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Gauge Theories as Distributions of Affine Defects

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Abstract

Geometric gauge theory provides a mathematical foundation for the most successful innovations in twentieth-century theoretical physics. The theories use symmetry groups, connections on fiber bundles, and concepts of parallel translation and curvature to model basic physics. More specifically, spacetime translations and projective conformal transformations model linear momentum; affine torsion, including torsion trace, models intrinsic angular momentum; conformal dilations model quantum wave mechanics; and unitary symmetry models elementary particles. The present work combines these basic gauge symmetries in a common framework as distributions of discrete defects in affine spaces, where each type of gauge field can be interpreted geometrically as a distribution of discrete defects in a regular lattice. The final section suggests in general terms research programs to search for distributions of discrete defects in regular lattices that underlie gauge theories.

Keywords

Gauge Theory, Affine Symmetry, Torsion, Conformal Symmetry, Unitary Symmetry, Inversion Symmetry

1. Introduction

In the sixteenth through twentieth centuries, a sequence of explosive advances propelled the progress of modern physics.

In the sixteenth century, Nicolaus Copernicus established the heliocentric theory of the solar system, which was later supported by more detailed measurements by Tycho Brahe [1]. In the early seventeenth century, Johann Kepler validated three explicit quantitative laws of planetary motion [2].

Later in the seventeenth century. Isaac Newton developed Newtonian mechanics using Euclidean geometry and differential and integral calculus. Newton's theory assumes the speed of light in a vacuum is infinite [3].

In the mid-19th century, J.C. Maxwell's theory of electrodynamics (ED) modeled all basic ED phenomena [4]. In 1887, experimental evidence was established that Maxwell's theory is correct in assuming a finite constant speed of light [5].

In 1905, Albert Einstein's special theory of relativity preserved Maxwell's experimentally-validated finite constant speed of light and altered Newtonian mechanics to make it agree with Maxwell's theory. He accomplished this by altered the definitions of Newtonian energy and momentum in terms of mass and velocity, particularly at high energies.

In the 1920s, Élie Cartan generalized Riemannian geometry by including affine torsion. Cartan's geometry, known to physicists as gauge theory, provided a unified framework for Maxwell's ED, Einstein's General Relativity (GR) (1915-1916), and the as-yet undiscovered gauge theories of high energy physics.

Quantum physics started the modern theory of microphysics. The first major advances were Planck's theory of black body radiation in 1900 [6], Einstein's explanation of the photoelectric effect in 1905 [7], Bohr's quantum theory of atomic structure in 1913 [8], followed by quantum mechanical wave equations by Schrodinger in 1926 [9] and Dirac in 1928 [10].

In 1922, Cartan suggested that Einstein extend GR to include affine torsion by dropping GR's ad-hoc assumption that torsion is zero. Cartan identified torsion as translational curvature and, moreover, related it via his second field equation, to the intrinsic angular momentum (a.m.) of a Cosserat fluid. In 1929, Einstein wrote to Cartan that he "didn't at all understand the explanations you gave me [about the role of torsion in geometry]; still less was it clear to me how they might be made useful for physical theory." [11]. Hubble's discovery in 1929 of cosmic expansion proved that Einstein's attempt to model a static universe was not realistic [12]. In the early 1960s, Kibble [13] and Sciama [14] established EC as a viable theory of gravity; they identified torsion with intrinsic a.m.

Maxwell's equations of classical electrodynamics (ED) unified electricity and magnetism in one field theory. However, ED was not recognized as a gauge theory until 1929 [15]. In ED, the electromagnetic potentials define a law of parallel translation whose curvature is the Faraday tensor; the structure group is the simplest compact unitary group U (1).

In 1954, Yang and Mills modeled the strong interaction between protons and neutrons in nuclei with SU (2) gauge theory [16]. In the early 1960s, Gell-Mann *et al*. introduced quantum chromodynamics, which models the strong interaction between quarks as an SU (3) gauge theory. In the late 1960s, Glashow and Salam unified the electromagnetic and weak forces into a single gauge theory based on the group U (1) × SU (2). In the early 1970s, the Standard Model of high-energy particle physics, based on the group U (1) × SU (2) × SU (3), unified all of these theories. The unitary theories deal with internal symmetries, not spacetime symmetries. The Standard Model remains the definitive theory of the three fundamental forces of high-energy particle physics.

After 1975, the pace of empirical validation of new theories slowed.

This historical background covers selected relevant historical topics in geometric gauge theory and concepts to which we contributed in 1975-2006. It does not cover recent advances in geometric field theory since roughly 1975. It makes no attempt to provide a comprehensive review of developments not made by us after 1976. We do not attempt to summarize work by others after about 1975.

2. General Relativity and Riemannian Geometry

We assume that reader is familiar with Riemannian geometry and General Relativity [17]-[20].

Among the most intriguing features of the geometric models is that the gauge theories can be interpreted as continuum approximations to distributions of discrete defects in affine lattices, analogous to affine defects in material science. This suggests discrete structures at a level below the wave equations of quantum physics.

The term "Post-Riemannian geometry" is defined in Google Search:

> "Post-Riemannian geometry refers to a broad field of study in mathematics that extends beyond the traditional framework of Riemannian geometry …"

Rotational curvature measures the ratio of rotation of a vector when parallel translated along a loop in the manifold, per unit area inside the loop, in the limit of small loops.

(1) Rotational Curvature = $\text{Limit}_{[\text{area inside loop path}\to 0]}$ (Rotational holonomy/(area inside the loop))

Figure 1 captures the basic relationship between curvatures and discrete holonomy.

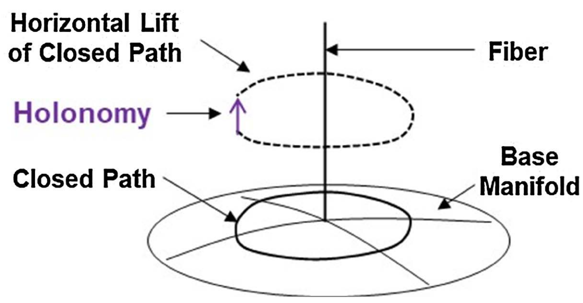

Figure 1. The fundamental picture of differential geometry.

2.1. Contravariant and Covariant and Representations of Curvature

We divert ourselves to discuss the concepts and notation for contravariant and covariant vectors and related concepts. See for example Wikipedia [21]. In multilinear algebra and differential geometry, A "contravariant" vector **v** is a tangent vector to the base manifold at a point.

A "covariant" vector **u** is the dual of a contravariant vector. That is, it is a linear map from the space of contravariant vectors to the real numbers (or more generally a field pf scalars).

We use conventional notation v^i for a contravariant vector (with an upper tensor index) and w_j for a covariant vector (a linear mapping from the space of contravariant vectors to the real numbers (or another scalar field).

The tangent space to the base manifold and the tangent space to the fiber space can be different associated vector spaces. These spaces can have identical dimension and different metrics and coordinates.

Examples:

Relativistic electromagnetic theory has base space dimension 4 and a unitary fiber SU (1) with real dimension 1.

Relativistic SU (2) unitary gauge theory has base space dimensions 4 and fiber space with real dimension3.

Relativistic SU (3) unitary gauge theory has base space dimensions 4 and fiber space with real dimension 8.

Conventional Riemannian geometry of 4-manifolds expresses the full curvature tensor as having four tensor indices.

The first antisymmetric pair of indices specify a plane in the base manifold in which the closed loop lies, as in **Figure 1**.

The other antisymmetric pair of indices specify a plane in the fiber in which the holonomy vector lies.

2.2. General Relativity and Rotational Curvature

In 1905 Einstein's theory of Special Relativity modified Newtonian physics to accommodate the constant finite speed of light [22]. Newton's theory of gravitation is not compatible with the finite constant speed of light or with Special Relativity. In 1915, Einstein's theory of General Relativity (GR) introduced Riemannian curvature into spacetime, so that Special Relativity, ED and gravitation of GR fit together [23]. In the author's opinion, the most intuitive introduction to GR dates from late twentieth century [24].

The metric of GR is the Lorentz metric. Its structure group is the Lorentz group, which is the homogenous (no translations) rotation group of the Lorentz metric. The associated fiber is the linear tangent space of spacetime. The curvature is Riemannian (rotational) curvature.

2.3. Geometric Interpretation of Disclinations

GR can be interpreted as representing gravitation as a very fine distribution of discrete defects in spacetime called disclinations. Below are three ways to visualize how curvature causes gravitation.

Each discrete disclination is like an angular wedge excised from a regular affine lattice. The edges of the cut are attached so that each horizontal layer is a cone, which has angular aperture less than 2π radians. The mass that causes the defect lies along the centerline of **Figure 2**.

A two-dimensional horizontal slice of **Figure 2** shows how a single disclination, and more generally continuous positive rotational curvature, can convert a straight trajectory into a curved orbit. Before making the cut, insert straight Line A that crosses the cut line and does not include the center point. Excise a sector as shown in **Figure 2**. Glue the two cut surfaces together. The remainder of Line A is a piecewise straight line with a bend of angle θ where it crosses the cut line. Mass near the center of the disk bends the trajectory to curve around the gravitational source.

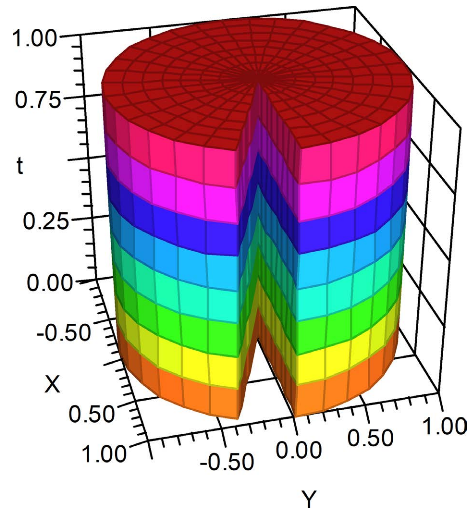

Figure 2. Graphical representation of a disclination associated with gravitation [25].

3. Einstein-Cartan Theory and Affine Torsion

3.1. Early History of EC

In 1922 E. Cartan proposed extending General Relativity (GR) by dropping GR's assumption that affine torsion (translational curvature) is zero [26]. To Cartan, torsion—translational curvature—is the natural companion of rotational curvature. Torsion extends the rotational affine symmetry of Riemannian geometry to include the translational symmetries of full affine group.

In 1929 Einstein wrote to Cartan that he "didn't at all understand the explanations you gave me [about the role of torsion in geometry]; still less was it clear to me how they might be made useful for physical theory."

In the 1960s, Tom Kibble and Dennis Sciama derived Einstein–Cartan theory (EC) from a variational principle. They identified the modified torsion with intrinsic angular momentum (a.m.). They argued that torsion is a natural extension of rotational curvature [27].

For the remainder of the twentieth century, EC was considered to be a speculative extension of GR that was unsupported by empirical evidence or derivation from accepted theories.

Acceptance of EC requires familiarity with the Riemann–Cartan extension of Riemannian geometry;

In the 1920s, Einstein tried to use torsion to create static cosmologies, which gave torsion a bad reputation.

Early EC employed torsion in all covariant derivatives, whereas it is necessary to distinguish base space indices from fiber indices, with torsion included only in covariant derivatives of fiber indices.

In 1975 Adamowicz showed that GR plus classical matter with spin yields the same linearized field equations as in EC [28].

In 1976 Hehl *et al* published a balanced review of EC [29].

In1976 Petti pointed out that only fiber indices include torsion in covariant derivatives [30].

In 2006, Trautman published a review article on EC. in which he stated that EC satisfied all known empirical tests of GR. EC remains an unproven speculation because there is no empirical evidence for it except the evidence for GR [31].

In 2013 Blagojevich and Hehl published a book on gauge theories of gravitation, in which they stated that only GR, EC and teleparallel gravity pass all known empirical tests of GR. EC remains an unproven speculation because it lacks empirical validation beyond empirical tests of GR. When torsion = 0, EC is identical to GR [32].

Calculations show that density of intrinsic a.m. required to cause observable effects in the early universe or near the centers of black holes is too small for contemporary instruments to detect. The problem might eventually be solved by more sensitive instruments [33] and/or by cleverer experiments such as effects of EC on quantum mechanical phase shifts [34].

3.2. Derivation of EC from GR

In 1986, Petti published a derivation of EC from GR [35]. An updated and expanded version of the original derivation was published in 2021 [36]. The second paper derives the key features of EC from classical GR in two ways.

Part I derives a discrete version of torsion and the spin-torsion field equation of EC from one Kerr solution in GR.

Part II derives the field equations of EC as the continuum limit of a distribution of many Kerr masses in classical GR.

In this work, the most relevant example of using different coordinate systems on a base manifold and fiber manifold is found in Derivation of EC from GR.] This article uses rotational coordinates (t, r, θ, ψ) to capture the rotation of the entire space, a Minkowski metric with coordinates (t, x, y, z) to capture the flatness of the fiber space, and a smooth transition between the two geometries. It also uses lower-case Greek tensor indices to indicate that spacetime tensors that are covariant differentiated without torsion, and lower-case Roman indices to indicate tensors whose covariant derivatives include torsion.

The key step in the derivation is to model torsion as the limit of {translational holonomy around a loop}/unit area of the loop], surrounding a Kerr mass, as the size of the loop approaches zero. Tensor calculus features, introduced into Macsyma software in the 1990s, facilitate performing the continuum equations of EC in Part II [37].

EC can model conservation of total a.m.; GR is genetically unable to do this because its momentum tensor must be symmetric. Therefore, EC is a necessary extension of GR to model conservation of total a.m. in gravitational theory. We believe this work is the first proof of a necessary extension of classical GR since 1915. However, there is still no accepted empirical evidence for presence of torsion or EC at this time.

The key concepts of the derivation of EC from GR are:

Riemann–Cartan geometry is the minimal extension of Riemannian geometry that describes exchange of intrinsic and orbital a.m.

EC removes some or all gravitational singularities in GR in regions of high spin density, specifically in black holes and Big Bang cosmological models.

EC introduces a spin contact force that is strong in regions with high spin density. This force is a viable candidate as a cause of cosmic inflation in the early universe.

EC can model conservation of intrinsic plus and orbital a.m. at the scale of high energy physics. See section 3.4 [38].

3.3. Intuitive Geometric Interpretation of Torsion

Torsion, which is translational curvature, is defined analogously to rotational curvature.

(2) Translational Curvature = $\text{Limit}_{[\text{area inside loop path}\to 0]}$ (Translational holonomy/(area inside the loop))

Torsion can be represented as a continuum distribution of discrete defects called dislocations in a spacetime lattice.

A screw dislocation in a regular lattice structure is like a parking garage ramp that connects two adjacent levels in a garage with a spiral ramp. A screw dislocation is present in all Fermions (**Figure 3**).

An edge dislocation is like the end of a level in parking garage where a railing prevents cars from driving over the edge (**Figure 4**).

Together, these dislocations enable spin-orbit coupling, which completes the law of conservation of a.m. in EC, neither of which GR can model. We conjecture

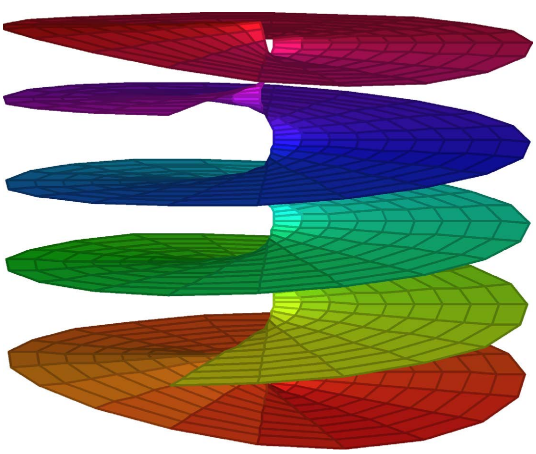

Figure 3. Graphic representation of a screw dislocation.

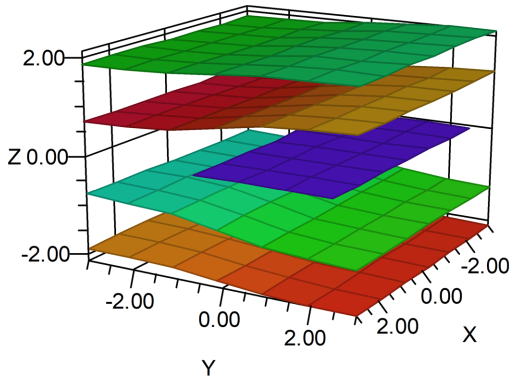

Figure 4. Graphic representation of an edge dislocation.

that edge dislocations enable conservation of a.m. in para-positronium, which intermediates annihilation of an electron and a positron with opposite z-spins. If the virtual bound state carries torsion, then the local law of conservation of a. m. works over the spacelike separation between the two leptons during annihilation (**Figure 5**).

3.4. Torsion Trace in Virtual Bound States

It appears that some virtual bound states carry affine torsion, which completes the law of conservation of a.m. For example, a virtual bound state positronium occurs in Fermion pair annihilation may carry torsion trace to complete the law of conservation of a.m. during pair annihilation of an electron and a positron with opposite z-spins. If the virtual bound state carries torsion, then the local law of conservation of a.m. can hold over the spacelike separation during annihilation [39].

- The left-handed screw dislocation on the left is depicted in Figures 3 and 5. Its torsion is $T_{yx}{}^{Z}$.
- The right-handed screw on the right is depicted in Figure 5 Its torsion is $T_{xy}{}^{Z}$.
- The edge dislocation has the orientation depicted in Figure 4. Its torsion is $T_{zy}{}^{Z}$.
- Above the screw and edge dislocations is a flat layer of affine lattice without defects, with part of the layer removed to expose the edge dislocation.

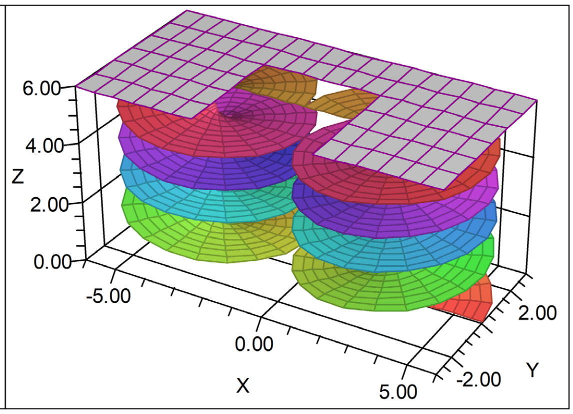

Figure 5. Two screws dislocations of opposite helicity joined by an edge dislocation.

The author is not aware of any other configuration in which torsion trace appears in basic physics.

3.5. Mach's Principle: Linear Momentum as Projective Translations at Conformal Infinity

The article "Momentum as Translations at Conformal Infinity" derives linear 4-

momentum in EC as the Noether current of translation symmetries in EC; by varying the action of EC with respect to translations in the Poincaré group] [40].

A "folk theorem" of GR is that linear momentum is the Noether current of translation symmetries in GR. The support for this "theorem" is that varying the action of GR with respect to a spacetime vectorfield yields the linear momentum tensor. However, the definition of a Noether current is that variation of the action with respect to a symmetry in the structure group yields a conserved current. Linear momentum is the only term in the field equations of GR that is not derived or interpreted with differential geometry. In EC, momentum is generated by an element of the structure group.

In this section, the structure group includes conformal inversions. This provides a constructive interpretation of Mach's Principle – that linear momentum is generated by interactions with large mass at conformal infinity. Consequently, linear momentum can be interpreted as a Noether current arising from a symmetry in the structure group.

4. Conformal Dilation Symmetry and Quantum Wave Mechanics

EC can express the quantum terms in a Lagrangian of the Klein–Gordon {KG} equation in terms of a modified spacetime metric [41]. Part I interprets the quantum terms in the Lagrangian as scalar curvature of conformal dilation covector nm that is proportional to ħ times the gradient of wave amplitude R. Part II replaces conformal dilation with a conformal factor ρ that defines a modified spacetime metric g' = exp (ρ) g, where g is the gravitational metric. Quantum terms appear only in metric g' and its metric connection coefficients. Metric g' preserves lengths and angles in classical physics and in the domain of the quantum field itself. g' combines concepts of quantum theory and spacetime geometry in one geometric structure. The conformal factor can be interpreted as the limit of a distribution of inclusions and voids in a lattice that cause the metric to bulge or contract. All components of all free quantum fields satisfy the Klein-Gordon equation, so this interpretation extends to all quantum fields. The main conclusions of this article are:

Quantum terms in relativistic quantum wave equations have the structure of scalar curvature of conformal dilation in the connection of spacetime, plus a divergence term that can be ignored in action integrals. Planck's reduced constant ħ appears only as a scaling factor in the conformal dilation. The complex number *i* does not appear in the Lagrangian, or the wave equations.

The conformal dilation can be integrated to define a conformal factor ρ that defines a modified spacetime metric g' = exp (ρ) g. The theory can be expressed with g' without conformal dilation. The metrics g' and g are identical outside the domain of quantum fields. The connection preserves g' in classical physics and in the domain of quantum waves.

Conformal dilation and the modified spacetime metric can be interpreted geometrically as the continuum limit of a distribution of inclusions and voids in a lattice.

Every field component of every free quantum field theory satisfies the KG equation, so the results can be extended to all quantum field theories. Each quantum field has its own amplitude and its own conformal factor ρ.

The metric g' straddles the fields of quantum theory and affine geometry. Such a hybrid construction may help to bridge the gap between gravitational theory and quantum theory.

This symmetry can be interpreted as describing intrinsic angular momentum as a distribution of discrete defects called inclusions and voids in a spacetime lattice, as in **Figure 6** and **Figure 7** below.

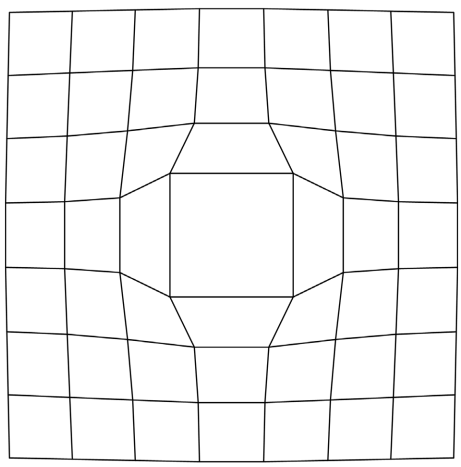

Figure 6. Schematic of an inclusion in a 2D lattice.

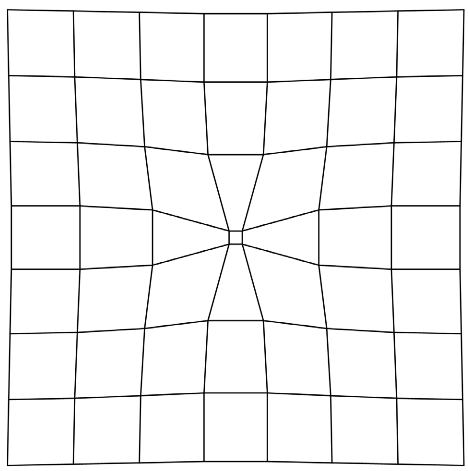

Figure 7. Schematic of a void in a 2D lattice.

5. Unitary Gauge Theories

Each gauge theory described so far has a structure group that is an extension of the symmetry group of real pseudo-Euclidean space. Except for GR, each theory has an isomorphism between tangents to the base manifold and a vector space of the associated fiber space, which introduces spacetime translations into the structure group. The spacetime underlying unitary gauge theories imports these structures to unitary gauge theories; but this is not the core of unitary gauge theories.

Unitary gauge theories are similar to post-Riemannian geometries in that they have a unitary connection that defines parallel translation of vectors in the unitary associated fibers along paths in the base manifold. Curvature in unitary gauge theory has the same basic definition as rotational (Riemannian) curvature and torsion (translational curvature).

(3) Unitary Curvature = $\text{Limit}_{[\text{area inside loop path} \to 0]}$ Unitary holonomy/(area inside the loop)

5.1. Classical Electrodynamics

The simplest unitary gauge theory is classical theory of electrodynamics (ED). The base manifold of ED is spacetime; the structure group is U (1), the fiber space F is a one-dimensional complex vector space; the 1-dimensional complex rotation group (the unit circle). The connection coefficients A_μ define the law of parallel translation of ED wave fields. The curvature tensor is the electromagnetic field, called the Maxwell tensor $F_{\mu\nu}$, which is the exterior derivative of the connection form.

5.2. Unitary Gauge Theory of High Energy Physics

Unitary gauge theory is the core of the Standard Model of high energy physics [42].

5.2.1. Elements of the Standard Model

Unitary gauge theories have:

Base manifold is (M, g) = spacetime M, dim (M) = n, with spacetime metric g that the connection preserves.

Structure group is a product of U (1), SU (2), and SU (3). It preserves the hermitian metric.

The connection form defines parallel translation of associated vectors and preserves the hermitian inner product.

Associated fiber H = complex vector space with dimension m, with hermitian inner product h. ("Hermitian" means ∀ u, v ∈ F, h (v, u) = h (u, v)* (where '*' means complex conjugate).

5.2.2. Intuitive Geometric Interpretation of Unitary Symmetry

Unitary symmetries suggest that in a neighborhood around each point of spacetime are internal symmetries that contain discrete defects of alignment. A physical analog for this structure is a crystal lattice with defects. Because of the misalignments, traversing a closed path in the crystal can generate holonomy of the crystal internal regular lattice, not affine holonomy of the affine structure of spacetime. In the theory of lattices, these misalignments are called "dispirations" (**Figure 8**).

The continuum limit of distributions of non-affine lattice defects, called dispirations by materials scientists, are non-affine connections, which form the mathematical foundation of Yang-Mills theories. Dispirations are defects in the orientation or structure of crystal basis elements which do not affect the affine or metric geometry of a lattice, as represented graphically in Figure 4. However, interpreting internal gauge symmetries as symmetries of crystal basis elements may not be equivalent to interpreting them as symmetries of compactified dimensions in all cases.

It is sensible to conjecture that all gauge theories are continuum limits of theories of lattice defects. In this view, differentiable structure and real topology are anthropocentric approximations to discrete structures. Differentiable structure consists of an infinitesimal flat affine structure at each point. Real numbers are the topological completion of the rational numbers, which arise from countably infinite subdivisibility of intervals.

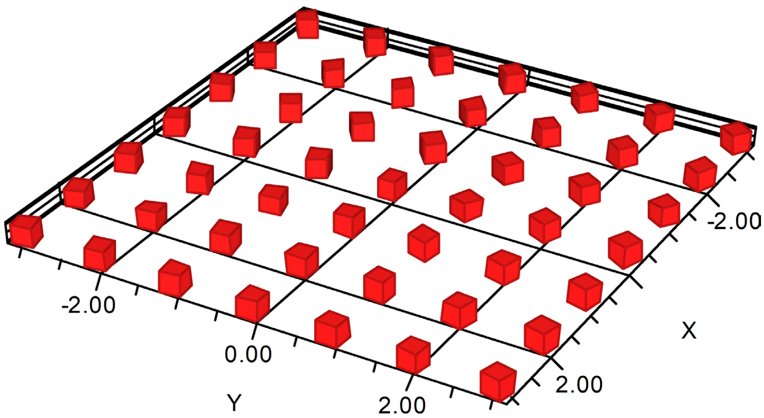

Figure 8. Graphic representation of a dispiration.

6. Proposed Discrete Geometric Foundation of Basic Physical Theories

The geometric structure of basic physical theories has two recurring themes [30] Why in the foundations of these theories do we encounter (1) real topology and differentiable structure, and (2) fiber bundles and connections that model distributions of discrete defects in lattice structures? Our answers to these questions support the conjecture that all physical theories exhibiting these basic structures are perturbation theories and theories of averages, approximating more detailed theories.

Topology attempts to formalize the concept of connectedness of sets; in particular boundary and dimension can be defined in terms of topology. Real topology is the simplest model of spacetime structure that allows classical macroscopic perceptions of spacetime–homogeneity, isotropy, scale invariance, subdivisibility–to be extended to the microscopic domain. Differentiable structure amounts to definition of an infinitesimal flat affine structure at each point on a topological manifold. Each fiber in a fiber bundle carries the perfect idealized symmetry of the theory. Connections on fiber bundles enable connecting the high-symmetry of local neighborhoods via parallel translation; curvature describes how the perfect symmetries are broken when parallel-translating between local fibers. Curvature describes the breaking of the idealized structure in each fiber as we translate between fibers.

In all these theories, curvature can be viewed as a continuum approximation to a distribution of discrete defects in a regular lattice [25].

In General Relativity, the defects are disclinations that model gravitation.

In addition to disclinations, Einstein-Cartan theory employs dislocations to include intrinsic angular momentum and exchange of intrinsic and orbital angular momentum [35] [36]. In addition, the hypothesized modeling intrinsic angular momentum in virtual particle interactions

In quantum wave mechanics, the defects are inclusion and voids that disturb the regular geometry of a lattice [41].

In unitary gauge theories, including electrodynamics as the simplest case, the defects are dispirations that are defects in the internal structure of each local neighborhood.

In all these theories, linear momentum can be interpreted as conformal inversion that makes a strange kind of connection between local momentum and momentum at an idealized infinity.

7. Potential Future Developments

We propose interpreting all these physical theories as continuum approximations to distribution of discrete defects in regular lattices. This suggests that future physical theories may treat differentiable structure and real topology as approximations to theories that focuses on distributions of discrete defects in regular lattices. The author believes it likely that the pattern of classical defects found in post–Riemannian geometry, and in real materials, is not a coincidence.

Below are some speculations about structures deeper than quantum theory that may be found from theories and experiments that are motivated by models from post-Riemannian geometry.

Gravitational force arises from distributions of discrete disclinations, which together create Riemannian curvature in and gravitational forces in GR.

Intrinsic a.m. in quantum theory arises from discrete distributions of two types of discrete dislocations in a regular lattice. Each Fermion contains at least one screw dislocation, and some or all virtual bound states contain edge dislocations that enable conservation of total a.m. in creation and annihilation of particle pairs. These dislocations complete the law of conservation of angular momentum.

The quantum terms in quantum wave fields arise from distributions of discrete inclusions and voids in regular lattices. Quantum wave fields are continuum approximations to the distribution of discrete inclusions and voids that, in post-Riemannian geometry, are conformal dilation fields. Projective conformal translations model linear momentum in a way that provides a constructive realization of Mach's Principle.

Unitary gauge theories model force potentials as dispirations of lattices.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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