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Assessing the Trade-Off between Voluntary and Forced Interventions to Control the Emergence of Recurring Pandemics—An Evolutionary Game-Theoretic Modeling

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Abstract

In this study, we aim to examine the dynamics of diseases by employing both voluntary and forced control strategies backed by evolutionary game theory (EGT). The impact of quarantine is investigated through our suggested framework provided that a partial adoption of voluntary vaccination is observed at the earlier stage. The combined and individual effect of dual preventive provisions are visualized through SEIR-type epidemic model. Additionally, the effect of coercive control policies' efficacy on individual vaccination decision is illustrated through the lens of EGT. We also consider the cost associated with vaccination and quarantine. The numerical simulations shown in our work emphasize how important it is to put quarantine rules in place to stop the spread of infection. These restrictions imposed by the government can be relieving, especially during times when a sizable section of the populace is reluctant to get vaccinated because of its ineffectiveness or excessive cost. We also show when and under what circumstances one policy works better than the other. How these policies' compliance rates should be calculated is therefore becomes a focal point of discussion. We support this claim by producing phase diagrams for three different evolutionary outcomes throughout our investigation and changing the two crucially important pick-up rate parameters, one connected with the quarantine policy and the other is related to the isolation policy, in various directions. We then additionally examine the efficacy and cost associated with different policy adaption. This model effectively highlights the importance of dual provisional safety in understanding public health issues by using the mean-field approximation technique, which aligns with the well-known imitation protocol known as individual-based risk assessment dy-

namics.

Keywords

Epidemic Modeling, Evolutionary Process, Vaccination Cost, Quarantine Rate, Social Payoff, Imitation Dynamics

Highlights

- The impact of forced interventions on compartmental epidemic model is presented using a game-theoretic framework.
- A thorough investigation shows the interplay between preemptive vaccination and quarantine policy in response to epidemic propagation.
- Cost and efficacy of several health interventions have been taken into account.
- Cheap yet effective vaccination plays an invaluable role in suppressing the outbreak of an epidemic.
- Forced provisions seem persuasive in restricting the rapid transmission of epidemics.

1. Introduction

In recent times, the well-being of our global population is being threatened by both new and recurring pandemic diseases. The occurrence and global dissemination of recurrent pandemics and severe epidemics pose significant threats to humanity. Consequently, comprehending and limiting the dissemination of infectious diseases is considered to be a pressing issue for our society. Recurring epidemics have profoundly detrimental effects on public health and impose substantial financial burdens on communities. The ensuing pandemic accentuates the necessity for comprehensive epidemic models to help shape public policy [1] [2]. Hence, it is crucial to assess potential approaches for effectively controlling the outbreak of these epidemic diseases. Implementing intervention measures to combat the spread of communicable emerging or re-emerging diseases often involves two common strategies: quarantining surmised susceptible or vaccinated people and isolating infected individuals whoever showing symptoms. However, it is quite important to note that these measures come with a partial cost for every individual within the society. Pre-emptive vaccination, as previously indicated, can be seen as an "individual-based provision' where each susceptible person determines whether or not to get vaccinated. Quarantine and isolation, in contrast, are seen as "public-based provisions' where the government urges people to participate regardless of their intentions. Investigating the effectiveness of these synthetic provisions in combination with several complementary measures to successfully control the spread of diseases is both important and educational. Vaccination has gained popularity as a highly successful method of disease prevention and eradication during the past few decades. Smallpox has been successfully eradicated, and other diseases such as various flu-like seasonal influenza, polio, measles, mumps, and rubella have significantly decreased. Latest vaccination programs have been crucial in modulating the prevalence of pediatric diseases including measles and whooping cough. Immunity brought on by vaccination may be permanent or may require periodic booster shots depending on the specific diseases involved and the effectiveness of each vaccine [3]-[5]. Additionally, by lowering the risks linked to the diseases, vaccination offers indirect protection to others who have not received it. They gain from this by having their worries about possible vaccination-related problems and side effects reduced, and they also save money on vaccination costs once herd immunity is established. It is crucial to remember that vaccines do not always confer full protection and can occasionally do so because the hosts natural immunity is fading. Additionally, erratic vaccination availability has the potential to lead to the reappearance and epidemic epidemics of specific infectious diseases. Quarantine and isolation are two frequently used imposed control interventions for treating infectious diseases, especially in situations of severe acute respiratory syndrome (SARS) [6]-[8]. Intervention like isolation is often considered as a better public health precaution whilst quarantine is morally more dubious. The two ethical issues with quarantine are that it confines people who might not be contaminated and that it frequently forces those who are not affected to be near those who are, increasing the likelihood that they will become infected later [9] [10]. Even a small number of sick people being removed from the main population helps the community's health, yet it may also violate peoples rights and freedoms. Biblical references to the exclusion of lepers and historical isolation of plague victims suggest that isolation--a successful intervention procedure for halting the spread of infectious diseases by reducing transmission to susceptible people have been one of the primitive infection control techniques. From the 15th to the 19th centuries, visiting ships suspected of bringing the plague were forbidden from interacting with other people in Mediterranean ports for a 40-day period. This restriction is what the name "quarantine" originally referred to. Quarantine used to mean holding and isolating people who were thought to be infected with diseases. The term "quarantine" refers to the enforced physical separation of healthy individuals who may have been exposed to an infectious disease, including restrictions on movement. It is worth clarifying the difference between terms like isolation and quarantine. The former refers to segregation and captivity of people who are found to be infected to prevent them from spreading the disease, while the latter infers to people who are suspected of doing so. Isolated people can be those who opt to skip out on school or work in the event of milder disorders. For more serious diseases, isolated people could include those who are required to obtain treatment in a hospital or those who live in remote areas without touch with those who are susceptible. However, in order for isolation to be effective, specific hospital cleanliness procedures are required in order to stop the transmission of infections within healthcare facilities, as various studies have examined [11]-[13]. The initial inquiry revolves around the adequacy of fundamental public health interventions vis-à-vis isolation (*i.e.*, separating symptomatic people from the healthy common population) and quarantine (*i.e.*, separating individuals who have been in contact with infected people yet do not exhibit symptoms), in controlling the transmission of the disease. If these foundational public health measures prove effective, the subsequent question arises as to when should both the isolation and quarantine be deployed? For instance, uncertainty prevails about which policy, amid isolation and quarantine, played the key role in preventing the transmission of SARS or whether both control measures were equally indispensable [5]. In order to effectively stop the transmission of infectious diseases, quarantine and isolation are used as measures to restrict contact between susceptible and infected persons. As evidenced by references in biblical scriptures on the exclusion of lepers, isolation of infected individuals was the major technique of managing communicable diseases in the past, before the development of modern medicine and scientific knowledge. This section provides a game for intervention that combines isolation, vaccination, and quarantine. The interventions include vaccination, seclusion, and quarantine. Our theoretical approach is fundamentally based on the idea that vaccination is an individuals voluntary strategy, influenced by Evolutionary Game Theory, whereas isolation and quarantine are universally enforced, regardless of an individuals purpose.

In the present era, the utilization of mathematical modeling has gained significant attention when it comes to examining the propagation and management of infectious diseases. Modeling approach takes the essential factors into consideration which influence the dissemination of epidemic, thereby aiding to forecast the disease transmission pattern over the course of time. To gain a more profound understanding of the dynamics of an epidemic disease, numerous scholars [14]-[25] have extensively explored meticulous mathematical frameworks involving nonlinear ODE-based mean-field approximation. These researchers have employed diverse control techniques to mitigate the scale of epidemic. Numerous mathematical models that examined the dynamics of vaccination with an emphasis on alternative vaccination schedules, varying hypotheses for vaccine uptake and efficacy, and the ensuing management of epidemics have already been published [14] [22] [26]. A few studies have also focused on creating methods for evaluating and quantifying vaccination efficacy as well as efficiency [22]. In order to gain a deeper understanding of the shifts in human behavior and decision-making processes that take place during an epidemic outbreak, a significant number of researchers [27]-[33] have already integrated the evolutionary game-theoretic approach into the dynamic exploration of mathematical epidemiology. Moreover, the utilization of statistical physics techniques has proven valuable in scrutinizing the progression of cooperation within social dilemma games [34] [35]. Lately, there has been a change in emphasis towards vaccination models that consider different levels of population awareness and the time delays caused by factors like latency or temporary immunity in the spread of infections. Moreover, the time it takes for people to react to the available information about the disease is also taken into consideration. Furthermore, complexity science and information systems can be vital in saving lives when traditional methods fail to accurately represent the actual behavior of the system [36]. To maximize its effectiveness, each vaccination program should be accompanied by relevant information campaigns that educate individuals about the importance of vaccination. These campaigns aim to curb the transmission of diseases and foster the desired level of herd immunity, thus maximizing the overall impact. As was the situation with the HPV vaccine in Romania and the MMR vaccine in the UK, negative media coverage has at times resulted in a significant decrease in vaccine acceptance or even a complete halt to the vaccination campaign [37]-[39]. Additionally, vaccine acceptance and success may be hampered by worries about potential side effects or false beliefs about vaccine efficacy. The majority of earlier studies on vaccination games heavily leaned on the multi-agent simulation (MAS) method, a potent tool for simulating intricate, sophisticated, and even 'realistic' events. Recently, Alam *et al.* [14] [22] devised analytical models utilizing a theoretical framework and MAS (Multi-Agent Systems) models. Their aim was to evaluate the complex processes involved in flawed vaccination policies when an alternative approach, such as an intermediate defense measure, exists within society. According to the authors of [40], implementing a combined control strategy for an SIR epidemic model that includes both isolation and vaccination can substantially reduce the required vaccine coverage to eliminate a disease. The quarantined or isolated people often suffer from various issues like physical inactivity, mental health, economic and social problems [41]-[43]. Their research also brings a novel feature of combined control that was not covered in [44], namely that the increase in variable population reduces the effect of isolation. Even if the efficacy of universal method is thought to be poor, Safi *et al.* [45] suggested that the quarantine and isolation policy be used in conjunction with an ineffective vaccination strategy can result in the complete eradication of disease. Ichinose *et al.* [46] found a reversal in the effect of social impact on voluntary vaccination depending on whether networks are heterogeneous or homogeneous. In another investigation focusing on the impacts of social status information, individuals were given the opportunity to utilize knowledge regarding the average social standing of their neighbors, as opposed to the social standing of a randomly selected individual neighbor [29].

According to study by various academics (for instance, [11]-[13]), in order for isolation to be effective, hospitals must maintain strict standards of sanitation. Numerous mathematical modeling studies have been conducted to evaluate the effectiveness of isolation and quarantine in preventing the spread of contagious diseases among human and animal populations [47]. Particularly, the 2003 outbreak of SARS prompted the development of several isolation and quarantine methods to stop the diseases spread. According to a deterministic model put up by Tray *et al*, the number of infections that can be prevented by quarantining is predicted to be extremely low as long as isolation is effective, but it rises sharply as effectiveness decreases. There have been numerous attempts to create mathematical models that accurately represent the dynamics of infectious disease transmission. These models were previously based on homogeneous networks, assuming a uniform transmission probability from infectious individuals to susceptible persons. This assumption is fundamentally flawed because physical interactions among individuals are diverse and rarely conform to a uniform pattern. Gradually, mathematical epidemiological models of the SEIR type have been used to evaluate the efficacy of various control techniques, such as policies of isolation and quarantine. Two SEIR-type epidemic models were described in a previous study [48], where the latent and infectious durations are assumed to have exponential and gamma distributions, respectively. In their assessment of the impacts of quarantine and isolation on the transmission of SARS, Gumel *et al.* discovered that an effective isolation policy played a more crucial role in reducing the spread of the disease compared to quarantine alone or a combination of quarantine and a less effective isolation policy [49]. By employing mean-field approximation, Alam *et al.* conducted a dynamic analysis of the SVEIR model to devise a simple yet effective approach for epidemic control management. This approach involves the simultaneous implementation of quarantine and isolation policies [5].

We are quite interested in performing a quantitative analysis into the contributions of two control mechanisms and the dynamics of a vaccination program, which is motivated by prior work in related disciplines. Our suggested mathematical model, which is based on an evolutionary game-theoretic methodology, is heavily concerned with evaluating the success of implementing isolation and quarantine regulations in current vaccination game models. Furthermore, we show the relevant areas and circumstances in which the isolation/quarantine strategy functions effectively while modifying the efficiency of the vaccination module.

The rest of the article is structured as follows. Section 2 introduces the epidemic game model and formulates the mathematical framework of the proposed model, briefly discusses the strategy updating protocols, payoff structure of the intervention strategies, and fundamentals of the evolutionary game theory. Section 3 reveals the disease dynamics considered in the recurring epidemic seasons, briefly covers parameter estimation and provides simulated results of the evolutionary outcomes. Finally, section 4 summarizes the key findings of this study and offers recommendations for future research.

2. Model Development

In our proposed model, we presume a population size that is limitless, optimally well-mixed, and devoid of any spatial organization. Additionally, based on their existing health status and the prevalence of the diseases, the population affected by the epidemic outbreak can be classified into different compartments, including susceptible (S), vaccinated (V), exposed (E), infected (I), and recovered (R). We also include new compartments, respectively quarantine (Q) and isolation (Is). Anyone who is susceptible to contracting the disease has been exposed to it. Individuals who have been exposed are frequently asymptomatic. A symptomatic exposed person may be examined as infected and vulnerable or immunized when asymptomatic. Thus, the asymptomatic exposed individuals are taken to quarantine. On the other hand, a person who is infected has disease symptoms, whereas a person who has recovered has developed immunity to the disease. An individual who has recovered is thought to be immune to re-infection. A symptomatic quarantined individual can later be examined as infected. A person who is infected is an exposed person is forcefully isolated from the general population, moving them from the infected (I) compartment to the isolation (Is) compartment. This precaution is made to reduce contact and stop the disease from spreading. An isolated person, on the other hand, is a person who has been identified as being infected and is as a result isolated from the general populace, frequently by being admitted to a hospital or rehabilitation facility. Policies involving quarantine and isolation are essential for controlling major epidemics because they successfully reduce contact between susceptible and infected populations. We assume that both susceptible (S) and vaccinated (V) individuals undergo a latent phase before changing to the exposed (E) stage in our theoretical model. The rate of infection transmission from susceptible (S) to the exposed (E) stage is represented as β (per day per person). A portion of those who are in the exposed (E) condition when symptoms first appear quickly go to the infected (I) state at a progression rate α. The remainder of the exposed (E) states asymptomatic persons are subsequently quarantined and transferred to the quarantine (Q) state at a constant quarantine rate ϵ. In contrast, at a constant isolation rate λ, a particular subset of infected people gets isolated and then are transferred to the isolation (Is) state. The disease recovery rate from infected compartment is indicated as γ_1 (per day), and the remaining afflicted people gradually move into the recovery condition (R). Additionally, those in the (Q) and (Is) states have the chance to recuperate at new rates of recovery known as γ_2 (day-1) and γ_3 (day-1), respectively. It is crucial to emphasize that not all exposed people will immediately recover. With a disease progression rate of α the majority of exposed people will inevitably move from the exposed state to the infected state. It is also true, though, that not every person who is exposed will unavoidably contract the disease if a quarantine policy is put in place. In fact, a portion of symptomatic exposed individuals will be forcibly quarantined at a rate known as ϵ before they have a chance to contract the disease. Individuals may still contract the infection even after being quarantined. There are two groups of people who are quarantined: immune people who confer perfect immunity with a probability of ζ (efficiency of quarantine) and non-immune people who do not develop quarantine-induced immunity with a complementary likelihood of $1-\zeta$. The proposed mathematical model thoroughly examines the interplay between the execution of quarantine-isolation measures (passive provision) and voluntary vaccination (active provision) during an epidemic outbreak. Different people may respond differently to vaccinations. The effectiveness of vaccination, indicated as e ($0 \le e \le 1$), is the proportion of recipients for whom the vaccine triggers an immunological response. Through numerous seasons, this efficacy value is consistent. Effective vaccination coverage is a key idea [14] [22] [26] and refers to the percentage of the population that gains immunity through vaccination. It is written as a variable with a positive value of x ($0 \le x \le 1$). Even if a person receives a vaccination and mounts an immunological response, this immunity could not last a lifetime. The diminishing immunity trait suggests that vaccinated people may gradually contract the disease again. Therefore, we divide preemptive vaccine recipients into two categories: vaccinated people, who have perfect immunity with a probability of e, and non-vaccinated people, who do not develop vaccine-induced immunity with a complementary chance of $1-e$. The basic reproduction number, R_0 ($R_0 = \frac{\beta}{\gamma}$), is defined throughout this work as the ratio of the infection spreading rate to the disease recovery rate.

2.1. Setup of the Proposed Model

We extend our model by including the new concepts: people who are vaccinated, people who are quarantined and people who are isolated, with a view to providing a full-fledged evolutionary game-theoretic framework. The seminal work of Kermack and McKendrick [50], who developed a deterministic SIR compartment model, has had some influence on this inclusion. Quarantine, isolation and imperfect vaccination are considered as different types of health interventions that can be implemented to restrain the rapid transmission of recurring pandemics. The dynamics of our proposed model is based on the assumption of unstructured population which we divide into seven mutually exclusive categories (see **Figure 1**), namely, Susceptible (S), Vaccinated (V), Exposed (E), Quarantined (Q), Infected (I), Isolated (Is), and Recovered (R).

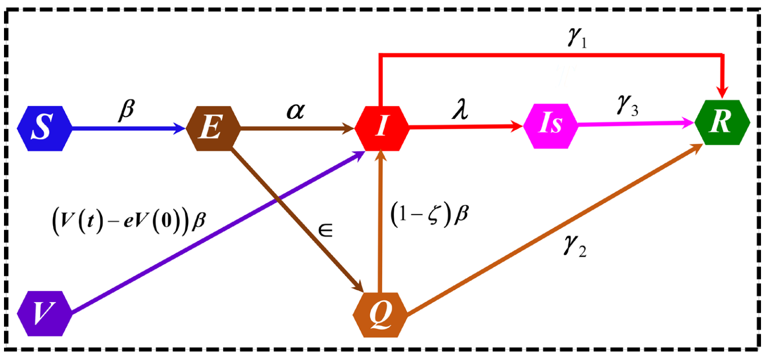

Figure 1. Schematic diagram of the proposed epidemic model.

The susceptible compartment consists of healthy people who have the potential to catch the disease through close contact with diseased persons. The rate of disease transmission may rise when vulnerable people come into touch with infected people. Quarantine and isolation are mandated measures, whereas vaccine providers actively participate in the vaccination campaign. However, because of the limits of these interventions, both the vaccinated and quarantined groups routinely interact with infected people, putting themselves at risk of infection. Based on our proposed models recovery rate, individuals who have totally recovered have developed immunity to the disease at γ_1, γ_2, γ_3 rates. Similarly, decreasing immunity reduces quarantines ability to completely protect against the same disease. So, to account for this, we need to propose a new measure termed "quarantine efficiency" Therefore, in order to keep things simple, our suggested approach takes into account two components, e and ζ, which respectively stand for the effectiveness of vaccination and the efficacy of quarantine ($0 \le e, \zeta \le 1$). In our dynamic configuration, all individuals can be divided into seven fractions at any given time t, designated as $S(t)$, $V(t)$, $E(t)$, $Q(t)$, $I(t)$, $Is(t)$, and $R(t)$. These fractions, in turn, stand for susceptible, vaccinated, exposed, quarantined, infected, isolated, and recovered individuals at any given time. We assume that $V(0)$, $Q(0)$, $Is(0)$ and indicate the initial fractions of immunized, quarantined and isolated people at the beginning of an epidemic season (at time $t = 0$). Meanwhile, $eV(0)$ and $\zeta Q(0)$ represent the proportion of people who receive the vaccination and the percentage of self-protectors who develop perfect immunity, respectively. However, the disease strain can still affect the ratios $(1-\zeta)Q(t)$ and $(V(t)-eV(0))$. As a baseline, we propose a set of governing non-linear ordinary differential equations (ODEs), entailing several preventive measures, that represent the overall dynamics of the epidemic paradigm. The set of governing differential equations of the proposed model are given below:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\beta S(t) I(t) \tag{1}$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -\beta\big(V(t) - eV(0)\big) I(t) \tag{2}$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\beta S(t) I(t) - \alpha E(t) - \epsilon Q(t) \tag{3}$$

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \epsilon E(t) - \beta(1-\zeta) Q(t) I(t) - \gamma_2 Q(t) \tag{4}$$

$$\frac{\mathrm{d}Is}{\mathrm{d}t} = \lambda I(t) - \gamma_3 Is(t) \tag{5}$$

$$\begin{aligned}\frac{\mathrm{d}I}{\mathrm{d}t} &= \alpha E(t) - \lambda I(t) + \beta\big(V(t) - eV(0)\big) I(t) \\ &\quad + \beta(1-\zeta) Q(t) I(t) - \gamma_1 I(t)\end{aligned} \tag{6}$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \gamma_1 I(t) + \gamma_2 Q(t) + \gamma_3 Is(t) \tag{7}$$

The anticipated set of initial conditions for the governing differential equations has the following structure: $S(0) = 1 - x - y \ge 0$, $V(0) = x \ge 0$, $E(0) \ge 0$, $Q(0) = y \ge 0$, $I(0) \ge 0$, $Is(0) \ge 0$, $R(0) \ge 0$. The fractions x, y, and $1 - x - y$, respectively, indicate the proportions of vaccinated, quarantined, and susceptible people at the start of an epidemic season. For the unstructured population, we use a mean-field approach to streamline the analysis. We assume that the population as a whole is equal to unity rather than considering the actual population size. As a result, each fraction related to a specific compartment keeps its value constant. As a result, the total population can be expressed numerically as follows:

$$S(t)+V(t)+E(t)+Q(t)+I(t)+Is(t)+R(t)=1$$

The system of dynamical equations has possible solutions that are all constrained on the range $[0,\infty)$ and positive for all $t \geq 0$. The model parameters β, α, e, ϵ, ζ, λ, γ_1, γ_2, and γ_3 are all positive as well. We categorize people into six groups depending on their health status and overall cost burden using a game-theoretic structure (see Table 1).

Table 1. Four types of people's proportions observed in the social equilibrium scenario (considered at the steady state situation).

Strategy/State	Healthy	Infected
Vaccinated (V)	HV	IV
Quarantined (Q)	HQ	IQ
Free-riders (F)	SFR	FFR

2.2. Game Payoff Structure

The exact expenses of quarantine can vary greatly based on a number of variables, including the length of quarantine, the location, the circumstances of individual, and governmental regulations. We consider that government pays the full cost of isolation. A normal epidemic season ends when everyone has moved to the healed state, R. Following thorough payoff analysis, individuals can modify or adjust their current practices. The outcome of an individual throughout an epidemic season quantifies his or her final benefit or loss in terms of social compensation is guaranteed, as well as costs. We utilize the relative cost of vaccination, $C_r = \frac{C_v}{C_i}$ ($0 \leq C_r \leq 1$), where C_r and C_i stands for the prices of vaccination and infection, respectively, to calculate everyones payoff. We can categorize all individuals into six classes using a game-theoretic approach based on their final health condition-either healthy or diseased-at the Nash equilibrium point. This equilibrium is defined as a local time scale within a single season, which is considered to be infinite, as shown in Table 2.

Table 2. Payoff structure projections at the end of each epidemic season.

Strategy/Status	Healthy	Infected
Vaccinated (V)	$-C_r$ (HV)	$-C_r-1$ (IV)
Quarantined (Q)	$-C_q$ (HQ)	$-C_q-1$ (IQ)
Free-riders (F)	0 (SFR)	-1 (FFR)

Deciding to get vaccinated is a crucial choice that hinges on weighing the costs of prevention against the possible dangers one may face during quarantine. By choosing vaccination, individuals can significantly reduce their health risks and contribute to the overall safety of their community, making it a proactive and responsible decision during uncertain times. Therefore, dependent on the epidemic incidence seen in the community, their choice may change throughout time for getting vaccinated. Therefore, using the average social payoff $\langle \pi \rangle$, average vaccinated payoff $\langle \pi_V \rangle$, average quarantined payoff $\langle \pi_Q \rangle$ and average non-vaccinated (defective) payoff $\langle \pi_F \rangle$ for the corresponding strategy modifications that are listed below, we can calculate the overall predicted payoff:

$$\langle \pi \rangle = -C_r HV - (C_r + 1) IV - (C_q + 1) IQ - FFR \tag{8}$$

$$\langle \pi_V \rangle = -\left(C_r HV + (C_r + 1) IV\right) / x \tag{9}$$

$$\langle \pi_Q \rangle = -\left(C_q HQ + (C_q + 1) IQ\right) / y \tag{10}$$

$$\langle \pi_F \rangle = -FFR / (1 - x - y) \tag{11}$$

2.3. Individual-Based Risk Assessment (IB-RA) Strategy Updating Rule

The most popularly used rule for strategy updating is based on the pair-wise Fermi function. By comparing payoffs in their original application of the agent-based vaccination game model, Fu *et al.* [51] determined the likelihood that players would adopt another players strategy by using the pair-wise Fermi function [32] [35]. This idea is utilized in the IB-RA strategy update. Here, a person randomly chooses one of his surrounding neighbors to make decision on whether or not to follow his neighbors existing strategy based on result obtained from Pair-wise Fermi function. In other words, if agent m chooses neighbor n for comparison, m will probabilistically copy n's strategy ($S_m \leftarrow S_n$).

$$Pr(S_m \leftarrow S_n) = \frac{1}{1 + \exp\left[\frac{-(\pi_n - \pi_m)}{\kappa}\right]} \tag{12}$$

where parameter $\kappa > 0$ defines the degree of selection, *i.e.*, the sensitivity of individuals to their payoff differences (selection pressure); a smaller value indicates that an individual is more sensitive to a payoff difference, where π_m is m's payoff assured in the preceding epidemic season. We used $\kappa = 0.1$ in the current study as a baseline value because it was commonly used as an example of a typical selection pressure in earlier research [29] [52] [53]. According to our present evolutionary worldview, there are six different types of people. There are three main tactics from which one can choose. Thus, based on the IB-RA update protocol, one of the twenty-four possibilities below cover up all the transition probability. The following table lists all transition probabilities based on the IB-RA strategy update rule:

$$Pr(HV \leftarrow HQ) = \frac{1}{1+\exp\left[-\frac{-C_q-(-C_r)}{\kappa}\right]},$$

$$Pr(HV \leftarrow IQ) = \frac{1}{1+\exp\left[-\frac{-C_q-1-(-C_r)}{\kappa}\right]}$$

$$Pr(HV \leftarrow SFR) = \frac{1}{1+\exp\left[-\frac{0-(-C_r)}{\kappa}\right]},$$

$$Pr(HV \leftarrow SFR) = \frac{1}{1+\exp\left[-\frac{-1-(-C_r)}{\kappa}\right]}$$

$$Pr(IV \leftarrow HQ) = \frac{1}{1+\exp\left[-\frac{-C_q-(-C_r-1)}{\kappa}\right]},$$

$$Pr(IV \leftarrow IQ) = \frac{1}{1+\exp\left[-\frac{-C_q-1-(-C_r-1)}{\kappa}\right]}$$

$$Pr(IV \leftarrow SFR) = \frac{1}{1+\exp\left[-\frac{0-(-C_r-1)}{\kappa}\right]},$$

$$Pr(IV \leftarrow FFR) = \frac{1}{1+\exp\left[-\frac{-1-(-C_r-1)}{\kappa}\right]}$$

$$Pr(HQ \leftarrow HV) = \frac{1}{1+\exp\left[-\frac{-C_r+C_q}{\kappa}\right]},$$

$$Pr(HQ \leftarrow IV) = \frac{1}{1+\exp\left[-\frac{-C_r-1+C_q}{\kappa}\right]}$$

$$Pr(HQ \leftarrow SFR) = \frac{1}{1+\exp\left[-\frac{0+C_q}{\kappa}\right]},$$

$$Pr(HQ \leftarrow FFR) = \frac{1}{1+\exp\left[-\frac{-1+C_q}{\kappa}\right]}$$

$$Pr(IQ \leftarrow HV) = \frac{1}{1+\exp\left[-\frac{-C_r+C_q+1}{\kappa}\right]},$$

$$Pr(HQ \leftarrow FFR) = \frac{1}{1+\exp\left[-\frac{-1+C_q}{\kappa}\right]}$$

$$Pr\left(IQ \leftarrow SFR\right) = \frac{1}{1+\exp\left[-\frac{0+C_q+1}{\kappa}\right]},$$

$$Pr\left(IQ \leftarrow FFR\right) = \frac{1}{1+\exp\left[-\frac{-1+C_q+1}{\kappa}\right]}$$

$$Pr\left(SFR \leftarrow HV\right) = \frac{1}{1+\exp\left[-\frac{-C_r-0}{\kappa}\right]},$$

$$Pr\left(SFR \leftarrow IV\right) = \frac{1}{1+\exp\left[-\frac{-C_r-1-0}{\kappa}\right]}$$

$$Pr\left(SFR \leftarrow HQ\right) = \frac{1}{1+\exp\left[-\frac{-C_q-0}{\kappa}\right]},$$

$$Pr\left(SFR \leftarrow IQ\right) = \frac{1}{1+\exp\left[-\frac{-C_q-1-0}{\kappa}\right]}$$

$$Pr\left(FFR \leftarrow HV\right) = \frac{1}{1+\exp\left[-\frac{-C_r+1}{\kappa}\right]},$$

$$Pr\left(FFR \leftarrow IV\right) = \frac{1}{1+\exp\left[-\frac{-C_r-1+1}{\kappa}\right]}$$

$$Pr\left(FFR \leftarrow HQ\right) = \frac{1}{1+\exp\left[-\frac{-C_q+1}{\kappa}\right]},$$

$$Pr\left(FFR \leftarrow IQ\right) = \frac{1}{1+\exp\left[-\frac{-C_q-1+1}{\kappa}\right]}$$

The current theoretical framework ensures that no opposing tendency arises when SB-RA is used instead of IB-RA in the strategy updating process. Therefore, the current work solely addresses the epidemic scenarios appeared from the IB-RA strategy updating rule. This update rule served as the foundation for all visual data used in the current investigation, as seen in the accompanying figures. Everyone has the option to adjust their strategy depending on the previous seasons payoff considered at the termination of each epidemic season. Thus, the global change in vaccine coverage x, as well as in quarantine coverage y, either increase or decrease, is unavoidable. Here, the independent variable, t, denotes the global timescale, or, to put it another way, the number of epidemic seasons that have passed. Taking everything into consideration just one set of dynamical equations needs to be developed for the pertinent instance.

The dynamical equation stating the rate of change of vaccinators:

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} = {} & HV \cdot HQ \cdot \left(Pr\left(HQ \leftarrow HV\right) - Pr\left(HV \leftarrow HQ\right)\right) \\ & + HV \cdot IQ \cdot \left(Pr\left(IQ \leftarrow HV\right) - Pr\left(HV \leftarrow IQ\right)\right) \\ & + HV \cdot SFR \cdot \left(Pr\left(SFR \leftarrow HV\right) - Pr\left(HV \leftarrow SFR\right)\right) \\ & + HV \cdot FFR \cdot \left(Pr\left(FFR \leftarrow HV\right) - Pr\left(HV \leftarrow FFR\right)\right) \\ & + IV \cdot HQ \cdot \left(Pr\left(HQ \leftarrow IV\right) - Pr\left(IV \leftarrow HQ\right)\right) \\ & + IV \cdot IQ \cdot \left(Pr\left(IQ \leftarrow IV\right) - Pr\left(IV \leftarrow IQ\right)\right) \\ & + IV \cdot SFR \cdot \left(Pr\left(SFR \leftarrow IV\right) - Pr\left(IV \leftarrow SFR\right)\right) \\ & + IV \cdot FFR \cdot \left(Pr\left(FFR \leftarrow IV\right) - Pr\left(IV \leftarrow FFR\right)\right)\end{aligned} \tag{13}$$

The dynamical equation representing the rate of change of quarantined individuals:

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}t} = {} & HQ \cdot HV \cdot \left(Pr\left(HV \leftarrow HQ\right) - Pr\left(HQ \leftarrow HV\right)\right) \\ & + HQ \cdot IV \cdot \left(Pr\left(IV \leftarrow HQ\right) - Pr\left(HQ \leftarrow IV\right)\right) \\ & + HQ \cdot SFR \cdot \left(Pr\left(SFR \leftarrow HQ\right) - Pr\left(HQ \leftarrow SFR\right)\right) \\ & + HQ \cdot FFR \cdot \left(Pr\left(FFR \leftarrow HQ\right) - Pr\left(HQ \leftarrow FFR\right)\right) \\ & + IQ \cdot HV \cdot \left(Pr\left(HV \leftarrow IQ\right) - Pr\left(IQ \leftarrow HV\right)\right) \\ & + IQ \cdot IV \cdot \left(Pr\left(IV \leftarrow IQ\right) - Pr\left(IQ \leftarrow IV\right)\right) \\ & + IQ \cdot SFR \cdot \left(Pr\left(SFR \leftarrow IQ\right) - Pr\left(IQ \leftarrow SFR\right)\right) \\ & + IQ \cdot FFR \cdot \left(Pr\left(FFR \leftarrow IQ\right) - Pr\left(IQ \leftarrow FFR\right)\right)\end{aligned} \tag{14}$$

3. Result and Discussion

3.1. Time Series Investigation for a Typical Single Season

First, we exclude any game-related considerations and restrict our discussion to epidemic dynamics alone. For the sake of simulating the model, we assume that a number of people (exact, $V(0) = 0.45$) use vaccine strategies. This implies that the vaccine provides a 45% coverage. For the time being, we will use the starting percentage of exposed, quarantined infected, isolated persons as $E(0) = Q(0) = I(0) = Is(0) = 0.0001$, which means there are only a few (nearly none) exposed, quarantined, infected, isolated people living in the population at the onset of an epidemic season. During a single epidemic episode, the disease spreads, reaches an equilibrium point known as the infection peak, and gradually disappears. To emphasize the stability of the epidemic model for future evolutionary processes, it is important to note that our analysis is limited to a single epidemic season before advancing to the evolutionary game-theoretic section. To demonstrate the suitability of our proposed model, we conducted a thorough sensitivity analysis at the equilibrium point using the time-series discretization approach. This analysis aims to determine how our model differs from the default one under various mathematical scenarios. First, we will illustrate how effectively quarantine and isolation measures are implemented to prevent the spread of the disease over a single epidemic season [54]. We assume that the set of parameters is comprised of $\beta = 0.8333$, $\gamma_1 = 0.3333$, $\gamma_2 = 0.2$, $\gamma_3 = 0.24$, $\alpha = 0.75$, $\epsilon = 0.4$, $\lambda = 0.05$, $\zeta = 0.2$, and $e = 0.75$. Here, two additional parameters ϵ and λ, which represent pick-up rates for quarantine and isolation policies, respectively, are included to quantify the amount of each type of policy needed to safeguard or restore public health. The coexistence of both quarantine and isolation policies occurs if both of these parameters take non-zero positive values. Incorporating quarantine is believed to be crucial yet suitable for reducing the transmission of COVID-19. The sooner the actions, the less destructive the impact [55] [56]. However, some quarantined COVID-19 patients may show critical symptoms even after completing quarantine due to a long median incubation period, essentially causing community transmissions [57]. The relative attribution of quarantine and isolation policies in the case of a policy that applies both of these measures can be established by the positive values given to each of the relevant parameters. At times quarantine is referred to as strict isolation that can be applied to confirmed infected cases [58]. On time-series line graphs for all conceivable scenarios, including no policy, quarantine policy, isolation policy, and both (quarantine and isolation) policy cases (portrayed in **Figure 2**), individuals' fractions lying in different compartments, such as susceptible, vaccinated, exposed, quarantined, infected, isolated, and recovered at a social equilibrium state, are shown. In all four cases, recovery rates often increase monotonically, with lowering susceptibility rates acting as the main driving force. A situation of social equilibrium results from a decline in the percentage of the population that has received vaccinations. Even people who receive vaccinations cannot be assured to be resistant to the disease they are spreading because vaccines aren't 100% reliable. Because vaccinated persons do not always remain healthy after receiving the vaccine, the proportion of those who have had vaccinations declines and the proportion of those who have become ill rises. In the absence of any governmental action, there is a noticeable increase in this tendency.

We display the infection graphs seen in all cases (see **Figure 3**) under the same framework once more to facilitate a more detailed study. For a single epidemic season depicted in **Figure 3**, when both strategies are played along produce the fastest rate of infection drop among the four choices, which amply validates their better performance. Quarantine or isolation are the best options for a combination policy when analyzing the quantitative spread of an infection. **Figure 4** shows the rate of decline of the infected fraction for various levels of isolation and quarantine. In contrast, when the parameters ϵ and λ are kept as constant, the reduction of infected fraction is slower. We establish that at the equilibrium point (shown in **Figure 5**), the infection dissipates more quickly and the fraction of recovered patients increases noticeably when the pre-emptive vaccination program and quarantine measures are utilized at an increased rate. We can verify

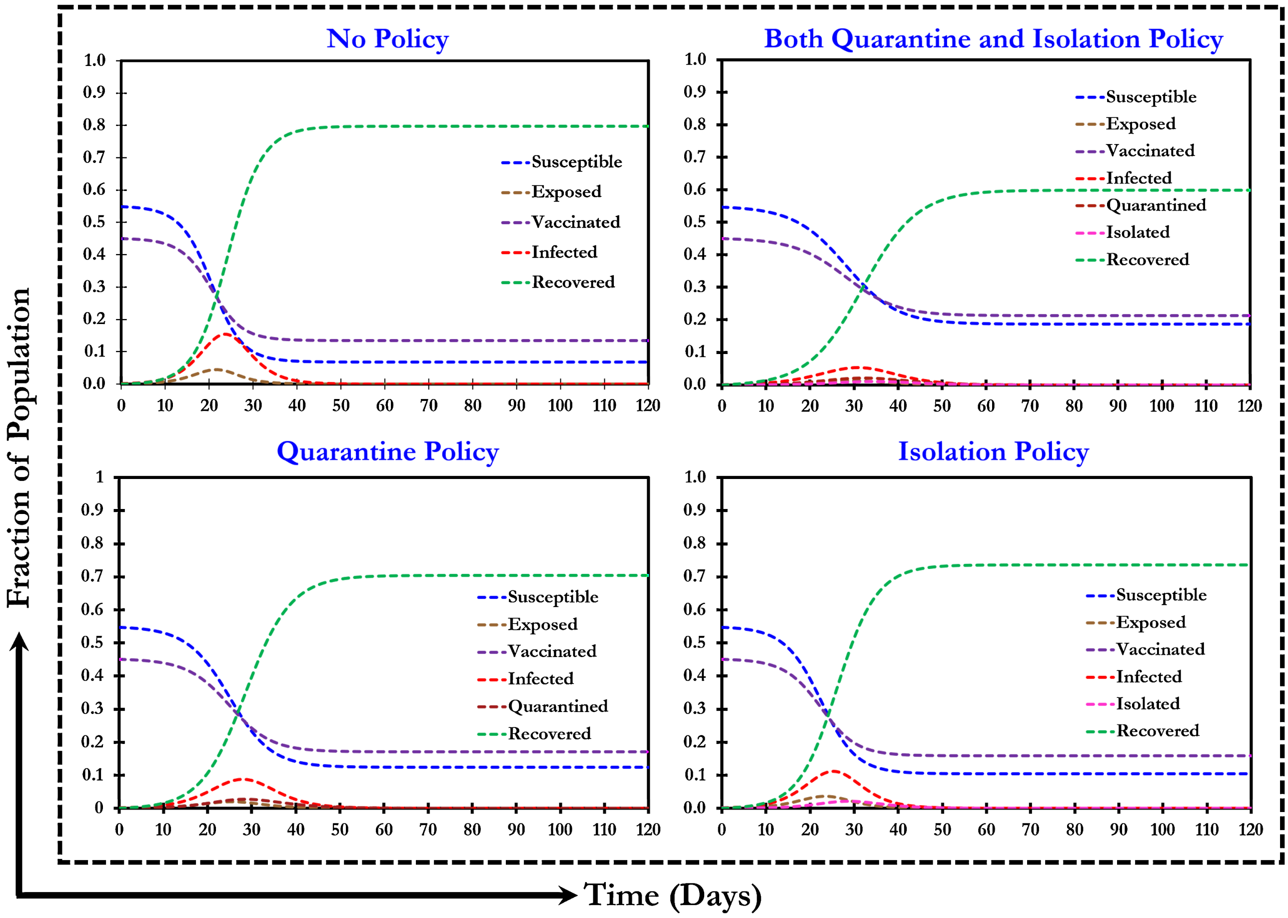

Figure 2. The proportion of individuals present in each of the compartments at local equilibrium using time evolutionary line graphs of four distinct schemes based on their policy selection: according to no policy, quarantine policy, isolation policy, both quarantine and isolation policy. When there is no policy condition, the maximum percentage of exposed people is seen. When both (quarantine and isolation) or quarantine policy is implemented, the exposed fraction expires significantly sooner. The baseline values of the parameters are taken to be $\beta = 0.8333$, $\gamma_1 = 0.3333$, $\gamma_2 = 0.2$, $\gamma_3 = 0.24$, $\alpha = 0.75$, $\epsilon = 0.4$, $\lambda = 0.05$, $\zeta = 0.2$. In order to keep things simple, we select $\epsilon = \lambda = 0.0$ to illustrate a no policy condition $\epsilon = 0.4$, $\lambda = 0.05$ for a joint policy condition, $\epsilon = 0.4$ and $\lambda = 0.0$ for a quarantine policy condition, and $\epsilon = 0.0$, $\lambda = 0.05$ for justifying an isolation policy condition.

that our proposed model for recurring seasons based on EGT is feasible by using time-series discretization to show the existence of this equilibrium. The *SVIR* (considering both effectiveness and efficiency mechanism) and *SVEQIIsR* models are contrasted in the Figure 6.

The initial infection rate was set as $I(0) = 0.0001$. It is evident that the epidemic state persists for a longer period of time to start decreasing in the *SVIR* model than in the *SVEQIIsR* model for the same starting infected fraction. Also, the *SVIR* models have higher infection peak. This result suggests that three techniques outperform one strategy. In Figure 7, we provide the time series trajectory of the disease propagation trend observed at three different transmission rates while holding the recovery rate constant for all three cases to demonstrate the effect of disease transmission rate on epidemic resurgence. It justifies the fact that the initial percentage of sick people has less of an impact on the epidemic dynamics

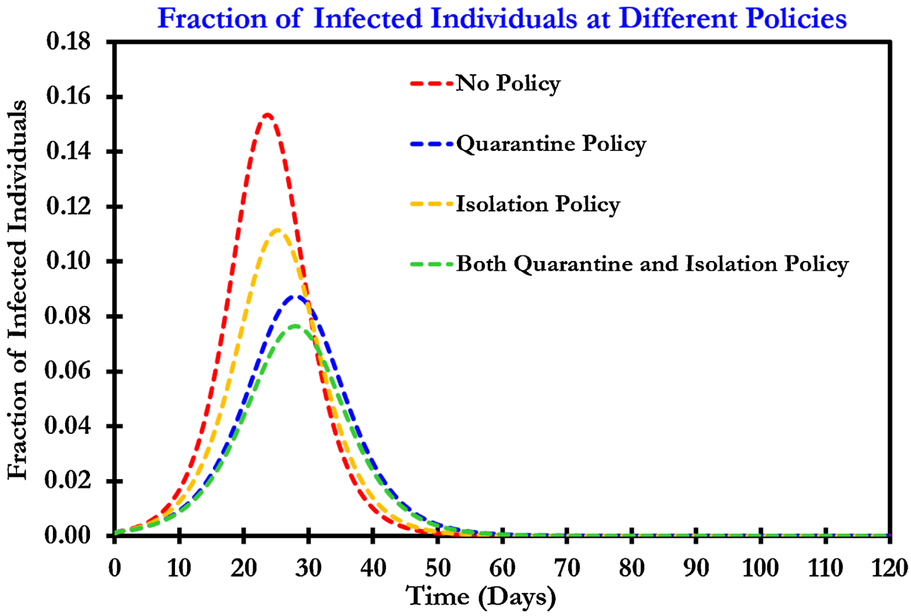

Figure 3. The trajectory of infected fraction produced from four different policy circumstances is completely controlled by two pick up rates, namely quarantine rate ϵ and isolation rate λ. The time development of the infected density is observed under different policies. We use $\epsilon = \lambda = 0.0$ for no policy, $\epsilon = 0.4$, $\lambda = 0.0$ for quarantine policy, $\epsilon = 0.0$, $\lambda = 0.05$ for isolation policy, and $\epsilon = 0.4$, $\lambda = 0.05$ for joint policy while generating the diagram. Positive constants are the initial values for each of the parameters, $\beta = 0.8333$, $\gamma_1 = 0.3333$, $\gamma_2 = 0.2$, $\gamma_3 = 0.24$, $\alpha = 0.75$, $\zeta = 0.2$. Additionally, the vaccines season-long efficacy is calculated as $e = 0.2$. For the situation of no policy, the infected proportion seen at its pick point is approximately 0.15. The infected fraction considerably decreases after the implementation of additional controls. We notice that the isolation strategy is inferior to the quarantine policy, whereas adopting both at once performs understandably well in limiting the amount of infection.

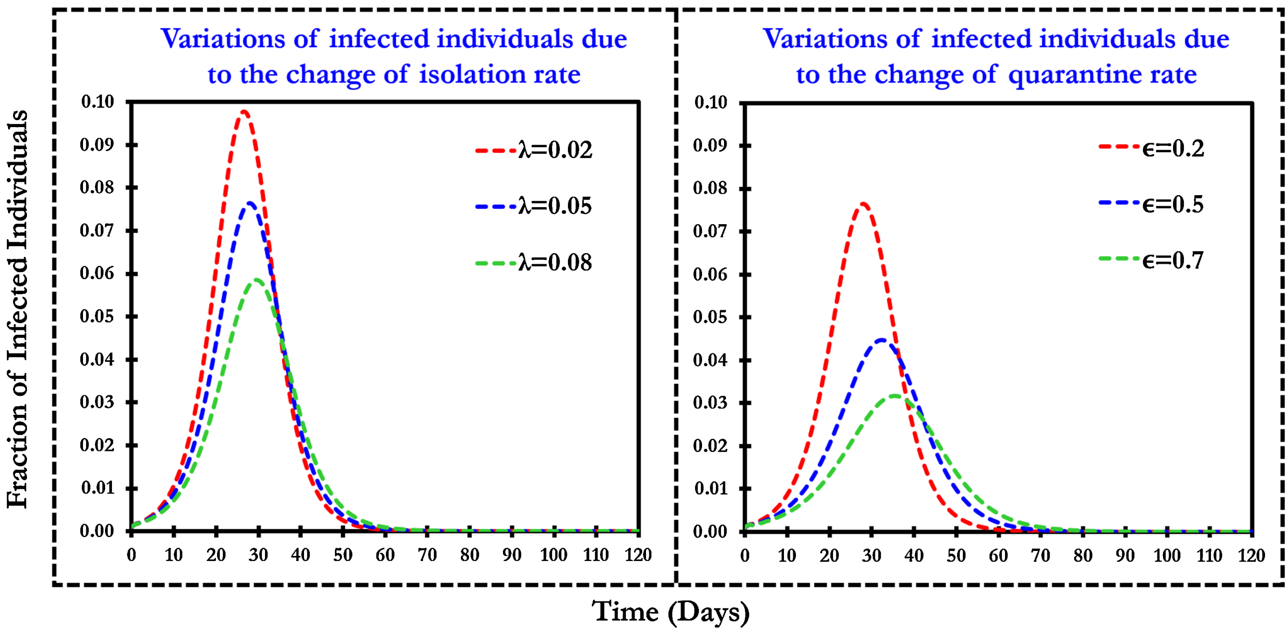

Figure 4. The seasonal variation in the rate of change of the infected fraction in response to changes in isolation and quarantine (by maintaining one rate constant, $\lambda = 0.05$ for the first one and $\epsilon = 0.4$ for the second one respectively) which implies that for the same rate of change the infected fraction decreases more quickly as quarantine rate increases. And that's a sure sign that quarantine is a more effective measure for controlling a pandemic than an isolation one.

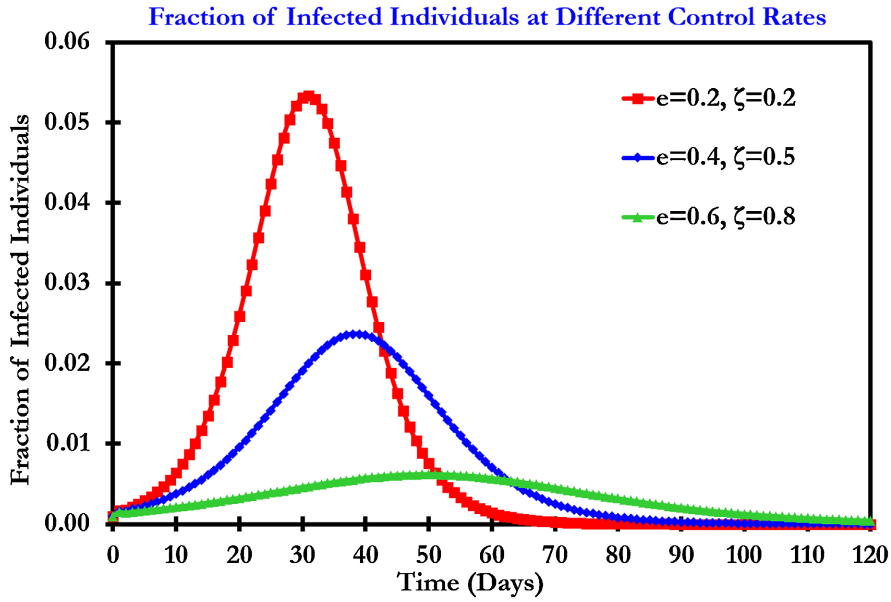

Figure 5. Fraction of infected individuals at the change of effectiveness and efficiency rate across time.

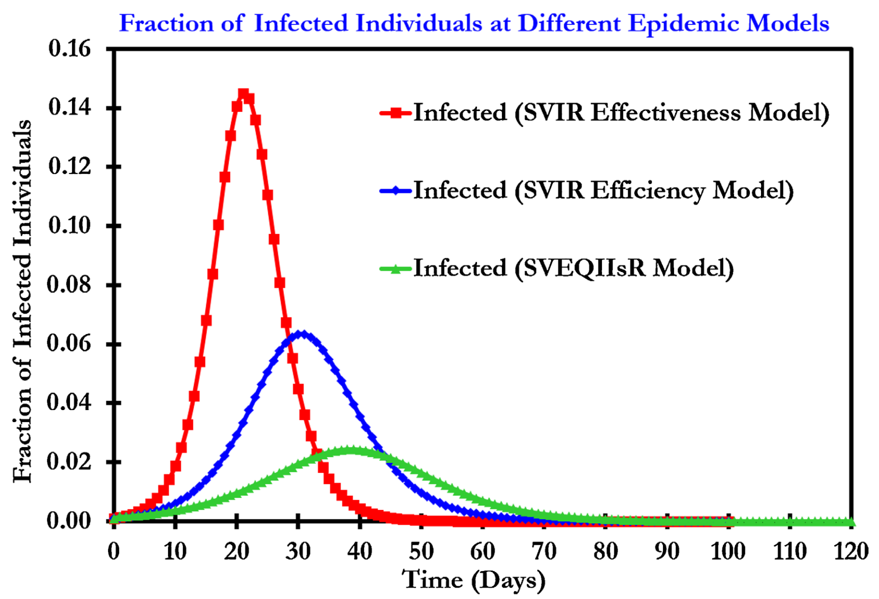

Figure 6. Comparison of the infectious ratios observed among the three epidemic models.

than the disease transmission rate. The infection peak can only be reduced when the authority enforces a proactive intervention effectively. This action is conventionally quantified by a higher sensitivity parameter [59] [60]. In sum, the chance of an epidemic spreading is greater the higher the rate.

3.2. Epidemic Threshold Analysis

We now turn our focus on employing an evolutionary game-theoretic approach for numerous epidemic seasons or generations as an advanced stage of our mathematical research to provide a general concept of the presence and severity of the

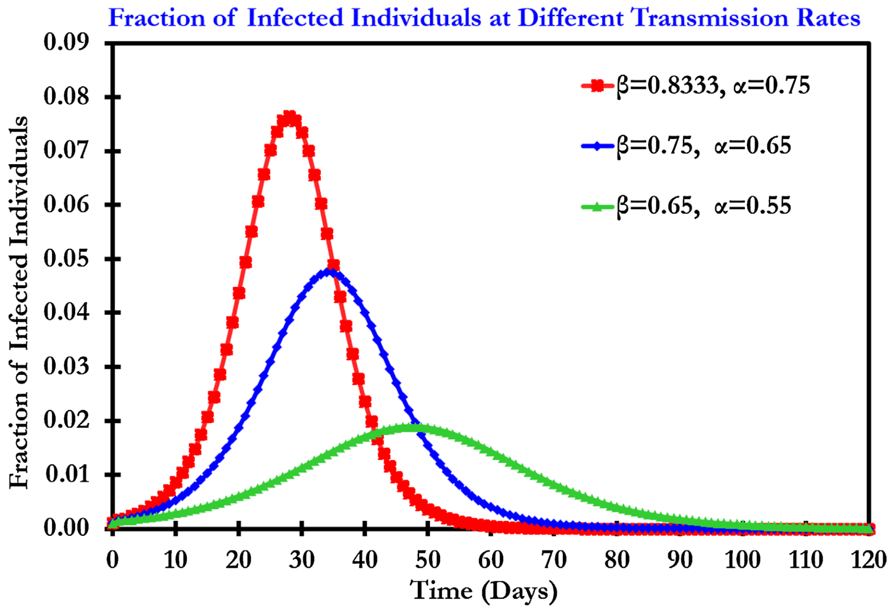

Figure 7. Counts of affected people over the time at various transmission rates.

epidemic disease over an extended period of epidemic resurgence. For better understanding of the complex mechanism, we would introduce three crucially important evolutionary parameter, namely, the final epidemic size (FES), vaccination coverage (VC), average social payoff (ASP) to calculate the amount of protection that each intervention program contributes to. In order to estimate the epidemic threshold (ET), or the critical FES, VC, and ASP as a function of vaccine efficacy, e, we draw a series of line graphs (**Figures 8-10**). In **Figure 8**, we redefine the cost of vaccination for a thorough case study by taking into account five different categories: extremely cheap ($C_v = 0.1$), cheap ($C_v = 0.3$), average ($C_v = 0.5$), costly ($C_v = 0.7$), and fairly expensive ($C_v = 0.9$) using the IB-RA strategy updating protocol. Maintaining generality, we determine the baseline values for the model parameters in the following manner: $\beta = 0.8333$, $\alpha = 0.75$, $epsilon = 0.4$, $\lambda = 0.05$, $\zeta = 0.2$, $\gamma_1 = 1/3$, $\gamma_2 = 0.32$, $C_q = 0.1$.

Keeping all the other values intact, we take $\epsilon = 0.0$, $\lambda = 0.0$ for no policy, $\epsilon = 0.4$, $\lambda = 0.05$ for joint (combined) policy, $\epsilon = 0.4$, $\lambda = 0.0$ for quarantine policy, and $\epsilon = 0.0$, $\lambda = 0.05$ for isolation policy in **Figure 10**. In **Figure 9**, we take the values same as **Figure 8** considering the cost of vaccination $C_v = 0.2$ as constant and solely varying the rate of quarantine as very low ($\epsilon = 0.1$), low ($\epsilon = 0.3$), average ($\epsilon = 0.5$), high ($\epsilon = 0.7$), and very high ($\epsilon = 0.9$). In **Figure 8**, the FES is declining and the ASP is rising as vaccine costs rises. The increasing VC is observed till the condition $e < 0.4$ is met. As the vaccination efficacy increases than 0.4 ($e > 0.4$), the VC is seen to be falling. Hence when we examine the situation with $e = 0.4$ in detail, we find that the VC threshold has two separate phases: a phase that is increasing and a phase that is dropping. When $e < 0.4$, the former is observed, meanwhile when $e \geq 0.4$, the later one is certain. Now let us look at the comparative epidemic prevalence scenario shown in **Figure 10**.

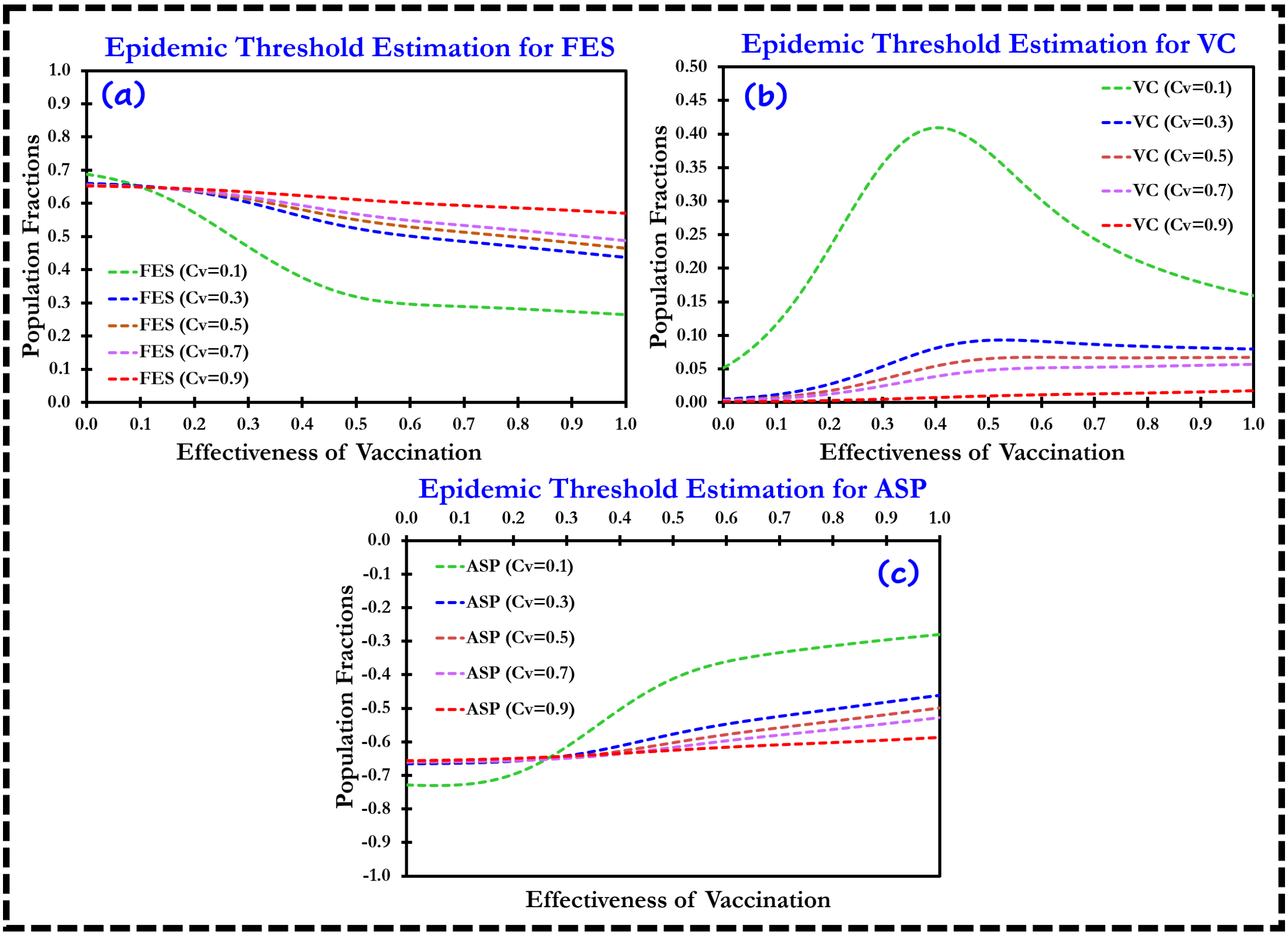

Figure 8. Effect of provisional efficacy in regulating the epidemic threshold to illustrate the relationship between vaccination cost and effectiveness using five different vaccination costs.

That is being said, no policy works better than the others in terms of suppressing disease as vaccine effectiveness starts to rise. To be more precise, no policy is followed by the supremacy of joint, isolation, and quarantine policies. Joint policy, on the other hand, works better during the pandemic phase (which is characterized by low vaccination efficacy), and isolation, quarantine, and no policies come next, accordingly. Again, in Figure 8, when vaccination costs are lower, $C_v = 0.1$, the FES can be effectively managed with comparatively lower vaccine efficacy. The enhanced degree of vaccination coverage is the primary component responsible for this. Actually, the more affordable and effective vaccinations encourage people to get vaccinated, which inadvertently contributes to the success of other coerced control measures. Consequently, across all possible intervention policies, a notable improvement in illness attenuation is seen. Generally speaking, the constant phase lengthens with rising vaccination costs, regardless of the policy used, and vice versa. The critical lines for FES to support the effect of provisional efficacy, e, in halting the progress of an epidemic are displayed in Figure 9. To keep things simpler, we assume that vaccination cost to be equally inexpensive and accessible keeping $C_v = 0.2$. Meanwhile, FES decreases and ASP disappears

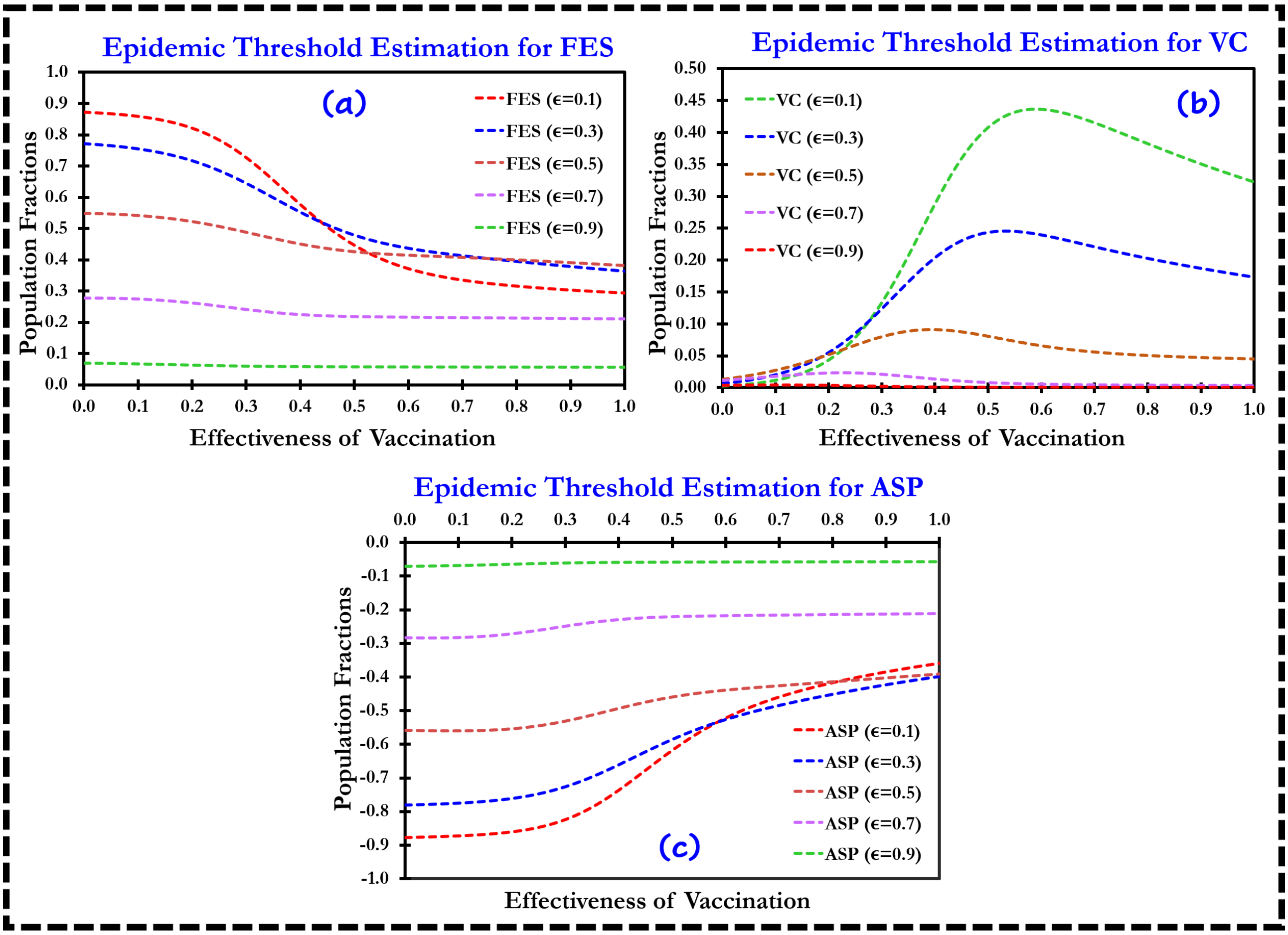

Figure 9. Effect of provisional efficacy in regulating the epidemic threshold illustrating the relationship between quarantine rate and effectiveness using five different quarantine rates.

when the quarantine rate rises up. At situations, when the vaccination efficiency is somewhat less than 0.45 ($e < 0.45$), the FES falls down quickly and the VC rises in the case of a reduced quarantine rate ($\epsilon = 0.1$). For decreasing FES, $\epsilon = 0.1$ performs better than $\epsilon = 0.3$ and $\epsilon = 0.5$, as vaccination efficacy increases.

3.3. Phase-Plane Analysis Using 2D Heat Maps

In order to determine how effective a voluntary pre-emptive vaccination program might be under various parametric circumstances, the evolutionary game-theoretic approach is applied for multiple generations, adopting the standard framework of the *SVEIR* epidemic model equipped with distinctive public health interventions. To quantify the eventual epidemic magnitude under a wide range of parametric circumstances utilized for the quarantine-isolation policy, we portray a set of 2D full-phase heat maps in Figure 11 that change the recovery rates and advancement rate. The results of subsequent theoretical and simulation studies fully support our analysis. For the purpose of debate, we also look at the social consequences on the effect of vaccination that enables immunized people slowing down the rapid transmission pace of the infection by utilizing the well-known

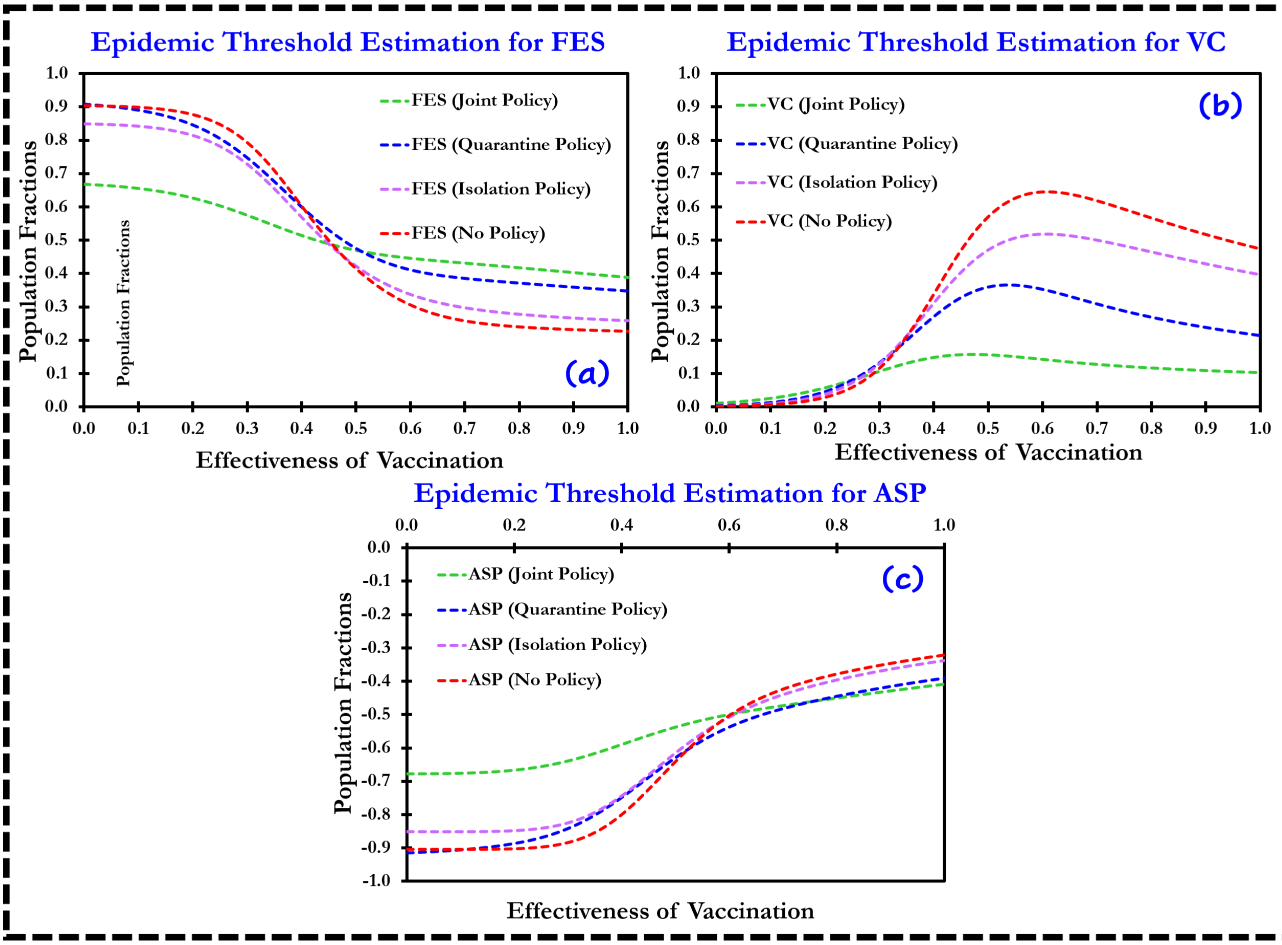

Figure 10. Analysis of epidemic thresholds for various intervention strategies.

vaccination game approach. We use two-dimensional heat maps, merging imitation protocol into disease dynamics, to combine an exhaustive sketch of the proposed model with game elements and to illustrate the dual effects of vaccination cost and effectiveness on evolutionary outcomes. For better understanding of the overall epidemic scenario, three potentially important evolutionary outcomes, namely the final epidemic size (FES), vaccination coverage (VC), and the average social payoff (ASP) have been presented throughout this study. We add two control parameters, λ and ϵ, which represent the contributions of isolation and quarantine policies, respectively, in order to acquire a thorough understanding. Each block has sixteen panels that compare the effects of isolation and quarantine measures when combined with pre-emptive vaccination policy. These panels show various combinations of the infection progression rate and recovery rates. In order to reflect all potential evolutionary outcomes, we consider $\alpha = 0.75$, $\beta = 0.8333$, $\gamma_1 = 0.3333$, $\gamma_2 = 0.35$, $\gamma_3 = 0.2$, $\zeta = 0.2$, and $C_q = 0.2$.

All of these evolutionary results were attained by employing the IB-RA strategy update protocols. It is important to note that the color bar representation in all heat maps used to indicate evolutionary outcomes adheres to a precise set of guidelines. When it comes to total infection size or FES, the phase transition from

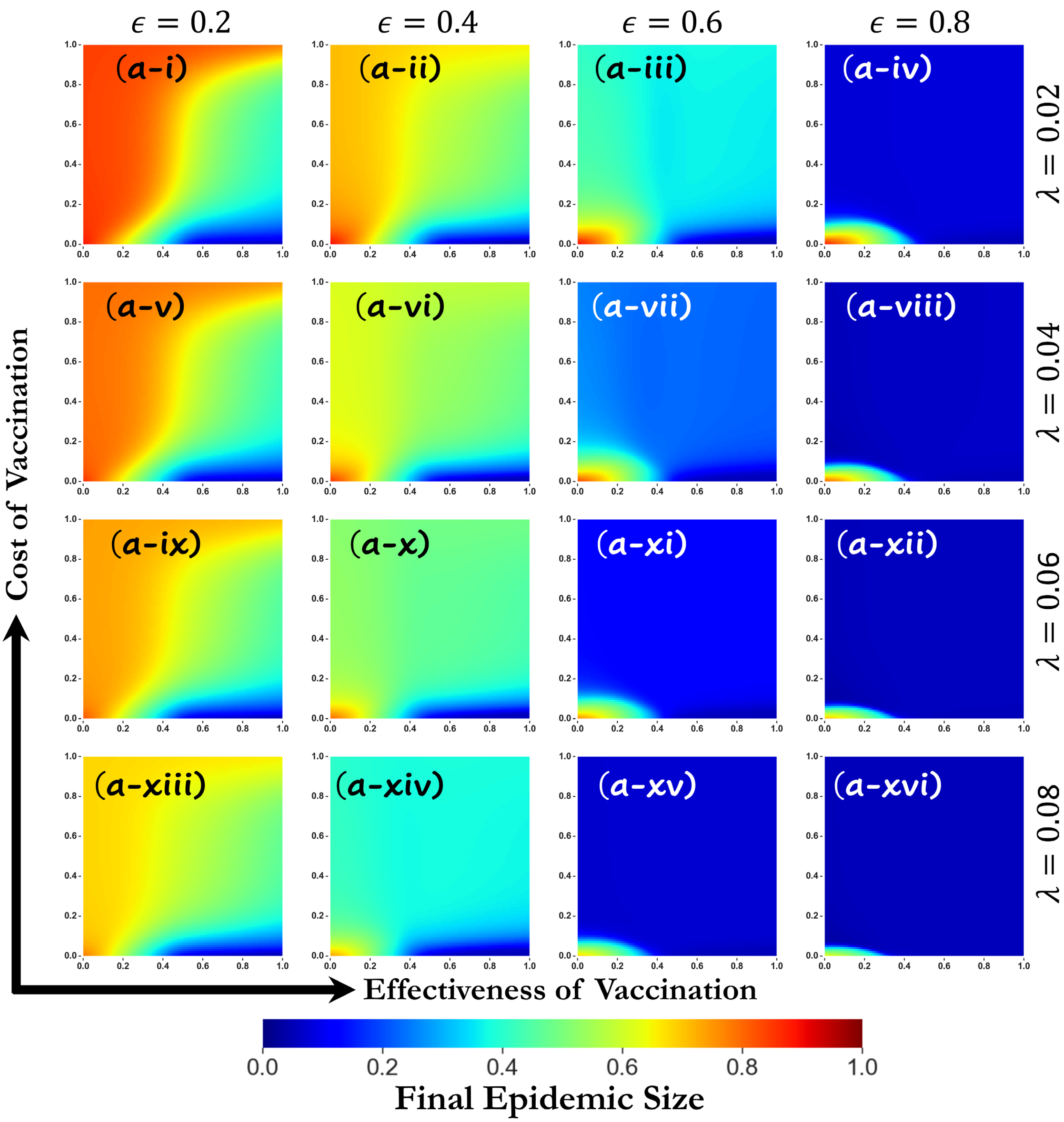

Figure 11. The estimated final epidemic size (FES) for various vaccine prices and efficacies. When implementing control policies the following parametric values have been considered: $\beta = 0.8333$, $\gamma_1 = 1/3$, $\gamma_2 = 0.35$, $\gamma_3 = 0.32$, $\alpha = 0.75$, $\zeta = 0.2$, $C_q = 0.2$. We employ two different pick-up rates: $\epsilon = 0.2, 0.4, 0.6, 0.8$ for a quarantine policy and $\lambda = 0.02, 0.04, 0.06, 0.08$ for an isolation policy.

deep blue to deep red shows that society is gradually been moving from better to worse situation. On the other hand, the phase transition from deep red to deep blue for outcomes like vaccination coverage, and average social payoff justifies the society is approaching towards a better state from the worse epidemic situation experienced earlier. All three evolutionary outcomes, FES, VC, and ASP are shown in Figures 11-19, utilizing the IB-RA strategy update rule, for representing the holistic scenarios of the epidemic over the recurring epidemic seasons. Notably the deep blue areas depict an equilibrium free of disease, while dark red areas locate places where a pandemic is underway (see FES heat map presented in Figure 11. At times when the majority of people prefer to rely entirely on free-riding

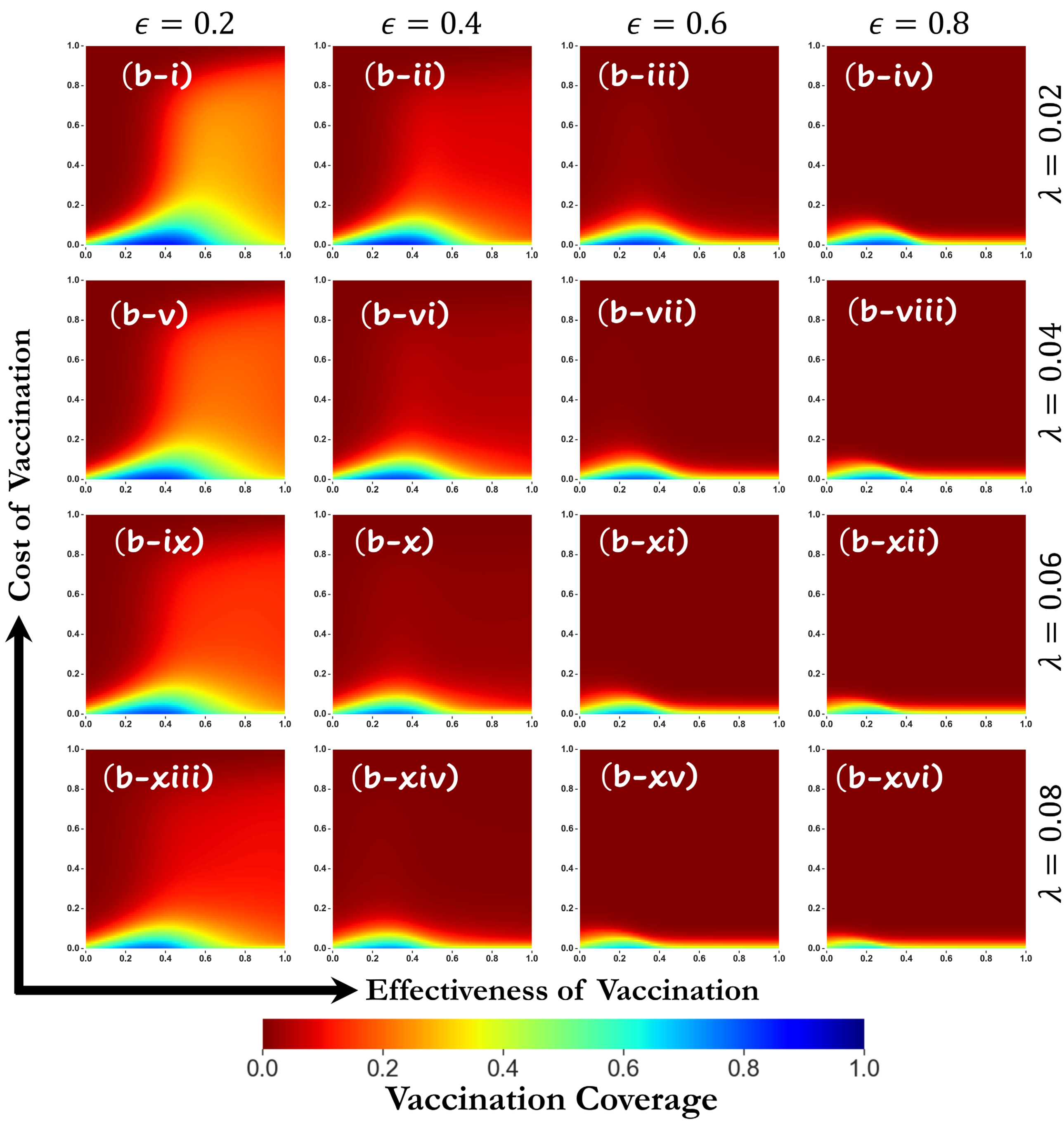

Figure 12. The estimated vaccination coverage (VC) for various vaccine prices and efficacies. When implementing control policies the following parametric values have been considered: $\beta = 0.8333$, $\gamma_1 = 1/3$, $\gamma_2 = 0.35$, $\gamma_3 = 0.32$, $\alpha = 0.75$, $\zeta = 0.2$, $C_q = 0.2$. We employ two different pick-up rates: $\epsilon = 0.2, 0.4, 0.6, 0.8$ for a quarantine policy, and $\lambda = 0.02, 0.04, 0.06, 0.08$ for an isolation policy.

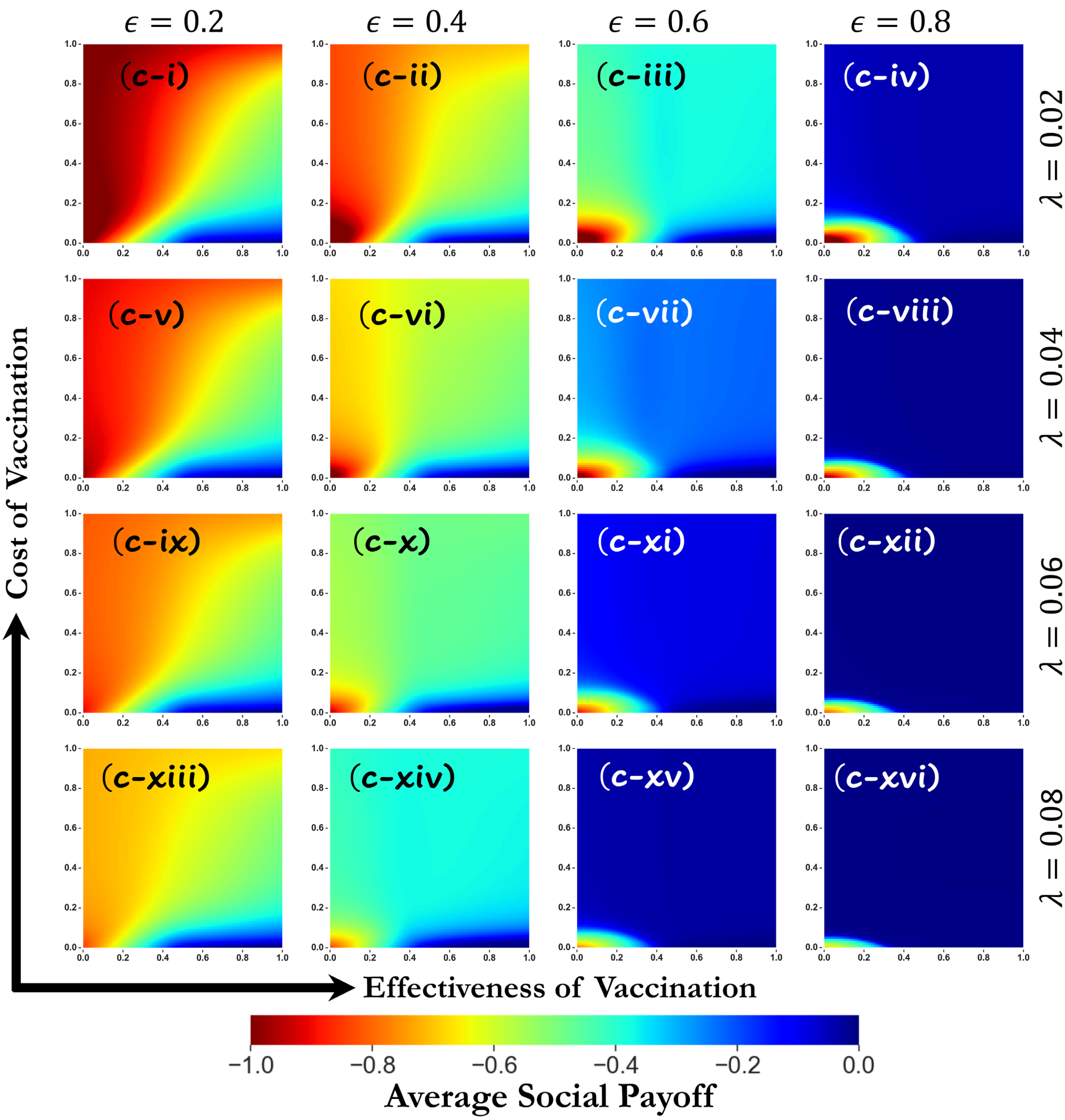

Figure 13. The estimated average social payoff (ASP) for various vaccine prices and efficacies. When implementing control policies the following parametric values have been considered: $\beta = 0.8333$, $\gamma_1 = 1/3$, $\gamma_2 = 0.35$, $\gamma_3 = 0.32$, $\alpha = 0.75$, $\zeta = 0.2$, $C_q = 0.2$. We employ two different pick-up rates: $\epsilon = 0.2, 0.4, 0.6, 0.8$ for a quarantine policy, and $\lambda = 0.02, 0.04, 0.06, 0.08$ for an isolation policy.

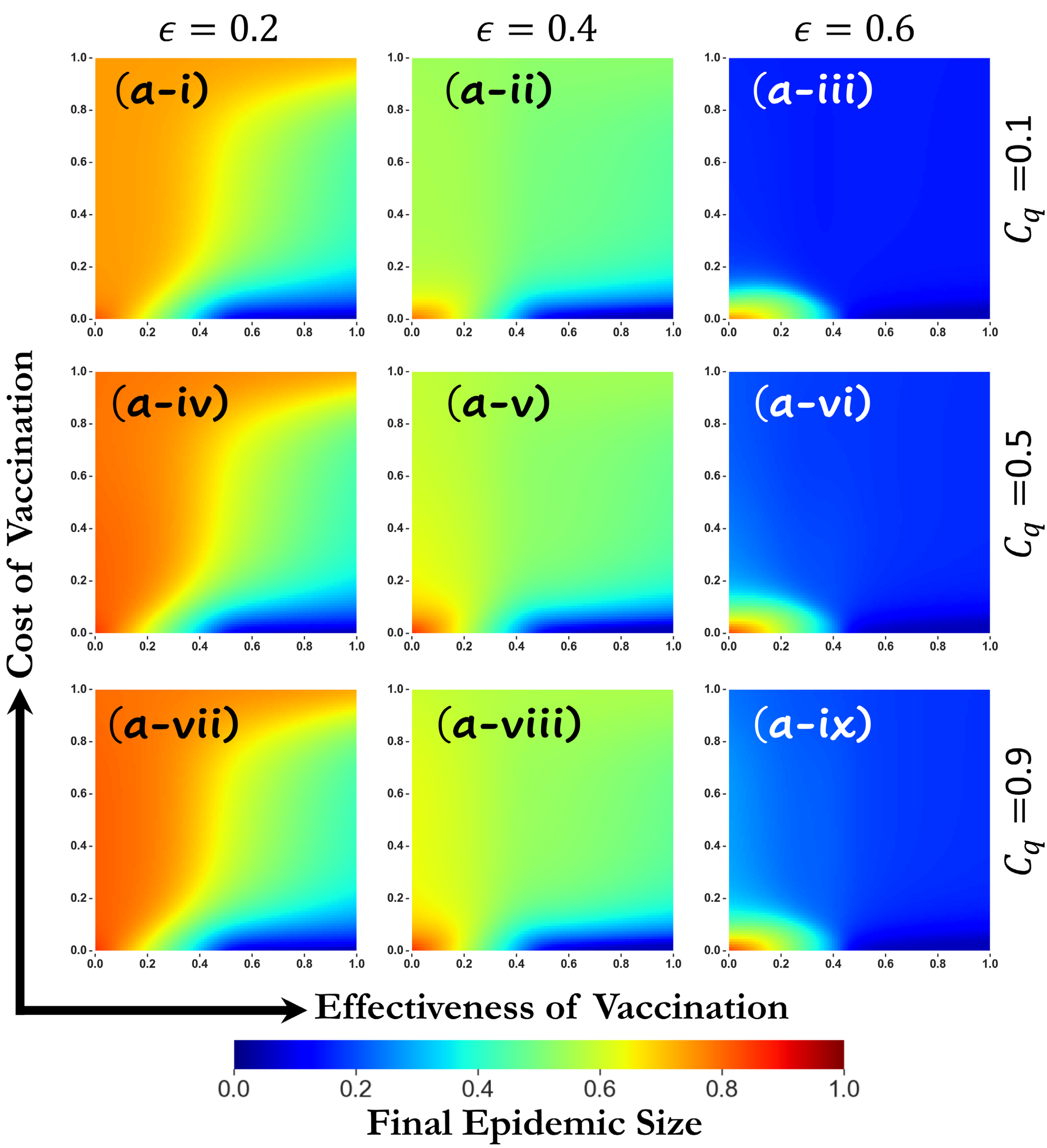

Figure 14. The impact of vaccine effectiveness and cost on final epidemic size. The heat maps are produced while maintaining constant values for the quarantine cost ($C_q = 0.1, 0.5, 0.9$) and quarantine rate ($\epsilon = 0.2, 0.4, 0.6$). Each of the nine panels in this graphic is the consequence of a unique set of quarantine rate and cost factors. If you reside in a region with low rates of quarantine and high quarantine costs, you are in the dark red area, which indicates how severe the epidemic outbreak is.

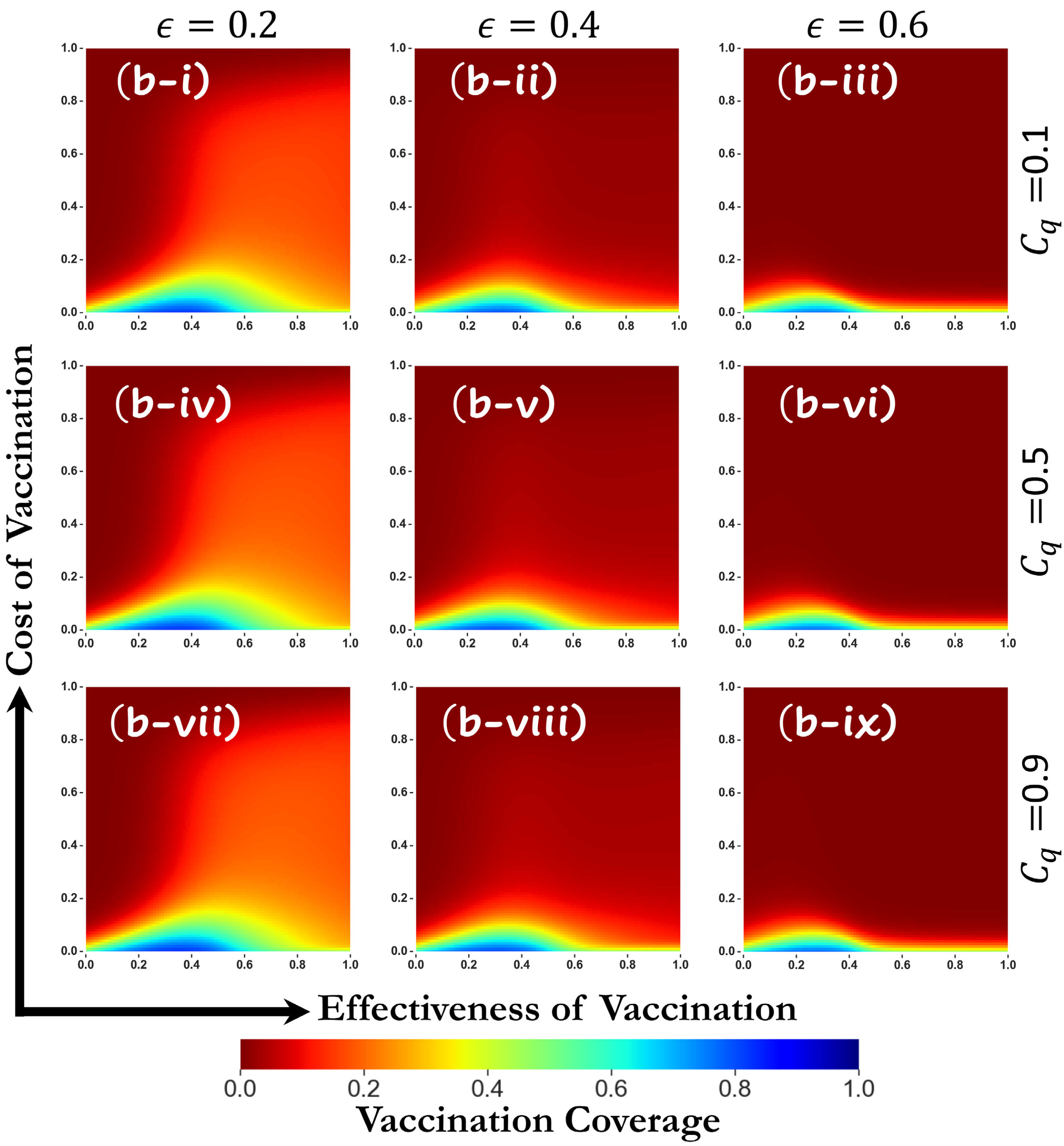

Figure 15. The impact of vaccine effectiveness and cost on vaccination coverage. The heat maps are produced while maintaining constant values for the quarantine cost ($C_q = 0.1, 0.5, 0.9$) and quarantine rates ($\epsilon = 0.2, 0.4, 0.6$). Each of the nine panels in this graphic is the consequence of a unique set of quarantine rate and cost factors.

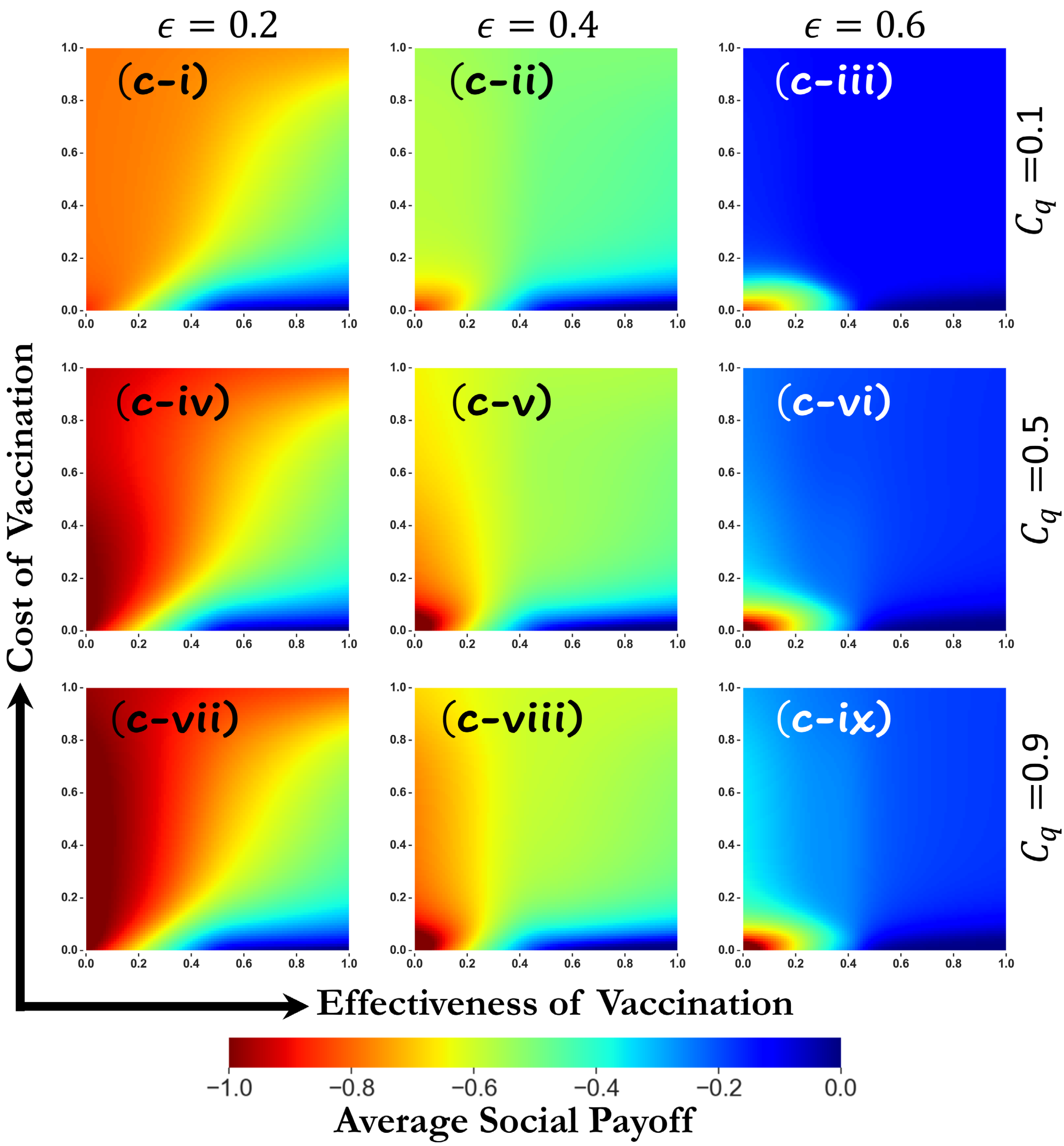

Figure 16. The impact of vaccine effectiveness and cost on average social payoff. The heat maps are produced while maintaining constant values for the quarantine cost ($C_q = 0.1, 0.5, 0.9$) and quarantine rates ($\epsilon = 0.2, 0.4, 0.6$). Each of the nine panels in this graphic is the consequence of a unique set of quarantine rate and cost factors.

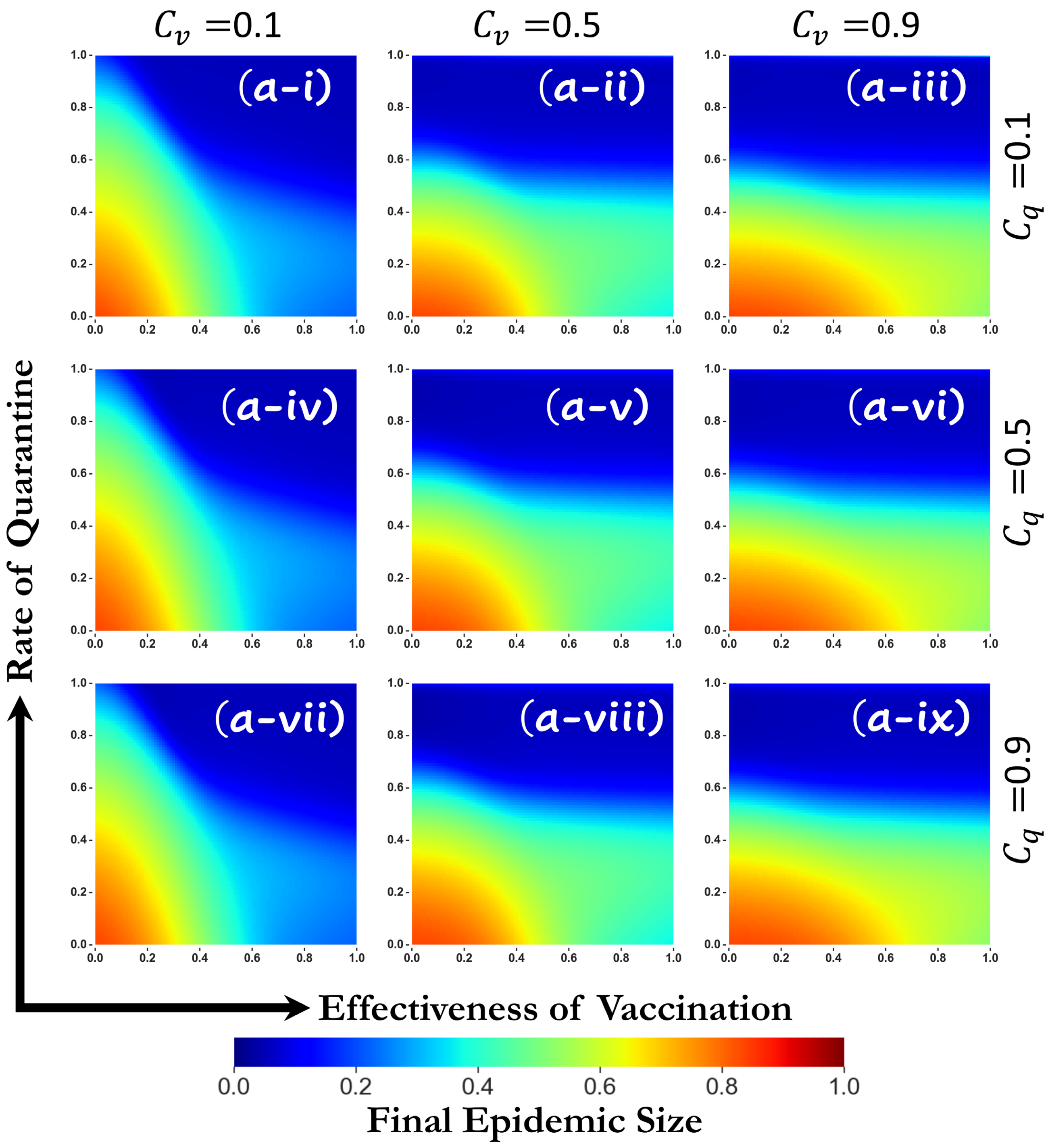

Figure 17. The impact of vaccine effectiveness and quarantine rate on final epidemic size. The heat maps are generated while maintaining fixed vaccination cost ($C_v = 0.1, 0.5, 0.9$) and quarantine cost ($C_q = 0.1, 0.5, 0.9$).

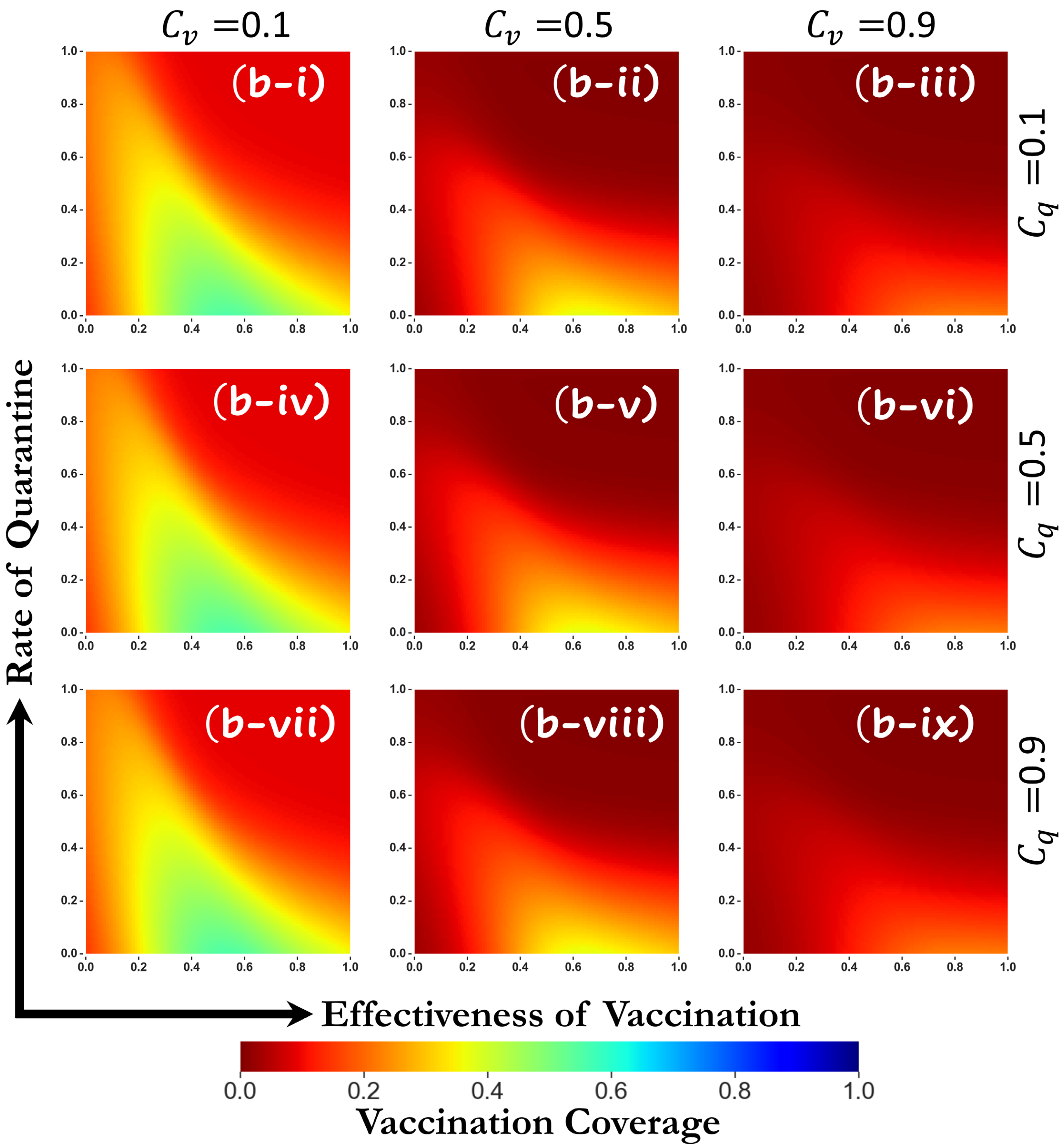

Figure 18. The impact of vaccine effectiveness and quarantine rate on vaccination coverage. The heat maps are generated while maintaining fixed vaccination cost ($C_v = 0.1, 0.5, 0.9$) and quarantine cost ($C_q = 0.1, 0.5, 0.9$).

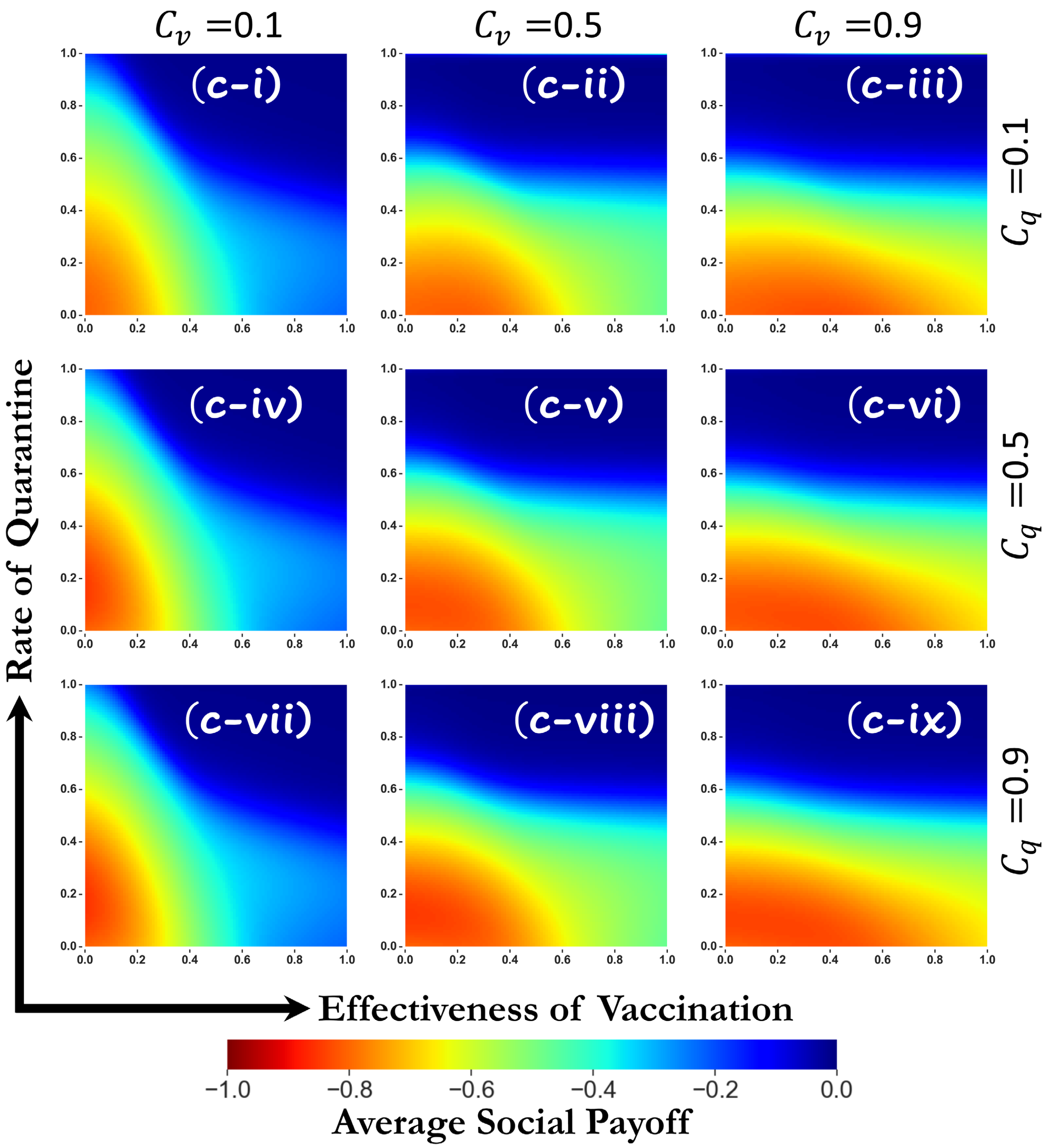

Figure 19. The impact of vaccine effectiveness and quarantine rate on average social payoff. The heat maps are generated while maintaining fixed vaccination cost ($C_v = 0.1, 0.5, 0.9$) and quarantine cost ($C_q = 0.1, 0.5, 0.9$).

other than adopting any of the preventative interventions, it inevitably brings a full-scale spreading of the disease at any given time. In reality, vaccination is often chosen as a promising intervention by individuals as long as vaccine efficacy is comparatively higher and its relative cost is lower. We can see if we vary isolation rate and quarantine rate, an increase in the quarantine rate led to blue areas faster than an increase in isolation rate.

For which it seems increasing quarantine rate is more effective measure for controlling an epidemic than isolation. This is because as the quarantine rate gets higher, the infection rate gets lower. Hence, less people are supposed to enter into the infected compartment. Consequently, a tiny number of people requires to be adopted the enforced isolation policy. That is why the isolation rate does not affect that much in case of higher isolation rate. Irrespective to cases, quarantine strategy delivers better disease attenuation than the partially immunized vaccination scheme in terms of epidemic size which is commonly observed in an epidemic-stricken society. Our theoretical experiment revealed the root cause is the higher advancement rate, which compels the government to implement the quarantine policy in order to protect the societal health issues. As a result, in these circumstances, the quarantine policy appears to perform better than isolation encountering the supremacy of quarantine. Typically a person's willingness to vaccination can be inferred from the vaccination coverage (VC) which is a measurement of the fraction of population must be immunized to control an epidemic. **Figure 12** offers a thorough explanation of VC confirmation for a wide variety of parameter settings. Due to vaccine's meager effectiveness and expensive cost in areas below the critical line, there is less hope for vaccinators to remain healthy adopting this very provision. On the other hand, if the cost of vaccination is slightly cheaper and effectiveness is relatively higher, it shows VC converging toward its maximum limit in contrast to the higher rates of policy cases, when VC diverges from its maximum limit.

This is because neither a quarantine nor an isolation policy will provide incentives to people for taking precautions as long as fairly high effectiveness and cheap cost are assured. In addition to encouraging pre-emptive vaccination, the governing health authorities must arrange sufficient resources and assistance, develop a comprehensive set of quarantine and isolation policies, and deal with this critical issue. With a well-implemented quarantine-isolation strategy, the severity of such diseases can be controlled even though there are no vaccinee or only a limited number of vaccinees present in a population. A phase diagram for ASP is shown in **Figure 13**, which aids in demonstrating why a quarantine-isolation policy must be put in place. Although the overall situation of ASP and FES are fairly similar, there are some significant variances above the critical line. The lack of support for the changing color gradient shown in the ASP map on the FES heat map may then raise a question. It may be worthwhile to look into the precise causes of this occurrence. A little change in the FES causes a significant difference in ASP, and a significant difference in the fraction of population that has received vaccinations also contributes significantly to higher social payoff. The effectiveness of vaccine versus the cost of vaccination heat maps are then adjusted for quarantine rate and quarantine cost. To do so, we choose a wide range of values for both the rate of quarantine ($\epsilon = 0.2, 0.5, 0.8$) and the cost of the quarantine ($C_q = 0.1, 0.5, 0.9$) for numerical simulation. Consequently, a bunch of heat maps for FES, VC, and ASP has been produced adopting the aforementioned parametric values depicted in **Figures 14-19**. The blue shading indicates that quarantine rate and quarantine cost both have contributions, and when particular conditions are satisfied, the phase shift takes place at the sparsely shaded region in light green. The change in quarantine rate is more significant since a slight increase in this factor results in the blue region, which suggests that the epidemic size is shrinking, whereas a slight increase in quarantine cost results slightly in the red region, which suggests that the epidemic size is increasing. Therefore, compared to other parameters, the effect of quarantine costs is less important. As a result, the authority can enforce quarantine as a strict preventative measure because it would not be expensive and its increasing rate would slow down the spread of pandemic. We have a maximum vaccination coverage at lower quarantine rate. When the rate is higher, we don't have adequate vaccination coverage. We get to see a strong average social payoff when the quarantine rate is higher and quarantine cost is lower. Through heat maps comparing vaccination efficacy to quarantine rate, our model also confirms that cost of quarantine has the least influence on evolutionary consequences thoroughly observed in **Figures 17-19**. The extent of the epidemic is reduced when the costs of quarantine and vaccination are kept at zero. The final pandemic magnitude grows as the price of the vaccination rises. The magnitude of the epidemic increased slightly as a result of higher quarantine costs, albeit between 0.4 and 0.8 there was no discernible difference. When the effectiveness of vaccine and efficacy of quarantine is higher, the growth of epidemic is slowed down. We have a maximum vaccination coverage at lower vaccination cost and moderate quarantine cost. When the vaccination cost is higher, we do not have adequate vaccination coverage, that is, a least presence of vaccinators in the society is inevitable. Additionally, we don not observe any noticeable change between quarantine cost of 0.5 to 0.9. Although for a certain range of vaccine efficacy, the VC gets higher, eventually the coverage falls down later on. From 19, we got to see a significantly higher average social payoff attained when both of the costs are nearing zero. In contrast, any notable change can not be observed when the cost values of quarantine ranging between $C_q = 0.4$ to $C_q = 0.8$. Understandably, as the vaccination cost rises up, ASP gradually turns towards the red region more quickly than that of with the rise in quarantine cost. Therefore, our proposed model justifies the fact that vaccination cost plays a more significant role in curtailing the growth of an epidemic compared to any other provisional cost burdens.

4. Conclusion

In conclusion, our current research looked at two main types of health intervention strategies for preventing the rapid emergence of recurrent pandemics and severe epidemics brought on by infectious diseases: voluntary provision (preemptive vaccination) and imposed control policies (quarantine and isolation). Incorporating mathematical epidemiology with the idea of the vaccination game, we carried out a thorough examination of the benefits of isolation and quarantine policies on disease prevention. Our suggested model thoroughly investigates the complex effects of these control measures and evaluates their suitability under various parametric conditions to lessen the severity of disease transmission, particularly in people who have chosen vaccination as a preventive measure. According to our study, a cumulative performance of both voluntary and imposed control policies works greatly in slowing down the rapid growth of an epidemic. Our proposed model performs significantly better in reducing the infected fraction of people than the usual SVIR (both effectiveness and efficiency) model. Both the forced policies should be enforced upon the people when an epidemic out breaks severely to achieve the most fruitful result. Our study also found that a higher quarantine rate is more notable than the rise in the isolation rate irrespective of quarantine cost. So, from the very beginning of the epidemic, quarantine should be more strongly enforced to minimize the loss. No policy outperforms other policies in case of increasing vaccination effectiveness. Also, vaccination cost plays more significant role than quarantine cost. We also found that it is of great importance that a more affordable and effective vaccination be introduced. However, because to the short-lived effectiveness of vaccination and the regular occurrence of epidemic outbreaks, there is an increasing need to find more long-term solutions for safeguarding the global society against viral diseases. The numerical simulations make it clear that implementing a quarantine-isolation policy might ease difficult conditions even in areas with dire circumstances. Our theoretical research implies that when the rate of disease transmission is high, a mix of regulations should be put into place. However, it is imperative that governments prioritize public health by putting any of these control strategies into action. In other words, keeping the overall epidemic magnitude at a reasonable level depends equally on both early vaccination and later control measures.

Authors' Contribution

Muntasir Alam conceptualized the model, carried out the numerical simulations to validate the proposed model, visualized the model outcomes, formatted the original draft, completed the formal analysis, and critically revised the manuscript. Sumaiya Jamila developed the proposed model, completed the numerical simulations, summarized the formal discussion, analyzed the outcomes, and prepared the original draft. Md. Kamrujjaman reviewed the manuscript and summarized the simulated results. Jun Tanimoto revised the manuscript, made the funding acquisition, and supervised the entire project.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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