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On the Unipolar Generator: An Experimental and Theoretical Study

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Abstract

When studying the phenomenon of the induced electromotive force, which originates from Faraday's unipolar inductor, the contrast between Faraday's view of the magnetic field dynamic lines and the theory of relativity is revealed. In order to remove this contradiction, this phenomenon was studied in depth, theoretically and experimentally, using an experimental setup similar to Faraday's. Calculations of the induced electromotive force, based on relativity on the one hand and on Faraday's view on the other were made with the help of measurements of the magnetic field components. Accurate magnetic field measurements are confirmed by analytical calculations. Precise-induced electromotive force measurements confirmed Faraday's view and contradicted the theory of relativity.

Keywords

Faraday's Experiment, Unipolar Generator, Homopolar Generator, Faraday's Inductor, Unipolar Induction

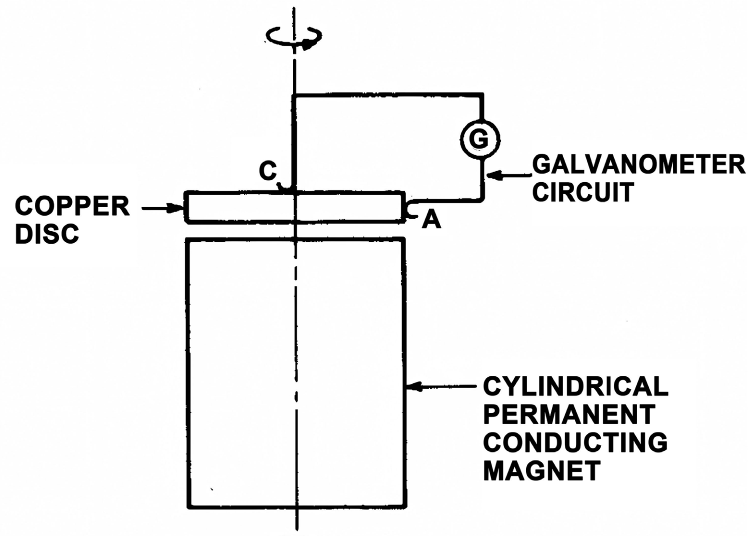
1. Introduction

The first homopolar generator was developed by Michael Faraday in 1831, during his experiments. It is frequently called the Faraday disc or Faraday wheel in his honor. It was the beginning of electrical generators which operate using a magnetic field.

This device consists of a conducting disk rotating in a magnetic field with one electrical contact near the axis and the other near the periphery. It has been used for generating very high currents at low voltages in applications such as welding, and electrolysis.

The magnetic field is provided by a permanent magnet, and the generator works regardless of whether the magnet is fixed to the stator or rotates with the disc. Before the discovery of the electron and the Lorentz force law, the phenomenon was inexplicable and was known as the Faraday paradox.

Faraday's apparatus and results for unipolar induction with a copper disc, a permanent magnet and a galvanometer circuit that makes sliding contacts with the disc, are described in [1] (par. 2, p. 157, The problem of whether magnetic lines of force rotate: 1831-1901), as follows (Figure 1)



(a)

Experiment	Disc	Magnet	Current
1	R	R	Yes
2	R	X	Yes
3	X	R^{-1} (or R)	NO

(b)

Figure 1. Faraday's apparatus (a) and results (b) for unipolar induction with a copper disc, a permanent magnet and a galvanometer circuit that makes sliding contacts with the disc at A and C (see Faraday[1], vol. I, 32 and 63). In table (b), X = No Rotation, R = Rotation, R^{-1} = Rotation Inverse to R.

"In experiment l, the disc and magnet were cemented together: in experiments 2 and 3, they were separate.[2] Faraday concluded: 'Hence, rotating the magnet causes no difference in the results, for a rotatory and a stationary magnet produce the same effect upon the moving copper[3] [there is] a *singular independence* of the magnetism and the bar in which it resides.'[4] Thus,

[1]Michael Faraday, *Experimental researches in electricity*, (3 vols., New York, 1965) vol. l, 24ff. A useful biography of Faraday is L, Pearce Williams, Michael Faraday: *a biography* (New York, 1971).

[2]Faraday describes experiment 3 in *Faraday's diary* (6 vols., London, 1932), vol. l, p. 402, w 257 (26 December 1831).

[3]Faraday (footnote 2), vol. l, 63, Section 218.

[4]*Ibid.*, 64, Section 220; italics in original.

in his opinion, the lines of force did not rotate with the magnet. This conclusion violated Ampère's theory of magnetism, in which the atomic current whirls were primitive, and so the lines of force should have rotated with the magnet. For Faraday, on the other hand, the lines of force were primitive. It is important to note Faraday's emphasis that experiments 2 and 3 were not inverses of one another.[5] Faraday recognized that there was involved in all three experiments a three-part system-magnet, disc and galvanometer circuit. Therefore, the experiment inverse to 2 was a rotation of the magnet and galvanometer circuit, and then a current would have been induced."

Just some of the published works that refer to this topic, and show the course of this research in its historical path until today, are in bibliographic references [2]-[16].

2. Description of the Experimental Setup

The experimental set-up includes an alternating current motor coaxially connected to a ring magnet and a brass conducting disc having the form of **Figure 2**. The metal brass shaft is fixed at one end to the motor shaft and at the other end to a bearing mounted on a plastic base. On this brass shaft are mounted the ring magnet and the conducting disk. Two cylindrical brushes are fixed on two plastic bases, and are pushed by springs so that they are in constant contact with the surface of the conducting disk during its rotation. One of these two brushes contacts the surface of the outer peripheral portion of the conducting disc while the other contacts the inner peripheral portion of the conducting disc which is close to the axis of rotation. Between the peripheral parts of the conducting disk with which the two brushes are in continuous contact are inserted four radial parts, which are illustrated in **Figure 2**.

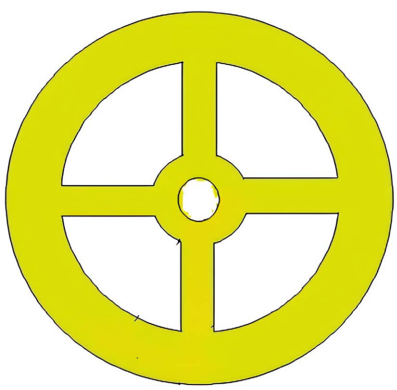


Figure 2. Conducting disk design.

The magnet used in the experimental setup is a ferrite ring magnet, the characteristics of which are:

[5]Faraday (footnote 8), vol. l, 402, Section 256.

- 190 mm outer diameter × 85 mm inner diameter × 23 mm thick.
- Each magnet's north and south poles are on opposite circular faces.

The engine features are:

- Power = 0.25 PS = 0.18 kw
- Voltage (AC) = 220 V
- Cosine ($\cos\varphi$) = 0.70

Between the conductor and the magnet is inserted a circular coaxial plastic sheet 3.5 mm thick and 10.5 cm in diameter, in contact with one of the two flat surfaces of the magnet (**Figure 3**). Another piece of plastic, which is circular in shape also, is in contact with the other flat surface and the cylindrical inner surface of the magnet, and has two bearings built into it. The magnet rests on the brass shaft through these two bearings. The thickness of the conducting disc is 3.5 mm and its diameter is 18.4 cm. The closest to the axis of rotation point of the circular contact surface of the brush, which is in contact with the surface of the outer peripheral part of the conducting disk, is 8.35 cm away from the axis of rotation. The corresponding distance of the most distant contact point of the brush, located near the axis of rotation, to the axis of rotation is 4 cm. The experimental setup gives the possibility of rotating the conductor with the magnet immobilized, the possibility of rotating the magnet with the conductor immobilized, and also the possibility of rotating the magnet and conductor simultaneously with the same frequency of rotation.

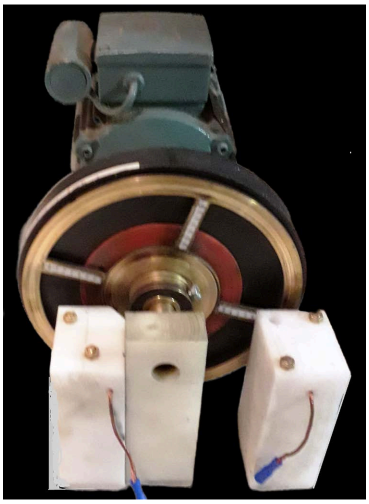


Figure 3. Experimental setup.

3. Analytical Calculation of the Magnetic Field

In the experimental setup of the present study, shown in **Figure 3**, the magnetic polarization of the permanent magnet ring is axial, so we will follow the corresponding analytical procedure for calculating the magnetic field, as stated in [17] (2. BASIC EXPRESSIONS, p. 72). Also the placement of the permanent magnet ring on the motor shaft has been done so that the flat surface of the south magnetic pole is the nearest to the conducting disk. Therefore, assuming that the surface of the north magnetic pole is in the plane $z=0$, as shown in **Figure 4**, and that the *z*-axis coincides with the axis of rotation with a direction from the magnet to the conducting disk, the direction of the magnetic field at each point of observation is opposite to that predicted in the analytical relations stated in [17] [18], where the surface of the south magnetic pole is in the $z=0$ plane.

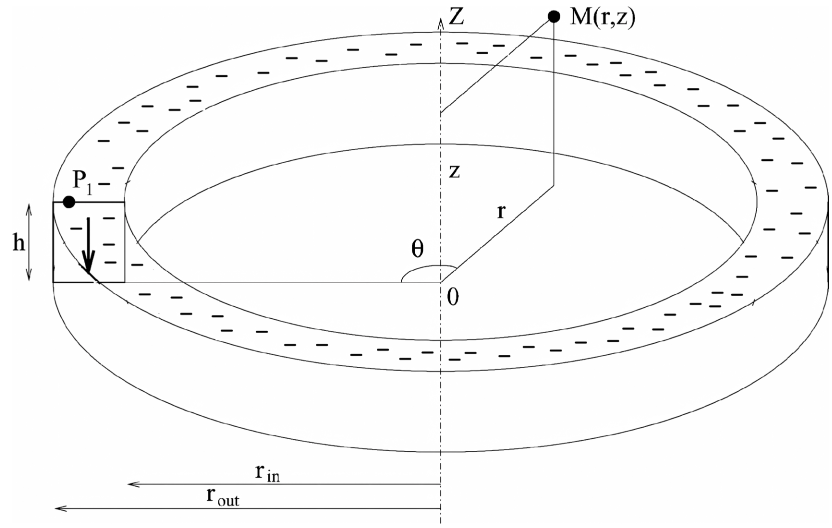


Figure 4. The geometric characteristics of the ring magnet. The axis of symmetry is *Z*, r_{in} is the inner radius and r_{out} is the outer radius. The magnetic polarization is axial.

Therefore, for the analytical calculations of the magnetic field of the present study, it is sufficient to replace σ^* with $-\sigma^*$ in all the aforementioned analytical relations, as follows.

$$\vec{H}^+(r,z)=\frac{-\sigma^*}{4\pi\mu_0}\int_{\theta=0}^{\theta=2\pi}\int_{r_1=r_{in}}^{r_1=r_{out}}\frac{\overrightarrow{P_{1+}M}}{\left|\overrightarrow{P_{1+}M}\right|^3}r_1\mathrm{d}r_1\mathrm{d}\theta$$

$$\vec{H}^-(r,z)=\frac{\sigma^*}{4\pi\mu_0}\int_{\theta=0}^{\theta=2\pi}\int_{r_1=r_{in}}^{r_1=r_{out}}\frac{\overrightarrow{P_{1-}M}}{\left|\overrightarrow{P_{1-}M}\right|^3}r_1\mathrm{d}r_1\mathrm{d}\theta$$

$$\vec{H}(r,z)=\vec{H}^+(r,z)+\vec{H}^-(r,z)$$

where

$$\overrightarrow{P_{1+}M}=(r-r_1\cos\theta)\hat{u}_r-r_1\sin\theta\hat{u}_\theta+(z-h)\hat{u}_z$$

$$\overrightarrow{P_{1-}M}=(r-r_1\cos\theta)\hat{u}_r-r_1\sin\theta\hat{u}_\theta+z\hat{u}_z$$

and $\hat{u}_r$, $\hat{u}_\theta$, $\hat{u}_z$ are the unit vectors of the cylindrical coordinate system.

Substituting θ with $\pi-2\beta$, integrating twice, and respecting angle variable symmetry, we obtained the 3D components of the magnetic field at the point $M(r,0,z)$. In this case, all integrals are transformed in the integration interval $0\le\beta\le\pi/2$ so that all results are expressed in terms of complete elliptic integrals.

$$\vec{H}_r(r,z)=\vec{H}_r^+(r,z)+\vec{H}_r^-(r,z) \quad (1)$$

$$\vec{H}_\theta(r,z) = \vec{H}_\theta^+(r,z) + \vec{H}_\theta^-(r,z) = 0 \tag{2}$$

$$\vec{H}_z(r,z) = \vec{H}_z^+(r,z) + \vec{H}_z^-(r,z) \tag{3}$$

where

$$\vec{H}_r^+(r,z) = \frac{-\sigma^*}{\pi\mu_0}\sum_{n=1}^{2}(-1)^{n-1}\frac{1}{k_n^+}\sqrt{\frac{r_n}{r}}\left[E(k_n^+) - \left(1 - \frac{k_n^{+2}}{2}K(k_n^+)\right)\right]$$

$$\vec{H}_r^-(r,z) = \frac{\sigma^*}{\pi\mu_0}\sum_{n=1}^{2}(-1)^{n-1}\frac{1}{k_n^-}\sqrt{\frac{r_n}{r}}\left[E(k_n^-) - \left(1 - \frac{k_n^{-2}}{2}K(k_n^-)\right)\right]$$

$$\vec{H}_\theta^+(r,z) = \vec{H}_\theta^-(r,z) = 0$$

$$\vec{H}_z^+(r,z) = \frac{-\sigma^*}{4\pi\mu_0(z-h)}\sum_{n=1}^{2}(-1)^{n-1}\frac{k_n^+}{\sqrt{rr_n}}$$
$$\left\{\left[\sqrt{r^2+(z-h)^2} - r\right]\left[\sqrt{r^2+(z-h)^2} - r_n\right]\Pi(h_1^+, k_n^+) + \left[\sqrt{r^2+(z-h)^2} + r\right]\left[\sqrt{r^2+(z-h)^2} + r_n\right]\Pi(h_2^+, k_n^+)\right\}$$

$$\vec{H}_z^-(r,z) = \frac{\sigma^*}{4\pi\mu_0 z}\sum_{n=1}^{2}(-1)^{n-1}\frac{k_n^-}{\sqrt{rr_n}}$$
$$\left\{\left[\sqrt{r^2+z^2} - r\right]\left[\sqrt{r^2+z^2} - r_n\right]\Pi(h_1^-, k_n^-) + \left[\sqrt{r^2+z^2} + r\right]\left[\sqrt{r^2+z^2} + r_n\right]\Pi(h_2^-, k_n^-)\right\}$$

$$k_n^{+2} = \frac{4rr_n}{(r+r_n)^2 + (z-h)^2},\quad k_n^{-2} = \frac{4rr_n}{(r+r_n)^2 + z^2}$$

$$h_1^+ = \frac{2r}{r + \sqrt{r^2+(z-h)^2}},\quad h_2^+ = \frac{2r}{r - \sqrt{r^2+(z-h)^2}}$$

$$h_1^- = \frac{2r}{r + \sqrt{r^2+z^2}},\quad h_2^- = \frac{2r}{r - \sqrt{r^2+z^2}}$$

$$r_1 = r_{in},\quad r_2 = r_{out}$$

4. Calculation of the Induced Electromotive Force Based on Relativity on the One Hand and Based on Faraday's View on the Other

In Feynman's lecture on "The Laws of Induction", in [19], in paragraph 17-2, Exceptions to the "flux rule", the following are stated.

> "Now we will describe a situation in which the flux through a circuit does not change, but there is nevertheless an emf. Figure 17-2 shows a conducting disc which can be rotated on a fixed axis in the presence of a magnetic field. One contact is made to the shaft and another rubs on the outer periphery of the disc. A circuit is completed through a galvanometer. As the disc rotates, the "circuit", in the sense of the place in space where the cur-

> rents are, is always the same. But the part of the "circuit" in the disc is in material which is moving. Although the flux through the "circuit" is constant, there is still an emf, as can be observed by the deflection of the galvanometer. Clearly, here is a case where the $\boldsymbol{v} \times B$ force in the moving disc gives rise to an emf which cannot be equated to a change of flux."

It is obvious that the calculation of the electromotive force, in the experimental setup we study in the present work, along a path of length L of a radial part of the conducting disk shown in **Figure 2** gives a result equal to W/q, where W is the work done by the Lorentz force to move a charged particle with charge q along the path of length L. Therefore, the electromotive force is given by the equation

$$V = \frac{1}{q}\int_L \vec{F} \cdot \mathrm{d}\vec{\ell} = \int_L \left(\vec{E} + \vec{v} \times \vec{B}\right) \cdot \mathrm{d}\vec{\ell} \tag{4}$$

where $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$ is the Lorentz electromagnetic force acting on the charge q and $\vec{E}$, $\vec{B}$ are the electric and magnetic field respectively.

Therefore, the calculation of the electromotive force requires the calculation of the Lorentz force $\vec{F}$ and the calculation of the Lorentz force requires the calculation of the fields $\vec{E}$ and $\vec{B}$. Also the calculation of the electromotive force in the rotating reference system is done in [20], (3.1. Electromagnetic force and *EMF* in the case of Faraday's unipolar inductor, p. 124) and in [21] [22], based on the general theory of relativity, where it is proved that for low speeds, such as those of the present experimental study, the calculation of the Lorentz force and the electromotive force in any differential line segment of the electric circuit does not require the assistance of the general theory of relativity, since the use of the special theory of relativity is sufficient, given that all terms that are due to the lack of inertia of rotating frame of reference are practically zero.

Let us consider two reference frames, one of which is the inertial frame of the laboratory denoted by S, and the other is the rotating reference frame denoted by S'. Given all the above, the transformations of the electromagnetic field are made based on the well-known equations of the special theory of relativity for low velocities, *i.e.* for velocities for which the Lorentz factor $\gamma = \left(1 - u^2/c^2\right)^{-1/2}$ is practically equal to 1,

$$\vec{E}'_{\parallel} = \vec{E}_{\parallel},\ \vec{E}'_{\perp} = \vec{E}_{\perp} + \vec{u} \times \vec{B} \tag{5}$$

$$\vec{B}'_{\parallel} = \vec{B}_{\parallel},\ \vec{B}'_{\perp} = \vec{B}_{\perp} - \frac{1}{c^2}\vec{u} \times \vec{E} \tag{6}$$

where $\vec{E}'$ and $\vec{B}'$ are the electric and magnetic field with respect to the rotating frame of reference, while the $\vec{E}$ and $\vec{B}$ are the electric and magnetic field with respect to the inertial system of reference of the laboratory. Also, in the symbolism of components of electromagnetic field that are parallel to the direction of the vector velocity $\vec{u}$ the subscript $\parallel$ has been added, while in the symbolism of components of electromagnetic field that are perpendicular to the direction of the vector velocity $\vec{u}$ the subscript $\perp$ has been added. The inverse transformations are given by the following equations.

$$\vec{E}_{\parallel} = \vec{E}'_{\parallel},\ \vec{E}_{\perp} = \vec{E}'_{\perp} - \vec{u} \times \vec{B}' \tag{7}$$

$$\vec{B}_{\parallel} = \vec{B}'_{\parallel},\ \vec{B}_{\perp} = \vec{B}'_{\perp} + \frac{1}{c^2}\vec{u} \times \vec{E}' \tag{8}$$

If we consider, as in Section 3, the unit vectors $\hat{u}_r$, $\hat{u}_\theta$, $\hat{u}_z$ of the cylindrical coordinate system, defining the axis Z perpendicular to the surface of the conducting disk in the direction towards the stationary part of the electric circuit in the laboratory, from where the conducting disk appears to rotate clockwise, the angular velocity vector is given by the relation $\vec{\omega} = -\omega \hat{u}_z$, and the vector velocity $\vec{u}$ is given by the equation

$$\vec{u} = \vec{\omega} \times \vec{r} = -\omega r \hat{u}_\theta \tag{9}$$

Therefore, according to the analytical relations of magnetic field in Section 3, the magnetic field has components only in the r and z directions of the cylindrical coordinate system, so the magnetic field is everywhere perpendicular to the direction of the vector velocity $\vec{u}$.

According to Faraday's view, the magnetic field does not follow the rotational motion of the magnet. In other words, Faraday's view is that, while the magnetic field originates from the magnet, it nevertheless does not rotate when the magnet rotates, because the reference frame of the magnetic field lines is at all times the inertial frame of reference of the axis of rotation of the magnet, *i.e.* the inertial reference system of the laboratory.

So let us now consider three different cases based on relativity and then according to this previously formulated view of Faraday.

4.1. First Case: The Magnet Is Stationary in the Laboratory and the Conducting Disk Is Rotating

When the magnet is stationary in the inertial reference frame S of the laboratory, the electric field, denoted by $\vec{E}$ in this reference frame, is zero. The magnetic field is denoted by $\vec{B}$ in the inertial frame of reference S. On a conduction electron, located on a differential surface of the conducting disc, is exerted a Lorentz electromagnetic force

$$\vec{F} = e\vec{v} \times \vec{B}$$

where $e = -1.602176634 \times 10^{-19}\ \text{Coulomb}$ is the charge of the electron and $\vec{v}$ is the velocity of the differential surface of conducting disc with respect to reference frame S, which comes from the rotational motion, so $\vec{v} = \vec{u}$.

In the reference system S', according to the relations (5) and (6) the following equalities are obtained

$$\vec{E}'_{\parallel} = 0$$

$$\vec{E}'_{\perp} = \vec{u} \times \vec{B}$$

$$\vec{B}' = \vec{B}$$

so, the Lorentz force in the rotating frame of reference S' is

$$\vec{F}' = e\left(\vec{E}' + \vec{v}' \times \vec{B}'\right)$$
$$= e\left(\vec{u} \times \vec{B} + \vec{v}' \times \vec{B}\right)$$

therefore, since the differential surface of the conducting disc is stationary at *S'*, the velocity is $\vec{v}' = 0$ and the force expression is the same as that of the Lorentz force in the laboratory reference system, *S*, as expected.

In the laboratory reference frame *S* the velocity of the aforementioned differential surface of the conducting disk is $\vec{v} = \vec{u} = -\omega r\hat{u}_\theta$, so

$$\begin{aligned}\vec{F} &= -e\omega r\hat{u}_\theta \times \left(\vec{B}_r + \vec{B}_z\right)\\ &= e\omega r\left(B_r\hat{u}_z - B_z\hat{u}_r\right)\end{aligned} \tag{10}$$

Consequently, two components of the electromagnetic Lorentz force arise, one radial and one perpendicular to the surface of the conducting disk. The component contributing to the developing electromotive force along a radial part of the conducting disk is given by the relation

$$\vec{F}_r = -e\omega r B_z\hat{u}_r \tag{11}$$

Since the charge e of the electron is negative and the component B_z of the magnetic field has negative values at the surface of the conducting disc, according to Equation (11) the radial component of the Lorentz force acting on the conduction electrons is directed towards the center of rotation, so according to the conventional direction of the electric current, the direction of the electric current is from the center to the periphery, *i.e.* equivalent to the direction of the electric current induced by a DC voltage electric source with its negative pole in axis of rotation and the positive one at the periphery.

The differential surface of a ring of radius r and differential width dr on the surface of conducting disc is equipotential, that is, all points of a circumference of radius r of the conducting disk, the center of which is located on the axis of rotation, have the same electric potential. This results from the lack of electromotive force along such a circumference. For example, in a differential line segment $\mathrm{d}\ell = r\mathrm{d}\theta$ of this circumference, the force exerted on a conduction electron due to the rotational motion, is given by the equality (10). The induced differential electromotive force along the aforementioned differential length is zero according to the equality

$$\mathrm{d}V = \frac{1}{e}\vec{F} \cdot \mathrm{d}\vec{\ell} = \omega r\left(B_r\hat{u}_z - B_z\hat{u}_r\right) \cdot \left(\mathrm{d}\ell\hat{u}_\theta\right) = 0$$

Also, the electromotive force along any closed path C that encloses the surface *S*, on the surface of the conducting disc, is zero. This follows from Stokes' theorem $\int_S\left(\vec{\nabla} \times \vec{F}\right) \cdot \hat{u}_z \mathrm{d}s = \oint_C \vec{F} \cdot \mathrm{d}\vec{\ell}$, because if we limit the closed path on the surface of the conducting disk, given that $\vec{F} = e\omega r\left(B_r\hat{u}_z - B_z\hat{u}_r\right)$, it follows that $\left(\vec{\nabla} \times \vec{F}\right) \cdot \hat{u}_z = 0$. This means that the potential difference, resulting from the induced electromotive force, between any two points on the surface of the conducting disc, is the same for every path connecting these two points and defined on that surface.

The induced electromotive force along a path L of a radial part of the conducting disk in this case is given by the equality

$$\begin{aligned} V &= \frac{1}{e}\int_L \vec{F}\cdot \mathrm{d}\vec{\ell} \\ &= \frac{1}{e}\int_{r_1}^{r_2} \vec{F}_r \cdot \hat{u}_r \mathrm{d}r \\ &= -\omega \int_{r_1}^{r_2} r B_z \mathrm{d}r \end{aligned} \quad (12)$$

where $r_2 - r_1 = L$ and $\mathrm{d}\vec{\ell} = \hat{u}_r \mathrm{d}r$.

Because the magnet is stationary in the laboratory, the relativistic effect of the equality (12) is the same as that predicted by Faraday's view, so this case of rotating conducting disc and the stationary magnet in the reference frame of laboratory, is not offered to examine the correctness of one or the other of these two views.

4.2. Second Case: The Conducting Disk and Ring Magnet Rotate Together

Let us consider the case where the angular velocity vector of the ring magnet and the conducting disc are the same. This is one of the cases of the experiment of the present study, where the ring magnet and the conducting disk, viewed from the side of the external part of the electric circuit, rotate clockwise with the same angular velocity. In this case, the relative velocities of all points of the conducting disk with respect to the reference system of the magnet are zero. The external part of the electric circuit, *i.e.* that part of the electric circuit which has as its ends the two brushes and through which the connection is made with the instrument for measuring the electric voltage, is stationary with respect to the inertial reference system of the laboratory. Also, the relative velocities of all points of the external part of the electrical circuit with respect to the reference system of the magnet are determined based on the opposite of the aforementioned angular velocity vector.

In the rotating frame of reference S', which is the frame of reference relative to which the magnet and the conducting disk are stationary, the electric field $\vec{E}'$ is zero, while the magnetic field $\vec{B}'$ is predicted for a ring magnet at rest. Therefore, the Lorentz force on a conduction electron located on a differential surface of the conducting disk, given that the velocity of the differential surface at S' is $\vec{v}' = 0$, as shown by the following equality is zero.

$$\vec{F}' = e\left(\vec{E}' + \vec{v}' \times \vec{B}'\right) = 0$$

In the inertial frame of reference S of the laboratory according to the relations (7) and (8) the following equalities are taken

$$\vec{E}_{\parallel} = 0$$

$$\vec{E}_{\perp} = -\vec{u} \times \vec{B}'$$

$$\vec{B} = \vec{B}'$$

then, since velocity of the differential surface, estimated in reference system *S*, is $\vec{v} = \vec{u}$, the Lorentz force exerted on the aforementioned conduction electron is given by the equality

$$\begin{aligned} \vec{F} &= e\left(\vec{E} + \vec{v} \times \vec{B}\right) \\ &= e\left(-\vec{u} \times \vec{B} + \vec{u} \times \vec{B}\right) \end{aligned}$$

and is zero with respect to reference frame *S*, as expected. Therefore, according to the theory of relativity, in this case is not induced electromotive force in the conducting disk.

However, also based on the theory of relativity, an electromotive force is induced in the stationary, with respect to laboratory, part of the electric circuit, since it appears to be rotating with respect to the reference frame of the magnet, with an angular velocity opposite to that of the conducting disk-magnet system observed in the laboratory reference system.

These relativistic results contradict the view of Faraday, who in his work on the unipolar inductor states that the measured electromotive force in the case where the magnet and the conductor rotate together is due to the fact that the rotating conductor intersects the lines of force of the magnetic field, because the magnetic field lines of force do not follow the rotational motion of the magnet. Therefore, according to Faraday's view, the electromotive force is induced in the conducting disk and is equal to that of the case studied in Section 4.1, while is not induced electromotive force in the stationary external part of the electric circuit.

Given all the above, this dichotomy should be completely removed. For this purpose, we calculated the induced electromotive force, making calculations based on the analytical equations of the components of the magnetic field. Also additional calculations were made using measurements of the magnetic field. All these calculations were done using mathematica and MATLAB. These results are then compared with the measured electromotive force.

But it should be emphasized that the induced electromotive force in the conducting disk according to Faraday's view is not equal to the induced electromotive force calculated according to general relativity in the stationary part of the electric circuit as we will see below. Therefore, in this case, the comparison between the theoretical values and the experimental results gives the answer regarding the agreement of one theoretical point of view or the other with the reality expressed by the experimental results.

4.2.1. Faraday Electromotive Force along a Radial Part of the Conducting Disc

The analytical expressions of the magnetic field of the ring magnet yield the expected cylindrical symmetry of this field. However, due to imperfections of the permanent magnet of the experimental setup used, the magnetic field of the experiment is not expected to be completely cylindrically symmetric.

The measuring instrument used to measure the components of the magnetic field is a TD8620 gauss meter with the following characteristics:

- Measurement Range: 0 - 2400 mT (2400 GS).
- Accuracy: 0 - 1000 mT (2%); 1000 mT - 2400 mT (5%).
- Range: 200 mT, 2000 mT (Automatic range switching).
- Power Supply: 9V battery.

The magnetic field on the surface of the conducting disc was measured along a radial part, taking one measurement per 5 mm. The values of these measurements are shown in the circled points in Figure 5. These obtained values of the magnetic field component which is perpendicular to the surface of the conducting disk, are negative because the surface of the south pole of the ring magnet is the closest to the conducting disk. The position of the surface of the conducting disk on the Z axis is $z_{01} = 29.5\text{ mm}$ and the analytical expression $B_z(r, z_{01})$ of the magnetic field along the radial part, for $\sigma^* = 220\text{ mT}$, gives the continuous curve of the graph in Figure 5. The values of the magnetic field of this continuous curve have been obtained from the analytical expression (3) using the mathematica program and then used to plot in MATLAB.

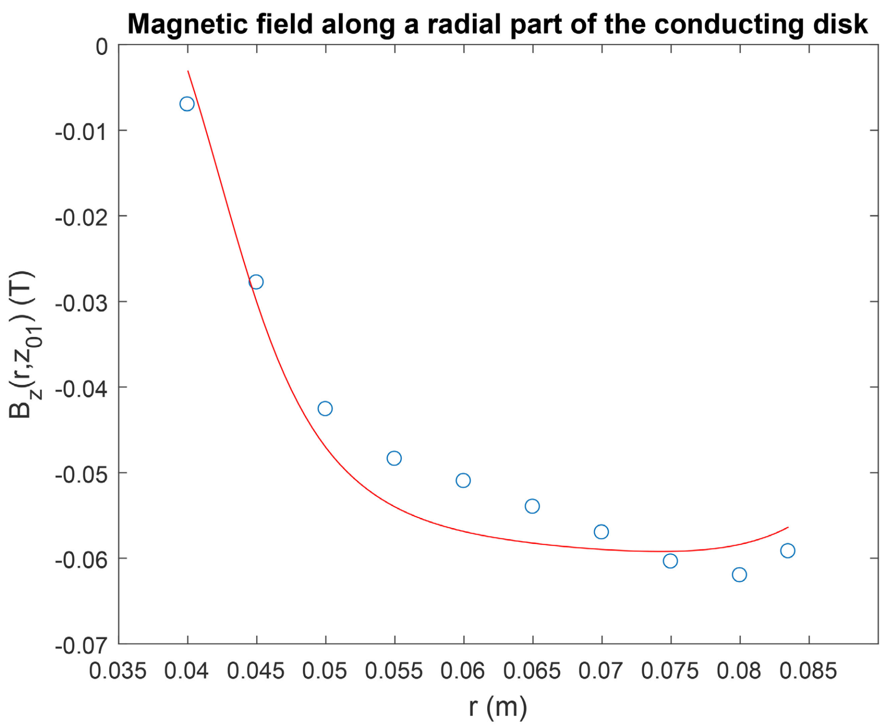


Figure 5. The continuous curve comes from the analytical relations of magnetic field, while the circular points come from measurements of the z-component of magnetic field, along a radial part of the conducting disc.

According to Faraday's view there is an induced electromotive force in this radial part because the magnetic field lines of force intersect during the rotation of the conducting disk. As mentioned previously, in Faraday's opinion this phenomenon is due to the fact that the dynamic lines of the magnetic field do not follow the rotational movement of the magnet. This induced electromotive force is calculated based on the relation (12) using either the analytical expression (3) and making use of the mathematica program, or the values of the magnetic field obtained from the measurements and making numerical integration with MATLAB. The value of the electromotive force obtained from the relation (12) and the analytical expression (3) is 22 mV, while the value of the electromotive force obtained from the relation (12) and the magnetic field measurements is 21.3 mV. The difference between the two values is due to the imperfections of the ring magnet which can be seen in Figure 5.

4.2.2. Faraday Electromotive Force Induced Entirely in the Conducting Disk

Due to the imperfections of the ring magnet, there is a small deviation of the measured values of the magnetic field along each radial part of the conducting disk compared to the corresponding values of the analytical relations of Section 3. These deviations cause differences in the induced electromotive forces of the four radial parts of the conducting disc. This system of four electromotive forces is "electrically equivalent" to an electric circuit of four electric sources of DC voltage, as shown in Figure 6, with electromotive forces E_1, E_2, E_3, E_4, with internal resistance r in each DC source, with all positive poles conductively connected to ensure equality of electric potential across the peripheral conductive connection ABCDA, and with the negative poles connected at point O. The potential difference between the common connection point O and the peripheral conductor ABCDA, is in this case equal to the average value of the individual electromotive forces, that is, it is expressed by the following relation.

$$V = \frac{1}{4}\left(E_1 + E_2 + E_3 + E_4\right) \tag{13}$$

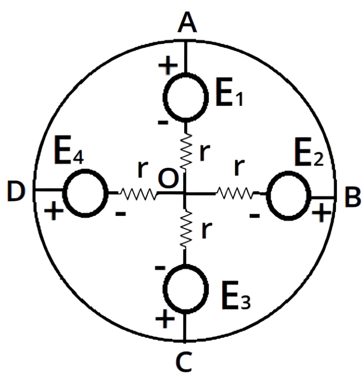


Figure 6. Circuit "electrically equivalent" to that of the conducting disk in the case where the conducting disk and ring magnet rotate together.

Therefore, in order to calculate the total induced Faraday EMF in the conducting disc, the average value of the induced EMFs of its four radial parts should be calculated. The analytical calculation of the EMF according to the relations (12), (3) does not require these extra calculations and the result is 22 mV, as mentioned in Section 4.2.1.

The calculation of the induced EMF in each of the radial parts of the conducting disc has been done following the procedure of Section 4.2.1, *i.e.* by measuring the component of the magnetic field which is parallel to the Z axis, along each of the four radial parts, but making five measurements at each position, per 5 mm, in order to reduce as much as possible the magnetic field measurement error due to poor placement of the measuring instrument at each position.

For convenience, we use the absolute values of the magnetic field measurements. We denote by $B_{zn_{ij}}$ the measured values of the z-component of the magnetic field. The index n expresses the serial number of the radial part, so $n = 1,2,3,4$. The index j expresses the serial number of the measurement at a certain radial distance, *i.e.* $j = 1,2,\cdots,M$, where $M = 5$, and the index i is the index of radial distance r, so $i = 1,2,\cdots,N$, where $N = 10$. As value $\overline{B_{zn_i}}$ we define the mean value of the z-component of magnetic field, of the radial part n, at the radial distance r_i, as follows

$$\overline{B_{zn_i}} = \frac{1}{M}\sum_{j=1}^{M} B_{zn_{ij}}$$

and the error of mean value is expressed as follows

$$\delta\overline{B_{zn_i}} = \frac{S_{n_i}}{\sqrt{M}}$$

where the term S_{n_i} is expressed as

$$S_{n_i} = \sqrt{\frac{1}{M-1}\sum_{j=1}^{M}\left|B_{zn_{ij}} - \overline{B_{zn_i}}\right|^2}$$

and it is the standard deviation. The induced EMF in the radial part n is calculated according to the relation (12), making a numerical integration as follows

$$\mathrm{E}_n = 2\pi f\left[\frac{1}{2}\sum_{i=1}^{N-1}\left(r_{i+1} - r_i\right)\left(r_i\overline{B_{zn_i}} + r_{i+1}\overline{B_{zn_{i+1}}}\right)\right]$$

where f is the rotation frequency. Therefore, including a radial distance estimation error equal to δr_i results in an error propagation in the numerical integration, determined by the relation

$$\delta\mathrm{E}_n = \sqrt{\sum_{i=1}^{N}\left[\left(\frac{\partial \mathrm{E}_n}{\partial\overline{B_{zn_i}}}\delta\overline{B_{zn_i}}\right)^2 + \left(\frac{\partial \mathrm{E}_n}{\partial r_i}\delta r_i\right)^2\right]}$$

where

$$\begin{aligned}\sum_{i=1}^{N}\left(\frac{\partial \mathrm{E}_n}{\partial\overline{B_{zn_i}}}\delta\overline{B_{zn_i}}\right)^2 = \left(2\pi f\right)^2\Bigg[& r_1^2\left(r_2 - r_1\right)^2\left(\delta\overline{B_{zn_1}}\right)^2 \\ &+ \sum_{i=2}^{N-1} r_i^2\left(r_{i+1} - r_{i-1}\right)^2\left(\delta\overline{B_{zn_i}}\right)^2 \\ &+ r_N^2\left(r_N - r_{N-1}\right)^2\left(\delta\overline{B_{zn_N}}\right)^2\Bigg]\end{aligned}$$

and

$$\sum_{i=1}^{N}\left(\frac{\partial \mathrm{E}_n}{\partial r_i}\delta r_i\right)^2=(2\pi f)^2\Bigg[\left[(r_2-2r_1)\overline{B_{zn_1}}-r_2\overline{B_{zn_2}}\right]^2(\delta r_1)^2$$
$$+\sum_{i=2}^{N-1}\left[r_{i-1}\left(\overline{B_{zn_{i-1}}}-\overline{B_{zn_i}}\right)+r_{i+1}\left(\overline{B_{zn_i}}-\overline{B_{zn_{i+1}}}\right)\right]^2(\delta r_i)^2$$
$$+\left[r_{N-1}\overline{B_{zn_{N-1}}}+(2r_N-r_{N-1})\overline{B_{zn_N}}\right]^2(\delta r_N)^2\Bigg]$$

The error propagation in estimating the potential difference given by the relation (13) is

$$\delta V=\frac{1}{4}\sqrt{\sum_{n=1}^{4}(\delta \mathrm{E}_n)^2} \tag{14}$$

From the obtained measurements of the z-component of the magnetic field in the four radial parts of the conducting disk, using MATLAB, the mean values at each radial distance and the corresponding errors were calculated, which were entered in Table 1 and Table 2, and also were calculated the errors $\delta \mathrm{E}_n$ for $n =$ 1, 2, 3, 4, and δV, considering an estimation error of the radial distance $\delta r_i = 0.5\ \mathrm{mm}$ for all measurements. The calculated value of the potential difference between the two brushes is

$$V = 20.0 \pm 0.4\ \mathrm{mV} \tag{15}$$

Table 1. Radial distances and corresponding mean values of the z-component of magnetic field along the four radial parts of the conducting disc.

i	r_i (cm)	$\overline{B_{z1_i}}$ (T)	$\overline{B_{z2_i}}$ (T)	$\overline{B_{z3_i}}$ (T)	$\overline{B_{z4_i}}$ (T)
1	4.0	0.007000	0.006200	0.006000	0.007600
2	4.5	0.027800	0.022400	0.026400	0.029600
3	5.0	0.042600	0.036600	0.039400	0.043400
4	5.5	0.048400	0.041400	0.043000	0.049000
5	6.0	0.051000	0.043000	0.044000	0.051800
6	6.5	0.054000	0.045000	0.046400	0.054200
7	7.0	0.057000	0.048000	0.049000	0.058200
8	7.5	0.060400	0.051200	0.053200	0.061800
9	8.0	0.062000	0.055200	0.056600	0.063800
10	8.35	0.059200	0.056000	0.057000	0.062400

Table 2. Radial distances and corresponding errors of mean values of the z-component of magnetic field along the four radial parts of the conducting disc.

i	r_i (cm)	$\delta\overline{B_{z1_i}}$ (T)	$\delta\overline{B_{z2_i}}$ (T)	$\delta\overline{B_{z3_i}}$ (T)	$\delta\overline{B_{z4_i}}$ (T)
	4.0	0.000447	0.000200	0.000000	0.000245
	4.5	0.000490	0.000400	0.000245	0.000245
	5.0	0.000400	0.000245	0.000245	0.000400

Continued

5.5	0.000245	0.000245	0.000000	0.000000
6.0	0.000000	0.000000	0.000000	0.000200
6.5	0.000000	0.000000	0.000245	0.000200
7.0	0.000000	0.000000	0.000316	0.000200
7.5	0.000245	0.000200	0.000200	0.000200
8.0	0.000000	0.000200	0.000245	0.000374
8.35	0.000200	0.000000	0.000000	0.000245

The difference between the value of V given by the relation (15) and the corresponding value of 22 mV calculated by the analytical procedure is due to the imperfections of the ring magnet, as extensively mentioned in Section 4.2.1.

4.2.3. Calculation of the Relativistic Electromotive Force along the External Part of the Electric Circuit

The assumption of the existence of an electromotive force along the external part of the electric circuit is a purely relativistic assumption, and as we have already mentioned it contradicts Faraday's view. Therefore the calculation of this electromotive force is done following the corresponding relativistic procedure.

As shown in **Figure 7** the external part of the electrical circuit consists of the line segments AB and CD which are of equal length, connected to the brushes and are parallel to the Z axis, and the radial straight line segments OA and OD whose ends, very close to the point O of the axis of rotation, are connected by a bipolar cable. The other end of the bipolar cable is connected to the instrument for measuring the potential difference.

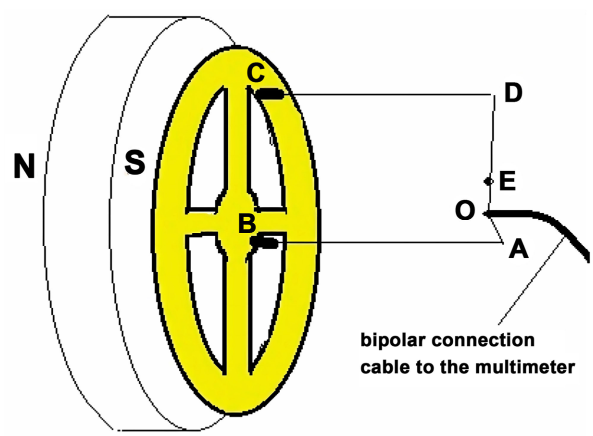


Figure 7. Drawing of a part of the experimental setup, including the magnet, the conducting disc up to its surface, the brushes, copper wires and bipolar cable that make up the external part of the electrical circuit. The plastic parts and springs have been removed from the drawing in order to show the geometry of the copper wires of the external part of the electrical circuit.

The total induced electromotive force along the bipolar wire is zero, because each differential line segment of it consists of a differential line segment of the input current conductor and a differential line segment of the output current conductor, and the Lorentz vector forces, which are exerted on the conduction electrons of these two differential parts, are equal. Therefore, the corresponding differential electromotive forces cancel each other.

The external part of the electric circuit, which is stationary with respect to the inertial reference frame of the laboratory, and is viewed by an observer which is stationary with respect to the magnet, is rotating with respect to him with an angular velocity given by the following equality.

$$\vec{\omega}' = \omega \hat{u}_z$$

that is $\vec{\omega}' = -\vec{\omega}$, because the vector angular velocity $\vec{\omega}'$ is opposite to that observed as vector angular velocity of rotation of disk-magnet system in the lab. Therefore, a differential line segment of the external part of the electric circuit moves with respect to an observer of the rotating reference system S', *i.e.* with respect to an observer of the reference system in which the magnet is stationary, with a velocity

$$\vec{v}' = \vec{\omega}' \times \vec{r} = \omega r \hat{u}_\theta$$

Since the electric field $\vec{E}'$ in the reference frame S' of the rotating magnet is zero, the Lorentz force acting on a conduction electron located in the aforementioned differential line segment is given by the relation

$$\vec{F}' = e\vec{v}' \times \vec{B}' = e\omega r \hat{u}_\theta \times \vec{B}'$$

In the inertial frame of reference S of the laboratory, according to the relations (7) and (8) we take the following equalities.

$$\vec{E}_{\parallel} = 0$$

$$\vec{E}_{\perp} = -\vec{u} \times \vec{B}'$$

$$\vec{B} = \vec{B}'$$

and since the velocity $\vec{v}$ of the aforementioned differential line segment in S is zero, the Lorentz force exerted to the conduction electron is given by the equality

$$\begin{aligned} \vec{F} &= e\vec{E} \\ &= -e\vec{u} \times \vec{B} \\ &= -e\left(\vec{\omega} \times \vec{r}\right) \times \vec{B} \\ &= e\omega r \hat{u}_\theta \times \vec{B} \end{aligned}$$

Therefore, the Lorentz force has the same value in the reference systems S' and S as expected. Based on the analytical relations for determining the components of the magnetic field in Section 3, the Lorentz force is given by the relation

$$\begin{aligned} \vec{F} &= e\omega r \hat{u}_\theta \times \left(\vec{B}_r + \vec{B}_z\right) \\ &= e\omega r\left(-B_r \hat{u}_z + B_z \hat{u}_r\right) \end{aligned} \tag{16}$$

The term $-e\omega r B_r \hat{u}_z$ of the Lorentz force in equality (16) is the cause of the induction of electromotive force along the parts of the external circuit which are parallel to the axis Z, while the term $e\omega r B_z \hat{u}_r$ is the cause of the induction of electromotive force along the radial parts.

In the line segment AB of the electric circuit, located at a distance from the axis of rotation equal to $r_{01} = 4\text{ cm}$, the component of the Lorentz force which is parallel to the direction of the Z axis is expressed by the following equation.

$$F_{01}(z)\hat{u}_z = -e\omega r_{01} B_r(r_{01}, z)\hat{u}_z \tag{17}$$

The corresponding component of the Lorentz force for the line segment CD of the electric circuit, located at a distance from the axis of rotation equal to $r_{02} = 8.35\text{ cm}$, is given by the equation

$$F_{02}(z)\hat{u}_z = -e\omega r_{02} B_r(r_{02}, z)\hat{u}_z \tag{18}$$

The component of the Lorentz force which is the cause of the induction of the electromotive force along the radial straight line segments OA and OD, which lie on the flat surface defined by the coordinate $z = z_{02} = 9.45\text{ cm}$, is given by the equation

$$F(r)\hat{u}_r = e\omega r B_z(r, z_{02})\hat{u}_r \tag{19}$$

Due to the existing cylindrical symmetry, with small deviations in the magnetic field of the ring magnet, if we define a radial straight line segment OE equal to OA, the calculated EMFs along these two line segments practically cancel each other. so as a total EMF along the line segments OA and OD only the EMF induced in line segment ED is obtained.

In order to determine the directions of the electric currents, in the conventional sense of the definition of the electric current, which tends to be created by the induced electromotive forces along the considered line segments of the external part of the electric circuit, we will graphically represent in principle the analytical functions $B_r(r_{01}, z)$, $B_r(r_{02}, z)$ and $B_z(r, z_{02})$ and the corresponding experimental points obtained from the magnetic field measurements. The measurements of the radial components of the magnetic field that have been done along the line segments AB and CD, and also the measurements of the components of the magnetic field which are parallel to the Z axis, and have been done along the radial straight line segment ED, are listed in **Tables 3-5**. The corresponding plots are in **Figures 8-10**.

Table 3. Coordinates z and corresponding measured values of the radial component of the magnetic field at a distance r_{01} = 4 cm from the axis of rotation, along the straight line segment AB.

z (cm)	B_r (mT)
2.95	38.0
4.45	9.0
5.45	3.0
6.45	1.0
7.45	0.5
8.45	0.1
9.45	−0.6

Table 4. Coordinates z and corresponding measured values of the radial component of the magnetic field at a distance r_{02} = 8.35 cm from the axis of rotation, along the straight line segment CD.

z (cm)	B_r (mT)
2.95	−28
4.45	−17
5.45	−12
6.45	−9
7.45	−7
8.45	−6
9.45	−5

Table 5. Radial distances and corresponding measured values of the z-component of the magnetic field, along the radial straight line segment ED, located on the flat surface defined by the coordinate $z = z_{02}$ = 9.45 cm.

r (cm)	B_z (mT)
4.00	−8.5
5.00	−8.8
6.00	−8.6
7.00	−8.0
8.00	−7.5
8.35	−7.2

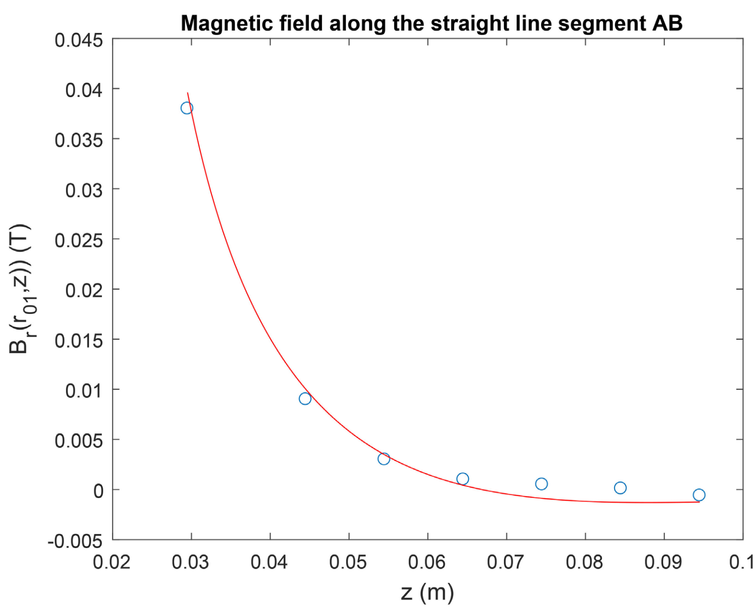


Figure 8. Values of the radial component of the magnetic field along the line segment AB of the external part of the electric circuit. The continuous curve comes from the analytical relations of the magnetic field, while the circular points come from the measurements of the radial component of the magnetic field along the line segment AB.

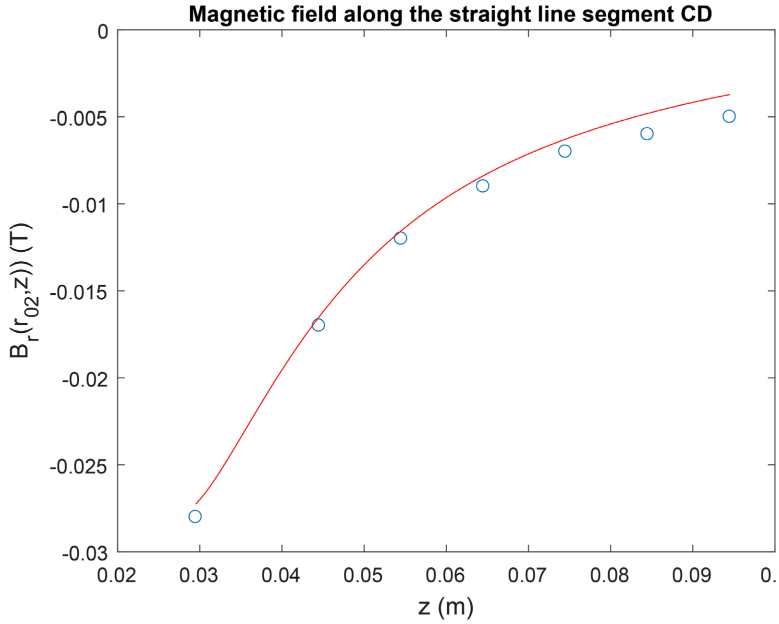


Figure 9. Values of the radial component of the magnetic field along the line segment CD of the external part of the electric circuit. The continuous curve comes from the analytical relations of the magnetic field, while the circular points come from the measurements of the radial component of the magnetic field along the line segment CD.

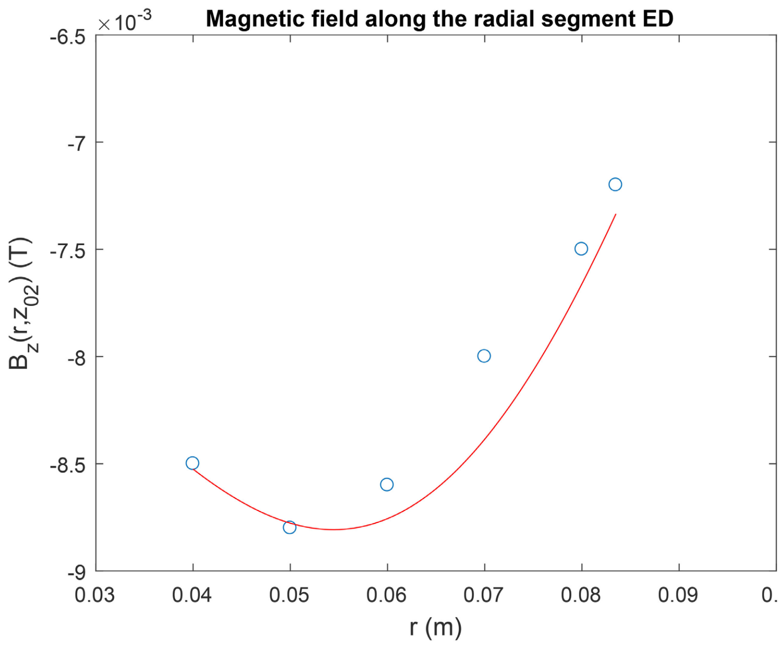


Figure 10. Values of the z-component of the magnetic field along the radial straight line segment ED of the external part of the electric circuit. The continuous curve comes from the analytical relations of the magnetic field, while the circular points come from the measurements of the z-component of the magnetic field along the radial straight line segment ED.

From Table 3 and Figure 8, it is found that along the straight line segment AB the values of the radial component B_r of the magnetic field, which give the most part of the induced electromotive force, are positive, so according to the relation (17) the Lorentz force exerted on the conduction electrons has mostly positive values and its direction is from point B to point A, excluding a small region of the line segment AB, whose contribution to the generation of the induced electromotive force is very small. Therefore, the conventional direction of the electric current that tends to be created is from point A to point B. We find out that this direction of the electric current agrees with the direction of the electric current generated according to Faraday's view due to the induced EMFs along the radial parts of the conducting disk.

From **Table 4** and **Figure 9**, it is found that along the straight line segment CD, the values of the radial component B_r of the magnetic field are negative, so according to the relation (18) the Lorentz force exerted on the conduction electrons has negative values and its direction is from point D to point C. Therefore, the conventional direction of the electric current that tends to be created is from point C to point D. We find out that this direction of the electric current agrees with the direction of the electric current generated according to Faraday's view.

From **Table 5** and **Figure 10**, it is found that along the radial straight line segment ED the values of the B_z component of the magnetic field are negative, so according to the relation (19) the Lorentz force exerted on the conduction electrons has positive values and its direction is from point E to point D. Therefore, the conventional direction of the electric current that tends to be created is from point D to point E. We again find out that this direction of the electric current agrees with the direction of the electric current produced according to Faraday's view.

Therefore, the absolute value of the induced relativistic EMF in the entire external part of the electric circuit will be equal to the sum of the absolute values of the induced EMFs along its individual parts, that is, according to the relations (17), (18) and (19), is given by the relation

$$\begin{aligned} V_{ext} &= \left|\frac{1}{e}\int_{z_{01}}^{z_{02}} F_{01}(z)\,\mathrm{d}z\right| + \left|\frac{1}{e}\int_{z_{01}}^{z_{02}} F_{02}(z)\,\mathrm{d}z\right| + \left|\frac{1}{e}\int_{r_{01}}^{r_{02}} F(r)\,\mathrm{d}r\right| \\ &= \left|-\omega r_{01}\int_{z_{01}}^{z_{02}} B_r(r_{01},z)\,\mathrm{d}z\right| + \left|-\omega r_{02}\int_{z_{01}}^{z_{02}} B_r(r_{02},z)\,\mathrm{d}z\right| + \left|\omega\int_{r_{01}}^{r_{02}} rB_z(r,z_{02})\,\mathrm{d}r\right| \end{aligned} \tag{20}$$

The absolute value of the numerical integration of the induced relativistic electromotive force on the entire external part of the electrical circuit was calculated by numerical integration using the mathematica program and MATLAB. The absolute value V_{ext} of the electromotive force obtained from the analytical expressions (1) and (3) is 15.4 mV, while the value obtained from the magnetic field measurements, making use of MATLAB and entering interpolated values at specific query points using "spline" as interpolation method, is 16 mV.

4.3. Third Case: The Magnet Rotates and the Conducting Disk Is Stationary with Respect to the Laboratory

In the case in which the conducting disk observed in the laboratory is stationary while the magnet rotates, according to Faraday's view is not induced electromotive force either in the conducting disk or in the external part of the electric circuit which is also stationary with respect to the laboratory. However, according to the theory of relativity, is induced electromotive force in the conducting disc and also in the external part of the electric circuit. The induced EMF in the external part of the electric circuit has already been calculated in Section 4.2.3. The value from the analytical calculation is 15.4 mV and the value from the magnetic field measurements is 16 mV.

In this case, the conducting disk rotates with respect to an observer of the frame of reference of the magnet, *i.e.* with respect to an observer of the rotating reference system S', with an angular velocity given by the equation

$$\vec{\omega}' = \omega \hat{u}_z$$

that is, $\vec{\omega}' = -\vec{\omega}$, as in Section 4.2.3. Therefore, the Lorentz force exerted on a conduction electron, located on a differential surface of the conducting disc, is given by the relation 16 and its radial component causing an electromotive force in the radial direction is given by the equation

$$\vec{F}_r = e\omega r B_z \hat{u}_r \tag{21}$$

which, however, is opposite to that given by the relation (11) of the first case, which is also applied to the second case, according to Faraday's point of view, for the calculation of the electromotive force induced in the rotating, with respect to the lab reference system, conducting disk. This means that, in the case considered in this section, the EMF induced in the radial parts of the conducting disc opposes the EMF induced in the external part of the circuit. Therefore, according to the relativistic calculations, the value obtained by the voltage measuring instrument should be equal to the difference between these two EMFs, with an opposite sign to that obtained by the same instrument in the first and second case, because the EMF induced in the conducting disc is in absolute value the highest. Specifically, according to the analytical calculation, the electric voltage is equal to (15.4 − 22) mV = −6.6 mV, while according to the calculation derived from the measurements of the magnetic field, the electric voltage is equal to (16 − 20) mV = −4 mV.

5. Measurements of the Induced Electromotive Force

The instrument for measuring the electrical voltage, which is connected at the other end of the bipolar cable of **Figure 7**, is a KEITHLEY 2000 multimeter. The rotation frequency of the motor shaft was measured with an MP5W-4N tachometer in combination with a SICK IME18 inductive sensor with characteristics: OUT: PNP/NO; 0 - 8 mm; 10 - 30 VDC; M18; IP67; 200 mA, and the measured value is 1490 rpm. The multimeter is connected through the serial port to the computer, so its operation management have been done by the computer, with a program written in the QBASIC programming language. In each run of the program 1024 values of the electrical voltage are recorded successively in the buffer of the instrument, which after the recording process are saved in a file on the hard disk of the computer. The first 980 consecutive values of the total 1024 recorded values are immediately usable in the first case because they correspond to exactly 11 rotation periods. Taking this data over an integer number of rotation periods is particularly useful for studying the voltage waveform during data analysis, as we will see next.

5.1. Measurement of the Induced EMF in the First Case

In the first case, in which the conducting disk is rotating and the magnet is stationary in the laboratory, an example waveform obtained is shown in Figure 11. The waveform shown in Figure 11 is due to two reasons. The dominant cause is the induced electromotive force, while the secondary is an additional signal due to the vibrations arising from the rotation. Vibrations are oscillations due to the rotation that result in induced sinusoidal voltages in the conducting disk, which appear in the diagram of Figure 11 as a signal consisting of the sum of all these sinusoidal voltages. In order to study the amplitudes of these sinusoidal voltages as a function of frequency we perform a Fast Fourier Transform (FFT) analysis of the waveform using the MATLAB, the result of which is shown in the plot of Figure 12. In this FFT analysis we get, as shown in Figure 12, the main frequency of 24.83 Hz, which is the rotation frequency of 1490 rpm, with an amplitude of 1.35 mV, as well as all the frequencies that are multiples of the main frequency with the corresponding amplitudes.

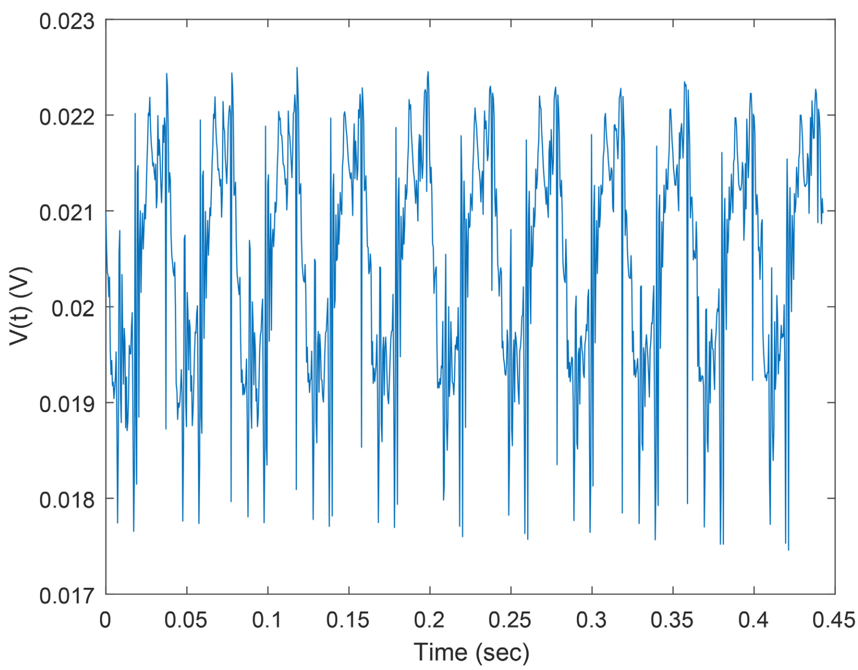


Figure 11. A waveform of the measured voltage $V(t)$ in the case where the magnet is stationary in the lab while the conducting disk is rotating.

Since the waveform of Figure 11 is obtained by an integer number of rotation periods, the calculation of the average value of 980 voltage values eliminates the contribution of the vibration of the main frequency as well as the contribution of its multiples.

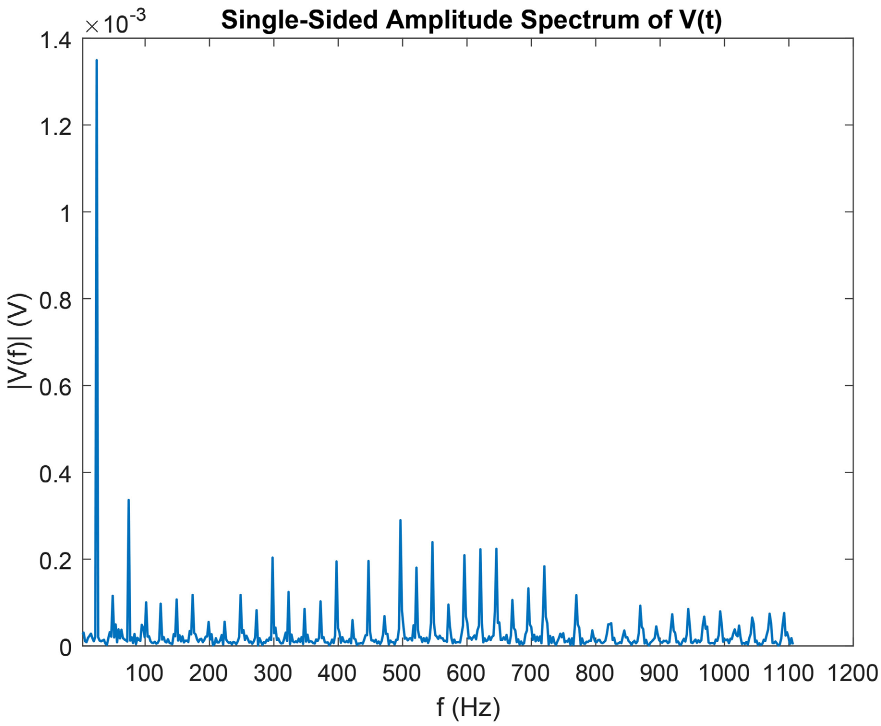


Figure 12. Plot derived from the FFT analysis of the $V(t)$ waveform in the case where the magnet is stationary in the laboratory while the conducting disk is rotating.

But in addition to the main frequency and its multiples, during the FFT analysis, a noise signal is also obtained, with a fluctuating amplitude of approximately 0.02 mV and frequencies that span the entire range of the obtained frequency spectrum.

In order to limit as much as possible, the introduced error due to noise, we consider the mean value of the 980 voltage values obtained in each run of the program as the value of one measurement and make five consecutive runs of the program. We then calculate the average value of these five measurements and the corresponding error of the average value, using MATLAB. The result is

$$V = 20.44 \pm 0.01\,\text{mV} \tag{22}$$

5.2. Measurement of the Induced EMF in the Second Case

In the second case, in which the conducting disk and the magnet rotate clockwise at the same angular velocity, observed from the external circuit side in the laboratory, an example waveform obtained is shown in Figure 13.

The waveform is different in this case because the ring magnet has been added to the rotating system, so the amplitudes corresponding to the frequency spectrum of the FFT analysis are different. The result of the FFT analysis for the waveform of Figure 13 is depicted in the plot of Figure 14.

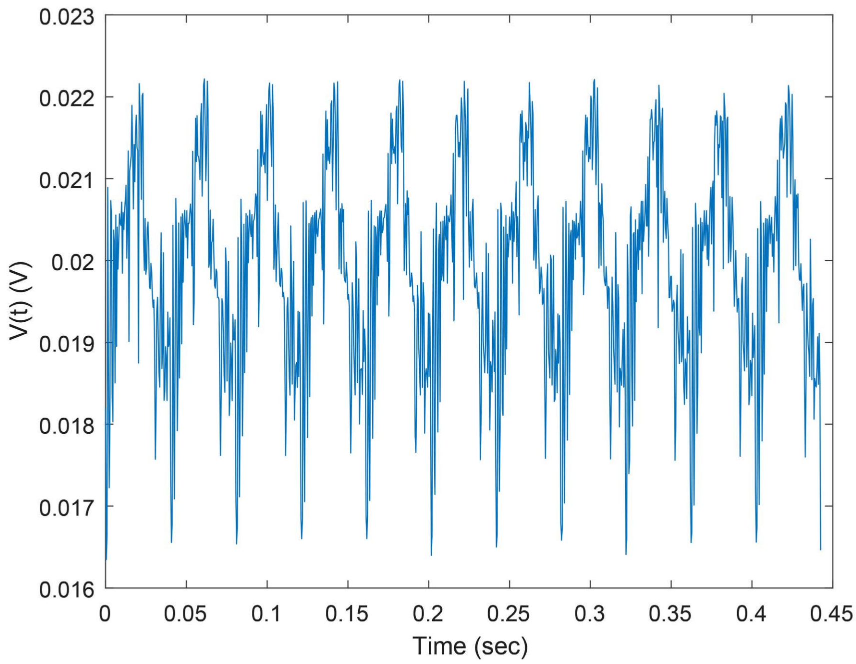


Figure 13. A waveform of the measured voltage $V(t)$ in the case where the magnet and the conducting disk rotate together.

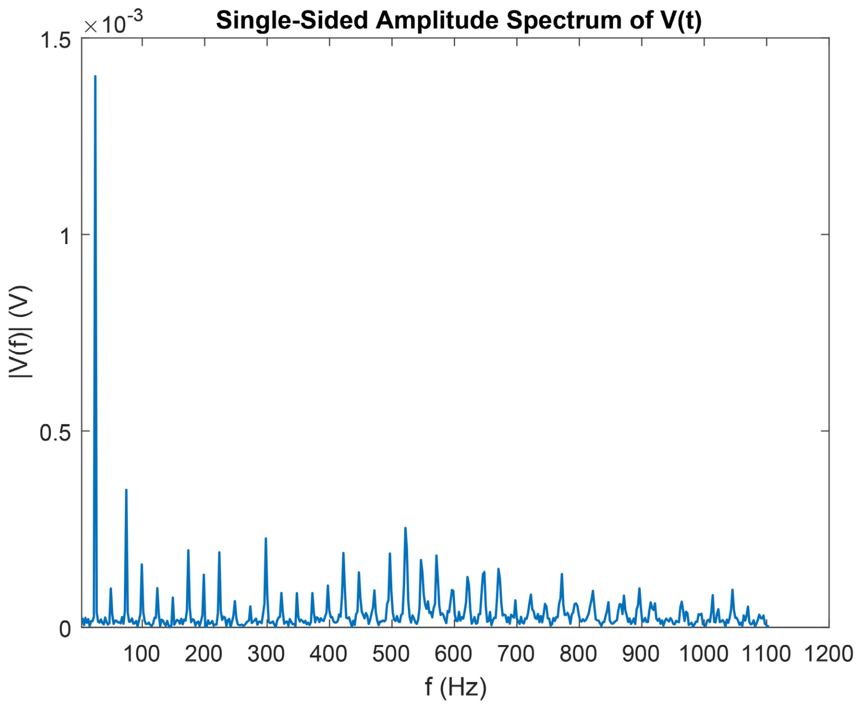


Figure 14. Plot derived from the FFT analysis of the $V(t)$ waveform in the case where the magnet and the conducting disk rotate together.

Following the same procedure as that of Section 5.1, for five consecutive runs of the aforementioned QBASIC program, we obtain a final result of measuring the induced EMF in this case which is

$$V = 19.86 \pm 0.02 \text{ mV} \tag{23}$$

This result agrees with that of the corresponding calculation of Section 4.2.2 given by the equality (15).

The difference between the results of the equalities (22) and (23), if examined from Faraday's point of view, is due to the fact that the first and the second cases are not completely equivalent when the magnetic field of the ring magnet used is not absolutely cylindrically symmetric. In the experimental setup of the present study, as we have already noticed, there are small deviations in the measured values of the magnetic field from those derived from the analytical calculations. Specifically, while in the second case, the radial parts of the conducting disk are constantly opposite the same positions of the surface of the south pole of the magnet during the rotation and are continuously traversed by the same dynamic lines of the magnetic field, in the first case the radial parts of conducting disk constantly change positions with respect to the surface of the magnet during the rotation. Therefore, in the first case the calculated electromotive force, the instant value of which can be obtained following the procedure of Section 4.2.2, is time-dependent, but with small deviations from the average value during the rotation.

5.3. Measurement of the Induced EMF in the Third Case

In the third case, in which the conducting disk is stationary with respect to the laboratory while the magnet is rotating, one of the obtained waveforms is shown in Figure 15.

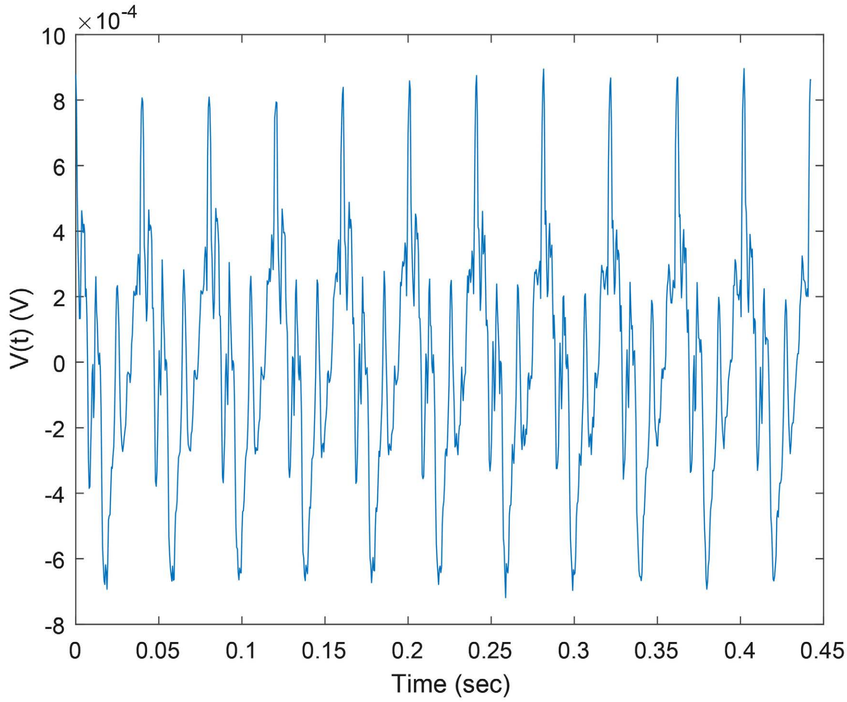


Figure 15. A waveform of the measured voltage $V(t)$ in the case where the conducting disk is stationary with respect to the lab, while the magnet is rotating.

The result of the FFT analysis for the waveform of **Figure 15** is depicted in the plot of **Figure 16**. Only the main frequency and some neighboring frequencies seem to have appreciable amplitude and this is due to the lack of participation of the conducting disk in the rotational motion.

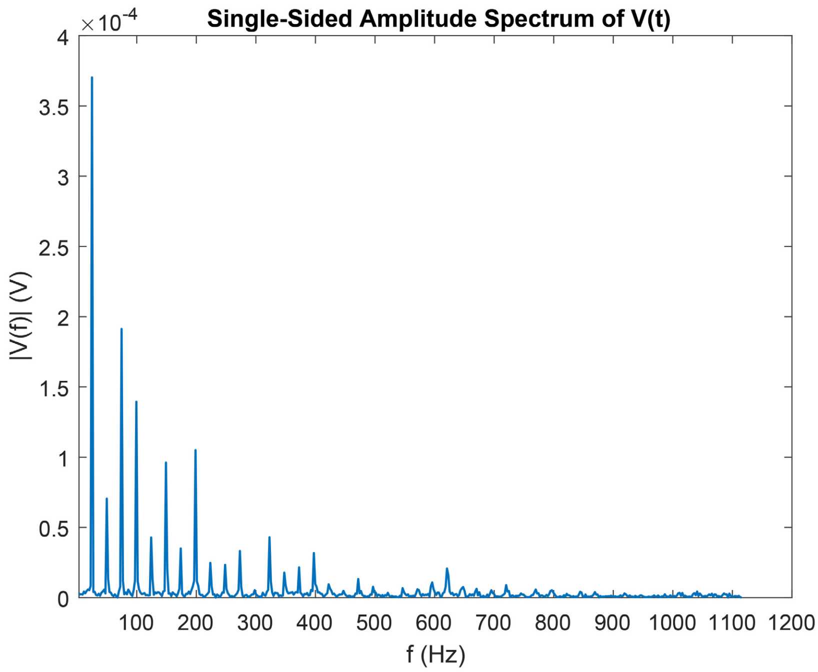


Figure 16. Plot derived from the FFT analysis of the $V(t)$ waveform in the case where the conducting disk is stationary with respect to the lab while the magnet is rotating.

Following the same process of calculating the induced EMF for each run of the program, as followed in Sections 5.1 and 5.2, the average values of EMF in five consecutive runs are as follows.

$$V_1 = -6.8210\times10^{-6}\text{ V}$$

$$V_2 = -1.8823\times10^{-5}\text{ V}$$

$$V_3 = -3.1928\times10^{-5}\text{ V}$$

$$V_4 = -2.8396\times10^{-5}\text{ V}$$

$$V_5 = -3.3790\times10^{-5}\text{ V}$$

These five values are of the order of magnitude of the error, so they are considered practically zero.

6. Summary Tables of Results

As already mentioned at the end of Section 4.1, the first case, *i.e.* the case of rotation of the conducting disk and immobility of the magnet in the reference frame of the laboratory, does not lend itself to examining the correctness of the theory of relativity or Faraday's view. Therefore, in order to examine the correctness of these two different considerations, based on the irrefutable reality of the experimental results derived from the measurements of the induced electromotive forces and the components of the magnetic field, we will summarize the calculations and the corresponding results of the measurements in the second and third case. **Table 6** is the consolidated table of results, which includes the results of calculations based on relativity and Faraday's view and measurements of induced electromotive forces, in the second case where the magnet and the conducting disk rotate together, and in the third case in which the conducting disk is stationary in the laboratory and the magnet is rotating.

Table 6. Summary table of results obtained from the data analysis of the experiment.

Cases	Relativity	Faraday's view	Measurement result
Second case	16 mV	20.0 ± 0.4 mV	19.86 ± 0.02 mV
Third case	−4 mV	zero	practically zero

In the experiment of the present study, and in all the experiments that are recorded in the international literature and have as a research objective the Faraday's unipolar generator, some of which are described in [2]-[4] [15], two common basic findings are stated, which are the following:

- The measured induced electromotive forces in the first and second cases are equal.
- The measured electromotive force in the third case is zero.

We can therefore say, with absolute certainty, that an experimental setup of a unipolar generator, in which a ring magnet with the same geometrical characteristics as the magnet of the experimental setup of the present study is used, but the magnetic field approximates in a fairly satisfactory degree the field of the relations 1 and 3 for $\sigma^* = 0.220\ \text{T}$, will give as measured value of induced EMF in the first and second cases the value resulting from the analytical calculation, which is equal to 22 mV, and zero measured value in the third case. **Table 7** is the summary table of results obtained from the analytical calculations and measurements obtained from such an experimental setup.

Table 7. Summary table of results obtained from a cylindrically symmetric magnetic field, originating from a ring magnet with $\sigma^* = 0.220\ \text{T}$ and geometric characteristics: 190 mm outer diameter, 85 mm inner diameter, and 23 mm thick.

Cases	Relativity	Faraday's view	Measurement result
Second case	15.4 mV	22 mV	22 mV
Third case	−6.6 mV	zero	zero

7. Discussion

A. Einstein in his paper entitled "On the Electrodynamics of Moving Bodies"[6], in

[6]This edition of Einstein's *On the Electrodynamics of Moving Bodies* is based on the English translation of his original 1905 German-language paper (published as Zur Elektrodynamik bewegter Körper, in Annalen der Physik. 17:891, 1905). Available: https://www.fourmilab.ch/etexts/einstein/specrel/specrel.pdf

II. ELECTRODYNAMICAL PART, Section 6. Transformation of the Maxwell-Hertz Equations for Empty Space. On the Nature of the Electromotive Forces Occurring in a Magnetic Field During Motion, he concludes with the phrase:

> "Furthermore it is clear that the asymmetry mentioned in the introduction as arising when we consider the currents produced by the relative motion of a magnet and a conductor, now disappears. Moreover, questions as to the 'seat' of electrodynamic electromotive forces (unipolar machines) now have no point."

This phrase alone is enough to show the lack of deeper understanding of the phenomenon we are examining in the present study, during the period of the early twentieth century. Because no attempt was made to interpret the phenomenon, not even a reference to the experiments carried out up to that time. If there was a scientific discussion, especially on the topic of the unipolar generator, then the non-relativistic character of this phenomenon would be seen.

Unfortunately, to this day, the prevailing view regarding the subjects we are discussing is that of classical relativistic electrodynamics, which however fails to interpret fundamental phenomena, such as that of the unipolar inductor.

In the present work, a new way of examining the unipolar generator effect is highlighted. This new way of examining the phenomenon is based on the analytical calculation of the components of the magnetic field in such a way as to confirm the corresponding measurements of the magnetic field. Even if no experiment is performed, one can consider a hypothetical experiment and based on analytical calculations determine the expected results according to Faraday's view, or according to the theory of relativity. Then using a simple experimental setup, like the one in the present study, he may establish which theoretical point of view agrees with the experimental results and which does not. He will find, for example, that in the case in which the conducting disk and the magnet rotate together, the resulting potential difference is greater than the relativistically expected, while it agrees with the potential difference which is expected according to Faraday's view. He will also find that when only the magnet rotates, while the conducting disk and the rest of the electrical circuit are at rest in the laboratory, the resulting potential difference is zero, which completely confirms Faraday's view and does not agree with the relativistically expected result, which is a non-zero potential difference.

8. Conclusions

According to the results presented in the summary tables of Section 6, it becomes clear that the phenomenon of Faraday's unipolar generator, based on the measurements, is in full agreement with the Faraday view, already described in the present study, while contradicting the predictions of the theory of relativity.

But this fact does not just affect an aspect of the theory of relativity, but the very basis of the relativistic electromagnetic theory. The physical phenomena of electromagnetism, as they had been studied with Faraday's experiments, cannot be interpreted on the basis of the theory of relativity when sources of electromagnetic fields in rotating frames of reference are involved, since in this case the relativistic calculations of electromagnetic fields and the Lorentz forces cannot give a result that is consistent with the physical reality of the measurements of these experiments.

Because of the great seriousness of this matter, a fundamental reformulation of classical electrodynamics is needed. Therefore, this must be the subject of an open scientific dialogue, among all members of the scientific community, *i.e.* among all researchers in the field of natural sciences and especially those who deal with issues of classical electrodynamics.

It is easy to carry out an experiment of unipolar induction in any physics laboratory at any university. This ease in combination with the proposed in the present study new procedure for examining the unipolar generator effect may contribute to the successful outcome of such a dialogue.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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