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Modeling and Simulation Analysis of Two-Wheel Steering System for Vehicles Based on Matlab/Simulink

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Abstract

Yaw rate and sideslip angle are critical indicators for evaluating vehicle handling stability. When a vehicle is driven at high speed, emergency steering by the driver may cause loss of control, leading to severe accidents such as rollover. This paper establishes a mathematical model of a two-wheel steering system to analyze vehicle handling stability. Using Matlab/Simulink, steady-state response curves under different front-wheel steering angles and speeds are compared to investigate the influence of yaw rate and sideslip angle on vehicle handling stability.

Keywords

Two-Wheel Steering, Yaw Rate, Sideslip Angle, Vehicle Handling Stability

1. Introduction

Vehicle handling stability, a crucial characteristic for safe driving, has long been a focus in automotive development. Factors affecting handling stability include yaw rate, sideslip angle, and tire slip ratio. To validate the rationality of vehicle control strategies, this paper selects yaw rate and sideslip angle as key indicators to establish a linear two-degree-of-freedom (2-DOF) vehicle model [1]. The simulation model is developed using Matlab/Simulink, and simulation analysis is conducted to verify vehicle handling stability [2].

2. Linear 2-DOF Vehicle Model

To facilitate comparison of handling stability under different simulation conditions, the following simplifications and assumptions are made: The front-wheel steering angle is taken as the input; the effects of the steering and suspension systems are ignored; the vehicle body moves parallel to the ground. The vehicle has two-degree-of-freedom: lateral motion along the y-axis and yaw motion around the z-axis. Assuming small driving forces and neglecting the influence of ground tangential forces on tire cornering characteristics and aerodynamic forces, the vehicle is simplified to a two-wheel motorcycle model.

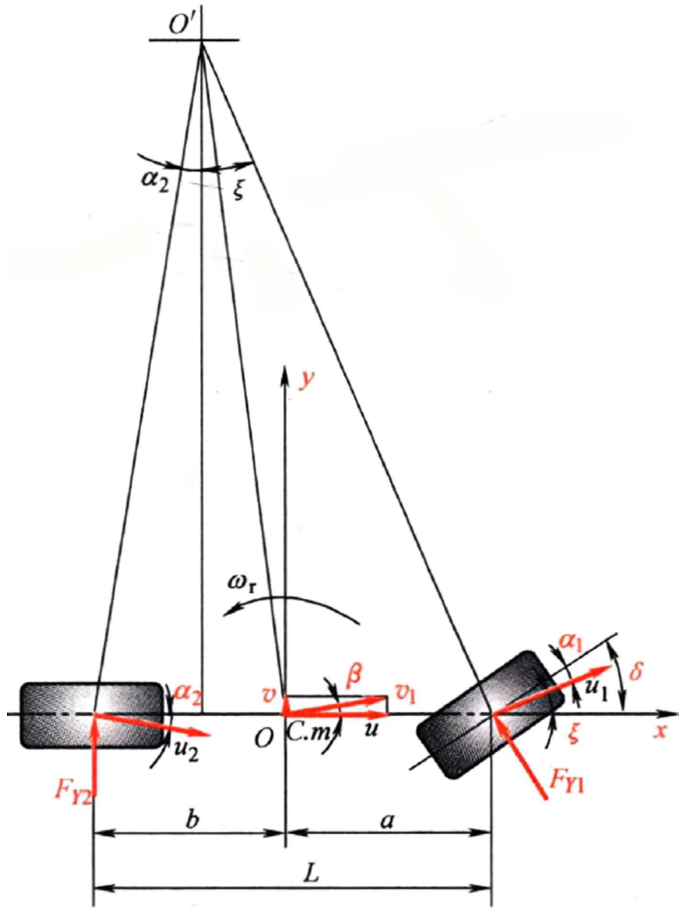


Figure 1. 2-DOF model.

As shown in **Figure 1**, the model consists of front and rear tires with lateral elasticity, supported on the ground, exhibiting lateral and yaw motions [3].

By aligning the origin of the vehicle coordinate system with the vehicle's center of mass (CoM), the absolute accelerations, angular accelerations, external forces, and moments are decomposed along the coordinate axes to derive the differential equations of motion.

The absolute acceleration components of the CoM in the vehicle coordinate system are determined as follows:

$$\begin{aligned} F_y &= ma_y = F_{y1}\cos\delta + F_{y2} \\ M &= I_{zz} \times B = aF_{y1}\cos\delta - bF_{y2} \end{aligned} \tag{1}$$

Here, F_{y1}、 F_{y2} are the lateral reaction forces (cornering forces) on the front and rear wheels, respectively; δ is the front-wheel steering angle.

The velocity variation along the ox-axis is:

$$\begin{aligned}\Delta ox &= (u+\Delta u)\cdot\cos\Delta\theta-(v+\Delta v)\cdot\sin\Delta\theta-u \\ &= u\cdot\cos\Delta\theta+\Delta u\cdot\cos\Delta\theta-v\cdot\sin\Delta\theta-\Delta v\cdot\sin\theta-u\end{aligned} \tag{2}$$

The velocity variation along the oy-axis is:

$$\begin{aligned}\Delta oy &= (u+\Delta u)\cdot\sin\Delta\theta+(v+\Delta v)\cdot\cos\Delta\theta-v \\ &= u\cdot\sin\Delta\theta+\Delta u\cdot\sin\theta+v\cdot\cos\Delta\theta+\Delta v\cdot\cos\Delta\theta-v\end{aligned} \tag{3}$$

For small $\Delta\theta$, higher-order terms are neglected: $\cos\theta\approx 1$, $\sin\theta\approx\theta$:

$$\begin{cases}\Delta ox = \Delta u - v\cdot\Delta\theta \\ \Delta oy = \Delta v - u\cdot\Delta\theta\end{cases} \tag{4}$$

Dividing by Δt and taking the limit, the absolute acceleration components of the CoM are:

$$\begin{cases}a_x = \dfrac{\mathrm{d}u}{\mathrm{d}t} - v\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \dot{u} - v\omega_r \\ a_y = \dot{v} + u\omega_r\end{cases} \tag{5}$$

Considering small, the cornering forces F_{y1} and F_{y2} are linearized:

$$F_y = k_1\alpha_1 + k_2\alpha_2$$

$$M_z = ak_1\alpha_1 - bk_2\alpha_2$$

Here, k_1 and k_2 are the front and rear tire cornering stiffnesses; α_1 and α_2 are the front and rear tire slip angles; a and b are the distances from the CoM to the front and rear axles.

With $\beta\approx\dfrac{v}{u}$, we derive:

$$\xi = \frac{v+\omega_r\cdot a}{u} = \beta + \frac{a\cdot\omega_r}{u} \tag{6}$$

The slip angles are:

$$\begin{aligned}\alpha_1 &= -(\delta-\xi) = \xi-\delta = \beta+\frac{a\cdot\omega_r}{u}-\delta_f \\ \alpha_2 &= \frac{v-b\omega_r}{u} = \beta-\frac{b\cdot\omega_r}{u}\end{aligned} \tag{7}$$

The force and moment equations become:

$$\begin{aligned}F_y &= k_1\left(\beta+\frac{a\omega_r}{u}-\delta\right)+k_2\left(\beta-\frac{b\omega_r}{u}\right) \\ M_z &= ak_1\left(\beta+\frac{a\omega_r}{u}-\delta\right)+bk_2\left(\beta-\frac{b\omega_r}{u}\right)\end{aligned} \tag{8}$$

Finally, get the 2-DOF differential equations are:

$$\begin{aligned}k_1\left(\beta+\frac{a\omega_r}{u}-\delta\right)+k_2\left(\beta-\frac{b\omega_r}{u}\right) &= m(\dot{v}+u\omega_r) \\ ak_1\left(\beta+\frac{a\omega_r}{u}-\delta\right)-bk_2\left(\beta-\frac{b\omega_r}{u}\right) &= I_z\dot{\omega}_r\end{aligned} \tag{9}$$

In the equation, I_z represents the moment of inertia of the vehicle about the z-axis, and $\dot{\omega}_r$ represents the yaw angular acceleration of the vehicle.

3. 2-DoF Vehicle Motion Differential Model

The motion differential equations for the two-degree-of-freedom vehicle model are derived by organizing the above formulas as follows:

$$\begin{cases} (k_1+k_2)\beta+\frac{1}{u}(ak_1-bk_2)\omega_r-k_1\delta=m(\dot{v}+u\omega_r) \\ (ak_1-bk_2)\beta+\frac{1}{u}(a^2k_1+b^2k_2)\omega_r-ak_1\delta=I_z\dot{\omega}_r \end{cases} \tag{10}$$

Transforming the above equations, we obtain:

$$\begin{cases} \dot{\omega}_r=\frac{a^2k_1+b^2k_2}{I_zu}\omega_r+\frac{ak_1-bk_2}{I_z}\beta-\frac{ak_1}{I_z}\delta \\ \dot{\beta}=\left(\frac{ak_1-bk_2}{mu^2}-1\right)\omega_r+\frac{k_1+k_2}{mu}\beta-\frac{k_1}{mu}\delta \end{cases} \tag{11}$$

The state-space representation of the above equations is:

$$\begin{bmatrix}\dot{\omega}_r\\ \dot{\beta}\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}\omega_r\\ \beta\end{bmatrix}+\begin{bmatrix}b_{11}\\ b_{21}\end{bmatrix}\delta$$

$$\begin{bmatrix}\omega_r\\ \beta\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\omega_r\\ \beta\end{bmatrix}+\begin{bmatrix}0\\ 0\end{bmatrix}\delta$$

The simplified state-space form is:

$$\begin{cases} \dot{x}=Ax+Bu \\ y=Cx+Du \end{cases}$$

where:

$$A=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}$$

$$B=\begin{bmatrix}b_{11}\\ b_{21}\end{bmatrix}$$

$$C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$D=\begin{bmatrix}0\\ 0\end{bmatrix}$$

and:

$$a_{11}=\frac{a^2k_1+b^2k_2}{I_zu}$$

$$a_{12}=\frac{ak_1+bk_2}{I_zu}$$

$$b_{11}=-\frac{ak_1}{I_z}$$

$$a_{21} = \frac{ak_1 - bk_2}{mu^2}$$

$$a_{22} = \frac{k_1 + k_2}{mu}$$

$$b_{21} = -\frac{k_1}{mu}$$

4. 2-DOF Vehicle Model Modeling and Simulation Analysis

Establish the corresponding Simulink simulation model based on the above equation, with the input being the front wheel steering angle and the outputs being the sideslip angle and yaw rate. The simulation model is shown in Figure 2.

Select specific parameters of a certain car, as listed in Table 1, and run the simulation model in the software. Through simulation analysis, compare the yaw

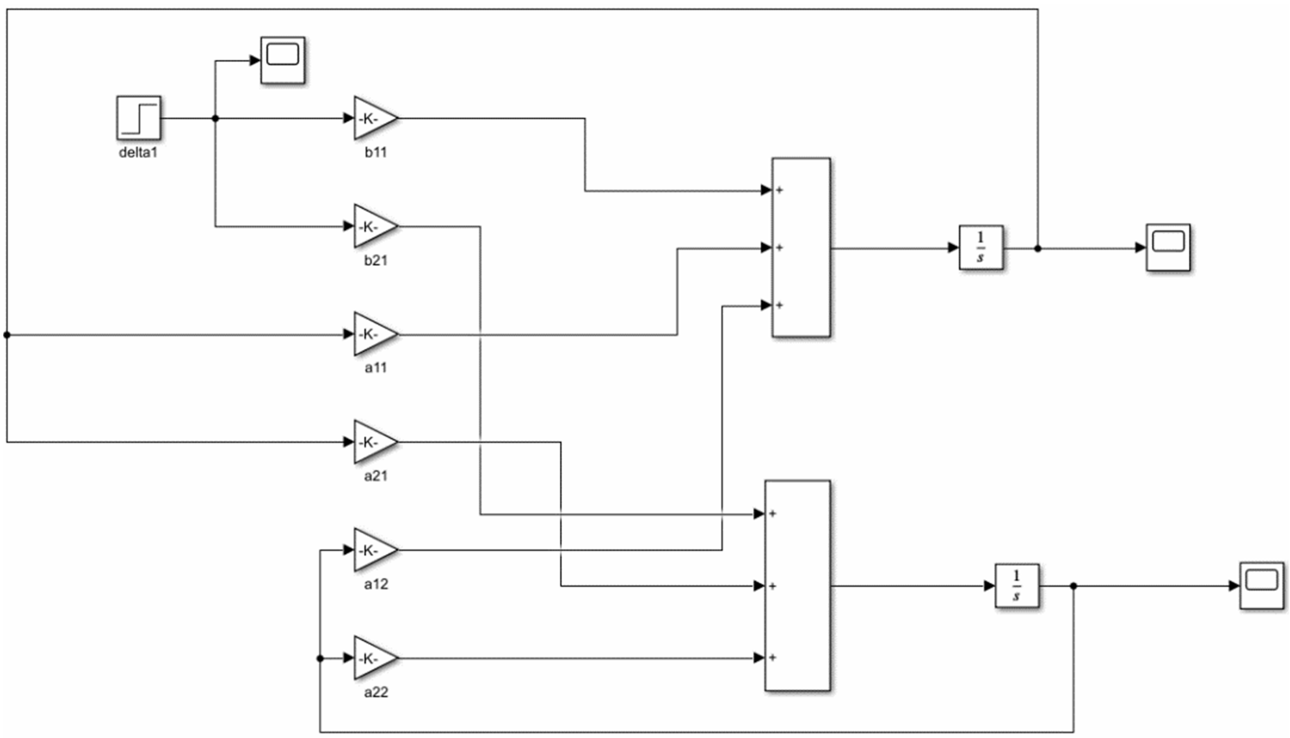


Figure 2. Two-degree-of-freedom vehicle simulation model.

Table 1. Specific parameters of a certain car.

Gross Weight (m)	2045
Moment of Inertia of the Vehicle about the z-axis (I)	5428
Distance from the Front Axle to the Center of Mass (a)	1.488
Distance from the Rear Axle to the Center of Mass (b)	1.712
Front Wheel Cornering Stiffness (Cf)	38,900
Rear Wheel Cornering Stiffness (Cr)	39,200

rate and sideslip angle under different front wheel steering angles and different vehicle speeds to assess the impact on vehicle handling stability [4].

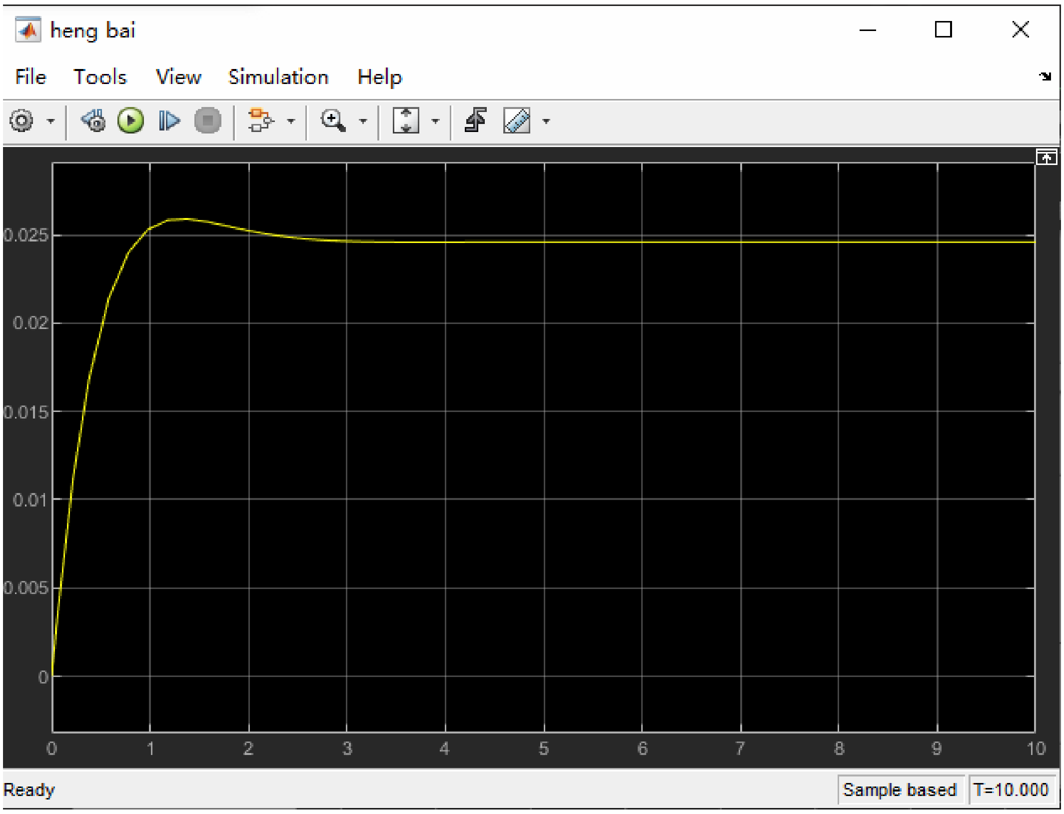
1) Comparison under Different Front Wheel Steering Angles

When the vehicle is traveling at a speed of 20 km/h, a step signal is applied to the front wheel steering angle at 0 seconds of simulation time, causing the front wheel steering angle to change from 0˚ to 2˚, 5˚, 10˚, 15˚, and 20˚, respectively, and remain at these angles. The yaw rate response curves are shown in **Figures 3(a)-(e)**, and the sideslip angle response curves are shown in **Figures 5(a)-(e)**.

From **Figure 3**, it can be observed that as the front wheel steering angle increases, the overshoot of the yaw rate also increases, rising from 0.025, 0.06, 0.12, 0.18, to 0.24, while the time required to reach a steady state slightly increases. From **Figure 4**, it can be seen that as the front wheel steering angle increases, the sideslip angle significantly increases, changing from −0.2, −0.43, −0.95, −1.4, to −1.85, with both the overshoot and stabilization time also increasing. Therefore, emergency steering maneuvers should be avoided during high-speed driving.

2) Comparison under Different Speed

When the simulation time is set to 0 s, a step signal is applied to the front wheels, causing them to rotate from 0˚ to 5˚ and maintain this angle. The vehicle is then set to travel at speeds of 20, 30, 40, 50, and 60 km/h. The yaw rate response curves are shown in **Figures 5(a)-(e)**, and the yaw angle response curves at the vehicle's center of gravity are shown in **Figures 6(a)-(e)**. From **Figure 5**, it can be



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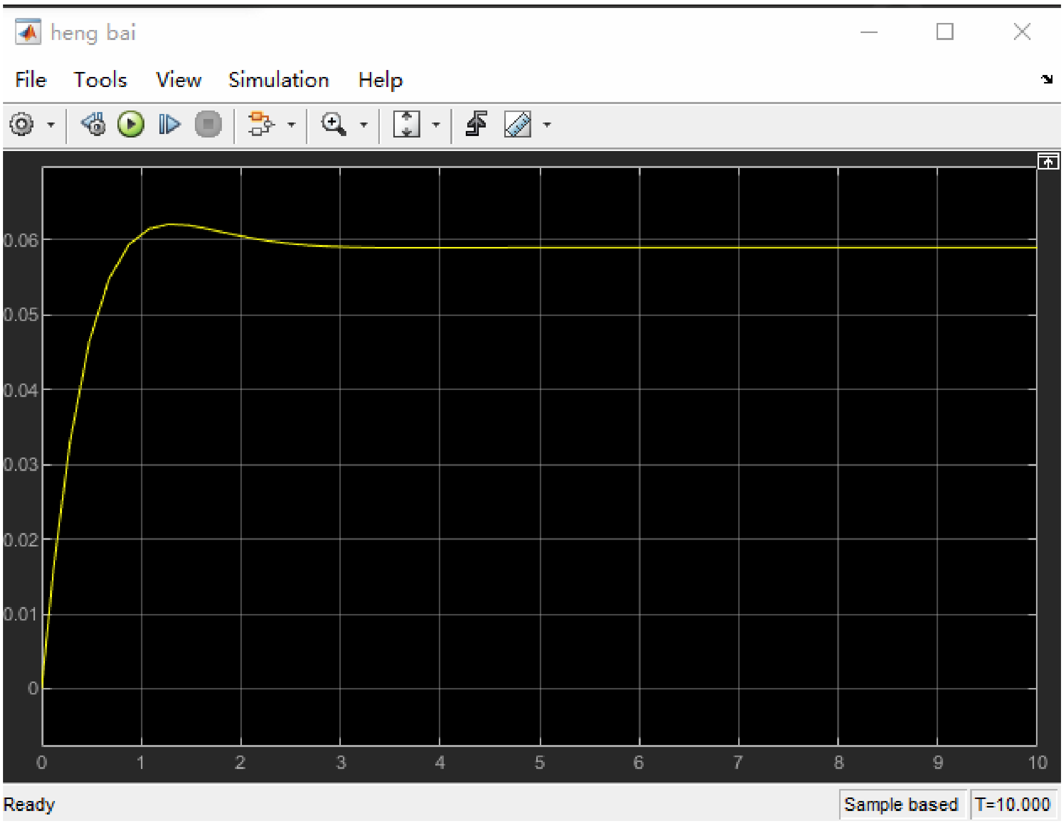
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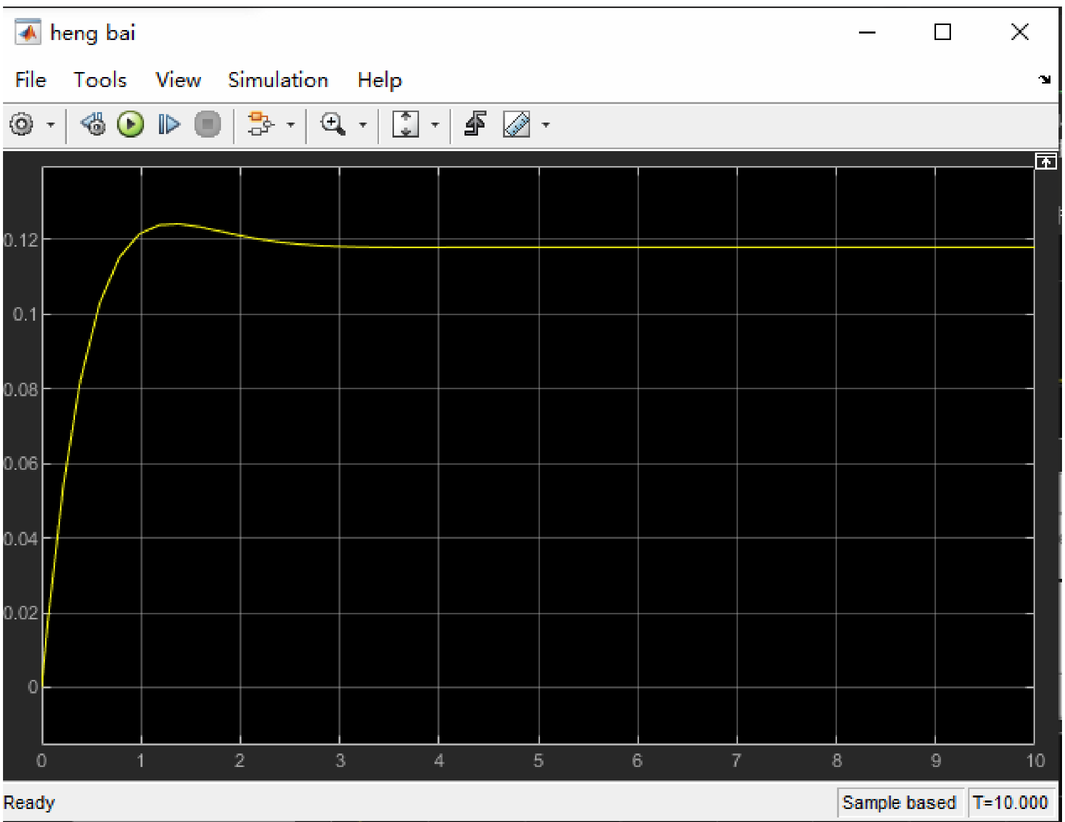
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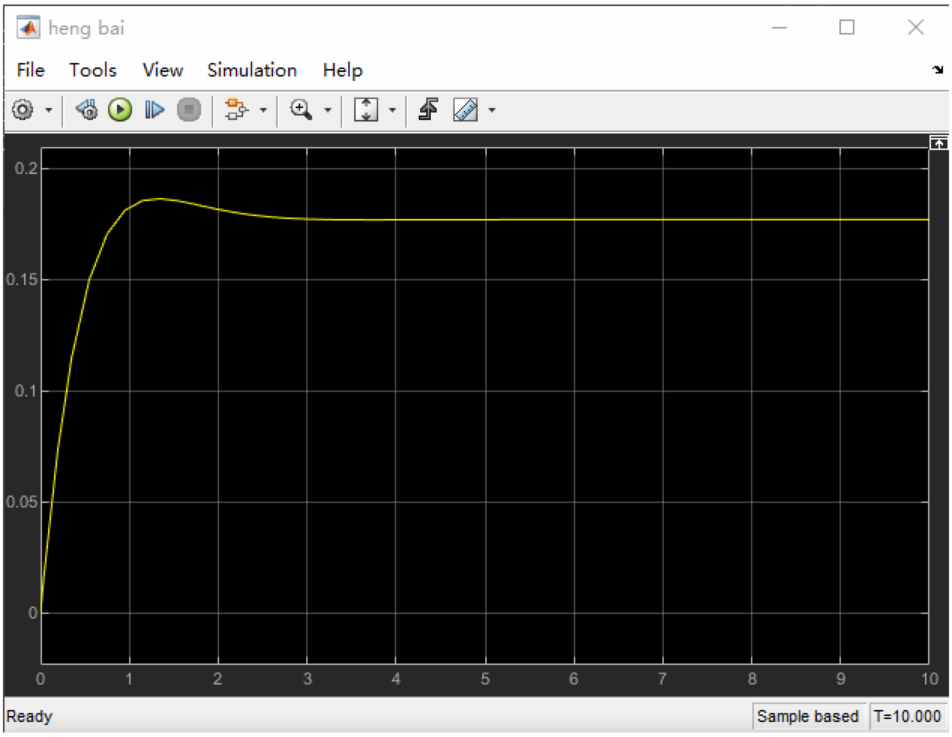

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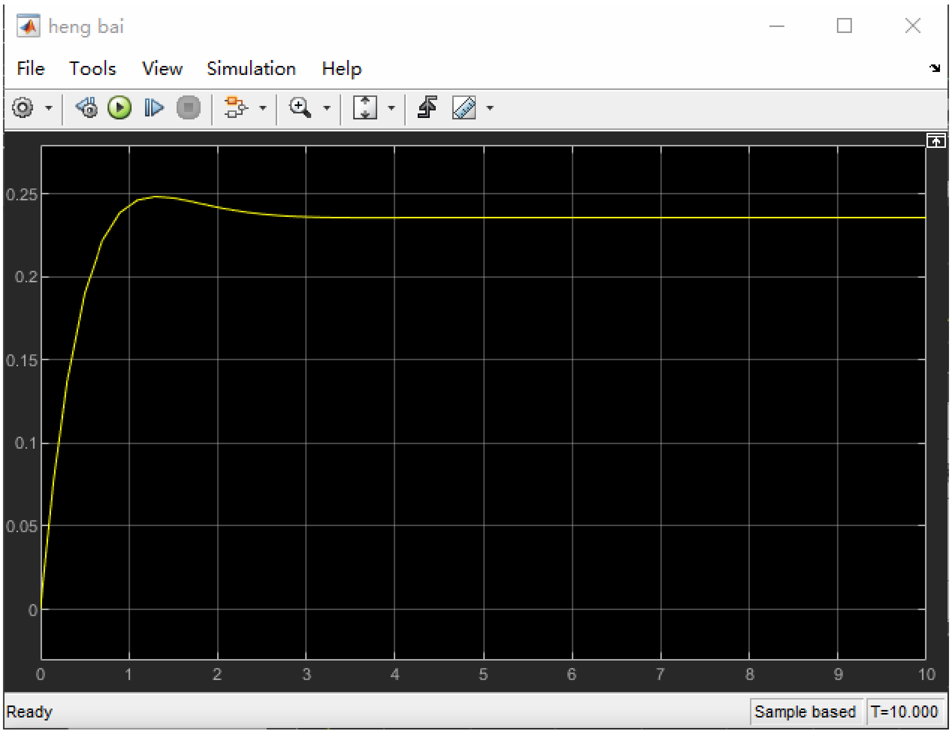
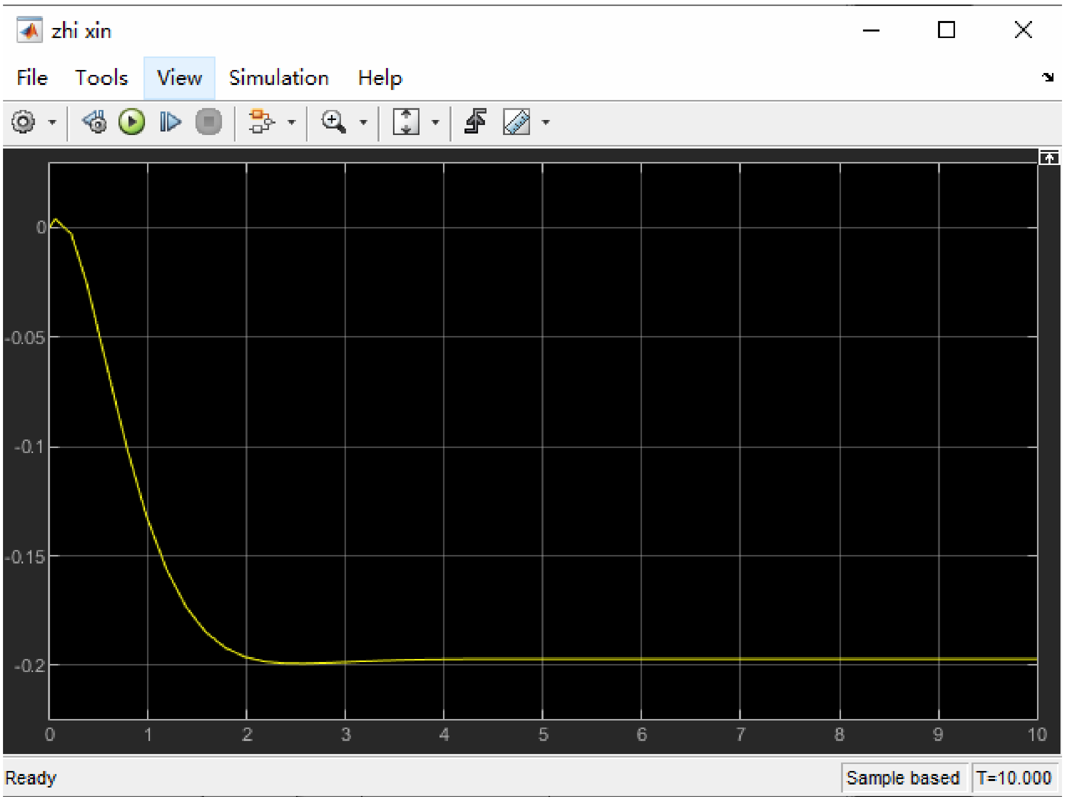


(e)

Figure 3. (a) Yaw rate response curve at a vehicle speed of 20 km/h with a front wheel steering angle of 2˚; (b) Yaw rate response curve at a vehicle speed of 20 km/h with a front wheel steering angle of 5˚; (c) Yaw rate response curve at a vehicle speed of 20 km/h with a front wheel steering angle of 10˚; (d) Yaw rate response curve at a vehicle speed of 20 km/h with a front wheel steering angle of 15˚; (e) Yaw rate response curve at a vehicle speed of 20 km/h with a front wheel steering angle of 20˚.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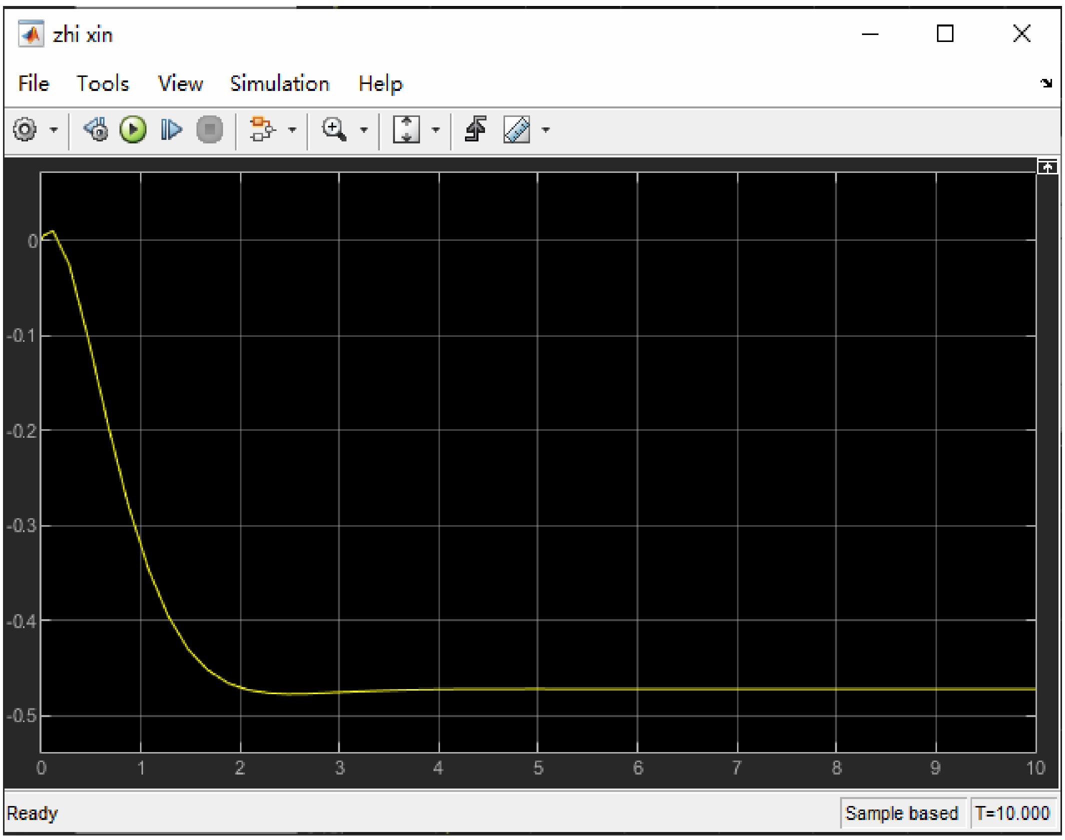
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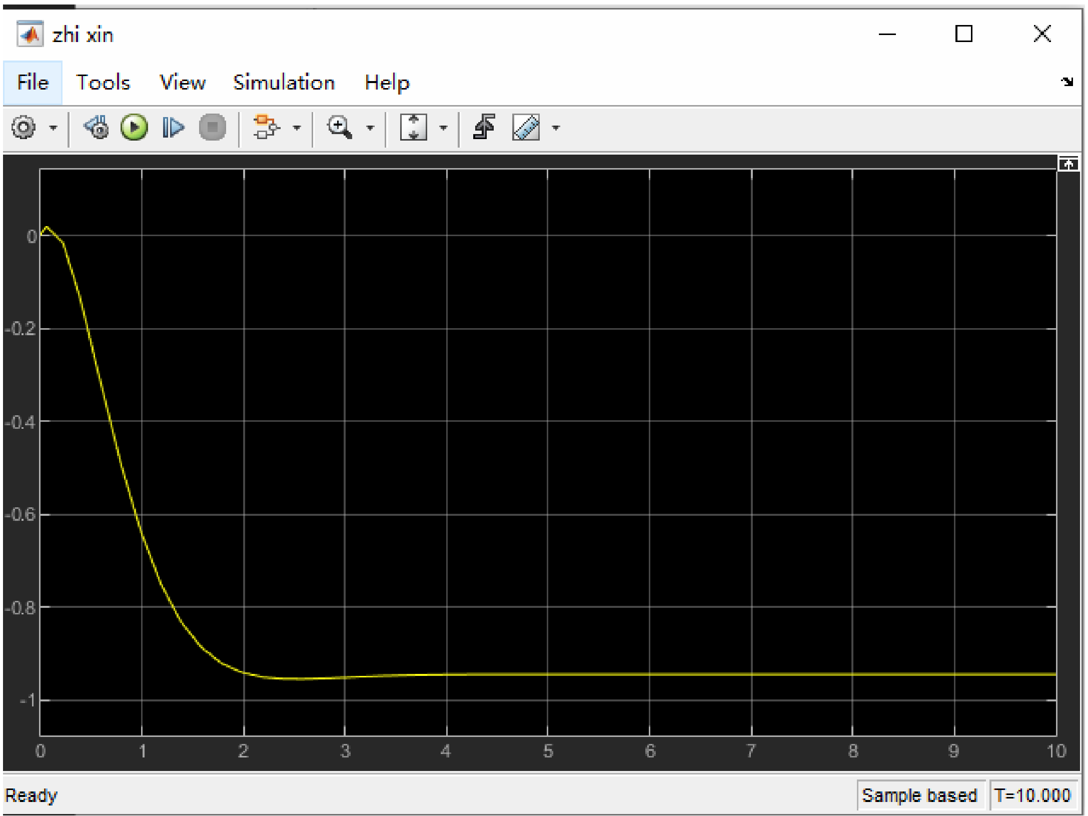
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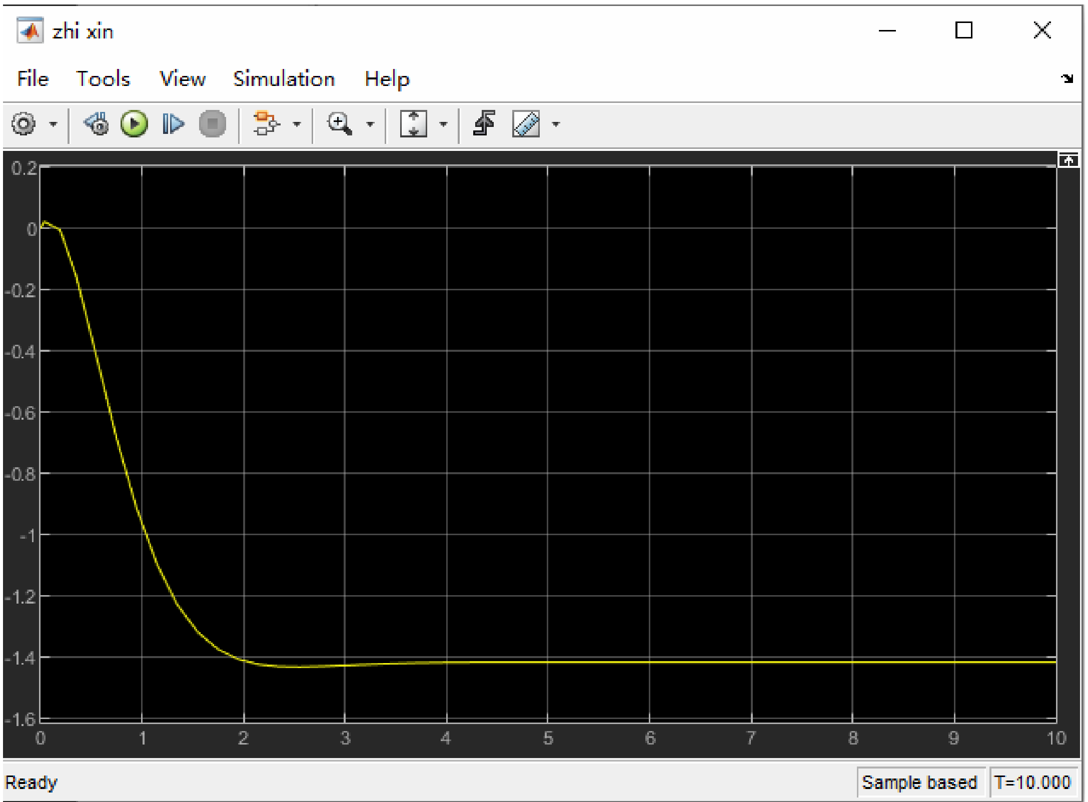
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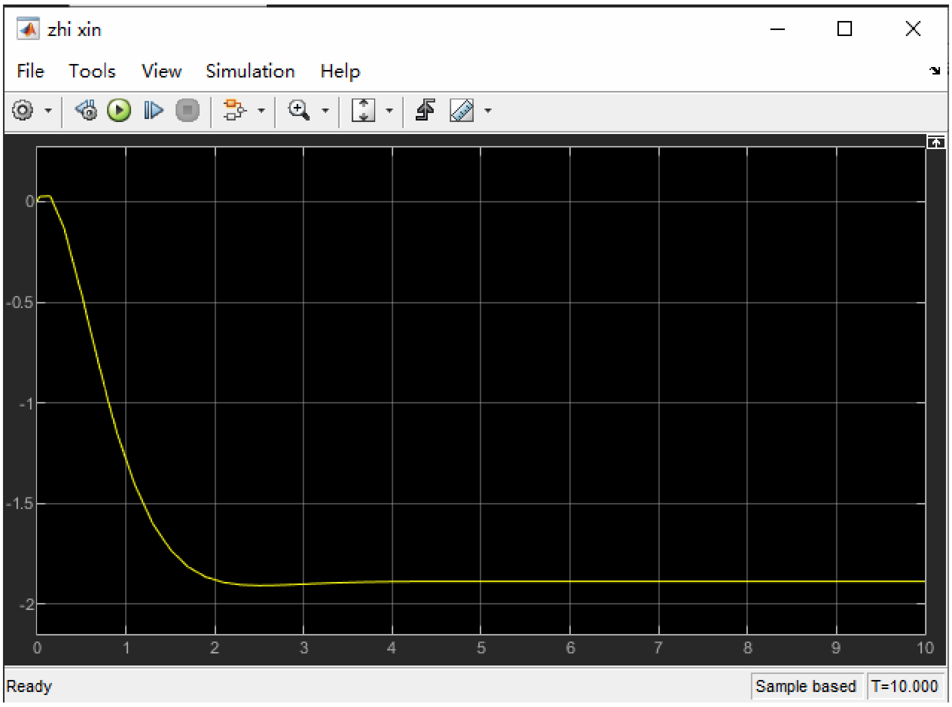

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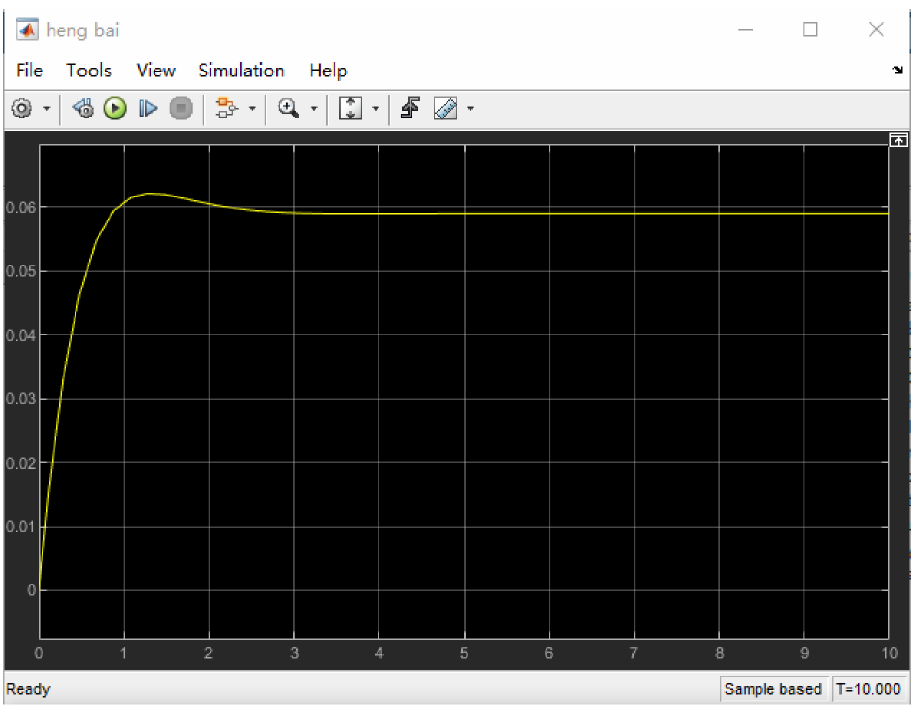
Figure 4. (a) Sideslip angle response curve at a vehicle speed of 20 km/h with a front wheel steering angle of 2˚; (b) Sideslip angle response curve at a vehicle speed of 20 km/h with a front wheel steering angle of 5˚; (c) Sideslip angle response curve at a vehicle speed of 20 km/h with a front wheel steering angle of 10˚; (d) Sideslip angle response curve at a vehicle speed of 20 km/h with a front wheel steering angle of 15˚; (e) Sideslip angle response curve at a vehicle speed of 20 km/h with a front wheel steering angle of 20˚.



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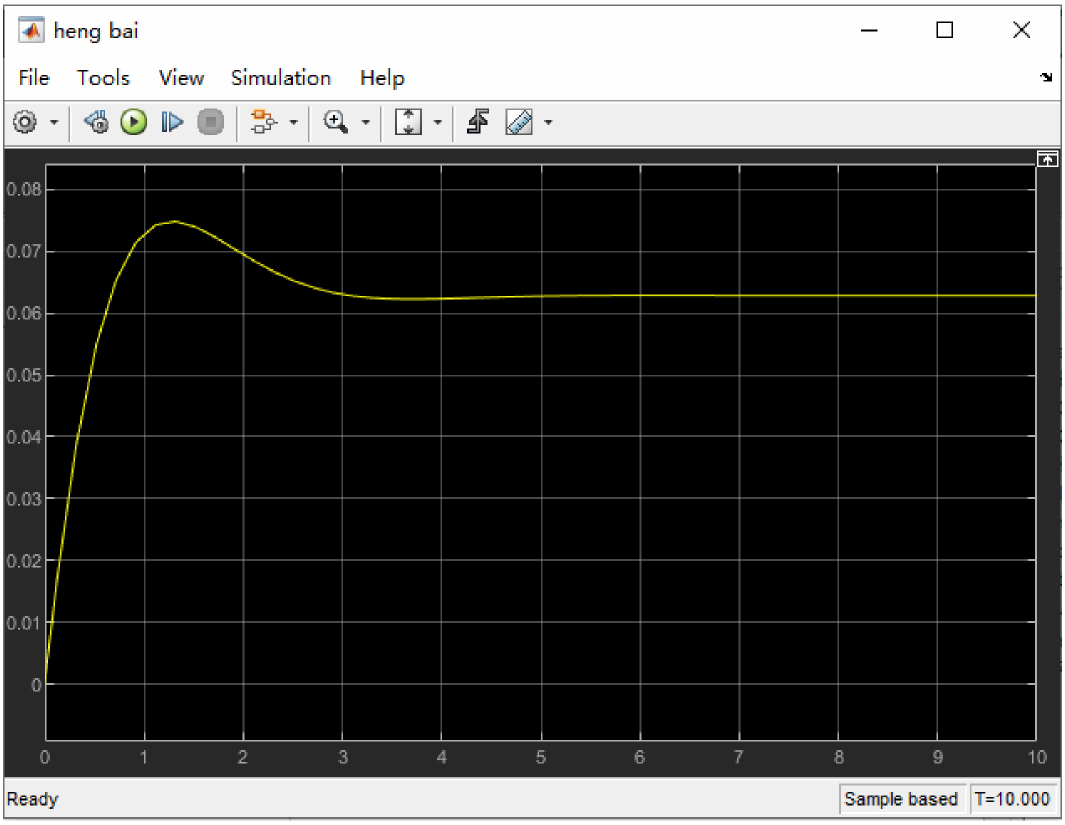
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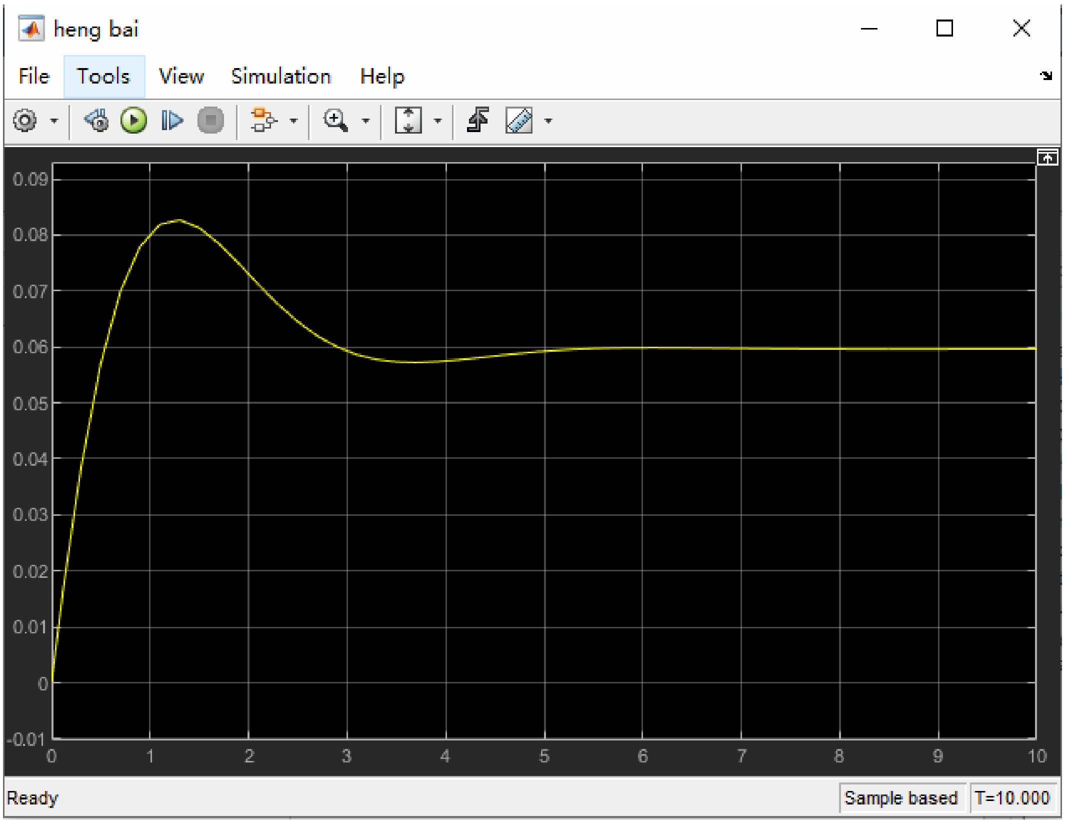
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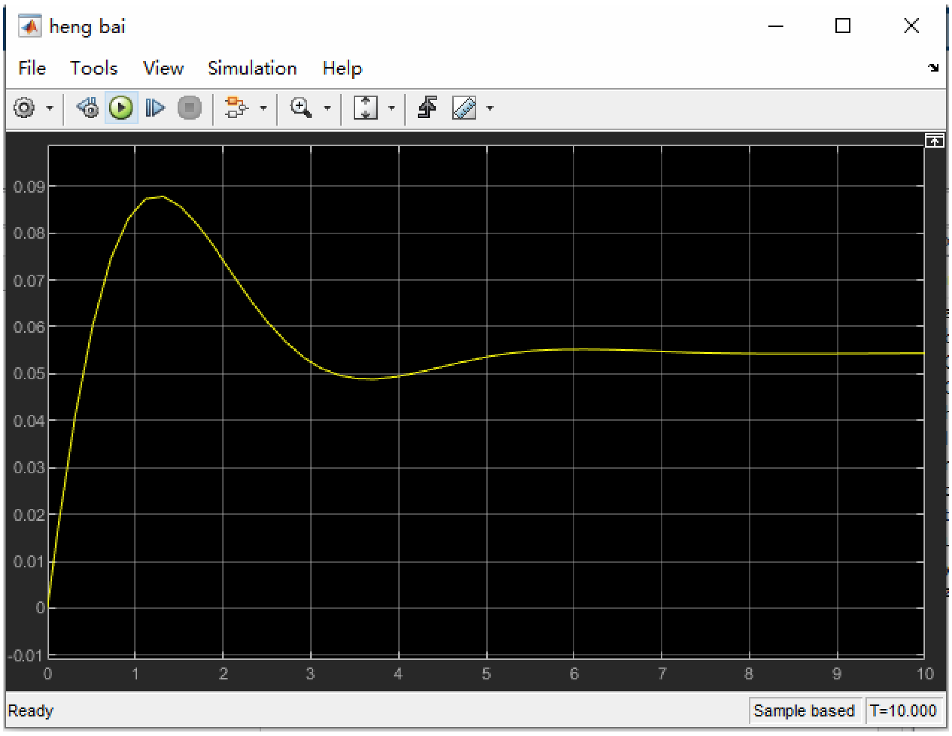

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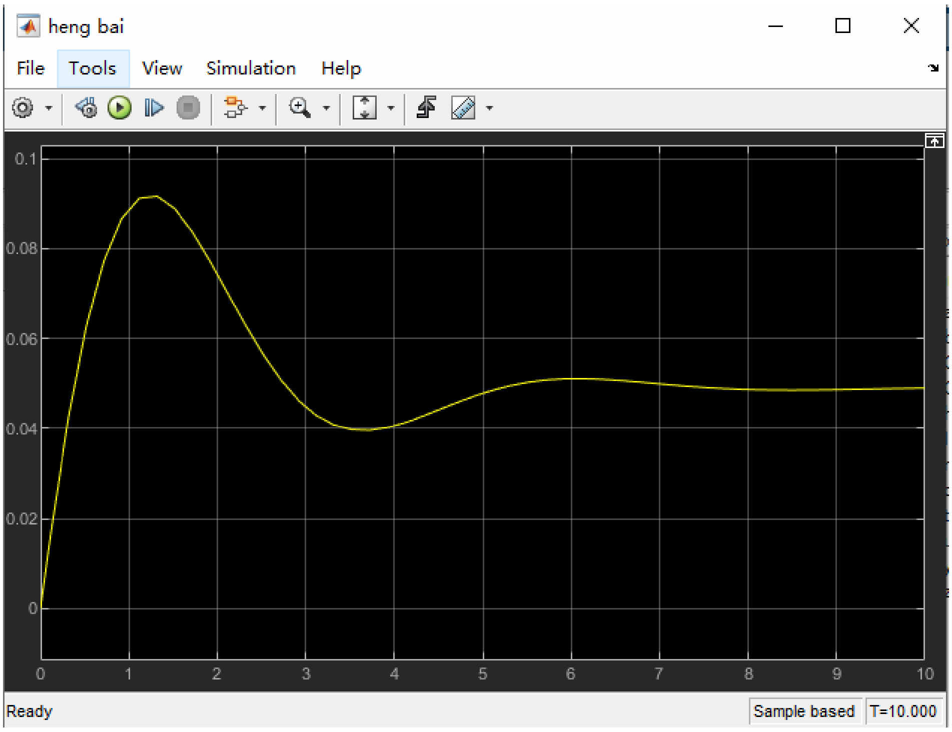

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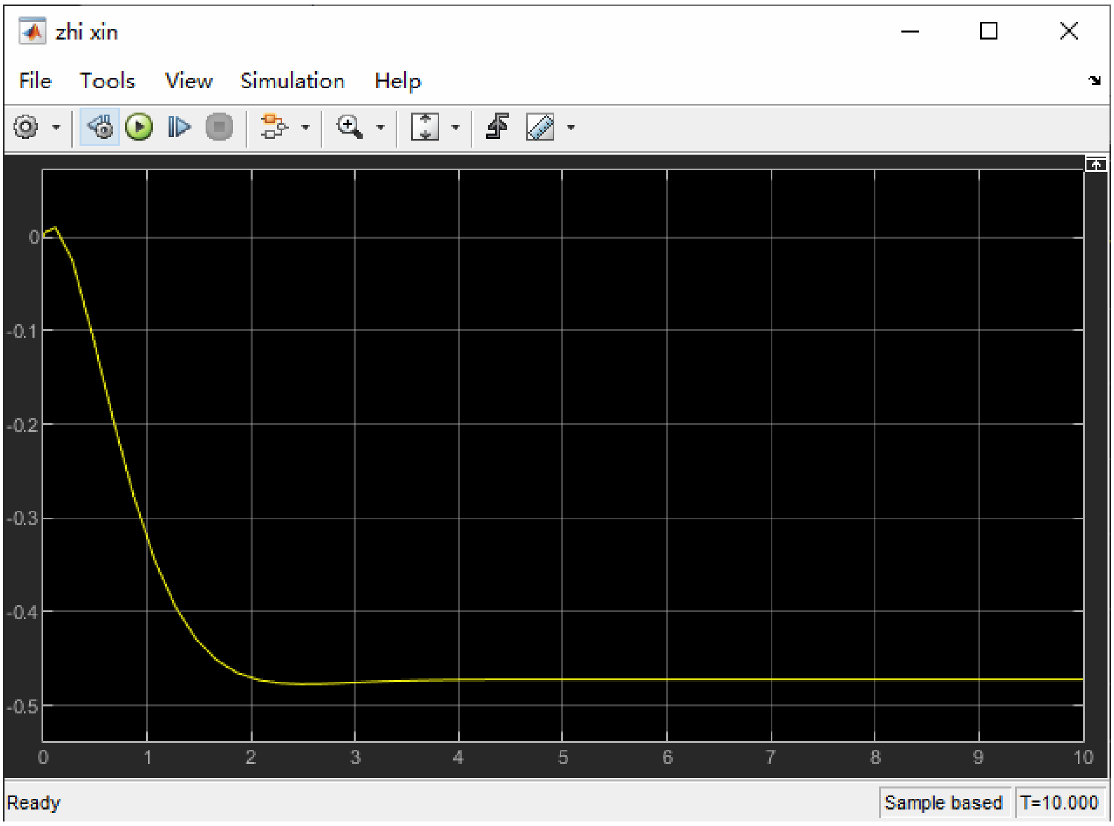
Figure 5. (a) Yaw rate response curve at a front wheel steering angle of 5˚ and a vehicle speed of 20 km/h; (b) Yaw rate response curve at a front wheel steering angle of 5˚ and a vehicle speed of 30 km/h; (c) Yaw rate response curve at a front wheel steering angle of 5˚ and a vehicle speed of 40 km/h; (d) Yaw rate response curve at a front wheel steering angle of 5˚ and a vehicle speed of 50 km/h; (e) Yaw rate response curve at a front wheel steering angle of 5˚ and a vehicle speed of 60 km/h.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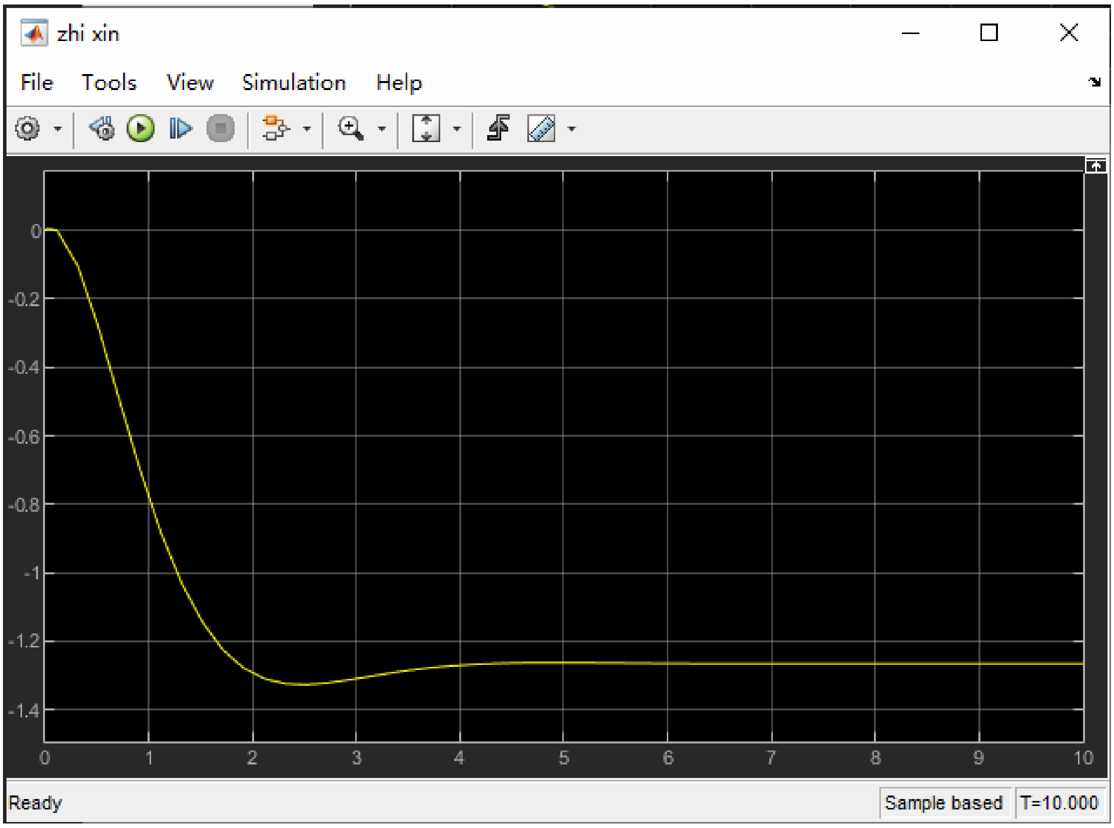
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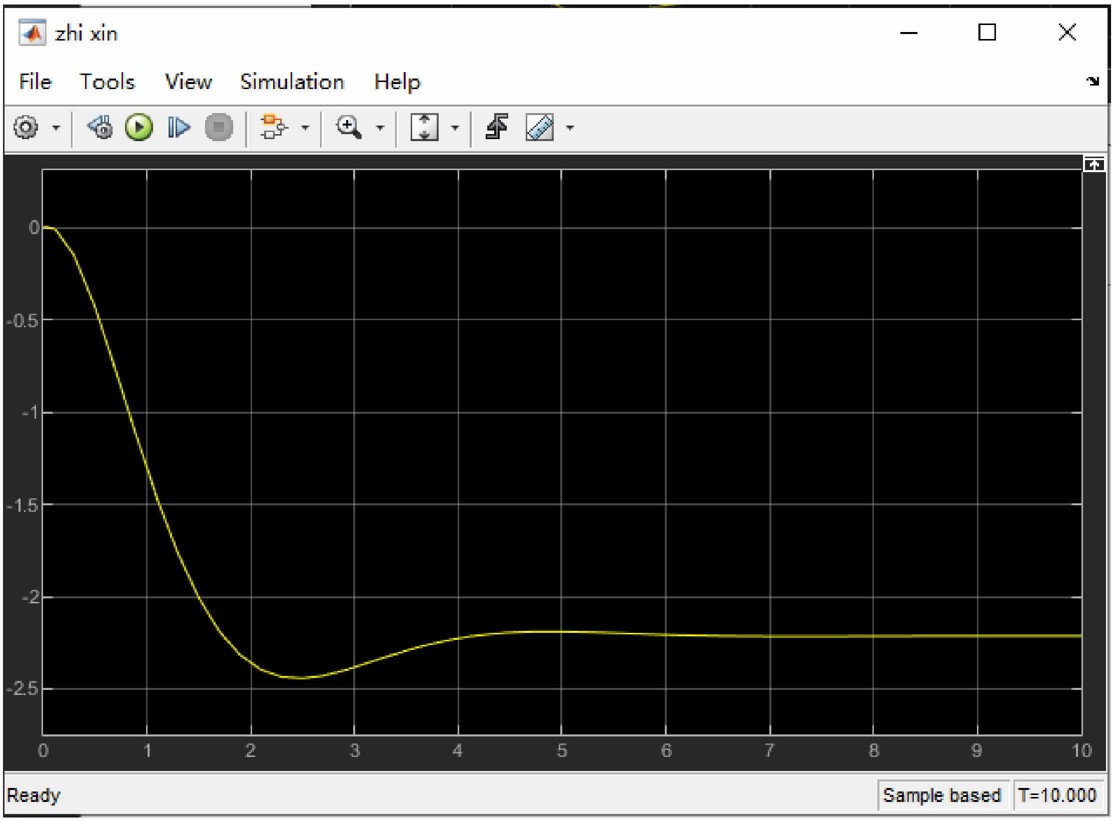
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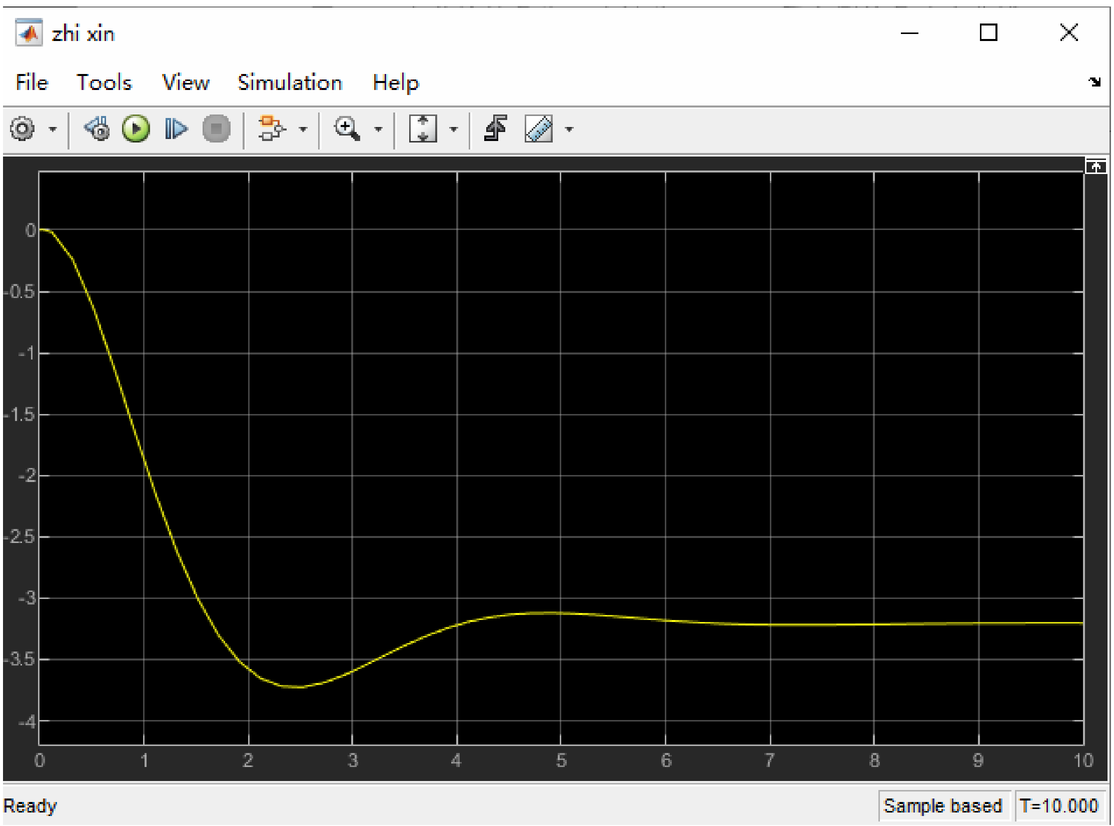
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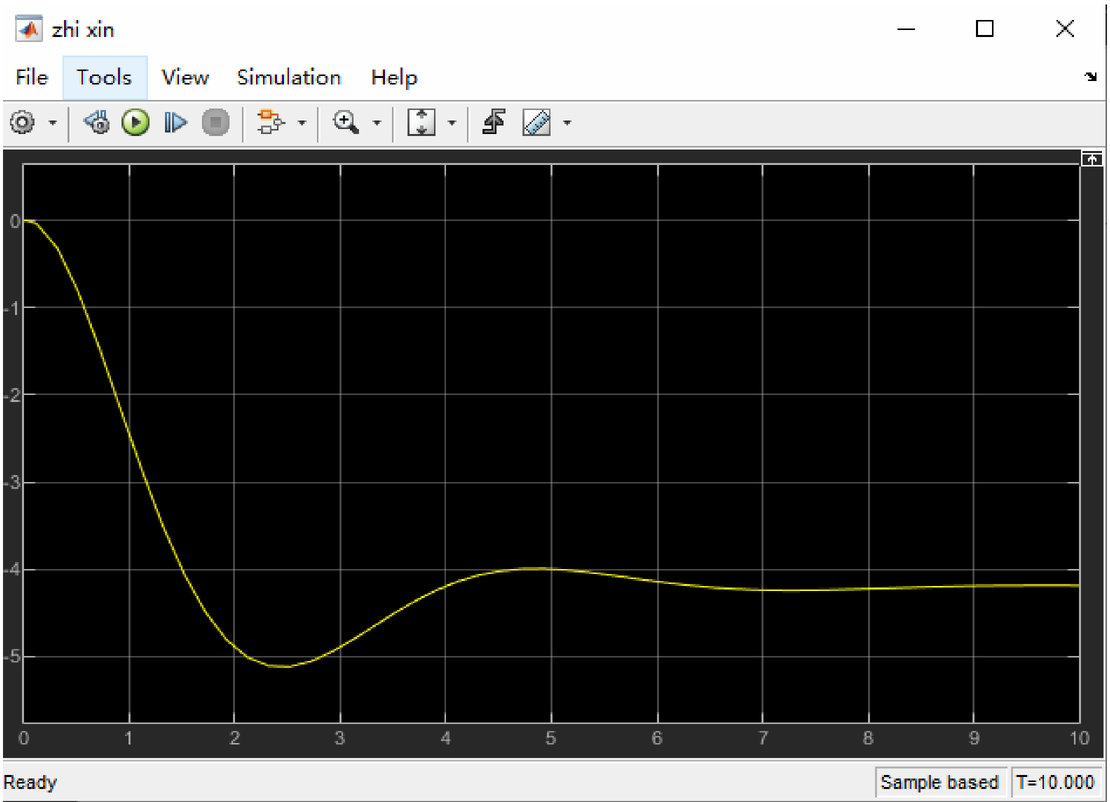

(e)

Figure 6. (a) Sideslip angle response curve at a front wheel steering angle of 5˚ and a vehicle speed of 20 km/h; (b) Sideslip angle response curve at a front wheel steering angle of 5˚ and a vehicle speed of 30 km/h; (c) Sideslip angle response curve at a front wheel steering angle of 5˚ and a vehicle speed of 40 km/h; (d) Sideslip angle response curve at a front wheel steering angle of 5˚ and a vehicle speed of 50 km/h; (e) Sideslip angle response curve at a front wheel steering angle of 5˚ and a vehicle speed of 60 km/h.

seen that as the speed increases, the overshoot of the yaw rate gradually increases, from 0.06, 0.062, 0.06, 0.055 to 0.05, and the time taken to reach steady state also increases. The stability of the vehicle decreases. When the vehicle's speed exceeds the critical stable speed, the center-of-gravity slip angle rapidly increases from −0.47, −1.28, −2.2, −3.2 to −4.2, surpassing the vehicle's stable critical value. The operational stability of the vehicle decreases, and there is a high likelihood of severe traffic accidents, such as vehicle skidding, which could endanger personal safety.

5. Conclusion and Discussion

Using the Matlab/Simulink simulation software, a step signal is applied as the front wheel steering angle input to the simplified two-degree-of-freedom linear vehicle model to obtain the response characteristics curves of the yaw rate and sideslip angle to the front wheel steering angle. By analyzing the curves under various operating conditions, the impact of the yaw rate and sideslip angle on vehicle stability is explained, as well as their intrinsic relationship with the front wheel steering angle and vehicle speed. This provides reference and insights for future research and exploration of vehicle stability.

The main objective of this paper is to verify the vehicle behavior under basic input conditions by simplifying the model, laying a foundation for subsequent more complex research and providing an important reference for understanding the basic characteristics of vehicle dynamics. At the same time, due to the high threshold and cost of vehicle hardware in the loop simulation platform tests, it is planned to introduce a comparison between experimental verification and simulation results in subsequent research to better ensure the reliability and scientific nature of the research.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Jin, Q.J. and Luo, Q. (2020) Analysis of Two-Degree-of-Freedom and Four-Degree-of-Freedom Vehicle Vibration Models Based on MATLAB. *Science and Technology & Innovation*, No. 17, 67-69.

[2] Tan, X.F., Cui, Y.H. and Wang, M.R. (2017) Dynamic Characteristic Analysis of Two-Degree-of-Freedom Vehicle Suspension. *Agricultural Equipment & Vehicle Engineering*, **55**, 49-51+88.

[3] Wang, X., Zhao, Q., He, F., *et al.* (2016) Simulation Analysis of the Handling and Stability of a Linear Two-Degree-of-Freedom Vehicle. *Forest Engineering*, **32**, 64-67+76.

[4] Liu, W. and Chen, H. (2010) Research on the Longitudinal Two-Degree-of-Freedom Control Method of Intelligent Electric Vehicles. *Journal of Shandong Jiaotong University*, **18**, 1-4.