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Optimization of Tubular Gas Heaters on Pellets with Recirculation of Gas-Air Mixture Using a Quasi-Two-Dimensional Model

Kostiantyn Dudkin[1], Vladislav Danishevskyy[2], Vyacheslav Irodov[3], Halyna Prokofieva[4], Leontina Solod[4], Valeria Tkachova[4]

[1]Collective Research and Production Enterprise "Energocomplex", Dnipro, Ukraine
[2]Department of Structural and Theoretical Mechanics and Strength of Materials, Prydniprovska State Academy of Civil Engineering and Architecture, Dnipro, Ukraine
[3]Department of Information Technologies of the Dnipro Technological University "Step", Dnipro, Ukraine
[4]Department of Heating, Ventilation, Air Conditioning, Heat and Gas Supply, Prydniprovska State Academy of Civil Engineering and Architecture, Dnipro, Ukraine
Email: hyaprok@gmail.com

Abstract

The article is devoted to decision-making in the control and design of autonomous heat supply systems with tubular gas heaters. The results of mathematical modelling and optimization of tubular gas heaters (TGN) are known. Tubular gas heaters are an extension of the term "infrared tubular gas heaters". The main elements are: a gas burner, a tubular heater inside which gas combustion products with air move, and a mechanical fan (supply or exhaust), which ensures the movement of the coolant inside the tubular part and its removal outside. There are a number of new technical solutions that expand the scope of tubular gas heaters, for example, tubular gas heaters on pellets. Mathematical models of tubular heaters on pellets and solutions to the problems of optimal design of tubular heaters of linear structure are known. Another possible structure of tubular gas heaters is heaters with recirculation of the heating gas-air medium. Optimisation of such pellet heaters has not been performed before, which determined the subject of this paper. The article is devoted to the presentation of the method of optimization of the design solution for tubular heaters taking into account recirculation under the existing constraints. The novelty of the optimization lies in the use of a quasi-two-dimensional mathematical model for the hydraulic circuit of the heater. An evolutionary search algorithm with binary choice functions is used for numerical search of solutions, for which convergence with probability 1 to the optimal solution is shown. The algorithm contains two consecutive functions: the function of so-

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lution generation and the function of solution selection. The function of solution generation is built largely independently of the content of the problem to be solved, while the function of selection is built in such a way that the resulting binary selection relation is completely determined by the requirement of finding the necessary solution. The resulting binary selection relation includes both the selection components of the available constraints and the basic optimiztion requirement.

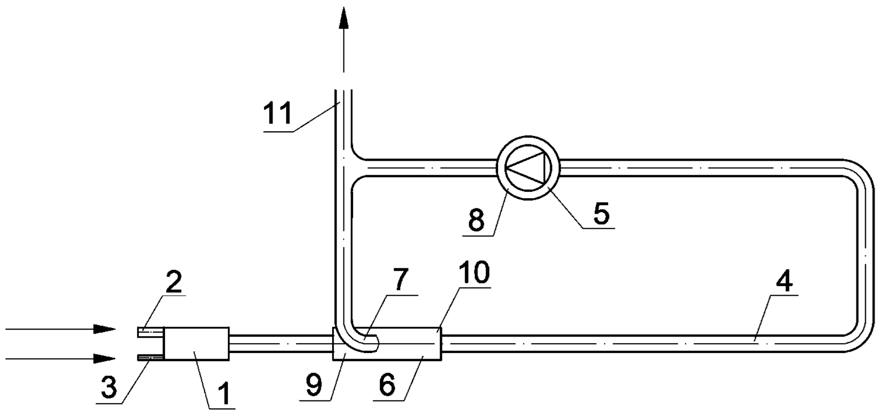
Keywords

Mathematical Model, Evolutionary Search, Binary Choice Relations, Tubular Gas Heaters on Pellets, Recirculation of Gas-Air Mixture

1. Introduction

Infrared tube gas heaters (ITGOs) are used for autonomous heating and heating, which consist of a gas burner, a tube heater—a pipe inside which the combustion products of gas and air move, reflectors of the radiant flow and a fan (supply or exhaust), thanks to which the coolant moves inside the pipe and is removed outside after its cooling. There is a lot of experience in the application of ITGOs [1]-[3]. The use of tubular gas heaters has been developed, for example [4] the use of tubular gas heaters as an industrial product, including partial recirculation of the gas-air mixture [5]. The term "tubular gas heaters" (TGH) is an extension of the concept of ITGOs, which consist formally of similar elements, but are used not only for infrared heating of surfaces, but also for heating air, water, obtaining water vapour [6]. As a rule, TGHs are used as a result of individual design of the heating and gas supply system, which requires appropriate methodological support and calculations for decision-making. Relatively recently in tubular gas heaters began to use instead of combustible gas—wood pellets [7], as a result of combustion processes of which a gas-air mixture is formed, which is a coolant of TGH. Mathematical models of tubular gas heaters with pellets and solution of the problem of optimal design of tubular gas heaters are known, for example [8]. Various technical solutions are known, including those for TGH with recirculation of the coolant. For such a technical solution, the calculation of parameters was performed on the basis of a one-dimensional mathematical model of the hydraulic circuit. Earlier, a quasi-dimensional mathematical model of a tubular gas heater was also proposed, which allows describing the change of external temperature on the heater surface, which is desirable for solving a number of problems. However, the solution of the optimisation problem in the design of tubular gas heaters with recirculation of the coolant and taking into account the quasi-dimensional model has not been previously carried out, which determined the purpose of this work. The decision-making problem is formulated using binary choice relations. For numerical solution of the problem, the original algorithm of evolutionary search with several branches of the process running in parallel was used. Due to such construction of the algorithm it is possible to control the search parameters, which provides convergence of the evolutionary search to the optimal solution with probability 1.

Today, one of the urgent problems is the decision-making to pellet tube heaters with recirculation of gas-air mixture (Figure 1).



1) pellet burner; 2), 3) air and pellet supply nozzles; 4) the main section of the tubular heater; 5) fan; 6) ejector; 7) nozzle of the ejector for the active medium; 8) fan outlet; 9) nozzle of the ejector for the passive medium; 10) outlet nozzle of the ejector; 11) a pipe for the exit of flue gases.

Figure 1. Tubular gas pellet heater with heat carrier recirculation.

The purpose of the study

The purpose of the research: to obtain algorithmic support for making optimal decisions in the processes of construction and design of autonomous heat supply systems for pellet tubular gas heaters with heat carrier recirculation.

The goal of the work: to develop an algorithm of evolutionary search with a binary relation of choosing the most attractive (optimal) solutions for tubular gas heaters with heat carrier recirculation.

2. Mathemanical Model of Pellet Tubular Gas Heaters

One-dimensional steady-state motion of the gas-air mixture inside the heater tube part is considered, l is a linear coordinate.

The equations of motion along the length of the tube heater are as follows

$$\mathrm{d}p = -\Lambda \cdot \mathrm{d}x/D \cdot \rho \frac{w^2}{2} + \mathrm{d}h \cdot \left(\rho_a - \rho\right) \cdot g \tag{1}$$

$$\mathrm{d}\rho = \left(\mathrm{d}\rho - \rho R \mathrm{d}T\right)/RT \tag{2}$$

$$\mathrm{d}w = \left(-\rho w \mathrm{d}F - wF\mathrm{d}\rho\right)/\rho F \tag{3}$$

where: ρ —density of the gas-air mixture; w —average linear velocity of the gas-air mixture moving along the radiating tube; F —cross-sectional area of the tube; P, T —absolute pressure and temperature of the gas-air mixture in the given cross-section of the radiating tube; R —gas constant depending on the composition of the gas-air mixture after complete combustion of combustible gas; $\mathrm{d}p$ —pressure drop of the gas-air mixture on the section of length $\mathrm{d}l$; Λ —friction coefficient.

$D(l), h(l), T(p,\rho), F(l)$ —known functions. There is an integral condition for the entire heater, which must be satisfied based on physical laws of the form. There is an integral condition for the entire heater, which must be satisfied based on physical laws of the form

$$\int_{l_i} \mathrm{d}p(l_i) = \Delta p = const \tag{4}$$

To make decisions, two criteria were used— $E_1(x)$ the criterion of the efficiency of the tubular heater, $E_{21}(x)$ —the criterion of fulfilling physical laws (4), $E_{22}(x)$ —the criterion of fulfilling the technical conditions for the operation of the heater along the entire length.

Heat Transfer Equations

$$\mathrm{d}Q_{1con} + \mathrm{d}Q_{1rad} = \mathrm{d}Q_2 = \mathrm{d}Q_{3con} + \mathrm{d}Q_{3rad} \tag{5}$$

where $\mathrm{d}Q_{1con}, \mathrm{d}Q_{1rad}$ —Convective and radiant heat flows on the inner surface of the pipe, $\mathrm{d}Q_2$ —heat flux by heat conduction through the pipe wall, $\mathrm{d}Q_{3con}, \mathrm{d}Q_{3rad}$ —Convective and radiant heat fluxes on the outer surface of the pipe, where

$$\mathrm{d}Q_2 = \pi D \mathrm{d}l \lambda / \delta \left(T_{wi} - T_{wo}\right) = \pi D \mathrm{d}l c_0 \varepsilon_w \left(T_{wo}^4 - T_o^4\right) 10^{-8} + \pi D \mathrm{d}l \alpha_2 \left(T_{wo} - T_o\right) \tag{6}$$

$$\mathrm{d}Q_2 = \pi D \mathrm{d}l \alpha_1 \left(T - T_{wi}\right) + \pi D \mathrm{d}l c_0 \varepsilon \left(T^4 - T_{wi}^4\right) 10^{-8} = \pi D \mathrm{d}l \lambda / \delta \left(T_{wi} - T_{wo}\right) \tag{7}$$

To obtain a quasi-dimensional mathematical model of the thermal and hydraulic modes of the TGN, it is proposed to consider the temperature dependence of the heater surface as a function of linear and angular coordinates:

$$\begin{gathered} T_{wi} = T_{wi}(l,\phi) \\ T_{wo} = T_{wo}(l,\phi) \\ T_o = const, \end{gathered} \tag{8}$$

where ϕ —angular coordinate of the heater surface; T_o —absolute ambient temperature.

$$\mathrm{d}Q_{3con} = \frac{D}{2} \int_0^{2\pi} \alpha_2 \left[T_{wo}(l,\phi) - T_o\right] \mathrm{d}\phi \mathrm{d}l \tag{9}$$

$$\mathrm{d}Q_{3rad} = \frac{D}{2} c_0 \int_0^{2\pi} \varepsilon_2 \left[T_{wo}^4(l,\phi) - T_o^4\right] \mathrm{d}\phi \mathrm{d}l 10^{-8} \tag{10}$$

A set of parameters affecting the temperature distribution around the heater perimeter due to convective motion can be identified and represented as follows

$\theta_{wi} = \psi\left(\theta_{wo}, Pr, Re, Gr\right)$, where θ_{wo} —average surface temperature of the heater, Pr, Re, Gr —Prandtl, Reynolds and Grasshoff numbers. On the basis of the experimental results, the regression dependence was obtained in the form of

$$\theta_{wi} = T_{wi}(\phi)/T_{wo} = k_1 + k_2 \left(Pr \cdot Gr\right)^{k_3} \cdot Re^{k_4} \cos(\phi) \tag{11}$$

where $T_{wi}(\phi)$ is the temperature on the surface of the heater cross-section with angular coordinate ϕ, T_{wo} is the average temperature across the heater cross-section, and k_1, k_2, k_3, k_4 are the regression constants. The regression dependence (11) is obtained as a result of minimizing the deviation of the calculated values of the surface temperature of the tubular heater from the experimental values presented in [9]. The error of this dependence on the experimental values is 0.07, i.e., the dependence corresponds to the process of complex heat transfer for a tubular heater and can be used for its mathematical modelling.

3. Methodology

Presentation of the Main Research Material

Following [10], optimisation is not just a tool in the arsenal of machine learning and artificial intelligence, it is the foundation upon which the efficiency and effectiveness of these fields results. Optimisation algorithms, discussed in the study [11], are key tools in machine learning and artificial intelligence. Each algorithm uses a different strategy to navigate the complex and treacherous terrain of the Rosenbrock function, a function known for its complex optimisation landscape characterised by narrow valleys and steep ridges. Gradient descent (GD) is one of the most fundamental and widely used optimisation algorithms, but the algorithm may overshoot the minimum, leading to divergence. Conversely, the algorithm can converge very slowly, requiring an impractical number of iterations to reach the minimum [12]. Stochastic Gradient Descent (SGD) is based on the principles of standard gradient descent, but introduces an element of randomness by updating the model parameters not on the entire dataset, but on a randomly selected subset of the data, called a minipartition. By using only a subset of the data, SGD significantly reduces the computational cost per iteration, which makes it particularly suitable for large-scale problems [13].

To solve the problem of calculating the thermal and hydraulic modes of operation of a tubular heater with heat carrier recirculation, the evolutionary search algorithm for the most attractive solutions was used. The basis of the algorithmic approach is the construction of algorithms for evolutionary search of solutions by binary choice relations. This approach is most fully described in [14].

It is consider a set Ω of elements (decisions) $x = \{x^1, x^2, \cdots, x^n\}$, where x^i—a scalar parameter (continuous or discrete). It is determined the binary choice relation R_S for elements of the set Ω.

It is meant that there is a rule (algorithm) according to one decision is "better" than another. It is determined that choice relation R_S is a no strictly order relation.

For subset X, $X \subset \Omega$ we denote the function of choice in the form

$$S(X) = \left\{x \in X \middle| \forall y \in [X \setminus S(X)], xR_S y\right\} \tag{12}$$

We shall assume that set $S(X)$ contains the concrete number of elements—N_{op}.

We shall that for the set Ω it was determined relation R_G with attachment function $\mu_{R_G}(x, y)$: $\Omega \times \Omega \to [0,1]$. Relation R_G will be termed generation relation.

For subset X, $X \subset \Omega$ we denote the function of generation in the form

$$G(X) = X \cup G_H(X),$$

$$G_H(X) = \left\{ y \in \Omega \middle| \exists x \in X, yR_G x, \mu_{R_G}(x, y) > 0 \right\} \tag{13}$$

We shall assume that set $G(X)$ contains the concrete number of elements—N_E.

The algorithm to search R_S—optimal solution can be represented as

$$X_k = S\left(G\left(X_{k-1}\right)\right), \quad k = 1, 2, \cdots \tag{14}$$

where X_k—the set of preferred solutions according to the binary choice relation R_S at the iterate step k, X_{k-1}—this is the same at the iterate step $k-1$. $G(X)$—the function of generation with relation of generation R_G. $S(X)$—the function of choice with the binary choice relation R_S.

The iterate algorithm (14)—is the general form of evolutionary search.

We will consider the decomposition

$$X_k = \bigcup_{j=1}^{N_b} X_{jk}, \quad X_{ik} \cap X_{jk} = \varnothing, \quad i \neq j \tag{15}$$

where X_{jk}—the set of preferred solutions according to the binary choice relation R_S at the iterate step k for the branch j of evolutionary search, N_b—the number of branches.

The algorithm (14) takes the form

$$X_{jk} = S\left(G\left(X_{jk-1}\right)\right), \quad j = \overline{1, N_G}, \quad k = 1, 2, \cdots \tag{16}$$

These iterate algorithms (14), (16)—are the general form of evolutionary search.

For the task of finding the most attractive solution for a tubular gas heater with recirculation, the binary selection relation can be represented as follows.

The dimensionless efficiency function in the form

$$F_1(x) = \eta \tag{17}$$

The general penalty function in the form of

$$F_2(x) = F_{21}(x) + F_{22}(x) \tag{18}$$

where

$F_{21}(x)$—dimensionless pressure loss throughout the closed heater circuit

$$F_{21}(x) = abs\left[\oint_{heater} \mathrm{d}p_i(x) \big/ abs\left(p_{\max} - p_{inlet}\right)\right] \tag{19}$$

$F_{22}(x)$—dimensionless exceeding the maximum permissible heat flux through the outer surface of the heater.

$$F_{22}(x) = abs\left(\Delta Q_{3con} + \Delta Q_{3rad} - \Delta Q_{lim}\right) / \Delta Q_{lim} \tag{20}$$

ΔQ_{lim}—maximum permissible heat flux through the outer surface of the heater.

$$\mathrm{d}Q_{3con} = \frac{D}{2}\int_{0}^{2\pi} \alpha_2 \left[T_{wo}(l,\phi) - T_o\right] \mathrm{d}\phi \mathrm{d}l \tag{21}$$

$$\Delta Q_{3rad} = \frac{D}{2} c_0 \int_{0}^{2\pi} \varepsilon_2 \left[T_{wo}^4(l,\phi) - T_o^4\right] \mathrm{d}\phi \Delta l 10^{-8} \tag{22}$$

The general relation R_{SS} for choosing between two possible solutions x and y is as follows:

$$\begin{aligned} xR_{SS}y = {} & \left[F_2(x) \le 0 \wedge F_2(y) > 0\right] \\ & \vee \left[F_2(x) > 0 \wedge F_2(y) > 0 \wedge F_2(x) \le F_2(y)\right] \\ & \vee \left[F_2(x) \le 0 \wedge F_2(y) \le 0 \wedge F_1(x) \ge F_1(y)\right] \end{aligned} \tag{23}$$

4. Numerical Results

The algorithm for evolutionary search for the most attractive solutions to the problem of optimising the parameters of a tubular gas heater with recirculation is presented in form (16).

The sought solution has the form $x = \{x^1, x^2, \cdots, x^6\}$ where x^1—power of pellet burner, x^2—total flow rate of fresh air supplied to the heater, x^3—diameter of the tubular part of the heater, x^4—diameter of the screen of the heater, x^5—total length of the tubular heater, including the recirculation part, x^6—length of the heater of the initial part with the screen.

The initial search parameters adopted are as follows

$$p_{\max} = 200\ \text{Pa}, \quad T_{wo} = 450^{\circ}\text{C}$$

$$\sigma(1) = 0.1, \sigma(2) = 10, \sigma(3) = 0.01, \sigma(4) = 0.01, \sigma(5) = 1, \sigma(6) = 1$$

$$N_E = 2, N_1 = 1, N_B = 3, N_P = 5$$

The permissible limits of variation of the parameters are as follows

$$x_{\min}^1 = 20, x_{\min}^2 = 120, x_{\min}^3 = 0.1, x_{\min}^4 = 0.25, x_{\min}^5 = 60, x_{\min}^6 = 3$$

$$x_{\max}^1 = 70, x_{\max}^2 = 200, x_{\max}^3 = 0.18, x_{\max}^4 = 0.35, x_{\max}^5 = 70, x_{\max}^6 = 6$$

The results of the evolutionary search for the solution of the problem of optimal design of a tubular heater with heat carrier recirculation are given in **Table 1**.

Table 1. Results of the evolutionary search for tubular heater with recirculation and 2 criteria.

Number of iterations	Argument values (parameters)						Functions (criteria)	
	x^1	x^2	x^3	x^4	x^5	x^6	F_1	F_2
Step 1 Branches 1, 2, 3	38.05292	178	0.121347	0.2899812	70	3	0.934281	0.846456
	39.47193	184.8262	0.123994	0.3498935	70	4.332048	0.940479	0.872469
	42.0129	172.3327	0.161610	0.2674288	68.91031	3	0.959257	1.42065
Step 2 Branches 1, 2, 3	31.84523	183.6643	0.1	0.35	69.60403	3.412462	0.913917	0.005766
	36.34251	200	0.105756	0.35	70	3	0.912229	0.142706
	44.32399	178.2993	0.149475	0.35	68.71887	4.826355	0.958499	1.254207

Continued

Step 3 Branches 1, 2, 3	31.84523	183.6643	0.1	0.35	69.60403	3.412462	0.909421	0
	33.03416	200	0.1	0.35	69.92034	3	0.896563	0
	29.27168	185.4664	0.137231	0.35	68.99318	5.626478	0.933428	0.759581
Step 5 Branches 1, 2, 3	29.73758	180.4266	0.1	0.35	69.74614	4.088115	0.914873	0
	28.81708	193.8471	0.1	0.35	69.84319	3	0.899616	0
	25.31072	183.625	0.1	0.35	69.30032	4.100157	0.905787	0
Step 12 Branches 1, 2, 3	29.34677	175.7512	0.1	0.35	68.98177	5.048008	0.918704	0
	29.6055	175.7642	0.1	0.35	68.51453	4.260027	0.919052	0
	29.54401	175.8815	0.1	0.35	68.45444	3.897392	0.918973	0

From the computational results, we can see that the evolutionary search converges quite quickly to a single solution, viz:

Heater power 30 kW, air flow rate 175 m^3/hour, heater diameter 0.1 m, screen diameter 0.35 m, total heater length 69 m, initial section length with screen—4 m.

The overall efficiency of the heater with recirculation reaches 91.9% while fully satisfying all constraints ($F_2 = 0$).

5. Discussion

It is of interest to compare the solution found for the recirculating tube heater on the one hand and the linear type gas tube heater on the other. The results of optimal calculation of the heater without recirculation are presented in Table 2.

Table 2. Results of the evolutionary search for tubular heater without recirculation and 2 criteria.

Number of iterations	Argument values (parameters)						Functions (criteria)	
	x^1	x^2	x^3	x^4	x^5	x^6	F_1	F_2
Step 8 Branches 1, 2, 3	20.2981	246.1759	0.1063	0.30688	65.51886	4.958077	0.8061	0.003290
	20	240.6709	0.1025	0.35	66.24492	5.20974	0.81849	0
	20	241.532	0.10673	0.337269	60.48888	5.06925	0.820997	0.007048
Step 20 Branches 1, 2, 3	2.017055	238.123	0.10185	0.346723	60.6891	3	0.82000	0
	2.00014	229.6907	0.10076	0.35	60.3744	4.505256	0.82884	0
	20	231.226	0.10120	0.347317	61.3520	45.67978	0.82655	0

As can be seen from the comparison of the two optimisation results—Table 1 and Table 2, there are differences in these results. First of all, at optimisation of the tube heater without recirculation the total power of the heater decreased, with recirculation +29.5 kW, without recirculation −20 kW. The fresh air flow rate increased slightly, instead of 175 m^3/hour, it is necessary to supply 230 m^3/hour. And the most important difference in achieving the overall efficiency of the tube heater: for the heater with recirculation—91.8%, and for the heater without recirculation—82%. These figures correspond to the efficiency of low-intensity infrared heaters [7], which are mass-produced, with a power of 15 - 50 kW, have a length of the heating part of the heater—6 - 15 metres, and the temperature of the heating pipe can reach 1000 ˚F (540˚C).

6. Concluding Remarks

Thus, it is convincingly demonstrated that the technical solution for the tubular heater provides improved heater efficiency compared to the linear configuration, as well as the simplicity and efficiency of using the algorithm for finding the most favoured solutions for selection by binary choice functions. The range of parameter variation can be specified based on the experimental study of gas emissions of pellet burner of tubular gas heater [7], namely:

$$x^1 \le 55\ \text{kW},\ x^2 \le 300\ \text{m}^3/\text{h},\ x^3 \le 150\ \text{mm}\ .$$

The validity of the obtained results is based on the reliability of the mathematical modelling of the pellet gas burner—this follows from [7], and the modelling and calculation of the linear part of the pellet heater is outlined in [15], which shows good agreement of the calculated temperature values with the experimental values for the tubular pellet heater located in the existing greenhouse.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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