

Global Dynamics in a Predator-Prey Model with Allee Effect

Tianjing Wang, Fuqin Sun

School of Sciences, Tianjin University of Technology and Education, Tianjin, China

Email: 598474669@qq.com

How to cite this paper: Wang, T.J. and Sun, F.Q. (2024) Global Dynamics in a Predator-Prey Model with Allee Effect. *Journal of Applied Mathematics and Physics*, 12, 2377-2385.

<https://doi.org/10.4236/jamp.2024.127142>

Received: June 7, 2024

Accepted: July 13, 2024

Published: July 16, 2024

Copyright © 2024 by author(s) and

Scientific Research Publishing Inc.

This work is licensed under the Creative

Commons Attribution International

License (CC BY 4.0).

<http://creativecommons.org/licenses/by/4.0/>



Open Access

Abstract

Since the last century, various predator-prey systems have garnered widespread attention. In particular, the predator-prey systems have sparked significant interest among applied mathematicians and ecologists. From the perspectives of both mathematics and biology, a predator-prey system with the Allee effect and featuring the Bazykin functional response has been established. For this model, analyses have been conducted on its boundedness, the properties of its solutions, the existence of equilibrium points, as well as its local stability and Hopf bifurcations.

Keywords

Bazykin, Allee Effect, Hopf Bifurcation

1. Model Construction

Predator-prey systems are almost ubiquitous in the real world. In recent years, the dynamic analysis of predator-prey models has attracted extensive research from many scholars as an important topic. In 1932, Allee, through experimental studies on the growth of goldfish populations, found that a higher density of goldfish was beneficial for the growth of the goldfish population. He proposed that when the population density is reduced to a certain level, it will be maintained at a very low level and tend towards extinction, which is known as the Allee effect [1] [2]. Subsequently, Stephens and others believed that the Allee effect is a positive correlation between the population's own fitness and its own number [3]. In recent years, many scholars have also conducted further extensive research on the Allee effect. They found that cooperative hunting can enhance the persistence of prey and can cause a strong Allee effect [4]-[6]. If $x(t)$ is used to represent the number of the prey population and $y(t)$ represents the number of the predator population, then the growth model of the prey with the Allee ef-

fect can be represented as.

$$\frac{dx}{dt} = rx \left(1 - \frac{x}{k} \right) (x - v)$$

r represents the natural growth rate of the prey, k represents the environmental carrying capacity, and v is the threshold overall level that represents the Allee effect, with $0 < v < 1$. Scholars have proposed various functional responses, among which Holling types I, II, and III have been widely discussed [7]-[9].

Holling I:

$$f(x) = \begin{cases} \frac{b}{a}x, & 0 < x < a \\ b, & x > a \end{cases}$$

Holling II:

$$f(x) = \frac{\alpha x}{\gamma + x}$$

Holling III:

$$f(x) = \frac{\alpha x^2}{\gamma + x^2}$$

In recent years, many scholars have conducted in-depth research on other models, but there have been relatively few studies on diffusive predator-prey models with Bazykin functional responses. The Bazykin functional response function is referred to as

$$f(x, p) = 1 / (1 + \alpha_1 x)(1 + \alpha_2 y)$$

Among them, α_1 and α_2 are positive constants [9]. The Bazykin functional response is used to describe the stabilizing force of predator saturation and the destabilizing force of competition for prey. Therefore, this paper studies a diffusive predator-prey model with an Allee effect and a Bazykin functional response, and the specific model is as follows:

$$\begin{cases} \frac{dx}{dt} = rx \left(1 - \frac{x}{k} \right) (x - v) - \frac{y}{(1 + \alpha_1 x)(1 + \alpha_2 y)} \\ \frac{dy}{dt} = \frac{\alpha y}{(1 + \alpha_1 x)(1 + \alpha_2 y)} - dy \end{cases} \quad (1.1)$$

2. Properties of Solutions

We start with the positiveness of solutions of system (1.1)

Let

$$\begin{aligned} \varphi_1(x, y) &= r \left(1 - \frac{x}{k} \right) (x - v) - \frac{y}{x(1 + \alpha_1 x)(1 + \alpha_2 y)}, \\ \varphi_2(x, y) &= \frac{\alpha}{(1 + \alpha_1 x)(1 + \alpha_2 y)} - d, \end{aligned}$$

And then we rewrite (1.1) as

$$\frac{dx}{dt} = x\varphi_1(x, y), \quad \frac{dy}{dt} = y\varphi_2(x, y)$$

Consequently,

$$x(t) = x(0)e^{\int_0^t \varphi_1(x(s), y(s)) ds} > 0, \quad y(t) = y(0)e^{\int_0^t \varphi_2(x(s), y(s)) ds} > 0$$

which implies the positiveness of the solution of (1.1).

Next, we prove the boundedness of solutions. From (1.1), we can construct a function involving $x(t)$ and $y(t)$

$$w(t) = \alpha x(t) + y(t)$$

Consequently,

$$\begin{aligned} \frac{dw(t)}{dt} &= \alpha r \left(1 - \frac{x}{k}\right) (x - v) - dy \\ &= \alpha r v x \left(1 - \frac{x}{k}\right) \left(\frac{x}{v} - 1\right) - dy \\ &= -\alpha r v x - dy + \frac{\alpha r}{k} x^2 (-x + k + v) \end{aligned}$$

Let

$$\delta = \frac{\alpha r}{k} x^2 (-x + k + v),$$

After calculation, it can be found that δ achieves its maximum value M at $x = \frac{2k + 2v}{3}$, then it can be concluded that

$$\frac{dw(t)}{dt} \leq M - \alpha r v x - dy \leq M - \delta w$$

Let

$$\delta = \min\{rv, d\}, \quad M = \frac{4}{27} \frac{\alpha v}{k} (k + v)^3$$

$$0 \leq w(x(t), y(t)) \leq \frac{\delta(1 - e^{-Mt})}{M} + w(x(0), y(0))e^{-Mt}$$

If $t \rightarrow +\infty$, $0 \leq w(t) \leq \delta/M$.

Consequently, the boundedness of $x(t)$ and $y(t)$ are derived. That is, both prey and predator are persistent from the biological angle.

Theorem 2.1: For the system (1.1) with arbitrary positive initial values $x(0)$, there is $\limsup_{t \rightarrow +\infty} x(t) \leq k$. When $x(0) < v$, then there is $\limsup_{t \rightarrow +\infty} x(t) = 0$.

Proof: Firstly, for any arbitrary positive initial value $x(0)$ of the system (1.1), it holds that $\limsup_{t \rightarrow +\infty} x(t) \leq k$.

Case 1: $x(0) < k$.

Assume the conclusion does not hold, that is, there exist t_1 and t_2 such that $0 < t_1 < t_2$, and it holds that $x(t_1) = k$, and for all $t \in (t_1, t_2)$ in the interval, it holds that $x(t) > k$. When $t \in (t_1, t_2)$, the first equation of system (1.1) implies that

$$x(t) = x(0)e^{\int_0^t \varphi_1(x(s), y(s)) ds}$$

among them

$$\varphi_1(x, y) = r\left(1 - \frac{x}{k}\right)(x - v) - \frac{y}{x(1 + \alpha_1 x)(1 + \alpha_2 y)}$$

We have

$$x(t) = x(0)e^{\int_0^t \varphi_1(x(s), y(s)) ds} = x(t_1)e^{\int_0^{t_1} \varphi_1(x(s), y(s)) ds}$$

There always exists an interval $(\partial, \partial + \Delta t)$ such that $v < k$, then

$$\begin{aligned} & r\left(1 - \frac{x}{k}\right)(x - v) - \frac{y}{x(1 + \alpha_1 x)(1 + \alpha_2 y)} \\ & < r\left(1 - \frac{x}{v}\right)(x - v) = \frac{r}{v}(v - x)(x - v) = -\frac{r}{v}(x - v)^2 \end{aligned}$$

Thus, for any $t \in (t_1, t_2)$, it holds that $\varphi_1(x(s), y(s)) < 0$, which implies $x(t) < x(t_1) = k$, a contradiction. Therefore, the conclusion stands, and it is proven.

Case 2: $x(0) > k$.

Because of $x(0) > k$, then from the first equation of the system, it can be deduced that

$$\frac{dx}{dt} = x(0)\varphi_1(x(0), y(t)) < 0$$

Similar to Case 1, by the same reasoning, we have $\limsup_{t \rightarrow +\infty} x(t) \leq k$.

Therefore, it is proven that for any $x(0)$, it holds that $\limsup_{t \rightarrow +\infty} x(t) \leq k$.

Case 3: When $x(0) \leq v$, by the same reasoning,

$$\frac{dx}{dt} = x(0)\varphi_1(x(0), y(t)) < 0$$

This proves $\limsup_{t \rightarrow +\infty} x(t) = 0$, which is proven.

Theorem 2.1 indicates that no matter how much the initial quantity of the prey is, when time tends towards positive infinity, it will ultimately be less than the environmental carrying capacity k . Furthermore, when the quantity of the prey is less than the Allee constant v , the Allee effect will play a primary role in the extinction of the prey. That is to say, even if predators do not prey on the prey, the prey itself will naturally become extinct.

3. Dynamic Analysis of the Model

This section first analyzes the existence of equilibrium points, followed by an analysis of their stability.

- 1) It is clear that the system has an equilibrium point $E_0 = (0, 0)$.
- 2) When $x \neq 0, y = 0$, let

$$rx\left(1 - \frac{x}{k}\right)(x - v) - \frac{mxy}{\alpha + \beta x + x^2} = 0$$

then

$$rx\left(1-\frac{x}{k}\right)(x-v)=0$$

$x=k$ or $x=v$, at this point, the system has two equilibrium points $E_1=(k,0)$, $E_2=(v,0)$.

3) When $x \neq 0, y \neq 0$, let

$$\frac{\alpha y}{(1+\alpha_1 x)(1+\alpha_2 y)} - dy = 0$$

then

$$x^* = \frac{\alpha - d - dy\alpha_2}{d\alpha_1(1+\alpha_2 y)}$$

From

$$\frac{\alpha y}{(1+\alpha_1 x)(1+\alpha_2 y)} - dy = 0,$$

we get

$$\frac{y}{(1+\alpha_1 x)(1+\alpha_2 y)} = \frac{dy}{\alpha}$$

Substitute it into the following equation

$$rx\left(1-\frac{x}{k}\right)(x-v) - \frac{y}{(1+\alpha_1 x)(1+\alpha_2 y)} = 0,$$

we get

$$rx\left(1-\frac{x}{k}\right)(x-v) = \frac{dy}{\alpha}$$

$$y^* = \frac{\alpha}{d} rx^* \left(1-\frac{x^*}{k}\right)(x^*-v)$$

Therefore, when condition $\alpha - d - dy\alpha_2 > 0$, $v < \frac{\alpha - d - dy\alpha_2}{d\alpha_1(1+\alpha_2 y)} < k$ is met,

the system has a unique equilibrium point.

The Jacobian matrix of system (1.1) is

$$J = \begin{pmatrix} r\left(1-\frac{x}{k}\right)(x-v) - \frac{rx(x-v)}{k} + rx\left(1-\frac{x}{k}\right) + \frac{\alpha_1 y}{(1+\alpha_1 x)(1+\alpha_2 y)} & -\frac{1}{(1+\alpha_1 x)(1+\alpha_2 y)} + \frac{\alpha_2 y}{(1+\alpha_1 x)(1+\alpha_2 y)^2} \\ -\frac{\alpha\alpha_1 y}{(1+\alpha_1 x)^2(1+\alpha_2 y)} & \frac{\alpha}{(1+\alpha_1 x)(1+\alpha_2 y)} + \frac{\alpha\alpha_2 y}{(1+\alpha_1 x)(1+\alpha_2 y)^2} - d \end{pmatrix}$$

Thus, by calculating the Jacobian matrix corresponding to each equilibrium point to analyze local stability.

3.1. The Stability of $E_0=(0,0)$

Let J_0 be the Jacobian matrix of the system at the equilibrium point E_0 .

$$J_0 = \begin{pmatrix} -rv & -1 \\ 0 & \alpha - d \end{pmatrix}$$

Thus, the two eigenvalues are $\lambda_1 = -rv, \lambda_2 = \alpha - d$. If $\lambda_2 < 0$, the system is stable at E_0 . Otherwise, the system (1.1) is unstable.

3.2. The Stability of $E_1 = (k, 0)$

Let J_1 be the Jacobian matrix of the system at the equilibrium point E_1 .

$$J_1 = \begin{pmatrix} -r(k-v) & -\frac{1}{1+\alpha_1 k} \\ 0 & \frac{\alpha}{1+\alpha_1 k} - d \end{pmatrix}$$

Thus, the two eigenvalues are $\lambda_1 = -r(k-v), \lambda_2 = \frac{\alpha}{1+\alpha_1 k} - d$. If $d > \frac{\alpha}{1+\alpha_1 k}$, the system is stable at E_1 . Otherwise, the system (1.1) is unstable.

3.3. The Stability of $E_2 = (v, 0)$

Let J_2 be the Jacobian matrix of the system at the equilibrium point E_2 .

$$J_2 = \begin{pmatrix} rv\left(1-\frac{v}{k}\right) & -\frac{1}{1+\alpha_1 v} \\ 0 & \frac{\alpha}{1+\alpha_1 v} - d \end{pmatrix}$$

Thus, the two eigenvalues are $\lambda_1 = rv\left(1-\frac{v}{k}\right), \lambda_2 = \frac{\alpha}{1+\alpha_1 v} - d$. the system (1.1) is unstable.

Since λ_1 is positive, according to the Routh-Hurwitz stability criterion, the system (1.1) is unstable at point E_2 .

3.4. The Stability of $E_3 = (x^*, y^*)$

Let J_3 be the Jacobian matrix of the system at the equilibrium point $E_3 = (x^*, y^*)$.

Then

$$\frac{y}{(1+\alpha_1 x)(1+\alpha_2 y)} = \frac{dy}{\alpha},$$

$$J_3 = \begin{pmatrix} r\left(1-\frac{x^*}{k}\right)(x^*-v) - \frac{rx^*(x^*-v)}{k} + rx^*\left(1-\frac{x^*}{k}\right) + \frac{d\alpha_1 y^*}{\alpha} - \frac{1}{(1+\alpha_1 x^*)(1+\alpha_2 y^*)} + \frac{\alpha_2 y^*}{(1+\alpha_1 x^*)(1+\alpha_2 y^*)^2} \\ -\frac{\alpha\alpha_1 y^*}{(1+\alpha_1 x^*)(1+\alpha_2 y^*)} & -\frac{d\alpha_2 y^*}{1+\alpha_2 y^*} \end{pmatrix}$$

Let

$$A = r\left(1-\frac{x^*}{k}\right)(x^*-v) - \frac{rx^*(x^*-v)}{k} + rx^*\left(1-\frac{x^*}{k}\right) + \frac{d\alpha_1 y^*}{\alpha}$$

$$B = -\frac{1}{(1+\alpha_1 x^*)(1+\alpha_2 y^*)} + \frac{\alpha_2 y^*}{(1+\alpha_1 x^*)(1+\alpha_2 y^*)^2} = -\frac{d}{\alpha(1+\alpha_2 y^*)}$$

$$C = -\frac{\alpha\alpha_1 y^*}{(1+\alpha_1 x^*)(1+\alpha_2 y^*)} < 0$$

$$D = -\frac{d\alpha_2 y^*}{1+\alpha_2 y^*}$$

The characteristic equation is $\lambda^2 - (A+D)\lambda + BC = 0$.

Therefore, according to the relationship between the roots and coefficients, the two characteristic roots λ_1, λ_2 of the equation satisfy

$$\lambda_1 + \lambda_2 = A+D, \lambda_1 \lambda_2 = BC \quad (1,2)$$

$$B = \frac{\alpha_2 y^*}{(1+\alpha_1 x^*)(1+\alpha_2 y^*)^2} \leq \frac{1+\alpha_2 y^*}{(1+\alpha_1 x^*)(1+\alpha_2 y^*)^2} = \frac{1}{(1+\alpha_1 x^*)(1+\alpha_2 y^*)}$$

Therefore, $B < 0, \lambda_1 \lambda_2 > 0$. Thus, if $\lambda_1 + \lambda_2 < 0$, that is, when $A+D < 0$, the two characteristic roots have negative real parts. At this time, the system $E_3 = (x_1^*, y_1^*)$ is locally asymptotically stable.

3.5. Hopf Bifurcation

From section 3.4, it is known that the local stability of the positive equilibrium point $E_3 = (x_1^*, y_1^*)$ of system (1.1) requires certain conditions to be met. Therefore, this subsection mainly analyzes the conditions under which a Hopf bifurcation occurs at the equilibrium point $E_3 = (x_1^*, y_1^*)$ in system (1.1), and the direction of the Hopf bifurcation.

Let

$$\ell(d) = r \left(1 - \frac{x^*}{k}\right) (x^* - v) - \frac{rx^*(x^* - v)}{k} + rx^* \left(1 - \frac{x^*}{k}\right) + \frac{d\alpha_1 y^*}{\alpha} - \frac{d\alpha_2 y^*}{1+\alpha_2 y^*}$$

Select the natural mortality rate of predators d as the bifurcation parameter.

Let $\ell(d) = 0$.

We have

$$d^* = \frac{r\alpha(1+\alpha_2 y^*) \{ (x^* - v)(k - 2x^*) + x^*(k - x^*) \}}{k(\alpha_1 y^*(1+\alpha_2 y^*) - \alpha\alpha_2 y^*)}$$

$$\frac{d(\lambda)}{d(d)} = \frac{\alpha_1 y^*}{\alpha} - \frac{\alpha_2 y^*}{1+\alpha_2 y^*} \neq 0 (d = d^*)$$

Then

$$d^* = \frac{r\alpha(1+\alpha_2 y^*) \{ (x^* - v)(k - 2x^*) + x^*(k - x^*) \}}{k(\alpha_1 y^*(1+\alpha_2 y^*) - \alpha\alpha_2 y^*)}$$

$$\lambda_1 \lambda_2 = BC$$

$$\text{and} = -\frac{\alpha\alpha_1 y^*}{(1+\alpha_1 x^*)(1+\alpha_2 y^*)} \left\{ -\frac{1}{(1+\alpha_1 x^*)(1+\alpha_2 y^*)} + \frac{\alpha_2 y^*}{(1+\alpha_1 x^*)(1+\alpha_2 y^*)^2} \right\} > 0$$

At this point, system (1.1) will undergo a Hopf bifurcation at the positive equilibrium point $E_3 = (x_1^*, y_1^*)$.

Next, by calculating the first Lyapunov coefficient to analyze the direction of the Hopf bifurcation.

For system (1.1), We make a transformation of $u = x - x^*$, $v = y - y^*$.

By performing a Taylor expansion on the system (1.1), we can obtain

$$\frac{du}{dt} = a_{10}u + a_{01}v + a_{11}uv + a_{20}u^2 + a_{02}v^2 + a_{30}u^3 + a_{03}v^3 + a_{21}u^2v + a_{12}uv^2 + P(u, v)$$

$$\frac{dv}{dt} = b_{10}u + b_{01}v + b_{11}uv + b_{20}u^2 + b_{02}v^2 + b_{30}u^3 + b_{03}v^3 + b_{21}u^2v + b_{12}uv^2 + Q(u, v)$$

$$a_{10} = r \left(1 - \frac{x^*}{k} \right) (x^* - v) - \frac{rx^* (x^* - v)}{k} + rx^* \left(1 - \frac{x^*}{k} \right) + \frac{d\alpha_1 y^*}{\alpha}$$

$$a_{01} = -\frac{d}{\alpha(1 + \alpha_2 y^*)}, \quad a_{11} = \frac{d\alpha_1}{\alpha}$$

$$a_{20} = -\frac{2r(x^* - v)}{k} + 2r \left(1 - \frac{x^*}{k} \right) - \frac{2rx^*}{k}, \quad a_{02} = \frac{d\alpha_2}{\alpha(1 + \alpha_2 y^*)^2}$$

$$a_{30} = -\frac{6r}{k}, \quad a_{03} = -\frac{d\alpha_2^2}{\alpha(1 + \alpha_2 y^*)^2}, \quad a_{21} = 0, a_{12} = 0$$

$$b_{10} = -\frac{\alpha\alpha_1 y^*}{(1 + \alpha_1 x^*)(1 + \alpha_2 y^*)} = \alpha y^* a_{11}, \quad b_{01} = -\frac{d\alpha_2 y^*}{1 + \alpha_2 y^*}, \quad b_{11} = \alpha a_{11},$$

$$b_{20} = \frac{\alpha\alpha_1^2 y^*}{(1 + \alpha_1 x^*)(1 + \alpha_2 y^*)^2}, \quad b_{02} = -\frac{d\alpha_2}{(1 + \alpha_2 y^*)^2},$$

$$b_{30} = -\frac{\alpha\alpha_1^3 y^*}{(1 + \alpha_1 x^*)^2 (1 + \alpha_2 y^*)^2}, \quad b_{03} = \frac{2d\alpha_2^2}{(1 + \alpha_2 y^*)^3}, \quad b_{21} = \frac{1 - \alpha_2 y^*}{(1 + \alpha_2 y^*)^3}, \quad b_{12} = 0$$

$$P(u, v) = \sum_{i+j=4}^{+\infty} a_{ij} u^i v^j, \quad Q(u, v) = \sum_{i+j=4}^{+\infty} b_{ij} u^i v^j$$

Thus, by calculating the first Lyapunov coefficient σ , we can obtain

$$\begin{aligned} \sigma = & -\frac{3\pi}{2a_{01}\Delta^{\frac{3}{2}}} \left\{ \left[a_{10}b_{10} (a_{11}b_{02} + a_{02}b_{11} + a_{11}^2) + a_{10}a_{01} (a_{20}b_{11} + a_{11}b_{02} + b_{11}^2) \right. \right. \\ & + b_{10}^2 (a_{11}a_{02} + 2a_{02}b_{02}) - 2a_{10}b_{10} (b_{02}^2 - a_{20}a_{02}) - 2a_{10}a_{01} (a_{20}^2 - b_{20}b_{02}) \\ & \left. \left. - a_{01}^2 (2a_{20}b_{20} + b_{11}b_{20}) + (a_{10}b_{01} - 2a_{10}^2) (b_{11}b_{02} - a_{11}a_{20}) \right] \right. \\ & \left. - (a_{10}^2 + a_{01}b_{10}) [3(b_{10}b_{03} - a_{01}a_{30}) + 2a_{10} (a_{21} + b_{12}) + (b_{10}a_{21} - a_{01}b_{21})] \right\} \end{aligned}$$

Let

$$\Delta = a_{10}b_{01} - a_{01}b_{10}$$

When $\sigma > 0$, system (1.1) undergoes a subcritical Hopf bifurcation; When $\sigma < 0$, system(1.1) undergoes a supercritical Hopf bifurcation.

4. Conclusion

This paper has studied a predator-prey model with Allee effects and featuring a Bazykin functional response. An analysis has been conducted on its boundedness, the nature of solutions, the existence of equilibrium points, as well as its local stability and bifurcation. The boundedness of the solutions was derived by constructing a function. Finally, the model was linearized, and the direction of the Hopf bifurcation was analyzed by calculating the Lyapunov exponents.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

- [1] Allee, W.C. and Bowen, E.S. (1932) Studies in Animal Aggregations: Mass Protection against Colloidal Silver among Goldfishes. *Journal of Experimental Zoology*, **61**, 185-207. <https://doi.org/10.1002/jez.1400610202>
- [2] Allee, W.C. (1931) Animal Aggregations, a Study in General Sociology. The University of Chicago Press. <https://doi.org/10.5962/bhl.title.7313>
- [3] Stephens, P.A. and Sutherland, W.J. (1999) Consequences of the Allee Effect for Behaviour, Ecology and Conservation. *Trends in Ecology & Evolution*, **14**, 401-405. [https://doi.org/10.1016/s0169-5347\(99\)01684-5](https://doi.org/10.1016/s0169-5347(99)01684-5)
- [4] Yan, S., Jia, D., Zhang, T. and Yuan, S. (2020) Pattern Dynamics in a Diffusive Predator-Prey Model with Hunting Cooperations. *Chaos, Solitons & Fractals*, **130**, 109428. <https://doi.org/10.1016/j.chaos.2019.109428>
- [5] Wu, D. and Zhao, M. (2019) Qualitative Analysis for a Diffusive Predator-Prey Model with Hunting Cooperative. *Physica A: Statistical Mechanics and Its Applications*, **515**, 299-309. <https://doi.org/10.1016/j.physa.2018.09.176>
- [6] Song, D., Li, C. and Song, Y. (2020) Stability and Cross-Diffusion-Driven Instability in a Diffusive Predator-Prey System with Hunting Cooperation Functional Response. *Nonlinear Analysis: Real World Applications*, **54**, 103106. <https://doi.org/10.1016/j.nonrwa.2020.103106>
- [7] Zhang, S. and Chen, L. (2005) A Holling II Functional Response Food Chain Model with Impulsive Perturbations. *Chaos, Solitons & Fractals*, **24**, 1269-1278. <https://doi.org/10.1016/j.chaos.2004.09.051>
- [8] Liu, X. and Chen, L. (2003) Complex Dynamics of Holling Type II Lotka-Volterra Predator-Prey System with Impulsive Perturbations on the Predator. *Chaos, Solitons & Fractals*, **16**, 311-320. [https://doi.org/10.1016/s0960-0779\(02\)00408-3](https://doi.org/10.1016/s0960-0779(02)00408-3)
- [9] Sarwardi, S., Haque, M. and Venturino, E. (2010) Global Stability and Persistence in Lg-Holling Type II Diseased Predator Ecosystems. *Journal of Biological Physics*, **37**, 91-106. <https://doi.org/10.1007/s10867-010-9201-9>