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Quantum Mechanics of a Quasi-Euclidean Space with Planck Length, Rotational Symmetry and Translational Symmetry

Yan Zhou[1,2*], Junyan Zhang[1*#], Ernst Meyer[2#], Xingkai Zhang[1], Jing Zhang[1]

[1]Lanzhou Institute of Chemical Physics, Chinese Academy of Sciences, Lanzhou, China
[2]Department of Physics, University of Basel, Basel, Switzerland
Email: #junyanzhang@licp.cas.cn, #ernst.meyer@unibas.ch

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Abstract

This work is focused on a quasi-Euclidean space with UV cutoff, IR cutoff and symmetries. Mathematical analysis reveals that the UV cutoff results in the minimum structures of space. Dominated by rotational symmetry, the structure should be a local one *in situ* or on a sphere. Investigations show that a 10D minimum structure is a non-local one with transformability between in-situ state and spherical state due to its special topology. Based on the quantum behaviors of the 10D structure controlled by translational symmetry, IR cutoff determines two long-range interactions with dimensionless constants of ~1/137.036 and ~1/1.628E+38, respectively.

Keywords

Planck Length, IR Cutoff, Symmetry, Fine Structure Constant (FSC), Quantum Gravity

1. Introduction

The fine structure constant (FSC, ~1/137.035999), which is a dimensionless quantity characterizing the strength of the electromagnetic interaction, has fascinated innumerable scientists since it appeared in 1916 [1]. Its value has been measured more and more precisely in the cosmos explored by humans [2]-[5], whereas its theoretical origin remains unknown till now. Although some interesting formulae have been proposed, such as $\left(137^2+\pi^2\right)^{\frac{-1}{2}}=1/137.036016$ [6],

*Authors contributed equally to this work.
#Corresponding authors.

$\left(4\pi^3+\pi^2+\pi\right)^{-1}=1/137.036034$ [7], etc., there is never a convincing solution that has both numerical consistency and sufficient theory for FSC. It's even unknown whether FSC is calculable in principle or is a non-calculable one determined by historical or quantum mechanical accident [8].

Nevertheless, there are still a considerable number of scientists who insist that FSC must have theoretical derivations [9]-[11]. If FSC is really a dimensionless constant with calculability, like another fundamental constant π defined by a radius and the semicircle it determines, then two prerequisites, including a natural object and its characteristic path relatively measured ~137.036, must be both present. As for the former, some mathematical models, including the point-like one, the lattice-like one, the string-like one, etc., have been assumed to be the natural objects [12] [13]. As for the latter, Euclidean spaces and manifolds have been involved to be the background for the research [14]. Although no precise solutions had been obtained directly, some attempts illuminate that the higher dimensional spaces beyond 3D/4D may be required to study some fundamental interactions [15] [16].

For the further pursuit of a calculable FSC, the present work chooses a generalized *n*D Euclidean space with Planck length as the background. Here the limit of Planck length turns the Euclidean space to a quasi-Euclidean space, resulting in an extremely small *space bubble*, inside which no distance is reasonably allowed. After taking such a *space bubble* as a natural object, its moving path in *n*D space then will be searched for. Definitely, this small *bubble* would obey quantum mechanics and some other basic principles of physics, such as conservation of mass, the minimum energy, etc. Besides, we should not forget the most important thing that they satisfy rotational symmetry and translational symmetry, since they are the parts of the vacuum, which is proven by all experiments to be of absolute symmetries, no matter when and where. This means geometrical limits will be strongly involved when considering the motion of such a *space bubble*. Therefore, we set up a quasi-Euclidean space with Planck length and symmetries, trying to quantize it via the space *bubble* and explore its quantum behaviors, not only physically but also geometrically.

2. Planck Units: Quantization of Quasi-Euclidean Space $\hat{\boldsymbol{P}}$

2.1. Quasi-Euclidean Space $\hat{\boldsymbol{P}}$

Compared to a general *n*D Euclidean space ${}^{n}\boldsymbol{P}$ (dimension symbols are marked in the upper left corner of a certain space $\boldsymbol{P}$ in this work), a special Euclidean space noted as ${}^{n}\hat{\boldsymbol{P}}$ is established here to be a Euclidean space carrying the Planck length λ_{P}, which determines the minimum distance in vacuum and comes from $\lambda_{\mathrm{P}}=\sqrt{\frac{G\hbar}{c^3}}=1.616252(81)\times10^{-35}\ \mathrm{m}$, where $\hbar$ is the reduced Planck constant, G is the gravitational constant and c is the speed of light in vacuum [17]. Therefore, space ${}^{n}\hat{\boldsymbol{P}}$ can be described as follows. On the one hand, it exists $\hat{x}\equiv x$ ($x\geq\lambda_{\mathrm{P}}$), showing that ${}^{n}\hat{\boldsymbol{P}}$ behaves as same as its corresponding Euclidean space ${}^{n}\boldsymbol{P}$ on the relatively macroscopic scale $\geq\lambda_{\mathrm{P}}$. On the other hand, it exists $\hat{x}=\lambda_{\mathrm{P}}$

($0 < x < \lambda_P$), showing that any 1D measure along a certain dimension of ${}^n\hat{\boldsymbol{P}}$ is never less than λ_P on the microscopic scale $< \lambda_P$, although ${}^n\hat{\boldsymbol{P}}$ remains a Euclidean space at the same time. E.g., for two adjacent points on a certain dimension of ${}^n\hat{\boldsymbol{P}}$, a blank interval measured λ_P occurs between them even they are originally infinitely close to each other on the macroscopic scale.

Based on the above commonalities and difference, relationship between quasi-Euclidean space ${}^n\hat{\boldsymbol{P}}$ and its corresponding Euclidean space ${}^n\boldsymbol{P}$ can be mathematically demonstrated as follows.

Commonalities (linearity):

$$\hat{x} = kx + a \tag{1}$$

$$\hat{x} = kx \quad (\text{same position with } a = 0) \tag{2}$$

$$\hat{x} = x + a \quad (\text{same measure with } k = 1) \tag{3}$$

Difference (Planck length):

$$\hat{x} = \lambda_P \cdot e^{\frac{i \cdot \hat{x}}{\bar{\lambda}_P}} \quad (\bar{\lambda}_P = \frac{\lambda_P}{2\pi}) \tag{4}$$

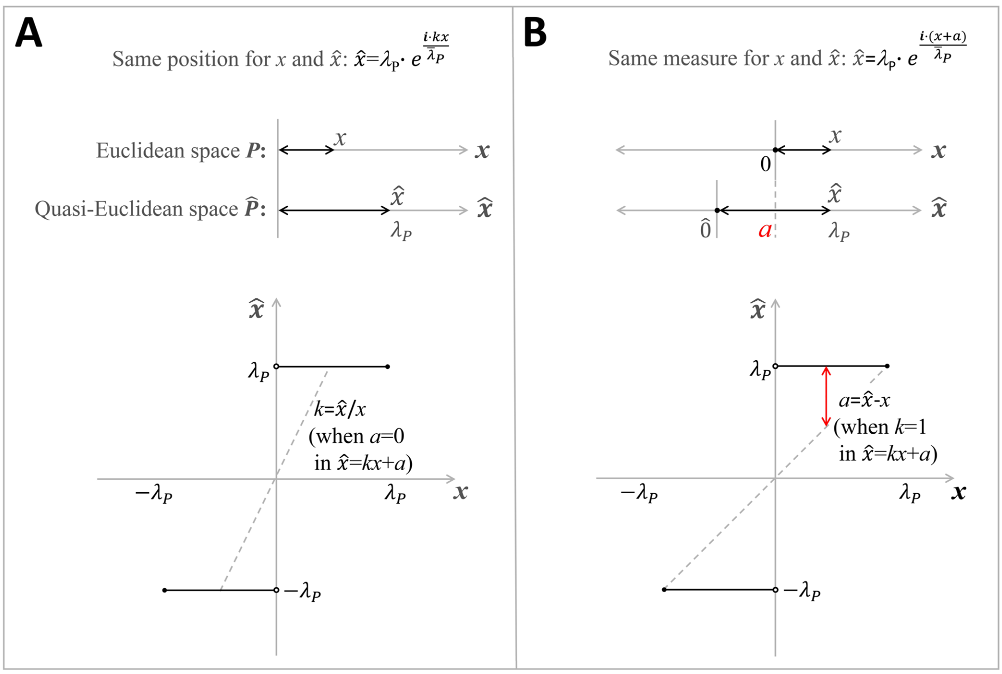

Figure 1. Difference between Euclidean space $\boldsymbol{P}$ and quasi-Euclidean space $\hat{\boldsymbol{P}}$ at Planck scale: $|\hat{x}| = \lambda_P$ ($0 < |x| < \lambda_P$) when x and $\hat{x}$ share the same position (A) or the same measure (B).

As two equivalent spaces at macro scale, ${}^n\hat{\boldsymbol{P}}$ and ${}^n\boldsymbol{P}$ always satisfy $k = 1$ and $a = 0$ simultaneously in Equation (1). However, the exclusive character of Planck length $\hat{x}_{min} = \lambda_P$ for ${}^n\hat{\boldsymbol{P}}$, as shown by Euler's formula in Equation (4), brings $\hat{x} \neq x$ (**Figure 1**) and leads to the alternatively satisfied $a = 0$ and $k = 1$ in Equation (2) and Equation (3), respectively. Briefly, it exists $\hat{x} = \lambda_P$ when $x \in (0, \lambda_P]$ (**Figure 1**). Besides, $\hat{x} = 0$ when $x = 0$ describes the special case where the minimum distance of $\lambda_{\rm p}$ is invalid since the two adjacent points coincide and act as one point. This special case demonstrates that it exists $\hat{x}=0$ when a single point is involved.

So quasi-Euclidean space ${}^{n}\hat{\boldsymbol{P}}$ can be mathematically defined in the space of non-negative part as

$$\hat{x}=x \quad (x\geq\lambda_{\rm p}) \tag{5}$$

$$\hat{x}=\lambda_{\rm p}\cdot{\rm e}^{\frac{\boldsymbol{i}\cdot kx}{\bar{\lambda}_{\rm p}}} \quad \text{or} \quad \hat{x}=\lambda_{\rm p}\cdot{\rm e}^{\frac{\boldsymbol{i}\cdot(x+a)}{\bar{\lambda}_{\rm p}}} \quad (0<x<\lambda_{\rm p}) \tag{6}$$

$$\hat{x}=0 \quad (x=0) \tag{7}$$

Obviously, unless considering the minimum distance at the extremely small scale, a quasi-Euclidean space ${}^{n}\hat{\boldsymbol{P}}$ is equivalent to its corresponding Euclidean space ${}^{n}\boldsymbol{P}$. So, ${}^{n}\hat{\boldsymbol{P}}$ is basically taken as a space that not only applies the axioms and definitions of Euclid space, but also applies all the physical principles in a general Euclidean space, being a normal background for experimental or theoretical physical objects, including a particle, a field, and so on.

2.2. Relationship between $\hat{P}$ and P

Based on the mathematical definition of quasi-Euclidean space $\hat{\boldsymbol{P}}$ in Equations (1)-(4), the relationship between $\hat{\boldsymbol{P}}$ and its corresponding Euclidean space $\boldsymbol{P}$ is obtained.

Non-commutativity in 1D. Equations (2)-(4) result in

$$k=-\boldsymbol{i}\bar{\lambda}_{\rm p}\frac{\rm d}{{\rm d}x};\quad k^{-1}=-\boldsymbol{i}\bar{\lambda}_{\rm p}\frac{\rm d}{{\rm d}\hat{x}} \tag{8}$$

$$[k,x]=-\boldsymbol{i}\bar{\lambda}_{\rm p};\quad \left[k^{-1},\hat{x}\right]=-\boldsymbol{i}\bar{\lambda}_{\rm p} \tag{9}$$

$$[\hat{x},x]=-\boldsymbol{i}\bar{\lambda}_{\rm p}\cdot x;\quad [x,\hat{x}]=-\boldsymbol{i}\bar{\lambda}_{\rm p}\cdot\hat{x} \tag{10}$$

where Equation (10) demonstrates the non-commutativity between $\hat{\boldsymbol{P}}$ and $\boldsymbol{P}$ in any one dimension (Part 1 in Supplementary information).

Commutativity in ≥2D. For any a pair of local ≥2D spaces $\hat{P}$ and P expanded respectively by orthogonal $\hat{x}_i,\hat{x}_j,\cdots$ and $x_i,x_j,\cdots$, it exists

$$\hat{P}=\prod\hat{x}_i\hat{x}_j\cdots=\prod x_j x_i\cdots=P \tag{11}$$

since the orthogonality results in $\left[x_i,\hat{x}_j\right]=0$, which guarantees Equation (11). Here ${}^{n}\hat{P}={}^{n}P$ ($n\geq 2$) in Equation (11) ensures the commutativity between $\hat{\boldsymbol{P}}$ and $\boldsymbol{P}$ in any ≥2D spaces.

Reciprocity. Results in Equations (8)-(10) also determine a special relationship of reciprocity between $\hat{x}$ and x for it always exists

$$\exists\, x=F(\hat{x}),\ \forall\, \hat{x}=F(x) \tag{12}$$

vice versa. This reciprocity leads to the interchangeability between $\hat{\boldsymbol{P}}$ and $\boldsymbol{P}$, demonstrating the equivalence for a certain object under two conditions, one is to measure it with reference to $\boldsymbol{P}$ when it lies in $\hat{\boldsymbol{P}}$, the other is to do so with reference to $\hat{\boldsymbol{P}}$ when it lies in $\boldsymbol{P}$.

Thus, the relationship between the quasi-Euclidean space $\hat{\boldsymbol{P}}$ and its corresponding Euclidean space $\boldsymbol{P}$ can be summarized into 3 points, including non-commutativity in 1D, commutativity in ≥2D, and reciprocity.

2.3. Properties of Quasi-Euclidean Space $\hat{P}$

Uncertainty and UV cutoff $\bar{\lambda}_{\mathrm{p}}$. According to the conclusions about non-commutativity [18], $\hat{x}$ and x, as a pair of non-commutative quantities represented in Equation (10), are relatively uncertain for they can't be determined simultaneously. Besides, Equation (9) results in the same minimum 1D measure of $\bar{\lambda}_{\mathrm{p}}$ (reduced Planck length $\bar{\lambda}_{\mathrm{p}} = \frac{\lambda_{\mathrm{p}}}{2\pi}$; Part 2 in Supplementary information) for both $\hat{x}$ and x, since

$$[k, x] = -i\bar{\lambda}_{\mathrm{p}} \Rightarrow (\Delta kx)^{+} \geq \frac{\bar{\lambda}_{\mathrm{p}}}{2};\ \left[\frac{1}{k}, \hat{x}\right] = -i\bar{\lambda}_{\mathrm{p}} \Rightarrow \left(\Delta\frac{\hat{x}}{k}\right)^{+} \geq \frac{\bar{\lambda}_{\mathrm{p}}}{2} \tag{13}$$

$$|\Delta\hat{x}| = (\Delta kx)^{+} - (\Delta kx)^{-} \geq \bar{\lambda}_{\mathrm{p}};\ |\Delta x| = \left(\Delta\frac{\hat{x}}{k}\right)^{+} - \left(\Delta\frac{\hat{x}}{k}\right)^{-} \geq \bar{\lambda}_{\mathrm{p}} \tag{14}$$

As a visual understanding of the above results, the minimum measurements may always be the same $\bar{\lambda}_{\mathrm{p}}$ for either of the two scenarios, one is to measure a math space with infinite scales by a physical ruler with the minimum scale $\bar{\lambda}_{\mathrm{p}}$, the other is to measure a physical space with the smallest length $\bar{\lambda}_{\mathrm{p}}$ by a math ruler with infinite scales. Here, the smallest 1D measure $\bar{\lambda}_{\mathrm{p}}$ is named UV cutoff for both $\hat{\boldsymbol{P}}$ and $\boldsymbol{P}$, when they are measured by each other.

Briefly, $|\Delta\hat{x}| \geq \bar{\lambda}_{\mathrm{p}}$ of the UV cutoff results in a 1D blank interval, directly leading to a quantized 1D for $\hat{\boldsymbol{P}}$.

Planck units $\hat{p}$. The minimum blank interval along any a dimension will naturally result in an *n*D minimum blank interval ${}^{n}\hat{p}$ or ${}^{n}p$. Algebraically, these two minimum spaces ${}^{n}\hat{p}$ and ${}^{n}p$ share the same minimum measure $\bar{\lambda}_{\mathrm{p}}^{n}$ because UV cutoff $\bar{\lambda}_{\mathrm{p}}$ requires any local space ${}^{n}\hat{P}$ or ${}^{n}P$ to satisfy

$${}^{n}\hat{P} = \prod \hat{x}_i\hat{x}_j \cdots \geq \bar{\lambda}_{\mathrm{p}}^{n} = \hat{p};\ {}^{n}P = \prod x_i x_j \cdots \geq \bar{\lambda}_{\mathrm{p}}^{n} = p \quad (x_i, x_j, \cdots > 0) \tag{15}$$

Geometrically, the boundary of an *n*D minimum interval ${}^{n}\hat{p}$ is determined by two factors, including the 1D condition and the *n*D condition. Regarding the 1D condition about UV cutoff $\bar{\lambda}_{\mathrm{p}}$, any distance $\rho < \bar{\lambda}_{\mathrm{p}}$ inside the boundary is forbidden. Let the boundary be determined by $\rho < \bar{\lambda}_{\mathrm{p}} \cdot \cos\alpha \cdot \sin\beta \cdot \sin\gamma \cdot \ldots$, which defines a boundary on a generalized *n*D sphere with center located at ($\frac{\bar{\lambda}_{\mathrm{p}}}{2}$, 0, $\frac{\pi}{2}$, $\frac{\pi}{2}$, ...) on the polar axis of a polar coordinates (**Figure 2(A)**). Obviously, any distance inside the boundary will result in $\rho < \bar{\lambda}_{\mathrm{p}}$ and the violation of the 1D condition. Regarding the *n*D condition that the minimum interval is of *n*D measure ${}^{n}\hat{p} \ngtr \bar{\lambda}_{\mathrm{p}}^{n}$, then ${}^{n}\hat{p}$ should be of another boundary on the surface of a generalized *n*D cube with side length $\bar{\lambda}_{\mathrm{p}}$, and any distance outside the cube is forbidden since it will result in an *n*D measure $> \bar{\lambda}_{\mathrm{p}}^{n}$ and the violation of the *n*D condition (Figure 2(B)). These two boundaries, including the spherical one and the cubic one, determine the inner and outer boundary for the minimum interval ${}^n\hat{p}$, respectively. Moreover, any intermediate between the outer one and the inner one is also a valid boundary for ${}^n\hat{p}$ because of the linearity of ${}^n\hat{\boldsymbol{P}}$. The above results indicate that it exists a series of boundaries for ${}^n\hat{p}$, and ${}^n\hat{p}$ has an uncertain boundary in nature (Figure 2(C)).

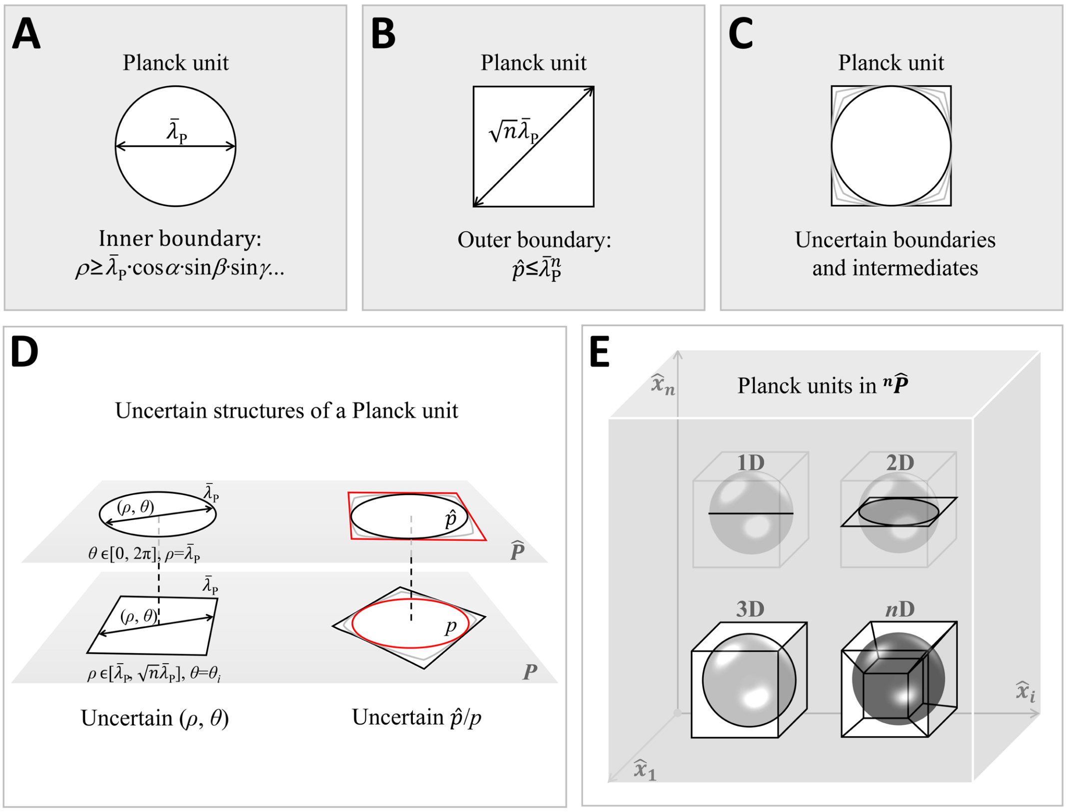

Figure 2. The uncertain boundary of a Planck unit: it can be a generalized circle without any distance shorter than $\bar{\lambda}_P$ inside (A), a generalized cube without any distances longer than $\sqrt{n}\bar{\lambda}_P$ outside (B), or an intermediate between them (C). The uncertain structure of a Planck unit and the uncertain relationship between $\hat{p}$ and p (D). *Space bubbles* of Planck units ${}^i\hat{p}$ located in subspaces of quasi-Euclidean space $\hat{\boldsymbol{P}}$ (E).

Consequently, the uncertain boundary results in the uncertain structure for ${}^n\hat{p}$. E.g., to represent a 2D minimum local space ${}^2\hat{p}$ by $\hat{p}(\rho, \theta)$, it always exists certain $\rho = \bar{\lambda}_P$ when θ is completely uncertain ($0 \leq \theta \leq 2\pi$), or uncertain ρ ($\bar{\lambda}_P \leq \rho \leq \sqrt{2}\bar{\lambda}_P$) when θ is of certainty (Figure 2(D)).

Here the minimum nD blank interval ${}^n\hat{p} = \bar{\lambda}_P^n$ with uncertain boundary and uncertain structure is named a Planck unit. In quasi-Euclidean space ${}^n\hat{\boldsymbol{P}}$, a Planck unit ${}^n\hat{p}$ generally defines a local space with the following characteristics. Firstly, it is a blank space with constant nD generalized volume $\bar{\lambda}_P^n$ but uncertain structure varying from a sphere with a diameter of $\bar{\lambda}_P$ to a cube with a side length of $\bar{\lambda}_P$. Secondly, the longest 1D distance of the boundary varies in interval of [$\bar{\lambda}_P$, $\sqrt{n}\bar{\lambda}_P$] (Figure 2(D)). Obviously, the Planck unit is a natural extension of the concept of the UV cutoff. Mathematical derivation indicates that ${}^{n}\hat{\boldsymbol{P}}$ can also be quantized by ${}^{n}\hat{p} = \bar{\lambda}_{\text{P}}^{n}$, just like its 1D subspace can be quantized by UV cutoff $\bar{\lambda}_{\text{P}}$. Considering the special case about $\hat{x} = 0$ (Section 2.1), an *i*D Planck unit ${}^{i}\hat{p}$ ($0 < i < n$) is also allowed when $\Delta\hat{x}_j = 0$ and $\hat{x}_j = c$ ($i < j < n$), demonstrating that ${}^{n}\hat{\boldsymbol{P}}$ can also be quantized in subspaces (**Figure 2(E)**). Therefore, quasi-Euclidean space ${}^{n}\hat{\boldsymbol{P}}$ can be redefined as such a special Euclidean space, which behaves as same as Euclidean space ${}^{n}\boldsymbol{P}$ at scale $\geq \lambda_{\text{P}}$ on the one hand, but does differently from ${}^{n}\boldsymbol{P}$ for its *i*D measure always satisfies ${}^{i}\hat{P} \geq \bar{\lambda}_{\text{P}}^{i}$ at scale $< \lambda_{\text{P}}$ on the other hand, although it remains a Euclidean space at the same time. This redefinition takes the original definition about 1D blank interval as a special case of 1D and results in a phenomenon that a serials of *space bubbles* exist in ${}^{n}\hat{\boldsymbol{P}}$ at the extremely small scale (**Figure 2(E)**).

IR cutoff *L*. For a generalized *n*D local space ${}^{n}\hat{P} > {}^{n}\hat{p}$ ($n \geq 2$), its *n*D generalized volume is of a constant measure of ${}^{n}\hat{P} \equiv {}^{n}P > \bar{\lambda}_{\text{P}}^{n}$, according to Equation (15). Besides, linearity of ${}^{n}\hat{\boldsymbol{P}}$ results in

$$\left(\hat{x}\right)_{\max} = \left(\sum_{1}^{n} \Delta\hat{x}_i\right)_{\max} = n\bar{\lambda}_{\text{P}}\Big|_{n\to\infty} \to +\infty \quad (\Delta\hat{x}_i = \bar{\lambda}_{\text{P}},\ \hat{x} > 0) \tag{16}$$

when $\hat{x}$ is taken as the linear summation of innumerous UV cutoffs between any adjacent point pairs. Obviously, $\left(\hat{x}\right)_{\max} \to +\infty$ satisfies uncertainty of ${}^{n}\hat{\boldsymbol{P}}$, when only one dimension along $\hat{x}$ is involved. Whereas the *n*D condition about the commutative ${}^{n}\hat{P}$ and ${}^{n}P$, as shown in Equation (11), should be involved when $\hat{x}$ is included in a certain local space ${}^{n}\hat{P}$ ($n \geq 2$). Considering the requirement aroused by the UV cutoff in Equation (17), the possible maximum and the minimum for 1D condition can be obtained as +∞ and $\bar{\lambda}_{\text{P}}$ (the mathematically reasonable solution about $\left|\hat{x}_i\right|_{\min} = 0$ is prohibited to ensure compliance with the physical principle of conservation of mass in the current 1D space), respectively, as shown in Equation (18). When the *n*D condition about the commutative volume is involved in Equation (19) as

$$\Delta\hat{x}_i \geq \bar{\lambda}_{\text{P}} \quad \text{or} \quad \Delta\hat{x}_i \leq -\bar{\lambda}_{\text{P}} \tag{17}$$

$$\left|\hat{x}_i\right|_{\max} \to +\infty, \quad \left|\hat{x}_i\right|_{\min} = \bar{\lambda}_{\text{P}} \tag{18}$$

$$\prod_{1}^{n} \hat{x}_i \equiv {}^{n}P \tag{19}$$

the allowed maximum of a certain $\hat{x}$ included in a local space ${}^{n}\hat{P}$ is

$$\text{IR cutoff:} \quad \left(\hat{x}_i\right)_{\max} = \frac{{}^{n}P}{\bar{\lambda}_{\text{P}}^{(n-1)}} \quad (\text{when } \hat{x}_j, \hat{x}_k, \cdots = \bar{\lambda}_{\text{P}}) \tag{20}$$

demonstrating the certain IR cutoff *L* for a general local space ${}^{n}\hat{P}$. To transform Equation (20) into $L = \frac{{}^{n}P}{\bar{\lambda}_{\text{P}}^{n}} \cdot \bar{\lambda}_{\text{P}}$, the physical or geometrical meaning for IR cutoff can be discovered to be the longest path for a Planck unit when its motion path covers the entire local space ${}^{n}\hat{P}$ uniformly and without overlap (Part 3 in Supplementary information).

Thus, properties of the quasi-Euclidean space $\hat{\boldsymbol{P}}$ can be summarized into 4 points, including uncertainty, UV cutoff $\bar{\lambda}_{\mathrm{p}}$, Planck unit $\hat{p}$ with certain measure $\bar{\lambda}_{\mathrm{p}}^{n}$ and uncertain structure varying from a generalized cube to a generalized sphere, and IR cutoff $L = \frac{{}^{n}P}{\bar{\lambda}_{\mathrm{p}}^{(n-1)}}$ as the longest path for a Planck unit ${}^{n}\hat{p}$ confined to a local space ${}^{n}\hat{P}$.

Section 2 defines a generalized quasi-Euclidean space $\hat{\boldsymbol{P}}$ with Planck length. Mathematical study discovers its uncertainty and clarifies the minimum structures of Plank units in it. As a micro-object with measure $\bar{\lambda}_{\mathrm{p}}$, a Planck unit might play the role of a quantum object when its motion inside $\hat{\boldsymbol{P}}$ is investigated at scale $\geq \bar{\lambda}_{\mathrm{p}}$, where $\hat{\boldsymbol{P}}$ behaves as a normal Euclidean background space. Consequently, a series of physical principles, including quantum mechanics, the minimum energy, conservation of mass, conservation of energy, etc., should be obeyed by a moving Planck unit. Besides, rotational symmetry and translational symmetry should be strictly satisfied by a Planck unit, since it is also part of the background space. Next, motion for such a Planck unit should be pursued, assuming that it can distinguish itself from the quasi-Euclidean space $\hat{\boldsymbol{P}}$ of the macro background. And IR cutoff L is expected to help in determining the ground state when the object is confined to a certain local space.

3. Planck Units under Control of Rotational Symmetry

3.1. A General Planck Unit Controlled by Rotational Symmetry

As part of space, a Planck unit $\hat{p}$ should satisfy rotational symmetry (abbreviated as RS).

RS states (I, R and T) and RS spaces (P_{I}, P_{R} and P_{T}). Strict RS bans Planck unit $\hat{p}$ from any radial displacement ($\mathrm{d}r = 0$) by infinite potential barrier V_r^{∞}, requiring $r = c$ mathematically. The solution results in two types of states, one is in-situ ($c = 0$), named state **I** (Figure 3(A)), the other is revolving around the in-situ position in the surface of a certain sphere ($c \neq 0$), named state **R**. Geometrically, the nD surface of an $(n + 1)$D sphere provides the simplest spherical space for an nD Planck unit ${}^{n}\hat{p}$ (Figure 3(B)). Considering the UV cutoff of quasi-Euclidean space $\hat{\boldsymbol{P}}$, the nearest sphere to state **I** is determined by $r = \bar{\lambda}_{\mathrm{p}}$, since a closer sphere is prohibited by the definition of UV cutoff $\bar{\lambda}_{\mathrm{p}}$. After normalizing the system by $\bar{\lambda}_{\mathrm{p}} = 1$, the simplest and nearest space for the revolving state **R**, symbolized as P_{R}, should be

$$P_{\mathbf{R}} : \sum_{1}^{n+1} \hat{x}_i^2 = 1 \tag{21}$$

here, the curved nD surface P_{R} is embedded in the $(n + 1)$D flat space, so state **R** actually takes the $(n + 1)$D space as its background, breaks away from the original nD background space of state **I**, and violates space conservation directly. Space conservation requires that ${}^{n}\hat{p}$ always moves in the same flat nD background space, and this then requires a flattened P_{R}, which is a finite plane P_{T} tangent to P_{R} (to show it more clearly, P_{T}'s tangent point is set up to be at the bottom of P_{R} to distinguish P_{T} from P_{I} in Figure 3(C), which represents the original nD background by a parallel space). Let $^n m$ be the generalized nD volume of an nD sphere with radius $r = 1$ (Part 5 in Supporting information), its generalized surface should be $(n - 1)$D space measured $(^n m)'$, and the flattened surface should be of measurement $\int_0^{2\pi} dx_1 \cdots \int_0^{2\pi} dx_{n-1} = (2\pi)^{n-1}$.

Logically, RS states **I**, **R** and **T** for $^n\hat{p}$ should be of spaces measured

$$P_{\mathbf{I}} = 1 \tag{22}$$

$$P_{\mathbf{R}} = \left(^{n+1}m\right)' \tag{23}$$

$$P_{\mathbf{T}} = (2\pi)^n \tag{24}$$

According to Equation (20), their IR cutoffs are $L_{\mathbf{I}} = 1$, $L_{\mathbf{R}} = \left(^{n+1}m\right)'$ and $L_{\mathbf{T}} = (2\pi)^n$, respectively. It should be noted that these spaces are of nD measurement while their IR cutoffs are of 1D measurement, and they share the same algebra expression only because of the normalization $\bar{\lambda}_{\mathrm{p}} = 1$.

So rotational symmetry requires Planck unit $^n\hat{p}$ exist at in situ state **I**, revolving state **R** or flattened revolving state **T** at the extreme condition, and these states are confined to certain spaces $P_{\mathbf{I}}$, $P_{\mathbf{R}}$ and $P_{\mathbf{T}}$ with certain IR cutoff $L_{\mathbf{I}}$, $L_{\mathbf{R}}$ and $L_{\mathbf{T}}$, respectively. Generally, there exists infinite potential V_r^{∞} to separate state **I** from the topologically connected $P_{\mathbf{R}}/P_{\mathbf{T}}$, resulting in localized Planck units with no transformability (Figure 3(C)). And such a background space $\hat{\boldsymbol{P}}$ without any non-local Planck units is taken to be a trivial one (a general case is shown in Part 6, Supplementary information).

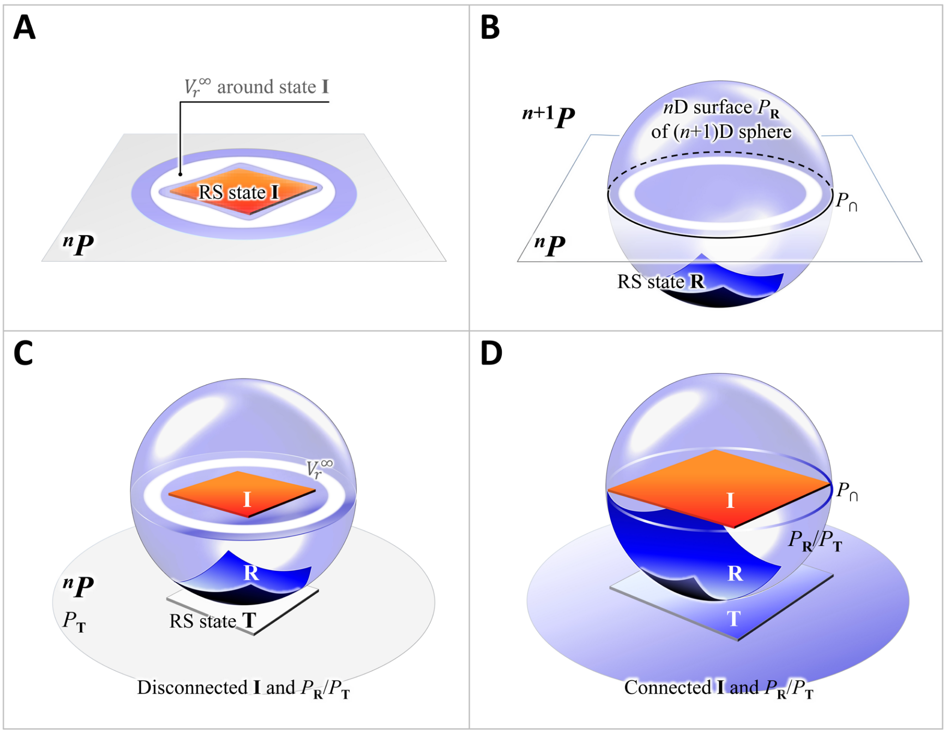

Figure 3. RS spaces of $^n\hat{p}$ at RS states of **I** (*in situ* state, **A**), **R** (revolving state, **B**) and **T** (tangent state, **C**). Transformable $^n\hat{p}$ with topologically connected **I** and $P_{\mathrm{R}}/P_{\mathrm{T}}$ (**D**).

3.2. Non-Triviality: Transformable Planck Units in High-Dimensional Spaces

If state **I** and space P_R/P_T are topologically connected, V_r^∞ will be invalidated, transformation between **I** and **R/T** will be possible, and the background space $\hat{\boldsymbol{P}}$ will become a non-trivial one since non-local Planck units in it are allowed (**Figure 3(D)**).

The topology of the RS spaces was investigated here. Assuming that a Planck unit at state **I** takes a generalized cube as its uncertain structure and takes diagonal $\sqrt{n}$ as its maximum 1D measurement, the increasing $\sqrt{n}$ will make itself possible to reach P_R/P_T in higher dimensional spaces. The possible points connecting **I** and P_R/P_T should belong to the intersecting space $P_\cap$ determined by P_I ($\subset {}^n\boldsymbol{P}$) and P_R ($\sum_1^{n+1} \hat{x}_i^2 = 1$) as

$$P_\cap : \sum_1^n \hat{x}_i^2 = 1 \tag{25}$$

which is an $(n-1)$D spherical surface enclosing an nD sphere (**Figure 3(D)**). Logically, the longest 1D measurement enclosed by $P_\cap$ is ${}^n m$, since the enclosed sphere measured ${}^n m$ is of IR cutoff $L = {}^n m$. Thus, state **I** will share points with $P_\cap$ when $\sqrt{n} \ge {}^n m$, meaning that the cube or some other intermediate reaches P_R/P_T topologically (**Figure 3(D)**).

Calculation demonstrates the relationship between $\sqrt{n}$ and ${}^n m$ in nD space (**Figure 4**), showing topology for **I** and P_R/P_T as follows. Firstly, 1 - 9D spaces are all trivial ones for $\sqrt{n} < {}^n m$, resulting in that their state **I** is always separated from P_R/P_T by V_r^∞ and state transformation is always forbidden. Secondly, 10D, 11D and 12D spaces are three non-trivial spaces with $\sqrt{n} > {}^n m$, where topological connection between **I** and P_R/P_T exists, potential V_r^∞ can be invalidated, and non-local Planck units are allowed. Lastly, ${}^n m < 1 < \sqrt{n}$ in ≥13D spaces makes these solutions meaningless for ${}^n m < 1$ violates the minimum local space ${}^n \hat{p} \ge 1$ (**Figure 4**).

Thus, Planck units ${}^{10}\hat{p}$, ${}^{11}\hat{p}$ and ${}^{12}\hat{p}$ are proven to be of non-trivial topology connections between their state **I** and P_R/P_T, discovering that transformation of a Planck unit is allowed in 10D, 11D and 12D spaces. Planck units ${}^{>12}\hat{p}$ inside quasi-Euclidean spaces >12D involve threshold space ${}^n m \le 1$, e.g., ${}^{13} m = 0.910$, which violates the definition of a Planck unit, resulting in the loss of reasonability for the current case. According to the monotonic decrease trend of the generalized formulae for ${}^n m$, spaces with higher dimensions are reasonably ignored (Part 5 in Supplementary information).

3.3. ${}^{10}\hat{p}$: RS Space Structures and Two Transforming Paths

${}^{10}\hat{p}$ is the first non-trivial Planck unit with transformability, since its *in-situ* state **I** has the possibility to reach the space of its revolving state **R**, resulting in the probability for itself to transform into the corresponding tangent state **T**. This possible transformation is determined by the geometrical characters of these different states, heavily depending on that **I** shares same point(s) with $\boldsymbol{P}_R$. Investigation in 3.2 discovers that there is no possibility for such a sharedness in a quasi-Euclidean space ${}^{n}\hat{\boldsymbol{P}}$ when $n \le 9$. Although any a Planck unit has the possibility to behave as a spherical one with completely uncertain θ but certain $\rho = \bar{\lambda}_{\mathrm{p}}$, which keeps itself always isolated from $\boldsymbol{P}_{\mathrm{R}}$, a possible sharedness still exists when it satisfies $\sqrt{n} \ge {}^{n}m$, where $\sqrt{n}$ is the possible maximum 1D distance for a Planck unit and ${}^{n}m$ equals to the maximum 1D distance inside the threshold space $P_{\cap}$. Essentially, algebraic relationship between $\sqrt{10}$ and ${}^{10}m$ determines that ${}^{10}\hat{p}$ is a transformable Planck unit with the lowest dimensional structure (Figure 4). Its non-triviality, however, changes motion of ${}^{10}\hat{p}$ in turn, since the topological connection changes its RS space structures and the transformation changes its movements.

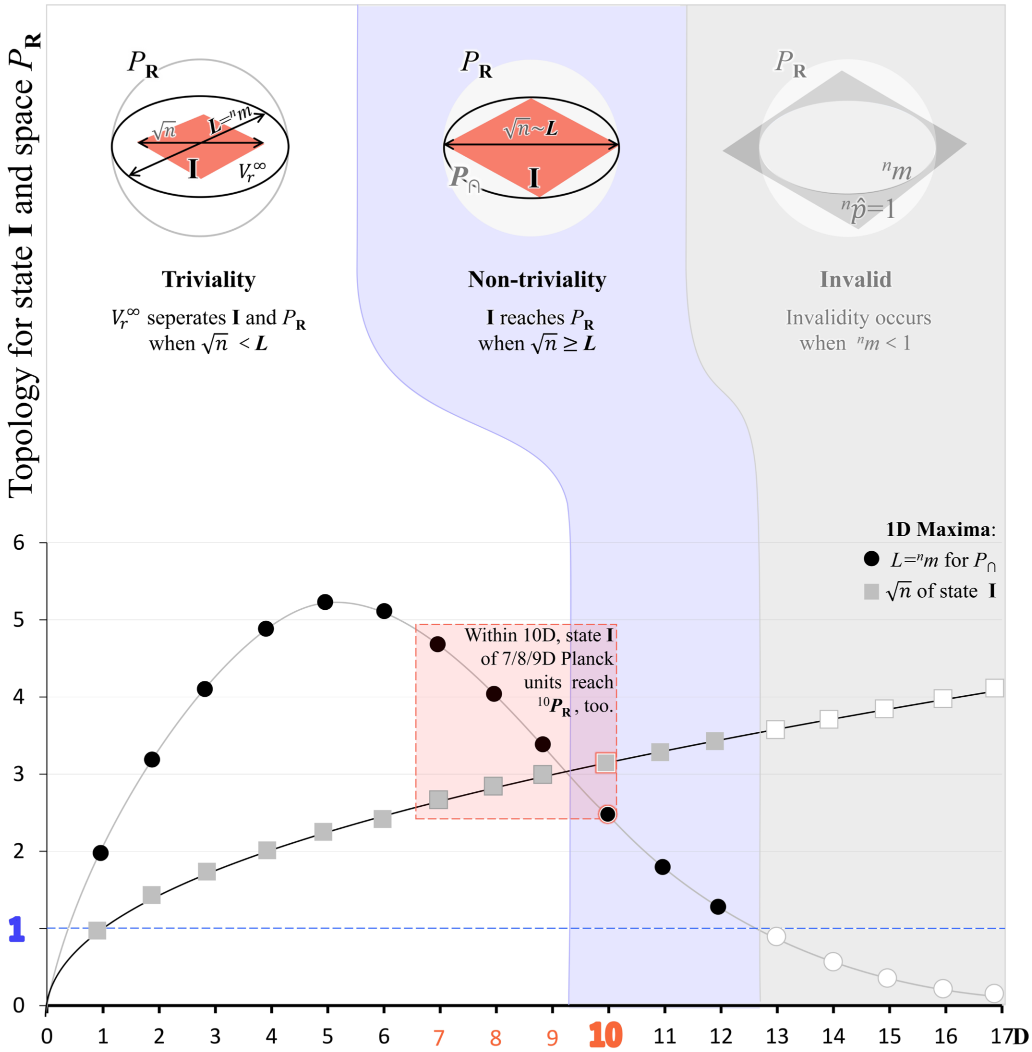

Figure 4. Topological relationship for state **I** and $P_{\mathrm{R}}/P_{\mathrm{T}}$ in nD space.

Changed P_{I} and P_{T}. Topological connection invalidates the infinitely potential V_r^{∞}, which is originally separating in-situ state **I** from the revolving space P_{R}, and then the disappeared V_r^{∞} makes state **I** exist actually in a space enclosed by $P_{\cap}$, which is the 10D sphere measured ${}^{10}m$ (**Figure 3(D)**). Besides, the topological connection makes P_R be flattened from a certain point on 9D space of $P_\cap$ and the corresponding P_T becomes $P_T = \int_0^{2\pi} x^8 dx \cdot \int_0^{2\pi} dx$. The above changes result in

$$ {}^{10}P_T = {}^{10}m = \frac{\pi^5}{120} \quad (26) $$

$$ {}^{10}P_T = \frac{(2\pi)^{10}}{9} \quad (27) $$

Two paths for I⇌R/T: the highest rotational symmetry (RS$^\uparrow$) and the minimum energy (E$_\downarrow$). Accordingly, spaces P_I and P_T of 10D Planck unit ${}^{10}\hat{p}$ are of IR cutoff $L_I = \frac{\pi^5}{120}$ and $L_T = \frac{(2\pi)^{10}}{9}$, respectively. And these two IR cutoffs might provide quantum paths for ${}^{10}\hat{p}$ at ground states. But as an RS state, movement of state **I** should also be measured equally along each dimension, meaning that P_I should be an equally expanded space of $\prod_1^{10} x_i = \frac{\pi^5}{120}$ where $x_1, x_2, \cdots, x_{10} = \left(\frac{\pi^5}{120}\right)^{\frac{1}{10}}$ and its observable IR cutoff should be $L_I = \left(\frac{\pi^5}{120}\right)^{\frac{1}{10}}$. As for state **R**, dimensional equivalence would not work here since curved P_R is of no dimensional equivalence intrinsically. Consequently, ${}^{10}P_R$ remains the same with $L_R = ({}^{11}m)'$, and so does ${}^{10}P_T$ with $L_T = \frac{(2\pi)^{10}}{9}$.

Thus, ${}^{10}\hat{p}$ at in-situ state **I** has two ways for its movement, one takes $L_I = \frac{\pi^5}{120}$ as its half wavelength when it satisfies the minimum energy, the other takes $L_I = \left(\frac{\pi^5}{120}\right)^{\frac{1}{10}}$ as its half wavelength when it satisfies the highest RS. The two principles, one is the minimum energy noted as $E_\downarrow$, the other is the highest rotational symmetry noted as $RS^\uparrow$, then determine the two different transforming paths for ${}^{10}\hat{p}$.

I⇌T: space structures for a transforming ${}^{10}\hat{p}$. Considering that ${}^{10}\hat{p}$ should remain in 10D space to satisfy conservation, its **R** state in P_R, which is a 10D generalized surface embedded in 11D background space (**Figure 3(B)**), is unobservable in ${}^{10}\hat{\boldsymbol{P}}$ for it violates the conservation. In fact, only state **I** and **T** for ${}^{10}\hat{p}$ are RS states with physical legality inside ${}^{10}\hat{\boldsymbol{P}}$, whereas state **R** is a virtual one just bridging **I** and **T** mathematically. Accordingly, the 9D space $P_\cap$ ($\sum_1^{10} \hat{x}_i^2 = 1$) in ${}^{10}\hat{\boldsymbol{P}}$ actually determines the topological connection for **I** and **T**, instead of P_R ($\sum_1^{11} \hat{x}_i^2 = 1$) embedded in ${}^{11}\hat{\boldsymbol{P}}$ (**Figure 3(B)**).

Taking 10D quasi-Euclidean space ${}^{10}\hat{\boldsymbol{P}}$ as the simplest background space with non-locality, critical space $P_\cap$ plays the role of threshold not only for ${}^{10}\hat{p}$, but also for its substructures in each subspace of ${}^{10}\hat{\boldsymbol{P}}$. Based on the maximum length ${}^{10}L_I = \frac{\pi^5}{120}$ allowed by $P_\cap$, transformability for each substructure in 1-9D proper subspaces had been also investigated. Results show that the 1D maximum $^{10}L_{\mathbf{I}} = \frac{\pi^5}{120} = 2.55$ allows Planck units $^{7}\hat{p}$, $^{8}\hat{p}$ and $^{9}\hat{p}$ to be transformable ones besides $^{10}\hat{p}$, since it exists $\sqrt{6} < 2.55 < \sqrt{7}$. This means that any ≥7D substructure is of transformability for its state **I** can exceed the threshold and connect to the corresponding state **T**. In other words, any ≤6D substructure never reaches the threshold space of $P_{\cap}$, being prohibited from transformation and maintaining triviality.

Let triviality be of priority to nontriviality and let x_i expand the i^{th} dimension of the iD subspace ($1 \le i \le 10$), the maximum space with triviality, noted as $^{t}\boldsymbol{P}$, should be 6D space expanded by x_1, x_2, …, x_6, and its complement space involving non-triviality, noted as $^{nt}\boldsymbol{P}$, should be 4D space expanded by x_7, x_8, x_9 and x_{10}. Then a map of space structures for $^{10}\hat{p}$ can be obtained for both state **I** and state **T** in each of the 1 - 10D subspaces (**Figure 5**).

Regarding the principle of the highest rotational symmetry ($\text{RS}^{\upharpoonleft}$), iD substructure $^{i}\hat{p}$ ($1 \le i \le 10$) at state **I** should be of $P_{\mathbf{I}}$ expanded by $x_1 = x_2 = \cdots = x_6 = 1$ and $x_7 = x_8 = \cdots = x_i = \left({}^{i}m\right)^{\frac{1}{i-6}}$, which meets $P_{\mathbf{I}} = 1$ ($i \le 6$) or $P_{\mathbf{I}} = {}^{i}m$ ($i \ge 7$), satisfies the dimensional equivalence as much as possible, and ensures the priority of the triviality in ≤6D. Correspondingly, $P_{\mathbf{T}}$ of $^{i}\hat{p}$ includes a trivial part and a non-trivial part measured $(2\pi)^6$ and $\frac{(2\pi)^{i-6}}{i-1}$, respectively. Taking 8D Planck unit or 8D substructure of $^{8}\hat{p}$ as an example, its $P_{\mathbf{I}}$ should be expanded by

$x_1 = x_2 = \cdots = x_6 = 1$ and $x_7 = x_8 = \left(\frac{\pi^4}{24}\right)^{\frac{1}{2}}$, and its $P_{\mathbf{T}}$ should be of IR cutoffs $(2\pi)^6$ and $\frac{(2\pi)^2}{7}$ in 6D trivial subspace and the other 2D non-trivial subspace, respectively. Similarly, $P_{\mathbf{I}}$ for $^{10}\hat{p}$ should be expanded by $x_1 = \cdots = x_6 = 1$ and

$x_7 = x_8 = x_9 = x_{10} = \left(\frac{\pi^5}{120}\right)^{\frac{1}{4}} = 1.26$, and its corresponding $P_{\mathbf{T}}$ should be of IR cutoffs $(2\pi)^6$ and $\frac{(2\pi)^4}{9} = 173$ in $^{t}\boldsymbol{P}$ and $^{nt}\boldsymbol{P}$, respectively (**Figure 5(B)**).

Regarding the principle of the minimum energy ($\text{E}_{\downharpoonleft}$), $^{n}\hat{p}$, as an nD Planck unit or nD substructure, would take IR cutoff L as its motion spaces in each of its own subspaces, acquiring $P = \prod_1^n {}^{i}L$ be its largest motion space. Taking 2D Planck unit or 2D substructure of $^{2}\hat{p}$ as an example, its proper substructure $^{1}\hat{p}$ takes $^{1}L_{\mathbf{I}} = 1$ and $^{1}L_{\mathbf{T}} = 2\pi$ as its longest paths for state **I** and **T**, respectively, and there must be $^{1}L_{\mathbf{I}} \,/\!/\, x_1$ and $^{1}L_{\mathbf{T}} \,/\!/\, x_1$. Besides, $^{2}\hat{p}$ itself takes $^{2}L_{\mathbf{I}} = 1$ and $^{2}L_{\mathbf{T}} = (2\pi)^2$ as its longest paths for state **I** and **T**, respectively, and there must be $^{2}L \subset {}^{2}\boldsymbol{P}$. Obviously, $^{2}P_{\mathbf{I}}$ and $^{2}P_{\mathbf{T}}$ have the maximum values of $^{2}P_{\mathbf{I}} = {}^{1}L_{\mathbf{I}} \times {}^{2}L_{\mathbf{I}}$ and $^{2}P_{\mathbf{T}} = {}^{1}L_{\mathbf{T}} \times {}^{2}L_{\mathbf{T}}$, respectively, if and only if $^{2}L \perp {}^{1}L$ and $^{2}L \,/\!/\, x_2$. Similarly, it exists $P_{\mathbf{I}} = \prod_1^{10} {}^{i}L_{\mathbf{I}} = \frac{8\pi^{16}}{4465123} = 161$ and $P_{\mathbf{T}} = \prod_1^{10} {}^{i}L_{\mathbf{T}} = \frac{(2\pi)^{55}}{3024} = 2.63 \times 10^{40}$ for

$^{10}\hat{p}$, when $^{10}\hat{p}$ is dominated by Principle $E_{\downharpoonleft}$.

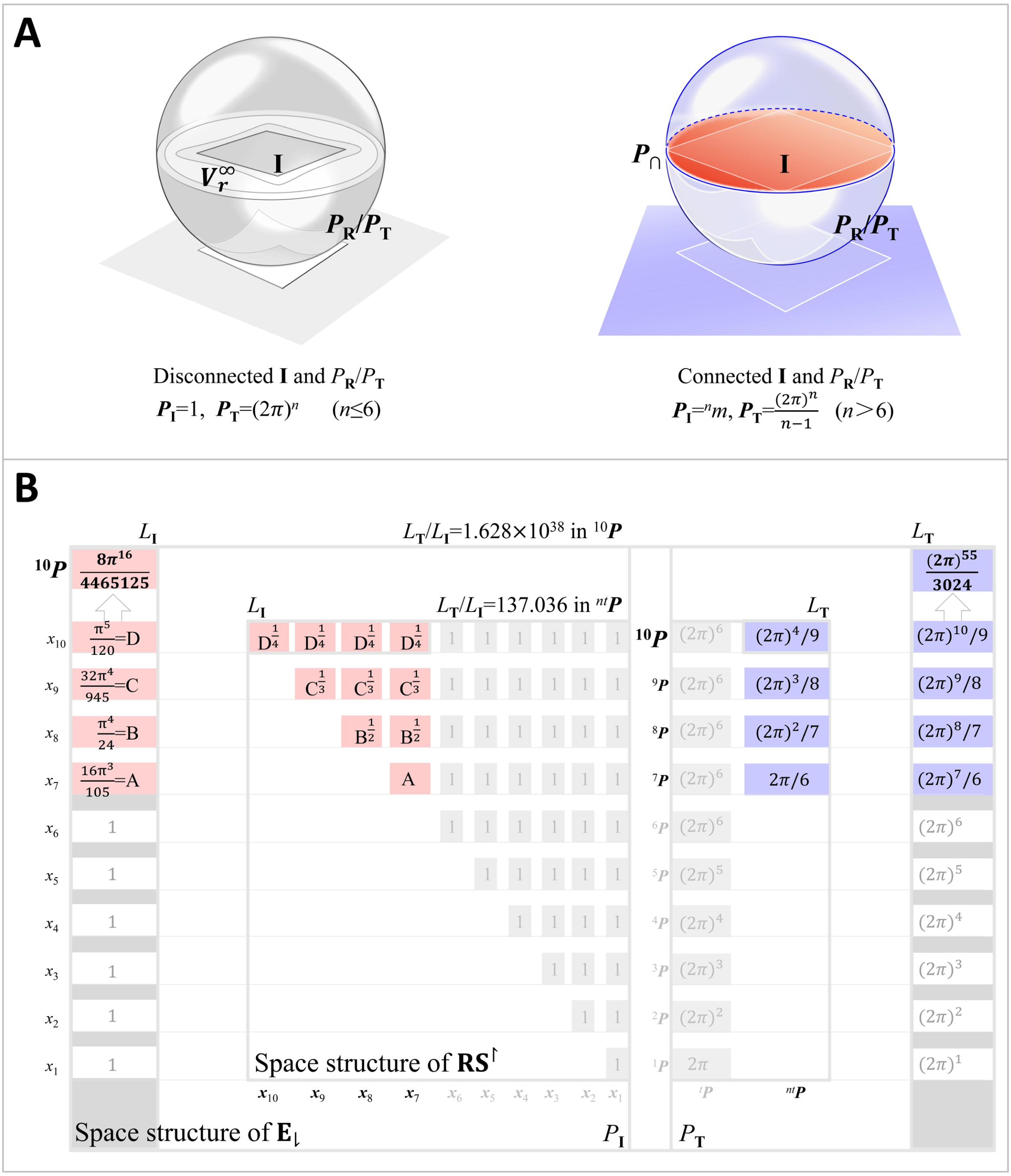

Figure 5. Non-transformability or transformability for a Planck unit $^{n}\hat{p}$ inside 10D (A). 10D space structure and two transforming paths for I⇌T: Path 1 with the highest rotational symmetry ($RS^{\upharpoonleft}$) and half-wavelength change ~137.036 times, while Path 2 with the minimum energy ($E_{\downharpoonleft}$) and half-wavelength change ~1.628 × 10^{38} times (B).

2 dimensionless constants. For $^{10}\hat{p}$ satisfying Principle $RS^{\upharpoonleft}$, it conserves triviality in ≤6D space of $^{t}\boldsymbol{P}$, meaning that the non-locality aroused by transformation only occurs in the 4D complement space of $^{nt}\boldsymbol{P}$. According to the map of the space structure (**Figure 5(B)**), ${}^{10}\hat{p}$ would take $L_{\mathbf{I}} = \left(\frac{\pi^5}{120}\right)^{\frac{1}{4}}$ and $L_{\mathbf{T}} = \frac{(2\pi)^4}{9}$ as its half wavelengths for its ground states, and ${}^{10}\hat{p}$ would be observed to be with wavelength change of $\alpha_1 = \left(\frac{L_{\mathbf{T}}}{L_{\mathbf{I}}}\right)^{-1} = 1/137.036082$ times during its transformation in 4D space ${}^{nt}\boldsymbol{P}$.

Similarly, ${}^{10}\hat{p}$ satisfying Principle $\mathrm{E}_{\downharpoonleft}$ would be observed to be with wavelength change of $\alpha_2 = \left(\frac{L_{\mathbf{T}}}{L_{\mathbf{I}}}\right)^{-1} = 1/\left(1.628008\times10^{38}\right)$ times during its transformation in global 10D space.

Section 3 investigated on a Planck unit dominated by rotational symmetry (RS). RS strictly defines the spaces for a Planck unit in-situ (state **I**) or in flattened spherical surface (state **T**), and these two states are generally isolated from each other, meaning that a Planck unit is always localized. Geometry study discovered that a 10D Planck unit acquired its transformability and non-locality when its two states **I** and **T** were topologically connected. Following the two principles, one was the highest rotational symmetry ($\mathrm{RS}^{\upharpoonleft}$), the other was the minimum energy ($\mathrm{E}_{\downharpoonleft}$), ${}^{10}\hat{p}$ presented two dimensionless constants of ~1/137.036 and ~1/(1.628 × 10^{38}) for its two transforming paths, respectively.

4. 10D Planck Unit under Control of Translational Symmetry

As part of 10D quasi-Euclidean space, Planck unit ${}^{10}\hat{p}$ should satisfy translational symmetry (TS) besides rotational symmetry (RS).

RS State T′. Besides $\boldsymbol{P}_{\mathrm{R}}$, RS also defines other concentric surfaces with $r > \bar{\lambda}_{\mathrm{p}}$ and results in innumerous state **R**′in infinite space. However, transformation between anyone of these **R**′states and the in situ sate **I** is forbidden because **R**′is not so close to **I**, which causes them to be separated by potential barrier V_r^{∞}. Consequently, it also exists innumerous state **T**′without transformability into state **I** (**Figure 6**). To satisfy principle of least action [19], a Planck unit at state **T**′should be of action $S = h$ for its ground state.

Constant velocity. Because of the translational symmetry, Planck units ${}^{10}\hat{p}$ in 10D background space should be identical objects, and their **T** states should be of constant velocity v.

Sharing-coupling effect. A system containing two centers A and B had been built up to explore the interaction between any two ${}^{10}\hat{p}$. Because of the constant velocity v of state **T**, one center of state $\mathbf{I}_{\mathrm{A}}$ shares $\mathbf{T}_{\mathrm{B}}$ as its state $\mathbf{T}'_{\mathrm{A}}$ at distance r ($r \gg 1$), since $\mathbf{T}_{\mathrm{B}}$ and $\mathbf{T}'_{\mathrm{A}}$ are indistinguishable for they are identical objects sharing the same velocity v. So are $\mathbf{T}_{\mathrm{A}}$ and $\mathbf{T}'_{\mathrm{B}}$ for another center of $\mathbf{I}_{\mathrm{B}}$. The two shared states **T** and **T**′spontaneously bring about energy difference of

$$\Delta E = \frac{hv}{2L_{\mathbf{T}}} - \frac{hv}{2L_{\mathbf{T}}} = \frac{hv}{2L_{\mathbf{T}}} - \alpha\cdot\frac{hv}{2L_{\mathbf{I}}}\cdot\frac{1}{r} \tag{28}$$

between them when they both have the least action h and take IR cutoffs as their half wavelengths at ground states. This effect caused by the shared **T** and **T**′states is named sharing-coupling effect (**Figure 6**).

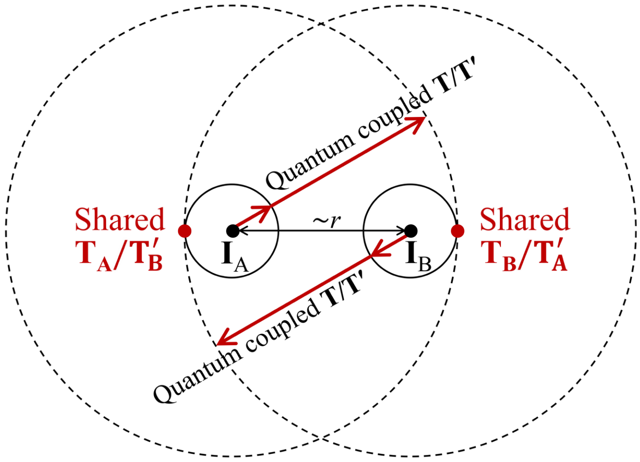

Figure 6. Shared state **T/T**′ sharing-coupling effect for a 2-center system.

Two long range interaction fields. The velocity v for state **T** or **T**′leads to their constant mass of 0 since $\frac{\mathrm{d}v}{\mathrm{d}k}=\frac{\hbar}{m_{\mathbf{T}}}=0$ requires $m_{\mathbf{T}}\equiv 0$ when dv = 0. So kinetic energy of **T** or **T**′should be protected by their constant velocity and mass, which results a kinetic energy difference of $\Delta T_{\mathbf{T}\text{-}\mathbf{T}'}=\frac{hc}{\lambda_{\mathbf{T}}}-\frac{\alpha hc}{2\pi r}$. Considering that the energy for each point in a vacuum should be always equal, there must be a potential energy different $\Delta U=-\Delta T_{\mathbf{T}\text{-}\mathbf{T}'}$ between $\mathbf{I}_A$ and $\mathbf{I}_B$ to balance the energy of the system, resulting in two long-range potential fields

$$\begin{aligned}\mathrm{RS}^{\upharpoonleft}: F &= \frac{1}{2L_{\mathbf{I}}}\cdot\frac{hc}{137r^2}\\ \mathrm{E}_{\downharpoonleft}: F &= \frac{1}{2L_{\mathbf{I}}}\cdot\frac{hc\times 10^{-38}}{1.628r^2}\end{aligned} \tag{29}$$

when the two dimensionless constants of α_1 = 1/137.036082 for $\mathrm{RS}^{\upharpoonleft}$ process or α_2 = 1/(1.628 008 × 10^{38}) for $\mathrm{E}_{\downharpoonleft}$ process had been brought in and state **T** took the light speed c as its constant velocity (Part 6 in Supplementary information).

Section 4 discovered the sharing-coupling effect determined by translational symmetry, obtaining a spontaneous potential field between any two shared **T/T**′ states in 10D space. According to the two transforming paths with dimensionless constants α_1~1/137.036 and α_2~1/(1.628 × 10^{38}), two long range interacting fields had been obtained accurately.

5. Conclusion and Discussion

Based on its three initial settings, including Planck length, rotational symmetry and translational symmetry, a quasi-Euclidean space is quantized by the extremely small structures of Planck units, and a 10D Planck unit is discovered to be the simplest Planck unit with two different transforming paths, resulting in two long-range interactions in a 4D subspace and in 10D global space, respectively.

The two constants, 1/137.036 for the transformation dominated by the highest rotational symmetry ($RS^{\uparrow}$) and $1/(1.628 \times 10^{38})$ for the transformation dominated by the minimum energy ($E_{\downarrow}$), are exactly equal to the fine structure constant (FSC = 1/137.035999) and approximately equal to the dimensionless gravitational constant ($\frac{Gm_P^2}{\hbar c} = \frac{1}{1.693 \times 10^{38}}$) [17], with deviations of ~10^{-6} and ~4%, respectively. But the two corresponding forces between any 2 Planck units are neither the electromagnetic interaction between any two static electrons nor the gravitation between any two protons, and the difference between them is $(2L_I/2\pi)$ times. Obviously, it suggests another pseudo space with clear geometrics. Besides the pseudo space, the 4D non-trivial space expanded by the 7th, 8th, 9th and 10thD, the positive or negative nature of the two long range interactions, the obvious deviation between $\alpha_2 \sim 1/(1.628 \times 10^{38})$ and the dimensionless gravitational constant, *etc*., are also puzzling. This work distinguishes the 10th dimension x_{10} from the other 3 ones of x_7, x_8 and x_9 since ${}^{7}\hat{p}$, ${}^{8}\hat{p}$ and ${}^{9}\hat{p}$ all have their own subspaces to conserve their triviality but ${}^{10}\hat{p}$ has not. The relationship between the non-trivial 4D subspace and the current 4D space-time need to be explored further. Although two constants for the two long range interactions have been obtained, no other useful information, such as that about their negativity or positivity, has been obtained.

In fact, $\alpha_1 = \frac{\left(\frac{\pi^5}{120}\right)^{\frac{1}{4}}}{(2\pi)^4/9}$ had ever been discovered before, and the 10D space with 6D + 4D structure had been investigated in other fields. *Wyler*'s constant of $\frac{9}{8\pi^4}\left(\frac{\pi^5}{2^4 \cdot 5!}\right)^{\frac{1}{4}}$, which is algebraically equal to α_1, was published in 1969 to try to solve FSC, but all the relevant works stopped abruptly here almost at the same time [20]. And string theory assumes that a micro string with six-dimensional curved structure moves in a 10D space in some cases [13]. The current work obtained a *bubble* moving in 10D spaces with its 6D substructure being locked into a 6D subspace for it behaves trivially and loses transformability here. Whereas the 10D → 6D + 4D in this work is obtained via a geometrical method for a *bubble-like* object.

As for the outlook for this work, we look forward to a future where a particle and a vacuum can be unified to prove the unity of the physical world.

Additionally, the newly defined symbols in this work have been listed in Part 7 of the Supplementary information.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Supplementary Information

1) Non-commutativity between $\hat{x}$ and x

Based on the mathematical definition of quasi-Euclidean space $\hat{\boldsymbol{P}}$ in Equation (2)-(4), relationship between $\hat{x}$ and its corresponding x in Euclidean space could be obtained.

To substitute Equation (2) into Equation (4) and to take derivative with respect to x on the both sides, it obtains

$$\hat{x} = \lambda_{\mathrm{P}} \cdot \mathrm{e}^{\frac{\boldsymbol{i}k\cdot x}{\bar{\lambda}_{\mathrm{P}}}} \tag{S1}$$

$$\frac{\mathrm{d}\hat{x}}{\mathrm{d}x} = 2\pi\boldsymbol{i}k \cdot \mathrm{e}^{\frac{\boldsymbol{i}k\cdot x}{\bar{\lambda}_{\mathrm{P}}}} = \frac{2\pi\boldsymbol{i}k \cdot \hat{x}}{\lambda_{\mathrm{P}}} \tag{S2}$$

And Equation (S2) leads to

$$k = -\boldsymbol{i}\bar{\lambda}_{\mathrm{P}} \frac{\mathrm{d}}{\mathrm{d}x} \tag{S3}$$

Then to substitute Equation (2) and (3) into Equation (4) and to take derivative with respect to $\hat{x}$ on the both sides, it obtains

$$kx = \lambda_{\mathrm{P}} \cdot \mathrm{e}^{\frac{\boldsymbol{i}\cdot\hat{x}}{\bar{\lambda}_{\mathrm{P}}}} \tag{S4}$$

$$\frac{\mathrm{d}x}{\mathrm{d}\hat{x}} = \frac{\mathrm{d}(x+a)}{\mathrm{d}\hat{x}} = \frac{2\pi\boldsymbol{i}}{k} \cdot \mathrm{e}^{\frac{\boldsymbol{i}\cdot(x+a)}{\bar{\lambda}_{\mathrm{P}}}} \tag{S5}$$

And Equation (S5) leads to

$$k^{-1} = -\boldsymbol{i}\bar{\lambda}_{\mathrm{P}} \frac{\mathrm{d}}{\mathrm{d}\hat{x}} \tag{S6}$$

Furthermore, Equation (S3) results in

$$\begin{aligned}[k, x]\hat{x} &= k(x \cdot \hat{x}) - x \cdot (k\hat{x}) \\ &= k(x) \cdot \hat{x} + x \cdot (k\hat{x}) - x \cdot (k\hat{x}) \\ &= -\boldsymbol{i}\bar{\lambda}_{\mathrm{P}}\hat{x}\end{aligned} \tag{S7}$$

which then leads to the non-commutativity of

$$[k, x] = -\boldsymbol{i}\bar{\lambda}_{\mathrm{P}} \tag{S8}$$

Similarly, Equation (S6) results in another non-commutativity of

$$\left[k^{-1}, \hat{x}\right] = -\boldsymbol{i}\bar{\lambda}_{\mathrm{P}} \tag{S9}$$

Based on Equation (S8) and (S9), relationship between x and $\hat{x}$ can be obtained as

$$[\hat{x}, x] = [k, x]x = -\boldsymbol{i}\bar{\lambda}_{\mathrm{P}} \cdot x \tag{S10}$$

$$[x, \hat{x}] = \left[k^{-1}, \hat{x}\right]\hat{x} = -\boldsymbol{i}\bar{\lambda}_{\mathrm{P}} \cdot \hat{x} \tag{S11}$$

demonstrating the non-commutativity between $\hat{x}$ and x along any a dimension.

2) On $|\Delta k \cdot \Delta x| \geq \frac{\bar{\lambda}_{\mathrm{P}}}{2}$ for $[k, x] = -\boldsymbol{i}\bar{\lambda}_{\mathrm{P}}$

For $[k,x] = -\boldsymbol{i}\bar{\lambda}_{\text{P}}$ in Equation (S8), let $|g(x)|^2$ be Gaussian for $\Delta x = a$, then it should exist

$$g(x) = \frac{1}{(2\pi)^{\frac{1}{4}}\sqrt{a}} e^{\frac{-x^2}{4a^2}} \tag{S12}$$

with $|g(x)|^2$ satisfying normalization condition of

$$\int_{-\infty}^{+\infty} |g(x)|^2 \, dx = 1 \tag{S13}$$

Let expansion coefficient c_p be

$$\begin{aligned} c_p &= \frac{1}{\sqrt{\bar{\lambda}_{\text{P}}}} \int g(x) e^{\frac{\boldsymbol{i}(k-k_0)x}{\bar{\lambda}_P}} dx \\ &= \frac{1}{\sqrt{\bar{\lambda}_{\text{P}}}} (2\pi)^{-\frac{1}{4}} \int_{-\infty}^{+\infty} \exp\left[-\frac{x^2}{4a^2} + \frac{i(k-k_0)x}{\bar{\lambda}_{\text{P}}}\right] dx \end{aligned} \tag{S14}$$

And let $\xi = x + 2a^2 i(k-k_0)/\bar{\lambda}_{\text{P}}$, Equation (S14) leads to

$$\begin{aligned} c_p &= \frac{1}{(2\pi)^{\frac{1}{4}}\sqrt{\bar{\lambda}_{\text{P}} a}} e^{\frac{-a^2(k-k_0)^2}{\bar{\lambda}_{\text{P}}^2}} \cdot \int_{-\infty}^{+\infty} \exp\left(-\frac{\xi^2}{4a^2}\right) d\xi \\ &= \sqrt{\frac{2a}{\bar{\lambda}_{\text{P}}}} (2\pi)^{-\frac{1}{4}} \exp\left[\frac{-a^2(k-k_0)^2}{\bar{\lambda}_{\text{P}}^2}\right] \end{aligned} \tag{S15}$$

showing that c_p is also Gaussian distribution with $\bar{c}_p = k_0$. So it exists

$$\overline{(\Delta k)^2} = \int_{-\infty}^{+\infty} (k-k_0)^2 c_p^2 dp \tag{S16}$$

Based on

$$\int_{-\infty}^{+\infty} x^2 e^{-ax^2} dx = \frac{\sqrt{\pi}}{2} e^{\frac{-3}{2}} \tag{S17}$$

it could be obtained that

$$\overline{(\Delta k)^2} = \frac{\bar{\lambda}_{\text{P}}^2}{4a^2} = \frac{\bar{\lambda}_{\text{P}}^2}{4\overline{(\Delta x)^2}} \tag{S18}$$

Equation (S18) leads to $|\Delta k \cdot \Delta x| = \frac{\bar{\lambda}_{\text{P}}}{2}$, which then results in $\Delta\hat{x} = 2 \times |\Delta k \cdot \Delta x| = \bar{\lambda}_{\text{P}}$ given the even symmetry for Gaussian. Considering Gaussian with the narrowest contribution, it generally exists $\Delta\hat{x} \geq \bar{\lambda}_{\text{P}}$.

3) On physical meaning of IR cutoff L: the longest length of a closed or open path (**Figure S1**).

4) A general cases for Planck unit with or without transformability.

Generalized surface of an $(n + 1)$D sphere is also an nD space, since it is expanded by orthogonal X_1, X_2, X_3, …, X_n, X_{n+1}, where $X_{n+1} = 0$ and $X_{n+1} \mathbin{/\!/} r$, according to the Frenet frame shown in **Figure S2(A)**. Accordingly, an nD Planck unit ${}^n\hat{p}$ can be located in both the curve surface $\hat{\boldsymbol{P}}$ s and the flat space ${}^n\hat{\boldsymbol{P}}$ (**Figure S2(B)**).

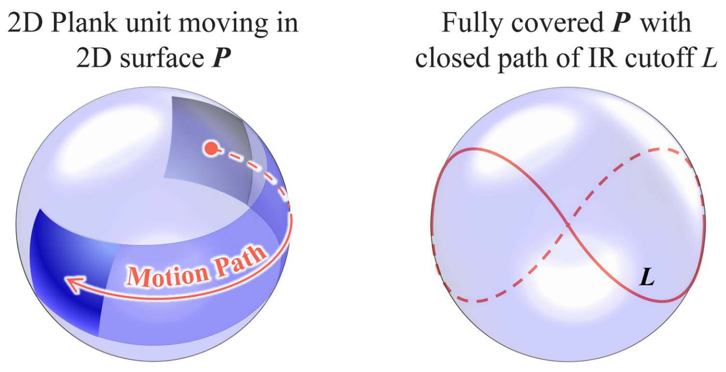

Figure S1. Physical meaning for IR cutoff L: the theoretically longest path for a Planck unit ${}^{2}\hat{p}$ confined to a local space ${}^{2}\hat{P}$ on a surface of a 3D sphere.

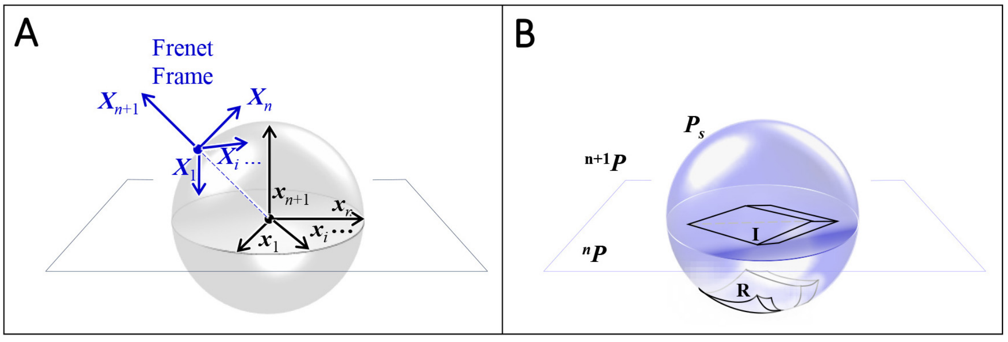

Figure S2. A general case for a Planck unit ${}^{n}\hat{p}$ at state **I** and state **R** (without transformability).

5) Calculation on generalized volume ${}^{n}m$ for a generalized sphere

For a general sphere determined by $\sum x_i^2 = 1$ (i = 1, Λ, n) in nD Euclidean space, its nD measurement ${}^{n}m$ should be

$${}^{1}m = \int_{-1}^{1} \mathrm{d}x = 2$$

$${}^{2}m = {}^{1}m \cdot \int_{-1}^{1} \sqrt{1-x^2}\,\mathrm{d}x = \pi = 3.141592654$$

$${}^{3}m = {}^{2}m \cdot \int_{-1}^{1} \left(\sqrt{1-x^2}\right)^2 \mathrm{d}x = \frac{4\pi}{3} = 4.188790205$$

$${}^{4}m = {}^{3}m \cdot \int_{-1}^{1} \left(\sqrt{1-x^2}\right)^3 \mathrm{d}x = \frac{\pi^2}{2} = 4.934802201$$

$${}^{5}m = {}^{4}m \cdot \int_{-1}^{1} \left(\sqrt{1-x^2}\right)^4 \mathrm{d}x = \frac{8\pi^2}{15} = 5.263789014$$

$${}^{6}m = {}^{5}m \cdot \int_{-1}^{1} \left(\sqrt{1-x^2}\right)^5 \mathrm{d}x = \frac{\pi^3}{6} = 5.167712780$$

$${}^{7}m = {}^{6}m \cdot \int_{-1}^{1} \left(\sqrt{1-x^2}\right)^6 \mathrm{d}x = \frac{16\pi^3}{105} = 4.724765970$$

$${}^{8}m = {}^{7}m \cdot \int_{-1}^{1} \left(\sqrt{1-x^2}\right)^7 \mathrm{d}x = \frac{\pi^4}{24} = 4.058712126\,; \quad \left({}^{8}m\right)^{\frac{1}{2}} = \left(\frac{\pi^4}{24}\right)^{\frac{1}{2}} = 2.014624562$$

$$^{9}m = {}^{8}m \cdot \int_{-1}^{1}\left(\sqrt{1-x^2}\right)^8 \mathrm{d}x = \frac{32\pi^4}{945} = 3.298508903;\ \left({}^{9}m\right)^{\frac{1}{3}} = \left(\frac{32\pi^4}{945}\right)^{\frac{1}{3}} = 1.488581281$$

$$^{10}m = {}^{9}m \cdot \int_{-1}^{1}\left(\sqrt{1-x^2}\right)^9 \mathrm{d}x = \frac{\pi^5}{120} = 2.550164040;\ \left({}^{10}m\right)^{\frac{1}{4}} = \left(\frac{\pi^5}{120}\right)^{\frac{1}{4}} = 1.263694308$$

$$^{11}m = {}^{10}m \cdot \int_{-1}^{1}\left(\sqrt{1-x^2}\right)^{10} \mathrm{d}x = \frac{64\pi^5}{10395} = 1.884103879$$

$$^{12}m = {}^{11}m \cdot \int_{-1}^{1}\left(\sqrt{1-x^2}\right)^{11} \mathrm{d}x = \frac{\pi^6}{720} = 1.335262769$$

$$^{13}m = {}^{12}m \cdot \int_{-1}^{1}\left(\sqrt{1-x^2}\right)^{12} \mathrm{d}x = \frac{128\pi^6}{135135} = 0.910628755$$

$$^{14}m = {}^{13}m \cdot \int_{-1}^{1}\left(\sqrt{1-x^2}\right)^{13} \mathrm{d}x = \frac{\pi^7}{5040} = 0.599264529$$

$$^{15}m = {}^{14}m \cdot \int_{-1}^{1}\left(\sqrt{1-x^2}\right)^{14} \mathrm{d}x = \frac{256\pi^7}{2027025} = 0.381443281$$

$$\cdots$$

And the general formulae for ${}^{n}m$

$$^{n}m = \frac{\pi^{n/2}}{(n/2)!} \quad (n = 2N)$$

$$^{n}m = \frac{2^{(n+1)/2} \cdot \pi^{(n-1)/2}}{1 \cdot 3 \cdot 5 \cdot \ldots \cdot n} \quad (n = 2N+1)$$

showing obviously the monotonic decrease trend aroused by $({}^{n+2}m/{}^{n}m) < 1$ for cases of $n > 12$.

Table S1 shows the longest 1D measurement $(x_m)_{\max} = {}^{n}m$ for an nD Planck sphere and diagonal $(x_{\mathrm{Ib}})_{\max} = \sqrt{n}$ for an nD Planck unit in Figure 4.

Table S1. IR cutoffs $L = x_{\max}$ for an nD Planck sphere ($L = {}^{n}m$) and the maximum length for an nD Planck unit at state **I** ($L = \sqrt{n}$).

Dim. (n)	1	2	3	4	5	6	7	8	9	10	11	…
${}^{n}m$	2.00	3.14	4.19	4.93	5.26	5.17	4.72	4.06	3.30	2.55	1.88	…
$\sqrt{n}$	1	$\sqrt{2}$	$\sqrt{3}$	$\sqrt{4}$	$\sqrt{5}$	$\sqrt{6}$	$\sqrt{7}$	$\sqrt{8}$	$\sqrt{9}$	$\sqrt{10}$	$\sqrt{11}$	…

6) Detailed solutions about the coupling fields

The maximum wavelengths for the two **T** states should be

$$\mathrm{RS}^{\upharpoonleft}: \lambda_{\mathbf{T}} = 2 \times {}^{nt}L \times \bar{\lambda}_{\mathrm{p}} = 8.91 \times 10^{-34}\ \mathrm{m} \quad (\text{in 4D } {}^{nt}\boldsymbol{P}) \tag{S19}$$

$$\mathrm{E}_{\downharpoonleft}: \lambda_{\mathbf{T}} = 2 \times {}^{10}L \times \bar{\lambda}_{\mathrm{p}} = 1.35 \times 10^{5}\ \mathrm{m} \quad (\text{in 10D } {}^{10}\boldsymbol{P}) \tag{S20}$$

The wavelength $\lambda_{\mathbf{T}} = 1.35 \times 10^5$ m for $\mathrm{E}_{\downharpoonleft}$ in ${}^{10}\boldsymbol{P}$ should be of

$L = \frac{(2\pi)^{34}}{6 \times 7 \times 8 \times 9} = 4.544998428 \times 10^{23}$ in ${}^{nt}\boldsymbol{P}$, with the maximum wavelength being

$$\mathrm{E}_{\downharpoonleft} \text{ of } {}^{nt}\boldsymbol{P}: \lambda_{\mathbf{T}} = 2\times {}^{nt}L \times \overline{\lambda}_{\mathrm{P}} = 2.34\times10^{-12}\ \mathrm{m}\quad (\text{in 4D } {}^{nt}\boldsymbol{P}) \tag{S21}$$

For a 2-body system, TS requires parity between the two symcenters of $\mathbf{I}_1$ and $\mathbf{I}_2$ (Figure 6), resulting in the shared $\mathbf{T}_1 \sim \mathbf{T}_2'$, $\mathbf{T}_2 \sim \mathbf{T}_1'$ and $\mathbf{T}_1' \sim \mathbf{T}_2'$ at the tangent point(s) and the interacting point(s) ($\mathbf{T}_1$ and $\mathbf{T}_2'$ are identical when $m_{\mathrm{T}} = 0$ and $\mathrm{d}v_{\mathrm{T}} = 0$, and so do $\mathbf{T}_2 \sim \mathbf{T}_1'$ and $\mathbf{T}_1' \sim \mathbf{T}_2'$), respectively (Figure S3).

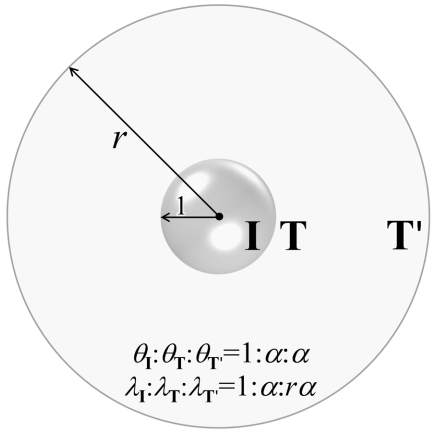

Figure S3. A potential field aroused by quantum coupled state $\mathbf{T}/\mathbf{T}'$.

So it should be $U_{\mathbf{T}_1} = U_{\mathbf{T}_2'} = U_{\mathbf{T}_1'} = U_{\mathbf{T}_2} = 0$ and $\frac{\mathrm{d}U_{\mathbf{T}}}{\mathrm{d}r} = 0$ for potential U of state $\mathbf{T}$. Based on $S_{\mathbf{T}} = S_{\mathbf{T}'} = h$, kinetic energy for state $\mathbf{T}$ satisfies

$$\Delta T_{\mathbf{T}} = T_{\mathbf{T}'} - T_{\mathbf{T}} = \frac{hc}{\lambda_{\mathbf{T}'}} - \frac{hc}{\lambda_{\mathbf{T}}} \tag{S22}$$

In the flat background space for state $\mathbf{I}$ and state $\mathbf{T}$, the conserved energy requires $\mathrm{d}\left(E_{\mathbf{I}} + E_{\mathbf{T}}\right) = 0$ and $\mathrm{d}\left(U_{\mathbf{I}} + T_{\mathbf{I}}\right) + \mathrm{d}\left(U_{\mathbf{T}} + T_{\mathbf{T}}\right) = 0$, resulting in

$$\Delta U_{\mathbf{I}} = -\Delta T_{\mathbf{T}} = \frac{hc}{\lambda_{\mathbf{T}}} - \frac{hc}{\lambda_{\mathbf{T}'}} = \frac{hc}{\lambda_{\mathbf{T}}} - \frac{\alpha hc}{2\pi r} \tag{S23}$$

between $\mathbf{I}_1$ and $\mathbf{I}_2$ since $\mathrm{d}U_{\mathbf{T}} = 0$ ($U_{\mathbf{T}} = U_{\mathbf{T}'} \equiv 0$) and $\mathrm{d}T_{\mathbf{I}} = 0$. Consequently, it exists

$$F_{\mathbf{I}} = \frac{\mathrm{d}\Delta U_{\mathbf{I}}}{\mathrm{d}\rho} = \frac{\alpha hc}{2\pi\rho^2}\quad (\rho = r\overline{\lambda}_{\mathrm{P}} \gg \overline{\lambda}_{\mathrm{P}}) \tag{S24}$$

Then a coupling field should be

$$F_{\mathrm{RS}^{\upharpoonleft}} = \frac{2.307076\times10^{-28}}{\rho^2}\ \mathrm{N}\quad (\alpha_1 = 1/137.036082) \tag{S25}$$

for the two ${}^{10}\hat{p}$ satisfying principle $\mathrm{RS}^{\upharpoonleft}$ in ${}^{nt}\boldsymbol{P}$.

And another coupling field should be

$$F_{\mathrm{E}_{\downharpoonleft}} = \frac{1.941959\times10^{-64}}{\rho^2}\ \mathrm{N}\quad \left(\alpha_2 = 1\big/\left(1.628008\times10^{38}\right)\right) \tag{S26}$$

for two ${}^{10}\hat{p}$ satisfying principle $\mathrm{E}_{\downharpoonleft}$ in ${}^{\mathrm{X}}\boldsymbol{P}$.

7) Symbol list

Symbols	Explanations
RS	rotational symmetry
TS	translational symmetry
${}^{n}\hat{p}$	the minimum *n*D local space in quasi-Euclidean space, *n*D Planck unit with uncertainty, where the uncertainty forbids $\hat{p}$ to be with certain measure and certain position simultaneously ($\hat{\rho}_{\theta}$ and $\hat{\theta}$ can't be of determinacy simultaneously)
I	in-situ state for a Planck unit $\hat{p}$ with RS, with uncertain structures, such as a generalized sphere with completely uncertain azimuth $0 \le \Delta\hat{\theta} \le 2\pi$ but certain 1D measure $x_i \equiv 1$, a generalized cube with certain azimuth $\Delta\theta$ but completely uncertain x_i varying within [$\bar{\lambda}_{\mathrm{P}}$, $\sqrt{n}\bar{\lambda}_{\mathrm{P}}$], or any an intermediate between the sphere and the cube
R	revolving state for a RS unit in Planck surface determined by $r = \bar{\lambda}_{\mathrm{P}}$
T/T ′	state for an RS unit in tangent plane of spherical surface determined by $r \ge \bar{\lambda}_{\mathrm{P}}$
${}^{n}\boldsymbol{P}$	*n*D orthogonal background space, also the super set of ${}^{1}\boldsymbol{P}$, Λ, ${}^{n}\boldsymbol{P}$
${}^{n}P$	a local *n*D space
${}^{t}\boldsymbol{P}$	≤6D subspace in 10D background, without non-triviality for any a Planck unit or a substructure expanded by x_1, Λ, x_6
${}^{nt}\boldsymbol{P}$	4D non-trivial subspace in 10D background, expanded by x_7, Λ, x_{10}
V_r^{∞}	infinite barrier to forbid an object from any radial displacement (demanding d*r* = 0)
L	IR cutoff of $(x_i)_{\max} = \frac{{}^{n}P}{\bar{\lambda}_{\mathrm{P}}^{(n-1)}}$ for a local space measured ${}^{n}P$
λ	wavelength of a Planck unit limited in a certain local space, where the unit always takes IR cutoff *L* as its motion path; when the path is not a closed loop, it exists $\lambda = 2L$ for the unit in ground state
${}^{n}m$	generalized volume for an *n*D sphere with normalized radius $r = 1$
$\mathrm{E}_{\downarrow}$	the minimum energy principle, results in a state with motion space as large as possible.
$\mathrm{RS}^{\uparrow}$	the highest RS principle, results in a state with RS as high as possible
α	ratio $\frac{L_{\mathbf{I}}}{L_{\mathbf{T}}}$ during transformation $\mathbf{I} \to \mathbf{T}$ for a Planck unit $\hat{p}$ in 10D background space, with value ~1/137.036 in ${}^{nt}\boldsymbol{P}$ when it satisfies Principle $\mathrm{RS}^{\uparrow}$, or ~1/(1.628 × 10^{38}) when it satisfies Principle $\mathrm{E}_{\downarrow}$