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Sub-Atom Particles and Magnetic Monopoles Spin Ice Condensed in Higgs-Field Portals

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Abstract

The study of magnetic monopoles continues to be a prominent and captivating topic in physics, particularly within the realm of physical materials. Recently, K. C. Tan, Hariom Jani, Michael Högen, and their collaborators (2023) reported groundbreaking discoveries, marking significant progress in this field. However, a sense of dissatisfaction persists among researchers regarding the current state of advancement. To address this, we propose a novel theoretical framework that explores magnetic monopoles through the lens of Higgs field portals. Our findings indicate that the spin of the magnetic monopole, $s = 1$, is intrinsically linked to the fundamental expression governing its behavior, with the two aspects being inseparable in practical terms. This theory offers a deeper understanding of the inherent nature of magnetic monopoles and provides a foundation for further exploration.

Keywords

Magnetic Monopoles, Higgs-Field, Magnetic Spin

1. Introduction

The *Dream Pool Essays*, written by the Chinese polymath and statesman Shen Kuo (1031-1095), contain an early reference to magnetic phenomena. Shen Kuo noted: "*We used magnets to sharpen their needles, and they could guide the South Pole. However, they often veered slightly eastward, not completely southward.*" In 1269, Petrus Peregrinus wrote in a letter that a magnet inherently possesses two poles: a North Pole and a South Pole. This concept was later expanded in the early 19th century by André-Marie Ampère, who proposed a hypothesis on the nature of magnetism. In 1931, British physicist Paul Dirac made a significant leap in this field by introducing the quantum theory of magnetic charge [1]. In his groundbreaking paper, Dirac proposed that if magnetic charges exist in the universe, all electric charges must be quantized—a principle known as the *Dirac quantization condition*.

While experiments confirmed that electric charge is indeed quantized, they did not definitively establish the existence of magnetic monopoles. This consistency, however, sparked widespread interest among physicists, leading to various experimental searches for magnetic monopoles. Efforts included using particle accelerators to artificially create monopoles, but these endeavors yielded no conclusive results. In 1975, American scientists conducting cosmic radiation studies with high-altitude balloons detected a peculiar track, which they initially interpreted as evidence of a magnetic monopole. Similarly, on February 14, 1982, Blas Cabrera, a physicist at Stanford University, reported detecting magnetic monopoles using superconducting coils. However, subsequent experiments failed to reproduce these findings, leaving the existence of magnetic monopoles unconfirmed.

Theoretical developments continued, and in 1994, American physicists Nathan Seiberg and Edward Witten further demonstrated the theoretical plausibility of magnetic monopoles. These advancements were built on Dirac's foundational work in 1931, which had established the theoretical connection between magnetic monopoles and the quantization of electric charge [2]. Since Dirac's seminal paper, numerous systematic searches for magnetic monopoles have been conducted. Notably, experiments in 1975 [3] and 1982 produced candidate events initially interpreted as evidence of monopoles. However, these results are now considered inconclusive [4]. As a result, the existence of monopoles remains an open question. Advances in theoretical particle physics, particularly in grand unified theories and quantum gravity, have provided increasingly compelling arguments supporting the existence of monopoles. String theorist Joseph Polchinski even described the existence of monopoles as "*one of the safest bets that one can make about physics not yet seen*" [5]. These theories are not inherently inconsistent with the current experimental evidence. In some models, magnetic monopoles are hypothesized to be extremely massive, making their creation in particle accelerators infeasible. Additionally, their rarity in the universe significantly reduces the likelihood of detecting them in particle detectors [5].

2. Quantum Field Theory of Higgs Field Phase Transition: Higgs Bosons as Inflatons

We present a calculation of Higgs masses based on the breaking of electroweak (EW) gauge couplings [6]. As shown in the overview of Fig. S1 in Ref. [6], the temperature—which corresponds to the Casimir temperature $T_{C'} = 0.00206\ \text{K}$ (for an atomic spacing of 0.1 nm; see **Figure 1**)—is represented as being located at the top of a sombrero (Mexican hat). This position signifies a state of high symmetry in the Higgs fields. Specifically, the expectation values of mass are constrained to the Higgs boson mass itself, with a critical angle $\theta_C \approx \tan^{-1}\infty = \pi/2$

situated at the edge of the sombrero. To analyze this high symmetry in the Higgs fields, we utilize Equation (11) from Ref. [6]
$\left(\left\langle \phi_{vac}^2 \right\rangle = \left\langle e^{i\theta} \middle| \phi_{vac}^2 \middle| e^{i\theta} \right\rangle, \phi_{vac} \to \phi'_{vac} = e^{i\theta}\phi_{vac}\right)$.
Therefore, $\theta = N \cdot 2\pi = 4\theta_C = 2\pi, N = 1$. Such leads

$$\begin{aligned} &\left\langle \phi_{vac}^2 \right\rangle = \left\langle e^{i\theta} \middle| \phi_{vac}^2 \middle| e^{i\theta} \right\rangle, \\ &\phi_{vac} \to \phi'_{vac} = e^{i\theta}\phi_{vac}\Big|_{\theta=2\pi}, \\ &\phi'_{vac} = \phi_{vac} \end{aligned} \tag{1}$$

Building on this framework, we make effective use of Equation (16) from Ref. [6]. In the context of axial calculations—considering principles of general covariance and background independence—we explore the implications for the Higgs field and its associated properties:

$$\begin{aligned} I_{\lambda=Higgs,x} &= \frac{2C'}{0.00206\ \mathrm{K}} \approx \lim_{C' \to e^{\frac{\theta}{2}}} \frac{2C'}{0.00206\ \mathrm{K}} \mathrm{K} \cdot \mathrm{mol} \cdot \frac{\mathrm{T}}{\mathrm{J} \cdot \mathrm{K}} = \frac{2}{(0.00206)(e^{\pi})} \mathrm{mol} \cdot \frac{\mathrm{T}}{\mathrm{J} \cdot \mathrm{K}}, \\ I_{\lambda=Higgs,y} &= \frac{2C'}{0.00206\ \mathrm{K}} \approx \lim_{C' \to e^{\frac{\theta}{2}}} \frac{2C'}{0.00206\ \mathrm{K}} \mathrm{K} \cdot \mathrm{mol} \cdot \frac{\mathrm{T}}{\mathrm{J} \cdot \mathrm{K}} = \frac{2}{(0.00206)(e^{\pi})} \mathrm{mol} \cdot \frac{\mathrm{T}}{\mathrm{J} \cdot \mathrm{K}}, \\ I_{\lambda=Higgs,-z} &= \frac{2C'}{0.00206\ \mathrm{K}} \approx \lim_{C' \to e^{\frac{\theta}{2}}} \frac{2C'}{0.00206\ \mathrm{K}} \mathrm{K} \cdot \mathrm{mol} \cdot \frac{\mathrm{T}}{\mathrm{J} \cdot \mathrm{K}} = \frac{2}{(0.00206)(e^{\pi})} \mathrm{mol} \cdot \frac{\mathrm{T}}{\mathrm{J} \cdot \mathrm{K}} \end{aligned} \tag{2}$$

By rotating the z-axis to the x-axis with $\pi/2$, as illustrated in Fig. S1 of Ref. [6], the non-oscillatory terms of $e^{\frac{\theta}{2}}$ are derived. Summarizing the above, the total orientations obtained in the scalar field theory Lagrangian are[1]:

$$\begin{aligned} I_{\lambda=Higgs}\,\mathrm{mol}^{-1} \cdot \mathrm{T}^{-1} \cdot \mathrm{C}^{-1} &= 3 \cdot \frac{2}{(0.00206\ \mathrm{K})(e^{\pi})} \\ &= \frac{125.8\ \mathrm{eV}}{\mathrm{K}} + \frac{0.06\ \mathrm{eV}}{\mathrm{K}} + O(\varepsilon^{i}) \neq \mathrm{const} \end{aligned} \tag{3}$$

where the idea of "Higgs bundle to be thought of as a flat holomorphic connection on a holomorphic vector bundle" is used, therefore we can see the rank 2 vector bundle: (125.8 eV/K, 0.06 eV/K). This type of wobbly bundles appears, such as the additions in Equation (3) undoubted. Then one may ask: what is the inertia of the Higgs boson? If we consider this in relation to the Higgs boson's specific mass, it can be expressed in terms of large energy scales, such as those relevant to the electroweak interaction. For instance, in Equation (3), the left-hand side (LHS) reflects orders 10^{-5} of magnitude, leading the right-hand side (RHS) to acquire values on the scale $\geq GK/c^2$. These values of Gega scales e.g. arrival $10^{11}\,K/c^2 \sim 10^{14}\,K/c^2$ depend on Lorentz factors and are influenced by the high velocities achieved in particle colliders like the Large Hadron Collider (LHC).

By reaching the energy scales associated with electroweak gauge couplings (>100 GeV), the excited state of the Higgs field is manifested, enabling exploration of the Higgs boson mass—first confirmed experimentally in 2012. The Higgs

[1]Equation (3) is an anti-Affine Higgs mechanism (*i.e.*, the famous Non-Abelian Higgs Mechanism) and obeys the non-Abelian gauge theory, where there is no affine limit. Obviously, it is not kept constant.

boson's mass is approximately 125 GeV based on current measurements. Meanwhile, radiation around 0.06 eV, which falls within the long-wavelength infrared (LWIR) band, is believed to be emitted into the surrounding vacuum, potentially offering insights into related physical processes. Equation (3) fits the results claimed by CERN (see Fig. 2, [7]). For the statements by stationary of Higgs boson mass we are in justification that the first term is valid fits Ref. [7] and the second term belongs to LWIR photon energy, and the higher order terms $O(\varepsilon^i)$ were missing in the early universe. The higher order terms of Equation (3) where $O(\varepsilon^i)$ exhibited as wavelength is surely smaller than it as mentioned by this paper. As one seen the second term (LWIR) therefore we can think that the first term of the formula "Higgs mass" due to the calculation of sombreros, it reveals that *Higgs bosons play the role of Inflatons*. Modestly these methods seem to be nonsense but are applicable under frameworks of calculations of the Higgs boson mass is absolutely suited.

Equation (3) aligns with the results reported by CERN (see Fig. 2, [7]). Regarding the stationary nature of the Higgs boson mass, we can justify that the first term in the equation is valid, as supported by Ref. [7]. Meanwhile, the second term corresponds to the energy of LWIR photons. Notably, the higher-order terms, denoted as $O(\varepsilon^i)$, were absent in the early universe. These higher-order terms in Equation (3), where $O(\varepsilon^i)$ represents a wavelength, are indeed smaller than the values discussed in the referenced paper. Given the second term (LWIR), it is reasonable to conclude that the first term of the formula, associated with the "Higgs mass," is related to the sombrero potential. This suggests that Higgs bosons may have played the role of inflatons during the inflationary phase of the universe. While these methods may initially appear unconventional, they are applicable within the theoretical frameworks used to calculate the Higgs boson mass, demonstrating their suitability for this context. The rotation-angles in Higgs fields appeared in Equations (1)-(3) that can be referred in **Figure 1**.

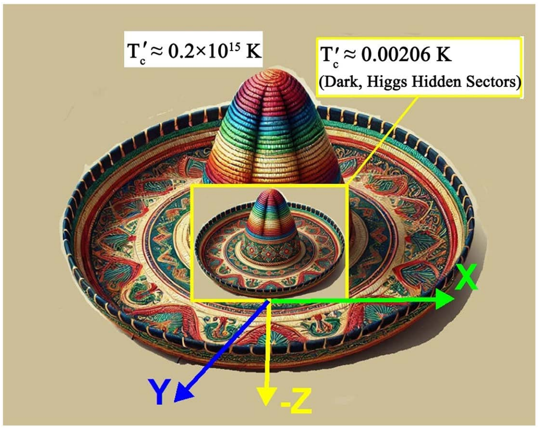

Figure 1. The Hat-in-Hat Representation: This figure extends the instability of Higgs fields clearly [12] (namely vacuum decay timescales are time-passed by 0.2×10^{15} K → 0.00206 K and this is longer than the current age of 13.787 ± 0.020 billion years). The dark portal in new physics is located within the dark section at the base of the larger hat.

Remark. Due to the Non-Abelian Higgs Mechanism, the term of $O\left(\varepsilon^{i}\right)$ can be rewritten as $O\left(\varepsilon^{i}\right)=\langle\phi_{RE}|\hat{O}|\phi_{IM}\rangle$, while the particle has not been located on the ground state in Higgs fields, namely the Higgs valley) $\left(|\phi_{RE}|\neq|\phi_{IM}|\right)$, so that $O\left(\varepsilon^{i}\right)=\langle\phi_{RE}|\hat{O}|\phi_{IM}\rangle=0$ (vanished). In Equation (3), after the inflaton epoch, $|\phi_{RE}|=|\phi_{IM}|$, therefore $O\left(\varepsilon^{i}\right)=\langle\phi_{RE}|\hat{O}|\phi_{IM}\rangle=1\,\mathrm{eV/K}$ (the electron and the mass is presented or a measurement). This is the famous Higgs oscillators, such as Higgs bundles discussed in the previous sections. The accuracy of Equation (3) is due to such mechanism of oscillators, since that its discipline follows all rules by Heisenberg uncertainty principle. In other words, the indicated *const* in Equation (3) can be regarded as $\frac{\hbar\omega_0}{2c^2}\mathrm{mol}^{-1}\cdot\mathrm{T}^{-1}\cdot\mathrm{C}^{-1}$ where $\omega_0>0$ is the threshold frequency. Based on this, all equations in this paper shall follow the same rule. Particularly, the above "Higgs bundles" such concept actually provides the potential and reliable evidence to a route for one to find the dark matters (DM). We discuss this issue later on.

To align with the data presented above, we illustrate the self-interactions at three points within gauge fields, as described by quantum field theory (QFT):

$$\begin{aligned}
L_{WWV} &= -ig\Big[\left(W_{\mu\nu}^{+}W^{-\mu}-W^{+\mu}W_{\mu\nu}^{-}\right)\left(A^{\nu}\sin\theta_W-Z^{\nu}\cos\theta_W\right)\\
&\quad +W_{\nu}^{-}W_{\mu}^{+}\left(A^{\mu\nu}\sin\theta_W-Z^{\mu\nu}\cos\theta_W\right)\Big],\\
L_{WWV} &= -ig\Big[\left(W_{\mu\nu}^{+}W^{-\mu}-W^{+\mu}W_{\mu\nu}^{-}\right)\left(A^{\nu}\sin\pi-Z^{\nu}\cos\pi\right)\\
&\quad +W_{\nu}^{-}W_{\mu}^{+}\left(A^{\mu\nu}\sin\pi-Z^{\mu\nu}\cos\pi\right)\Big],\\
L_{WWV} &= -ig\left[W_{\mu\nu}^{+}W^{-\mu}Z^{\nu}-W^{+\mu}W_{\mu\nu}^{-}Z^{\nu}+W_{\nu}^{-}W_{\mu}^{+}Z^{\mu\nu}\right]
\end{aligned} \tag{4}$$

By using Einstein's summation convention:

$$L_{WWV}=-ig\left[W_{\mu\nu}^{+}W^{-\mu}Z^{\nu}-W^{+}W_{\nu}^{-}Z^{\nu}+W_{\nu}^{-}W^{+}Z^{\nu}\right]=-igW_{\mu\nu}^{+}W^{-\mu}Z^{\nu} \tag{5}$$

If $\nu=0$,

$$L_{WWV}=-igW_{\mu}^{+}W^{-\mu}Z^{0},\mu=1,2,3 \tag{6}$$

Similarly, we present the self-interactions at three points within gauge fields in the context of QFT-electrons:

$$\begin{aligned}
L_{WWV} &= -ig\Big[\left(W_{\mu\nu}^{+}W^{-\mu}-W^{+\mu}W_{\mu\nu}^{-}\right)\left(A^{\nu}\sin\theta_W-Z^{\nu}\cos\theta_W\right)\\
&\quad +W_{\nu}^{-}W_{\mu}^{+}\left(A^{\mu\nu}\sin\theta_W-Z^{\mu\nu}\cos\theta_W\right)\Big],\\
L_{WWV} &= -ig\left[\left(W_{\mu\nu}^{+}W^{-\mu}-W^{+\mu}W_{\mu\nu}^{-}\right)\left(A^{\nu}\sin\frac{3\pi}{2}-Z^{\nu}\cos\frac{3\pi}{2}\right)\right.\\
&\quad \left.+W_{\nu}^{-}W_{\mu}^{+}\left(A^{\mu\nu}\sin\frac{3\pi}{2}-Z^{\mu\nu}\cos\frac{3\pi}{2}\right)\right],\\
L_{WWV} &= -ig\left[-A^{\nu}W_{\mu\nu}^{+}W^{-\mu}+A^{\nu}W^{+\mu}W_{\mu\nu}^{-}-A^{\mu\nu}W_{\nu}^{-}W_{\mu}^{+}\right]
\end{aligned} \tag{7}$$

Using Einstein summation convention:

$$L_{WWV}=-ig\left[-A^{\nu}W_{\mu\nu}^{+}W^{-\mu}+A^{\nu}W^{+}W_{\nu}^{-}-A^{\mu}W^{-}W_{\mu}^{+}\right], \tag{8}$$

If $\nu = 0$,

$$L_{WWV} = -ig\left[-A^0 W_\mu^+ W^{-\mu} + A^0 W^+ W^- - A^\mu W^- W_\mu^+\right], \mu = 1,2,3 \tag{9}$$

If $\mu = 1$,

$$\begin{aligned}
L_{WWV} &= -ig\left[-A^0 W_\mu^+ W^- + A^0 W^+ W^- - A W^- W_\mu^+\right], \mu = 1, \\
L_{WWV} &= ig\left[A^0\left(W_\mu^+ W^- - W^+ W^-\right) + A W^- W_\mu^+\right], \mu = 1, \\
L_{WWV} &= ig\left[A^0\begin{pmatrix} W_\mu^+ & W^- \\ W^+ & W^- \end{pmatrix} + A\begin{pmatrix} W_\mu^+ & 0 \\ 0 & W^- \end{pmatrix}\right], \mu = 1, \\
& J_\mu^{em} A^\mu, \mu \equiv \nu = 0 \\
L_{WWV} &= i\underbrace{g}_{=0} A^0\begin{pmatrix} W_\mu^+ & W^- \\ W^+ & W^- \end{pmatrix} + igA\begin{pmatrix} W_\mu^+ & 0 \\ 0 & W^- \end{pmatrix}, \mu = 1
\end{aligned} \tag{10}$$

Yields

$$L_{WWV} = igAW^- W_\mu^+, \mu = 1 \tag{11}$$

With QFT-reactions shown as **Figure 2**. Notice that, in Equation (11) where A represents the neutral current and is interpreted as photon(s).

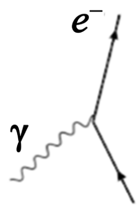

Figure 2. QFT-reactions as shown.

Based on the same principles, the electron mass is calculated as follows. Using Equation (2) and substituting the angle of spontaneous symmetry breaking (SSB, see **Table 1**) with $3\pi/2$ from Higgs fields in this paper, we derive that for six axial $\langle x, y, -z; -x, -y, -z\rangle$, the inertia of an electron is represented as:

$$I_{\lambda=electron}\,\mathrm{mol}^{-1}\cdot\mathrm{T}^{-1}\cdot\mathrm{C}^{-1} = 6\cdot\frac{2}{(0.00206\,\mathrm{K})\left(\mathrm{e}^{3\pi/2}\right)} \approx \frac{52.33\,\mathrm{eV}}{\mathrm{K}} \tag{12}$$

Table 1. Landau's ideas about M and Energy.

Expressions	Significances
$E(M) = a_0 + a_1 M + a_2 M^2 + a_3 M^3 + \cdots$	Landau's polynomial expansion in M. The expression shows the continuous symmetry.
$E(M) = a_2 M^2 + a_4 M^4$	The odd-terms and a_0 are hidden, *i.e.*, the Higgs continuous symmetry is broken (SSB). Moreover, this reveals a critical exponent $\delta = 3$ where T_C occurred (refer to Landau theory).
$E(M) = \alpha M^2 + \beta M^4, \alpha < 0, \beta > 0$	Sombreros in the Higgs field potentials are described with a scalar z-axis T_C, representing the Higgs valley.

At high temperatures, the highly symmetrical Higgs fields were broken in the early universe. Electrons were situated in the state $T = 10^9\ \text{K}$, such that Equation (12) becomes:

$$I_{\lambda=electron}\,\text{mol}^{-1}\cdot\text{T}^{-1}\cdot\text{C}^{-1}\cdot 10^9\ \text{K} \approx \gamma 0.523\ \text{MeV} = \gamma\cdot\left(0.511\ \text{MeV}\right) + E_{K',e^-} \quad (13)$$

This implies that the electron rest mass is approximately 0.511 MeV ($c^2 \equiv 1, \gamma \approx 10^5$). It is particularly recommended to refer to the above solution. For calculating the three flavors of neutrinos at $T = 10^7\ \text{K}$ (the temperature of solar nuclear fusion), one can uniquely determine the axial neutrino mass as:

$$\begin{aligned} I_{\lambda=\nu}\frac{\text{mol}^{-1}\cdot\text{T}^{-1}\cdot\text{C}^{-1}\cdot 10^9\ \text{K}}{10^2} &= \frac{1}{100}\cdot\frac{2}{\left(0.00206\ \text{K}\right)\left(\text{e}^{\pi}\right)}, \\ I_{\lambda=\nu}\frac{\text{mol}^{-1}\cdot\text{T}^{-1}\cdot\text{C}^{-1}\cdot 10^9\ \text{K}}{10^2} &\approx \gamma\cdot\left(0.42\ \text{eV}\right) \approx \gamma\cdot\left(0.401\ \text{eV}\right) + E_{K',\nu}, \gamma \approx 10^7 \end{aligned} \quad (14)$$

Here, $0 + \pi = \pi$ corresponds to $-\hat{z}$, and this unique axis is identified as the left-chiral neutrino. Meanwhile, γ is represented as the Lorentz factor.

Neutrinos case: The self-interactions at three points in gauge fields are as follows:

$$\begin{aligned} L_{WWV} &= -ig\left[\left(W^+_{\mu\nu}W^{-\mu} - W^{+\mu}W^-_{\mu\nu}\right)\left(A^\nu \sin\theta_W - Z^\nu \cos\theta_W\right)\right. \\ &\quad \left. + W^-_\nu W^+_\mu\left(A^{\mu\nu}\sin\theta_W - Z^{\mu\nu}\cos\theta_W\right)\right], \\ L_{WWV} &= -ig\left[\left(W^+_{\mu\nu}W^{-\mu} - W^{+\mu}W^-_{\mu\nu}\right)\left(A^\nu \sin 0 - Z^\nu \cos 0\right)\right. \\ &\quad \left. + W^-_\nu W^+_\mu\left(A^{\mu\nu}\sin 0 - Z^{\mu\nu}\cos 0\right)\right], \\ L_{WWV} &= -ig\left[\left(-Z^\nu W^+_{\mu\nu}W^{-\mu} - W^{+\mu}W^-_{\mu\nu}\right) - Z^{\mu\nu}W^-_\nu W^+_\mu\right], \end{aligned} \quad (14.1)$$

Using Einstein summation convention:

$$L_{WWV} = -ig\left[\left(-Z^\nu W^+_{\mu\nu}W^{-\mu} - W^+W^-_\nu\right) - Z^\mu W^- W^+_\mu\right] \quad (14.2)$$

If $\nu = \mu = 0$,

$$L_{WWV} = -ig\left[-Z^0\cdot 0\right] = 0 \quad (14.3)$$

This implies that neutrinos do not interact with any cosmic matter at any time. Below, shows the naturalness of 1/2, and this will be helpful for the naturalness of magnetic monopoles in Equation (25) later on.

Schrödinger's Cat in Axion Scenarios: To establish a rational link to the properties of quantum field theory (QFT) for our calculations, we examine axion masses within the QFT frameworks. Using Equation (14), the hypothesis of a QFT-axion mass (e.g., 1.52 eV) is applied. Consequently, the one-axial mass is obtained as[2]:

$$I_{\lambda=A^0}\frac{\text{mol}^{-1}\cdot\text{T}^{-1}\cdot\text{C}^{-1}\cdot 10^9\ \text{K}}{10^2} = \frac{1}{100}\cdot\frac{\text{K}\cdot 1\ \text{eV}}{\left(0.00206\ \text{K}\right)\left(\text{e}^{\theta}\right)} = 1.52\ \text{eV} \quad (15)$$

Yields[3]

[2]Such as $O\left(\varepsilon^i\right) = \langle\phi_{IM}|\hat{O}|\phi_{RE}\rangle = 1\ \text{eV/K}$ (mentioned previously) in the valley of Higgs fields.

[3]Also follows the government (constraint) of Higgs bundles.

$$\theta \approx 2.273^{\circ} \approx 0.0126\pi, -\hat{z}(50\%) \tag{16}$$

with mixing *x*-ray photons ranged in ($T \approx T_{C'}$)

$$152.58\ \text{eV},\quad \hat{z}(50\%) \tag{17}$$

Interestingly, Equations (16) and (17) align with Schrödinger's cat thought experiment, where axions are represented as Nambu-Goldstone bosons. It is well known that the effective field theory (EFT) for dynamical axions in Weyl semimetals (WSMs) has been established. The pseudo-scalar axion excitation is precisely labeled around $T_{C'}$, as stated in this paper. Initially, the axion mass is zero; however, after continuous symmetry breaking, the axion acquires mass along the $-\hat{z}$-axis (*i.e.*, the Higgs valley). In this context, the axion mass is projected at an angle described by Equation (16) along the z-axis (or equivalently, in the *xy*-plane). On the other hand, this situation results in the axion mass returning to the scalar potential v of the Higgs field. In doublet $SU(2)_L$ case:

$$\langle H \rangle = \langle \psi_{i,j} | \hat{H} | \psi_{i,j} \rangle = \begin{cases} \left(\frac{1}{\sqrt{2}}\right)^2 \psi_{\gamma}^{*} \begin{pmatrix} 0 \\ v \end{pmatrix} \psi_{A^0} = 0, \text{initial} \\ \left(\frac{1}{\sqrt{2}}\right)^2 \psi_{A^0}^{*} \begin{pmatrix} 0 \\ v \end{pmatrix} \psi_{\gamma} = 0, \text{initial} \\ \left(\frac{1}{\sqrt{2}}\right)^2 \psi_{A^0}^{*} \begin{pmatrix} 0 \\ v \end{pmatrix} \psi_{A^0} = \frac{1}{2} \begin{pmatrix} 0 \\ v \end{pmatrix}, \text{broken} \end{cases} \tag{18}$$

If added the complex potential $i\phi$:

$$\langle H \rangle = \langle \psi_{i,j} | \hat{H} | \psi_{i,j} \rangle = \left(\frac{1}{\sqrt{2}}\right)^2 \psi_{A^0,\gamma}^{*} \begin{pmatrix} 0 \\ v + i\phi \end{pmatrix} \psi_{A^0,\gamma} = \frac{1}{2} \begin{pmatrix} i\phi \\ v \end{pmatrix}, \text{mixing} \tag{19}$$

This convincingly supports the correctness of the mixed states described in Equations (16) and (17). Notably, for these mixed states, the axion is not expected to require any form of "burying" (hiding). Most importantly, the naturalness of axions is characterized by a magnitude of 1/2, which is connected to the new concept named "half-bury" (refer to **Table 2**). See the important justification below.

***Justification*:**

Equations (15) to (17) perchance provide a route to explore axion-dark matters. Because of the axions following all rules of Higgs bundles, and their peculiar physical behaviors in the valley of Higgs fields, on some context. Besides, due to the work done by "US Cosmic Visions: New Ideas in Dark Matter 2017: Community Report Marco Battaglieri (SAC co-chair), Alberto Belloni (Coordinator), Aaron Chou (WG2 Convener), *et al.* p-42 (2017)" in which they state the followings:

"*GaAs has a direct gap of* 1.52 *eV, and thus a DM particle can scatter off a valence-band electron exiting it into the conduction band.*"

Through comparing with the previous discussion (e.g. Equation (15)), therefore we confirm the claim that: the axions are actually the dark matters[4].

[4]Section 2 is challenging yet intriguing, as this paper calculates the value of the Higgs boson mass, 125.8 GeV/c^2. Specifically, 125.09 GeV/c^2 (as measured by CMS and ATLAS) includes a statistical error ±0.57 and a systematic error ±0.11, both of which were announced by CERN in 2012. Above concepts due to Higgs fields can be useful for later discussion and results.

Table 2. Masses of particles and their relevance in Quantum Field Theory (QFT).

Sub atoms	Masses ($c^2=1$)	Possible Relationships	Buries
$W^{\pm}$	80.385 ± 0.015 GeV	$m_{H^0} \approx \frac{m_{W^+}+m_{W^-}+m_{Z^0}}{2}+0.11$	*xy*-plane of the sombrero[a].
Z^0	91.1876 ± 0.0021 GeV	(Divided by 2, based on the symmetry of being half-buried. Units: GeV)	*z*-axis of the sombrero[b].
H^0	125.8 GeV		c

[a]The symmetry of masses and charges is partially hidden in the Higgs valley. [b]The symmetry of masses is also partially hidden in the Higgs valley. [c]Presence of *x*-, *y*-, *z*-axes after $W^{\pm}, Z^0$ are half-hidden.

3. Results and Discussion

1) Magnetic Monopoles Overwhelmingly Revealed

Based on the analysis in Section 2.1, and using Equation (3), we find that the second term (representing the energy of the inflaton at its temperature scale) is denoted as

$$0.06 \text{ eV/K} \tag{20}$$

Specifically, the inflaton is hidden within the Higgs field sectors, with an energy scale of approximately 1 K. According to Equation (3), the value 0.06 eV corresponds to a Lagrangian of scalar fields. Recognizing that an inflaton contributes its energy to magnetic monopoles during the cosmic inflation epoch, we find that within the indicated region of Higgs hidden sectors[5], this calculation leads to a magnetic monopole with a Curie temperature around 713 K. Based on the previous deductions, the Magnetic Monopole Curie Temperature is further explored through:

$$\begin{gathered} E = \underbrace{0.06 \text{ eV}}_{\text{Inflaton}} = \underbrace{k_B T}_{\text{Higgs Hidden Sector}}, \quad T \neq T_{C'} = 0.00206 \text{ K} \\ T_{C,monopole} = \frac{0.06 \text{ eV}}{k_B} = \frac{9.6 \times 10^{-21} \text{ J}}{1.38 \times 10^{-23} \text{ J/K}} \approx 7.0 \times 10^2 \text{ K} \end{gathered} \tag{21}$$

where $T_{C'}$ means the Casimir temperature. These matters are clearly identified as α-Fe_2O_3 in experiments, as reported in the work done by Tan, K. C., Jani, H., Högen, M., *et al.* (2023), through the examination of tables detailing Curie points. The magnetic monopole remains hidden within specific energy scales, emerging around 7.0×10^2 K. Due to G. F. Giudice's principle of naturalness, hence

$$\sim \left(\frac{0.06 \text{ eV}}{\text{K}}\right) \times 713 \text{ K} \tag{22}$$

Divided by k_B and then produces

$$\frac{0.06 \text{ eV}}{k_B} \times 713 \sim \left(7.0 \times 10^2\right)^2 \text{ K} = 4.90 \times 10^5 \text{ K} \tag{23}$$

[5]See: Gunion, John; Haber, Howard; Kane, Gordon; Dawson, Sally (2000). The Higgs Hunter's Guide (illustrated, reprint ed.). Westview Press. ISBN 9780738203058.

For an electron, this corresponds to the Lorentz factor $\gamma \approx 10^5$ (refer back to Equation (12)). We suggest that a heavy ion traveling at such speeds could enable the detection of potential magnetic monopoles. Recalling Equation (12), it is evident that the factor 2 $(\uparrow\uparrow)$ degenerates due to spin flipping within a spin-ice system $(\uparrow\uparrow,\downarrow\downarrow)$. This occurs because the electron's axial configuration is fixed in the Higgs field (as expected), and the system can be positioned in a portal within the Higgs hidden sectors. These sophisticated methods suggest that the nature of the magnetic monopole can be entirely reflected in an undefined material—a "sea" of activated electrons, such as Fe-*xy* or Mg-*xy* (where *xy* represents as the unknown properties of atomic components)—through electron configurations in a pure spin state $(\uparrow\uparrow)$ or its opposites. Following sections will elaborate on this concept. The foundational work on spin-ice can be referenced in Ref. [8] [9]. In the next section, we will discuss the magnetic monopole in greater detail.

Dirac Quantization Conditions: The Derivative Magnetic Charge

Represented in symbolic of ρ_m form with spin angular frequency (1/sec) for $S = 1$:

$$0.06\ \text{eV} = 1.05\times10^{-34}\ \text{J}\cdot\text{s}\cdot\left(\frac{1}{\text{s}}\right)\rho_m, 1\ \text{m} \tag{24}$$

This implies that the magnetic monopole ρ_m passes through a vacuum flux per second per unit length. In CGS units (refer to **Appendix A** for the calculation), we derive the following remarkable expression:

$$\rho_m \approx 1\ \text{dyn}\cdot\left(\frac{\text{m}}{\text{eV}}\right) + \frac{1}{2}\ \text{dyn}\cdot\left(\frac{\text{m}}{\text{eV}}\right) \tag{25}$$

It is evident that the second term is sensitive to the Dirac quantization condition $\frac{2q_e q_m}{\hbar c}$. Consequently, it is denoted as $n = 1$, representing the ground state of the magnetic monopole. Meanwhile, the first term corresponds to the first excited state of the magnetic monopole $(n = 2)$. Furthermore, this leads to the following remarkable expression[6]:

$$\rho_m = \pm n'\cdot O\left(\varepsilon^i\right) + \underbrace{2\cdot\frac{1}{2}\ \text{dyn}\cdot\left(\frac{\text{m}}{\text{eV}}\right)}_{\text{F.E.S. with } n=2} + \underbrace{1\cdot\frac{1}{2}\ \text{dyn}\cdot\left(\frac{\text{m}}{\text{eV}}\right)}_{\text{G.S. with } n=1}, n' > 2 \tag{26}$$

We observe that the vastly diminished higher-order terms ($n' > 2$, which evidently fall within macroscopic scales) are associated with positive and negative polarizations. This accurately reflects the general structure of magnets: N, S/S, N. Based on this observation, we cautiously hypothesize that the term $n' > 2$ represents a singular point—a barrier[7] that separates bulk regions of magnets. This

[6]Also obeys the famous Non-Abelian Higgs Mechanism.

[7] $n' > 2$ is inherently included in any Grand Unified Theories (GUTs). For instance, this is demonstrated in the work done by the authors in Ref. [10] later on. Specifically, by referencing their Equation (11) and applying the natural logarithm, the ground state of the magnetic monopole is undoubtedly represented as $\rho_m \approx \frac{1}{2} \pm n\cdot O\left(\varepsilon^i\right)$.

hypothesis is supported by the infinitesimal term $\varepsilon^{i} \to 0$, which converges to zero and is related to inflatons (*i.e.*, spatially flat configurations). This sensitivity suggests that a world-line must be eliminated to allow for the division of magnets, in accordance with the principles of physics.

2) Inflations: The Magnetic Monopole MgO-Fe_2O_3 Hidden in The Portal of Higgs Hidden Sectors

The magnetic monopole is embedded within the scenario of a Higgs portal, and this has been strongly validated. Upon further detailed analysis, we find that

$$\begin{aligned} &E = hc/\lambda, \\ &0.06\ \text{eV} = 12400\ \text{eV}\left(0.1\ \text{nm}\right)/\lambda, \\ &\lambda_{LWIR} = \frac{12400\ \text{eV}\left(0.1\ \text{nm}\right)}{0.06\ \text{eV}} \approx 20.6\ \mu\text{m} \end{aligned} \tag{27}$$

Using the ΛCDM model at t_0. Therefore, obviously

$$\lambda_{CBM,micro.} \sim 1\ \text{m} \tag{28}$$

Interestingly, during the inflationary epoch, the size of the universe was approximately 2 meters (waves are two directional propagated).

Justification: The higher-order terms in Equation (3), where $O\left(\varepsilon^{i}\right)$ represents the wavelength, are undoubtedly smaller than the values discussed in this paper. These terms were initially overlooked. As observed, the second term relates to the LWIR region near the Cosmic Microwave Background (CMB). Consequently, we can infer that the first term of the formula for the "Higgs mass", derived from calculations involving the sombrero potential, indicates that Higgs bosons act as inflatons. And, the inflaton energy is due to the portion of 0.06 eV of Higgs bosons. See Equation (3).

In Search of Materials for the Magnetic Monopole:

The Curie temperature of the magnetic monopole, grounded in the Higgs mechanism as discussed in previous sections, is explored through Equation (21), where $T_{C'}$ represents the Casimir temperature as discussed in the published paper (2024). The results are as follows:

a) Around 700 K, under the condition of inflaton energy at 0.06 eV, the previously referenced file on the inertial Curie temperature[8] in Higgs fields highlights the need to identify corresponding materials. This investigation leads to intriguing observations.

b) By inspecting tables of Curie points (Buschow 2001, p. 5021, tables mentioned by Buschow), the first author identifies **MgO-Fe_2O_3** as the corresponding material for a magnetic monopole at a temperature of 713 K. Additionally, research conducted by the University of Oxbridge and Singapore Team (2024) concludes that hematite **(Fe_2O_3)** is a favored candidate.

c) These materials are discoveries of interest but remain significantly distinct from the unique existence of a fundamental magnetic monopole, as predicted by

[8]See Ref. [6] about the definition of the inertial Curie temperature.

Paul Dirac. They may be classified as candidates but exclude the specific theory of spin-ice.

3) The Conversion: Two Photons and One Magnetic Monopole

Einstein's photon energy can be expressed as following: where E is the energy of the photon, h is Planck's constant. Taking the natural logarithm on both sides under consistent units gives:

$$E = \frac{hc}{\lambda} \tag{29}$$

Furthermore,

$$\begin{aligned} \log E &= (\log h + \log c) - \log \lambda, \\ \log h + \log c &= \log E\lambda \end{aligned} \tag{30}$$

Equation (30) exhibits the implicit. In the previous study by the authors, using Equation (61) as part of Model II: Small Size Instanton Contribution (Belén Gavela *et al.*, 2019, p. 103 of Ref. [11]), it was found that:

$$\frac{1}{3}\log_{10}\left(6.626\times10^{-34}\right)\frac{1}{10\times7.06} \approx -0.15 \tag{31}$$

It appears to exhibit a slope, though a positive value is also possible. This aspect can be attributed, at least in part, to the findings in Ref. [11], particularly the red lines. The suggested approach involves applying derivative rules. Let the Running Couplings be represented on the *y*-axis, and the RGE scale μ (in GeV) on the x-axis. Following the implicit differentiations, the slope is given as:

$$-\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\mathrm{d}x/\mathrm{d}y\right)^{-1} \tag{32}$$

The new expression becomes:

$$\frac{1}{3}\log_{10}\left[\left(6.626\times10^{-34}\right)\right]^{-1}\frac{1}{10\times7.06} \approx 0.15 \tag{33}$$

The curve fitting at the LHS point 0.14 (*y*-axis) on p-103 in Ref. [11] indicates that the term $\ln\hbar$ can represent any magnetic monopole. Based on the above computations, the terms

$$\log h + \log c = \log E\lambda \tag{34}$$

It can be correctly aligned with the following decayed reactions[9]:

$$\rho_m \to \gamma\gamma, E \approx 0.06\ \mathrm{eV}, \tau \approx 10^{-14}\ \mathrm{s} \tag{35}$$

This aligns, in some contexts, with the predicted phenomena involving world-lines, suggesting that the conversion between photons and magnetic monopoles is possible. Consequently, magnetic monopoles may potentially be produced in large accelerators or colliders (e.g., heavy ions traveling at speeds approaching the speed of light, C, and nearing the world-line). It is worth noting that Equation (33) provides a reliable mechanism for Spontaneous Symmetry Breaking (SSB), particularly during the 1 TeV cosmic epoch. This mechanism facilitates the generation of

[9]Amounts of magnetic monopoles are estimated around 5.22×10^3, and are a few amounts in cosmology.

tables of physical constants (e.g., the electric charge, Boltzmann constant, etc.). The process involves substituting the physical constants into brackets for computation. This suggests that fundamental physical constants can be broadly reproduced within the framework of new physics concepts.

4) Graphenes: 't Hooft Condition 2D-Graphenes with QFT-Axions

We demonstrate that the momentum space situated within Higgs fields (represented by the Mexican hats or sombreros) is

$$\left.\begin{aligned}&\sqrt{k_x^2+k_y^2}\pm k_z=0,\\&\sqrt{k_x^2+k_y^2}=0=k_z, A^0(WSMs)\to\gamma\gamma\\&-k_z=\sqrt{k_x^2+k_y^2}=0, \gamma\gamma\to A^0(WSMs), T=0.00206\text{ K}\end{aligned}\right\}\text{Breakings} \tag{36}$$

The Landau Level and Dirac Matters both indicated:

$$\begin{aligned}&E_n=\hbar\omega_c\sqrt{|n|}, |n|=1/9,\\&E_{1/3}=\hbar\omega_c/3\end{aligned} \tag{37}$$

The fraction $\delta^{-1}=1/3$ (the Ginzburg-Landau's critical exponent $\delta=3$) complies with the Fractional Quantum Hall Effect (as shown in Equation (3.15) of the paper titled *Axions and Superfluidity in Weyl Semimetals*). Moreover, this value adheres to the principle of naturalness concerning UV divergence, characterized by the slope $\hbar/c$ (see Note 1 below).

Recalling Equation (12), it is modified to:

$$\begin{aligned}&I_{\lambda=electron}\,\text{mol}^{-1}\cdot\text{T}^{-1}\cdot\text{C}^{-1}\overset{(\text{Break. by }\mu_5\to\infty)}{=}6\cdot2\cdot\left((0.00206\text{ K})\prod_{\mu_5=1}^{\infty}\text{e}^{\mu_5\cdot3\pi/2}\right)^{-1}\\&=\underbrace{\overbrace{2}^{\text{Weyl nodes}}}_{(2b^0,2b)}\cdot\overbrace{6}^{\text{Dirac points}}\cdot\left(\frac{1}{0.00206\text{ K}}\right)\lim_{\mu_5\to\infty}\frac{1}{\text{e}^{\mu_5\cdot3\pi/2}}\\&=0\end{aligned} \tag{38}$$

With the electron-hole pairs:

$$+/-\Leftrightarrow-/+ \tag{39}$$

Refer to p. 12 of the paper *Axions and Superfluidity in Weyl Semimetals*, where $\text{e}^{i\frac{\phi}{\hbar}}\equiv\text{e}^{-\mu_5 3\pi/2}$ is associated with anomalous symmetry and is explicitly referred to as the Chern number. Additionally, $1/\text{e}^{\mu_5\cdot3\pi/2}$ plays the role of 't Hooft's large N expansion ($N\equiv\text{e}^{\mu_5\cdot3\pi/2}$)[10].

Note 1: Refer to CERN, Bevela *et al.* (2019), for curves showing weak correlations involving UV divergence and $\hbar$:

The slope m of the secant lines is

$$m=\frac{\text{d}y}{\text{d}x}=\frac{\hbar}{c}=\frac{1.05\times10^{-34}\text{ J}\cdot\text{sec}}{3\times10^{8}\text{ m/sec}}=\frac{1.05\times10^{-34}\text{ J}\cdot\text{s}^2}{3\times10^{8}\text{ m}}\approx\left(\frac{1}{3}\right)\frac{10^{-42}\text{ J}\cdot\text{s}^2}{\text{m}}\to0 \tag{40}$$

[10]The possible breaking described above corresponds to $U(1)^{ch}$. Notably, 't Hooft proposed that a 't Hooft anomaly in a continuous symmetry can be calculated both at high-energy and low-energy scales, yielding the same result. $\text{e}^{i\frac{\phi}{\hbar}}\equiv\text{e}^{-\mu_5 3\pi/2}, \phi=-i\mu_5 3\pi\hbar/2, \mu_5=1,2,3,\cdots$ is Chern-Simons action.

This is evidently consistent with the naturalness of 1/3. In Equation (40), C and $\hbar$ nearly lie on a horizontal line, demonstrating the naturalness of $1/g = 1/3$ with precision. This resolves the issues of 't Hooft's strong interactions, particularly in the context of the indicated $\lim N \to \infty, g^2 N = \textit{fixed}$ (*i.e.*, $g = 3$) for hadrons. As stated by 't Hooft [13]: "*For hadrons*, *N is probably equal to three*."

The Dirac Matters: Electrons are situated at six Dirac points, where they remain massless. These material electrons, present in any graphenes (classified as a semimetal), are widely recognized as Dirac matters.

4. Conclusions

The long-standing mystery of magnetic monopoles is theoretically addressed in this paper. Utilizing the concept of Higgs field hidden sectors (*i.e.*, the Casimir temperature located within Higgs portals), we uncover key insights. The spin of the magnetic monopole is entirely $s = 1$, and its magnetic charge is overwhelmingly expressed as $\rho_m \approx 1\,\text{dyn}\cdot(\text{m/eV}) + \frac{1}{2}\,\text{dyn}\cdot(\text{m/eV})$. Our extrapolation suggests that the corresponding temperature for the magnetic monopole is approximately 700 K. Based on this, and through further investigation, we identify that MgO–Fe_2O_3 is concealed within the Higgs field hidden sectors. Furthermore, this material aligns well with other experimental results reported (2023). And on the other hand, we even have found that the axions are dark matters successfully.

Abbreviation

SSB means "Spontaneous Symmetry Breaking".

Data Availability Statement

1) Equations (4) to (14) exhibit one that the method by Higgs field hidden sectors can be effective to any sub-atom particles (exceptions of photons) for the relevant calculations (e.g. masses and the Lagrangian).

2) The confidential data in Equations (3) and (17) are used. Particularly, in Equation (3), where 0.06 eV is hidden in the dark Higgs field hidden sectors.

3) The data calculated by later Equations (B.1) to (B.3) are confidential.

4) The data exhibited by Equations (3) and (35) are confidential.

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Author Contribution Statements

The first author contributes main ideas of construction of the frames of this theory. The second author plays the role of supervisor and is responsible for the plot. The authors contribute equally.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix

A. Calculation: Finding Magnetic Monopoles

Using the principle of energy conservation for the portal of Higgs hidden sectors, and eventually let the units be expressed as *dyn* to fit the definition of the magnetic charge:

$$\begin{aligned}
&0.06\left(1.6\times10^{-19}\,\mathrm{J}\right)=\left(1.05\times10^{-34}\right)\mathrm{J}\rho_m,\\
&\rho_m=\frac{0.06\left(1.6\times10^{-19}\right)}{1.05\times10^{-34}}=0.0914\times10^{15},\ \text{SI units}\\
&\rho_m\cdot10^{7}\,\mathrm{erg}=0.0914\times10^{20}\,\mathrm{dyn}\cdot10^{2}\,\mathrm{cm},\ \text{CGS units}\\
&1\,\mathrm{dyn}\approx10.94\times10^{-20}\rho_m\cdot\left(10^{7}\,\mathrm{erg}/10^{2}\,\mathrm{cm}\right),\\
&1\,\mathrm{dyn}\approx10.94\times10^{-20}\rho_m\cdot\left(6.24\times10^{18}\,\mathrm{eV}\right)/\mathrm{m},\ \text{SI units for fractions}\\
&\rho_m\approx1\,\mathrm{dyn}\cdot\left(\frac{\mathrm{m}}{\mathrm{eV}}\right)+\frac{1}{2}\,\mathrm{dyn}\cdot\left(\frac{\mathrm{m}}{\mathrm{eV}}\right)
\end{aligned}\tag{A.1}$$

B. Lists of 1 TeV Cosmic Epoch (SSB)

Spontaneous Symmetry Breaking: The physical constants of elementary particles align with the plots from *Model II*: *Small Size Instanton Contribution* (work done by Belén Gavela *et al.*, on p-103 (2019) as mentioned previously). The computations yield the following four sets of values:

$$\frac{1}{3}\log_{10}\left(\Gamma^{-1}\right)\frac{1}{10\times7.06}\Bigg|_{\Gamma=(e,\mu_B,m_e,k_B)}\approx\left(0.089,\ 0.11,\ 0.14,\ 0.108\right)\tag{B.1}$$

where e is the electric charge, μ_B is the Bohr magneton, m_e is electron mass, and k_B is the Boltzmann constant and Equation (B.1) follows the implicit differentiations. The average is obviously computed as

$$\bar{x}=\frac{0.089+0.11+0.14+0.108}{4}\approx0.112\tag{B.2}$$

This corresponds to the region located on the RGE scale μ (x-axis) in GeV, with $10^2<\bar{x}<10^4$ expressed in GeV units. This demonstrates that the elementary particles associated with physical constants emerged at specific energy scales, particularly during the Spontaneous Symmetry Breaking (SSB) phase of the 1 TeV cosmic epoch.

Additionally, the other physical constants are believed to align with the calculations mentioned above, falling within the permitted errors. Interestingly, if one incorporates the gravitational constant (in SI units) and applies the unconditional elimination method:

$$\begin{aligned}
&\frac{1}{3}\log_{10}\left(\Gamma^{-1}\right)\frac{1}{10\times7.06}\Bigg|_{\Gamma=G_N}\approx0.04^{+},\sim10^{11}\,\mathrm{GeV}\\
&\frac{1}{3!}\log_{10}\left(\Gamma^{-1}\right)\frac{1}{10\times7.06}\Bigg|_{\Gamma=G_N}\approx0.02^{+},\sim10^{18}\,\mathrm{GeV}
\end{aligned}\tag{B.3}$$

This leads to the point 0.02^{+}, which is located in the region of the RGE scale μ (x-axis) ~10^{18} in GeV and corresponds to the speed of light C (see Ref. [10]). This implies that the four fundamental forces of the cosmos can converge at 10^{18}-scale energies (in GeV). While this idea is widely known, this paper provides a rational link to this concept by utilizing ideas presented in Ref. [10]. This reference only unifies the cosmic three forces but not includes the gravitational force, since that the G_N running points have never appeared at any data from CERN, and due to the reason of the much small parameter as angle of G_N such that this negligible angle is not taken to consideration. To unify cosmic four fundamental forces one has to think another ways.

C. Conjecture

According to Equation (35) and like $\ln \hbar$ represented as magnetic monopoles in GUTs [10], therefore, $\chi \sim \ln\left(\frac{10^{-14}\,\text{s}}{1\,\text{s}}\right) \approx -32$, we guess that the inflationary epoch ended at $10^{\chi}\,\text{s} \sim 10^{\ln\left(10^{-14}\right)}\,\text{s} \approx 10^{-32}\,\text{s}$.