theoretically, $\Delta x \Delta p \ge \frac{1}{2}\hbar$ merely represents the minimum dispersion of the wavefunction.

The orthodox interpretation of quantum theory (i.e., the Copenhagen interpretation) includes Born's probability density interpretation of the wavefunction, wave-particle duality, the uncertainty principle, and wavefunction collapse. However, the physical significance of the wavefunction ψ goes far beyond probability density. For example, the current four-vector $q^\mu = \psi^+ \alpha^\mu \psi$ can represent the precise four-dimensional current distribution of a unit charge, exhibiting additivity. Additionally, the Lagrangian of the system is also an additive scalar, indicating that quantum theory has a deep mathematical structure. Microscopic particles are not point particles, and before defining their coordinates and momenta, the uncertainty principle has no actual physical meaning. In fact, the wavefunction or spinor field itself completely describes all the properties of particles, and the spinor field equation already contains all the content of Newtonian mechanics and quantum theory [20].

The so-called contradiction between quantum mechanics and general relativity is a false problem, primarily arising from misunderstandings of quantum theory. Quantum mechanics is a theory of spinor fields, and its core content is solving the eigenstates of the wavefunction. Spacetime and matter interact, each possessing a Lagrangian, which together form the total Lagrangian of the system (5.2). Thus, the dynamic equations are well-defined, and the solving process is regular—where is the contradiction? Therefore, the so-called "quantum gravity" problem is largely an artificially constructed difficulty due to misunderstandings of fundamental theories.

In summary, the essence of various physical theories and their relationships can be outlined as follows [14] [20]:

1) Relativity: Describes the geometry of spacetime with strong mathematical properties, where spacetime interacts with matter fields through the metric or frames.

2) Spinor theory: Describes fundamental particles in a holographic nature, logically encompassing both quantum mechanics and classical mechanics, interacting with other fields through the form of (3.21).

3) Classical mechanics: A integral form of the Noether charge of spinor fields, determined by symmetry and conservation laws.

4) Quantum mechanics: A specialized field theory of spinor fields, primarily involving the computation of energy eigenstates.

5) Electrodynamics: A field theory of four-dimensional gauge potentials, where the electromagnetic field strength is a bivector in geometric algebra. The dynamics of strong interaction potentials should be similar to electrodynamics, except that as in (5.4), it includes a "mass term".

From this perspective, nonlinear spinor equations can fully describe fundamental particles, suggesting that the "mystery" of quantum theory mainly stems from misunderstandings of its mathematical structure. In a certain sense, nonlinear spinor fields and fundamental particles are different expressions of the same concept [20]. The four-dimensional gauge potential is the only interaction potential that does not affect the inertial mass of fermions, making it compatible with classical mechanics. The compatibility of bosons reflects the fact that interaction potentials are linearly superimposable. The structure of spacetime, spinor fields, and four-dimensional gauge potentials can be unified within Clifford algebra, exhibiting hypercomplex number structures. Microscopic particle motion has two forms: one is translational motion that follows classical mechanics, and the other is transition motion between energy levels, characterized by quantum mechanics. Thus, classical mechanics is not the limit of quantum mechanics as $\hbar \to 0$ but rather the dynamics of the Noether charge of spinor fields.

5.3. Other Difficult Issues

Below, we provide a brief analysis of several commonly debated "major issues". Due to space limitations, our discussion here is only suggestive, with the primary goal of drawing colleagues' attention to the possible biases in our paradigms of understanding the world. In the Standard Model of elementary particles, the Higgs field endows fundamental particles with mass, essentially acting as an interaction field stronger than both gravity and electromagnetism [29]-[31]. However, while gravity and electromagnetism shape the grand cosmic structures and the diverse material world, the influence of the Higgs field remains elusive and difficult to directly perceive—this presents an intriguing paradox. The Lagrangian of the Standard Model does not directly correspond to any realistic physical system but is instead constructed as a phenomenological model based on subjective $SU(2)$ and $SU(3)$ relations to fit experimental data. As a result, the conclusions of the Standard Model are not accountable to any real physical system and effectively bypass the need to explain this paradox.

The Standard Model appears more like a phenomenological framework based on experimental data rather than a fundamental theory derived from first principles. As analyzed in ([16], Ch6.3), under short-range strong interaction potentials, spinor fields can exhibit a rich particle spectrum closely resembling the observed hadron spectrum [28] [32]. This suggests that the Standard Model may merely be a data-fitting model, and one with less-than-ideal fitting accuracy [33].

The unification of gravity and electromagnetism is often regarded as a crucial topic in theoretical research. However, from a physical perspective, this is actually a pseudo-problem. If we treat the electromagnetic and gravitational fields as a composite system, the characteristic field of electromagnetism is the vector potential A_μ, while that of gravity is the metric tensor $g_{\mu\nu}$ or the vierbein γ_μ. These two subsystems obey their respective Lagrangians, and the total Lagrangian of the system is simply their sum, with interactions between them manifesting through the metric [14]. Taking the variation with respect to each set of characteristic field variables yields their respective dynamical equations, collectively forming a complete description of the system's structure and evolution. Thus, "unification" does not mean that these two forces must originate from the same theoretical framework, but rather that their fundamental differences and interaction mechanisms should be properly understood [16] [28].

String theory attempts to achieve unification by extending the number of spacetime dimensions, hypothesizing that the universe is not 1 + 3 dimensional but exists in higher dimensions. However, if extra dimensions affect the behavior of fundamental particles and introduce new interactions, then in theory, one should be able to observe Lorentz symmetry violation, as Lorentz symmetry violation accumulates over time. Yet, no significant anomalous effects have been detected experimentally to date, posing a major challenge to the testability of string theory. Furthermore, there is no inherent symmetry between spinor fields and vector fields, suggesting that bosons and fermions are unlikely to exhibit supersymmetry. Many current unification models are based on subjective assumptions, reflecting a tendency toward "designing the universe". Such an approach carries immense risk, with an almost negligible probability of success. A more appropriate research direction should be to match experimental data through the construction of fundamental fields on a solid foundation of first principles rather than forcefully constructing a theoretical framework ([16], Ch1).

Spinor modes satisfy normalization conditions, implying their conservation. Fermions described by spinor fields should therefore exist eternally. The annihilation of particle-antiparticle pairs is, in essence, a close binding under electromagnetic interaction rather than true disappearance. Short-lived particles are generally composite states or excited states rather than fundamental particles. The decay of particles is essentially the disintegration of composite particles, which is fundamentally no different from chemical reactions.

The free constants in physics may be incalculable, with their values seemingly "optimized" by nature to ensure a stable and controllable cosmic structure. For example, the fine-structure constant $\alpha \approx 1/137$ ensures the high precision of quantum mechanics' linear theory, allowing atomic energy levels to be expanded as power series with rapid convergence, while also limiting the number of possible elements to no more than 137. When an atomic nucleus's charge number exceeds 137, its wave function ceases to have well-defined energy eigenstates ([34], p. 226). These numerical constraints hint at a deeper physical mechanism, warranting further investigation.

6. Conclusions

This paper discusses three first principles of physics: the principle of relativity, the principle of least action, and the principle of regularity. By examining coordinate transformations, we elucidate the objectivity of spacetime and the subjectivity of coordinate systems as mathematical constructs. The article introduces Clifford geometric algebra as a mathematical framework for the principle of relativity, further revealing the mathematical structure of physical theories. The principle of least action is demonstrated to be an intrinsic characteristic of physical systems, fully encompassing their dynamical properties, and is validated in specific problems such as the compound pendulum equation, galactic dynamics, and elasticity theory. Additionally, this paper analyzes the fundamental distinction between the finite and the infinite, pointing out that infinity is merely a variable for analysis rather than an actual numerical value. When physical equations yield solutions involving infinite energy densities, the theory must be modified to ensure self-consistency.

These first principles not only form the foundation of physics but also reveal the profound symmetry and universality of its mathematical structures. They continue to provide valuable guidance in frontier topics such as dark matter and dark energy, as well as the interpretation of quantum theory. Scientific progress stems from the continuous exploration and deepening of first principles. Ancient China once led the world in technology and engineering, but due to a lack of pursuit in fundamental principles and logical systems, it failed to develop modern science. Today, a similar pattern persists in the domestic academic community, where empirical and pragmatic traditions still dominate. New theoretical frameworks and ideas are often met with excessive caution, if not outright conservatism. This path dependency has already cost us numerous historical opportunities, and without a shift in mindset, breakthroughs in fundamental science will remain elusive. Only through bold inquiry and relentless exploration of new first principles can we drive science toward deeper levels and uncover the fundamental laws of the unknown world.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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