Scientific Research Publishing

Journal of Applied Mathematics and Physics, 2024, 12, 2582-2593
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

A Study on Multivariable Interactions Concerning Radar Cross Section Reduction through Geometric Attributes

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How to cite this paper: Sharp, E. (2024) A Study on Multivariable Interactions Concerning Radar Cross Section Reduction through Geometric Attributes. *Journal of Applied Mathematics and Physics*, **12**, 2582-2593.
https://doi.org/10.4236/jamp.2024.127153

Received: June 20, 2024
Accepted: July 23, 2024
Published: July 26, 2024

Abstract

This resolution 5 (2^{5-1} factorial) study aimed to ascertain an understanding of the interactions between different geometries on the resulting Radar Cross Section (RCS) of a target. The results of the study are in line with the general understanding of the impact different geometries have on RCS but show that geometries can also influence the variance of measured RCS, and typical attributes that reduce RCS increase the variance of the measured RCS. Notably, an increased angle between the front face of a plate and the direction of the radar signal decreased RCS but increased the variance of the RCS measured.

Keywords

Radar Cross Section, RCS, Geometrical Attributes, Radar, Stealth

1. Background and Research

On February 26th, 1935, Robert Watson-Watt and Arnold F. Wilkins first successfully demonstrated the use of radar technology to detect aircraft [1]. This was a revolutionary breakthrough, specifically in the military sector as it allowed defenses to detect intruding enemy aircraft, and to accurately track/target them. Radar technology was made possible by transmitting electromagnetic waves towards metallic objects and observing the echoed returns of these waves to the receiver [2].

Since the conception of radar, many methods of anti-radar stealth have been researched with the hopes of avoiding enemy detection. The main basis of this research was finding ways to decrease the radar cross section of an aircraft, which is often referred to as RCS. RCS is a measurement of the equivalent area of an object seen by a radar [3]. By having a low enough RCS, military fighter jets can go undetected by enemy radars and establish air dominance. Air dominance is a vital factor in the outcomes of military interactions, and since the invention of the Lockheed F-117A Nighthawk, the first operational stealth fighter jet, the skies have been ruled by stealth technology. By having greater stealth technology, an unmatched military advantage can be created.

Two main methods are used for reduction of RCS: absorption and shaping. Absorption includes using certain materials, paints, or coatings that absorb radar signals, while shaping concerns designing aircraft (or whatever product that is concerned) so that the radar transmission is scattered rather than reflected directly back to the transmitter [3]. Many equations have been derived to predict the radar cross section of different geometric shapes, but all are derived from the following equation for the RCS of a conducting square plate with a physical area A from the normal direction [3]:

$$\sigma = \frac{4\pi A^2}{\lambda^2} \tag{1}$$

where σ = RCS, A = the actual area of the plate, and λ = the wavelength of the transmitted signal. While Equation (1) represents a perfectly flat plate, this is not a common occurrence in industry, as the metallic geometries on an aircraft are hardly ever flat. A more accurate equation for the RCS of a square plate angled to normal is:

$$\sigma = \frac{4\pi A^2}{\lambda^2} * \left[\frac{\sin\left(k * a * \sin(\theta)\right)}{k * a * \sin(\theta)} \right] \tag{2}$$

where

$$k = 2\pi/\lambda \tag{3}$$

And a = the side length [3].

Reduction by shaping techniques commonly used on stealth aircraft includes the rounding of edges, as spheres inherently have a low RCS, and the increase of the angle between faces and the normal direction from the radar sensor, as faces that have a further angle from the normal direction of the signal transmission will scatter the signal in further directions from the receiver [4]. These two techniques are effective due to a phenomenon called specular reflection, where light, and thus radar signals, are reflected off reflective materials in just one direction. This direction is determined by the angle between the incoming light beams and the direction normal to the reflective surface (called the angle of incidence). The angle of reflection is symmetrical to the angle of incidence across the normal of the reflective surface [5]. Thus, by having rounded edges, the wave shaped radar will be scattered in a multitude of directions as it comes in contact with the round geometry rather than a flat plane. Furthermore, by increasing the angle between the faces and the normal direction of the radar sensor (which is the direction that the radar signals will be traveling), the angle of incidence is increased, and thus the angle of reflection is increased further from the direction of the sensor.

Additionally, a study published in the Journal of Applied Mathematics and Physics included a simulation to calculate the bistatic RCS of a dielectric hemisphere. The simulation concluded that the two highest measurements of RCS at varying angles were directly to the side of the dielectric hemisphere. Thus, the highest measurements of RCS were highest when the flat edges of the bottom of the hemisphere were most visible to the radar transmitter and receiver [6]. With this in mind, research behind the interactions between these variables, as well as the distance from the radar, the number of faces, and the curvature of the corners of a given panel have been scarcely studied.

2. Background and Research

The purpose of the following experiment is to better understand the impact of various geometric variables and their interactions on the resulting RCS of a target specimen. Due to limited access to materials and sensors, an HC-SR04 sensor was used rather than a radar sensor. The HC-SR04 sensor is an ultrasonic sensor that works in a similar manner, as it transmits a frequency and bases its reading on the echoed frequency returned to the receiver. The only difference is that sound is being sent and received, rather than electromagnetic radio waves [7]. Additionally, the use of an Ultrasonic sensor allows for the use of non-conductive materials for the testing specimens. Non-conductive materials, such as plastics, tend to be penetrated by radar signals, whereas conductive materials fully reflect them [8]. This will provide more accurate results for the experiment, as the test specimens are made of PLA on a 3d printer. Despite the use of ultrasonic, the signal received will act in the same way as if radar was used, and thus a test concerning just the geometry and distance of the targets is accurate and applicable.

16 different test specimens were printed on a LulzBot TAZ-Pro 3d printer in the same batch using the same settings. These print settings can be found in **Appendix 1**. The 16 specimens represent the 16 runs present in a 2^{5-1} experimental setup, allowing for five different control variables and a resolution of five. The five variables are the number of intersecting faces (A), the radius of the top edges formed between the faces (B), the angle of the faces from the normal direction of the sensor (C), the distance from the radar transmitter/receiver (D), and the radius of the corners of the panel (E) (**Figure 1**).

Although each control variable is continuous, only their high and low levels were represented. The levels of these variables were selected to represent the intersecting edges of metal panels used to manufacture an aircraft. Because of this, the number of faces was limited to 2 and maximized to 4, and the angle of the faces from the direction normal to the sensor was kept to 100 and 120 degrees. The radius of the top edges was 0 inches to 0.25 inches, as anything more than 0.25 inches made it so that a distinctive edge between the faces was no longer discernable. The radius of the corners was kept to 0 inches and 0.2 inches, as anything more than 0.2 inches was deemed to alter the surface area too drastically which would harm the accuracy of the resulting measurements. Lastly, the distance from the radar is 6 inches on the low side, and 12 inches on the high side. This was used as it accurately fit the scale of the testing specimens and the test apparatus used to hold them. The following table shows the levels of each control variable for the specimens (**Table 1**, **Figure 2**).

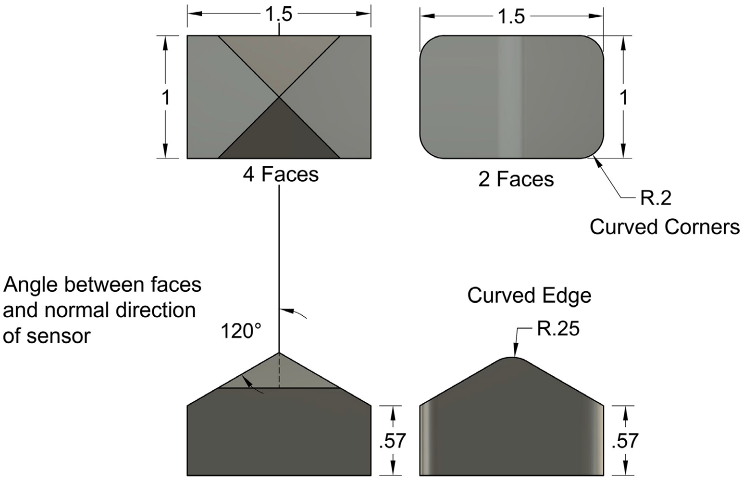

Figure 1. Control variables/test specimens illustrated.

Table 1. Specimen control variable levels.

Specimen Label	Number of Faces (A)	Top Edge Radius (B) (Inches)	Angle of Faces From Normal direction of Sensor (C) (˚)	Distance from Radar (D) (Inches)	Corner Radius (E) (Inches)
1	2	0	100	6	0.2
2	4	0	100	6	0
3	2	0.25	100	6	0
4	4	0.25	100	6	0.2
5	2	0	120	6	0
6	4	0	120	6	0.2
7	2	0.25	120	6	0.2
8	4	0.25	120	6	0
9	2	0	100	12	0
10	4	0	100	12	0.2
11	2	0.25	100	12	0.2
12	4	0.25	100	12	0
13	2	0	120	12	0.2
14	4	0	120	12	0
15	2	0.25	120	12	0
16	4	0.25	120	12	0.2

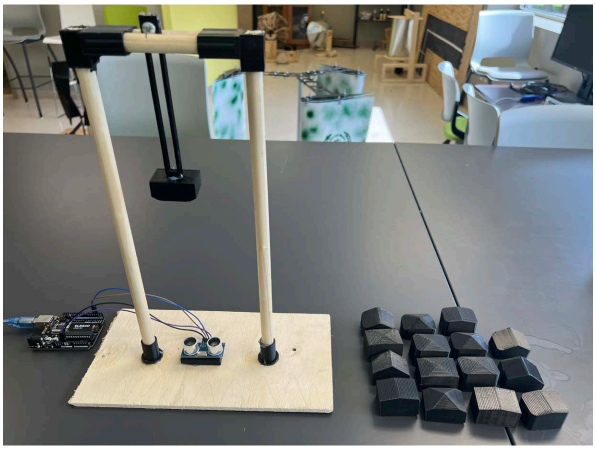

Figure 2. Testing setup.

One value was recorded for each specimen using an Arduino interfaced with the sensor. Note that because this is only a value that measures the strength of the signal sent back, this only represents the relative size of the RCS for this experiment, and not the actual RCS of the specimens. Although the actual RCS value would need much more equipment to measure, this still gives an accurate representation of how the size of the RCS would change based on the independent variables present. Because this is only a representation of RCS, no units were recorded for the measurements, and they were rather treated as raw unitless readings from the Arduino. The order of all 16 tests was randomized to ensure no variable of fatigue or learning curve was present. The sensor was coded to record one measurement every second. Before the testing was initiated, it was ensured that there was no background noise present that may have altered the results. All tests were conducted in the same room (STEAM 203 at Jacksonville University) in the same hour to ensure no outside noise variable was present, or any temperature/humidity fluctuations that would have impacted the data. After each round of experimentation was initiated, the sensor was given 20 seconds to stabilize on one reading, as it was found that the initial readings seemed to oscillate before settling on a final reading. After each individual reading, the specimen was removed, and the sensor was given 20 seconds to rest before the next specimen was attached. After all the data was collected, it was analyzed, and the results were studied.

3. Results and Discussion

Using a 2^{5-1} multivariable experimental framework, the magnitude of the impact of each variable and the impact of the interactions between each multi-variable factor were averaged for all 16 specimens. This average effect was then divided by two in order to represent the low and high levels of each variable, resulting in a coefficient for each of the 6 variables, and each of the 9 multivariable interactions, that represented the magnitude and direction of impact due to each changing variable on the resulting RCS of the specimen. Furthermore, these coefficients were then ranked in ascending order from 1 to 15, where after 0.5

was subtracted from their rank, and the resulting value was divided by 15. This final value represented the normal probability of each coefficient occurring naturally within the study (due to natural variance not the independent variables). Because only one replicate was taken for each specimen, ANOVA was not possible on this data set. Instead, a Daniels Plot was created, which plotted each coefficient of the variables and their interactions with their expected probabilities to find which coefficients did not follow a normal distribution, and thus had a statistically significant impact on the data.

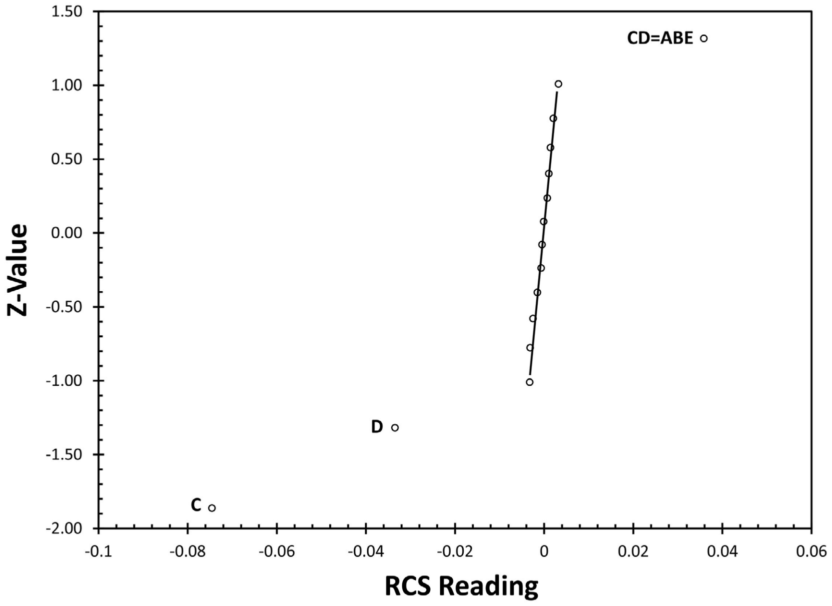

Figure 3. Daniels plot.

From **Figure 3**, it can be concluded that the variables and interactions between variables that had a statistically significant impact on the data are:

(C) the angle of the faces from the normal direction of the sensor, (D) the distance from the radar, and (C * D = ABE).

Using the coefficients used in the Daniels plot for the significant coefficients mentioned above, an equation can be derived to predict the resulting RCS signal returned in this experiment.

$$\text{RCS} = -0.0745 * \text{C} - 0.0334 * \text{D} + 0.0358 * \text{C} * \text{D} \tag{4}$$

Despite C * D also representing A * B * E in this experiment, it is likely that only the interaction between C * D had an impact on the data, as only those independent variables had an impact, and neither A, B, nor E had a statistical impact.

Although Equation (4) cannot be used to predict the actual RCS of an object because just raw readings were taken not RCS measurements, it can be used to estimate the overall relative strength or weakness of the reflected signal from a rectangular panel. This estimation can be based on the geometric attributes of the edges, faces, and corners facing the transmitted radar signal, and the distance from the radar. Note that the actual value for each geometric attribute should not be used in these equations, but rather a −1 or 1 to represent the high or low values of the control variables within the scope that which they were measured. Moreover, the findings of coefficient C, the angle of the faces from the normal direction of the sensor having the greatest magnitude of impact upon the resulting measured RCS is in line with data collected by other researchers. Specifically, a journal published in the Chinese Journal of Aeronautics 23 includes figures that confirm specimens with an increased angle from the normal direction of the radar sensor, and thus a greater angle of incidence, have a decreased RCS [9]. Furthermore, a study published in the 36th edition of the same journal shows that distance is also a key factor in detectability and thus the size of RCS detected by a radar system [10]. In many cases, depending on the frequency of the radar signal being transmitted, the probability of detection of a stealth craft is decreased from 100% to 0% if the distance of the radar to the craft is increased from 0 to 350 km. This is in line with the findings concluded above, showing that distance is also a key determining factor of RCS. Despite these affirmations, a study conducted on the bistatic RCS measurements of a cylindrical object made of metasurface arrays (a class of manufactured materials that allow manipulation of phase, amplitude, and polarization state of incident rays) found that despite manipulation of the rays, the rounded portions of the cylindrical objects tended to have the lowest RCS, meaning the signals were being scattered by the curvature of the objects [11]. This is not aligned with the results above that showed curvature having no statistical impact upon the RCS. This is likely because the varying levels of curvature in the experiment conducted for this paper were not differentiated enough. Further analysis should be conducted using more differentiated edge curvatures.

To ensure no outside variance was present due to fatigue or learning curve, a plot of the residuals with the run order was created using equation 3 to calculate the expected RCS values for each specimen.

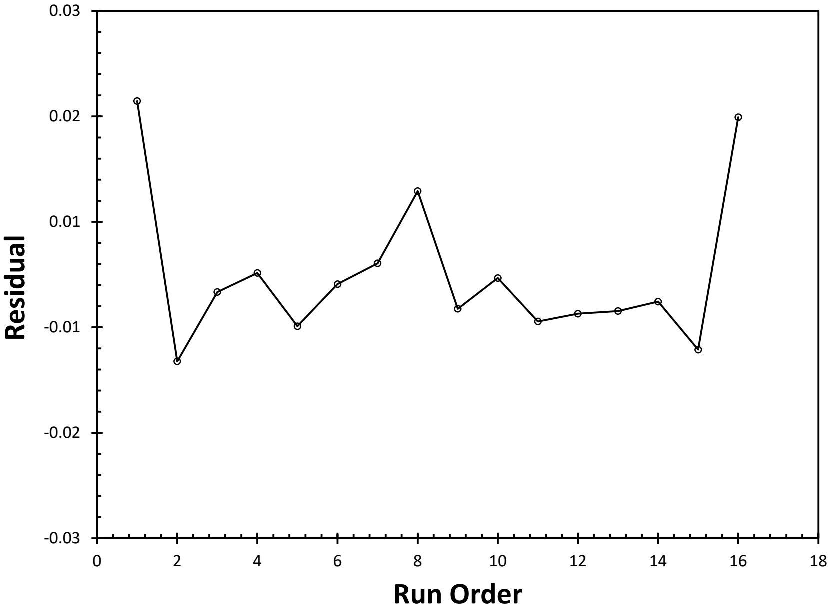

Figure 4. Run order vs. residual.

Through analysis of **Figure 4**, it can be concluded that no element of fatigue or learning curve is present to influence the retrieved data and alter the results. Furthermore, the residuals for each level of each variable were analyzed to conclude whether any factor was present that would cause a change in the independent variable that would sway the variance of the data.

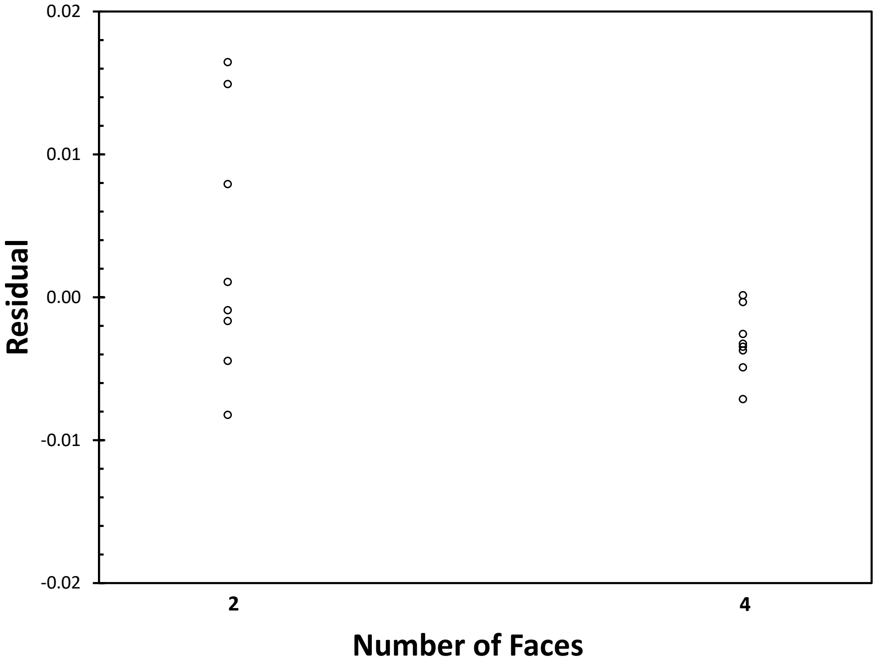

Figure 5. Number of faces vs. residual.

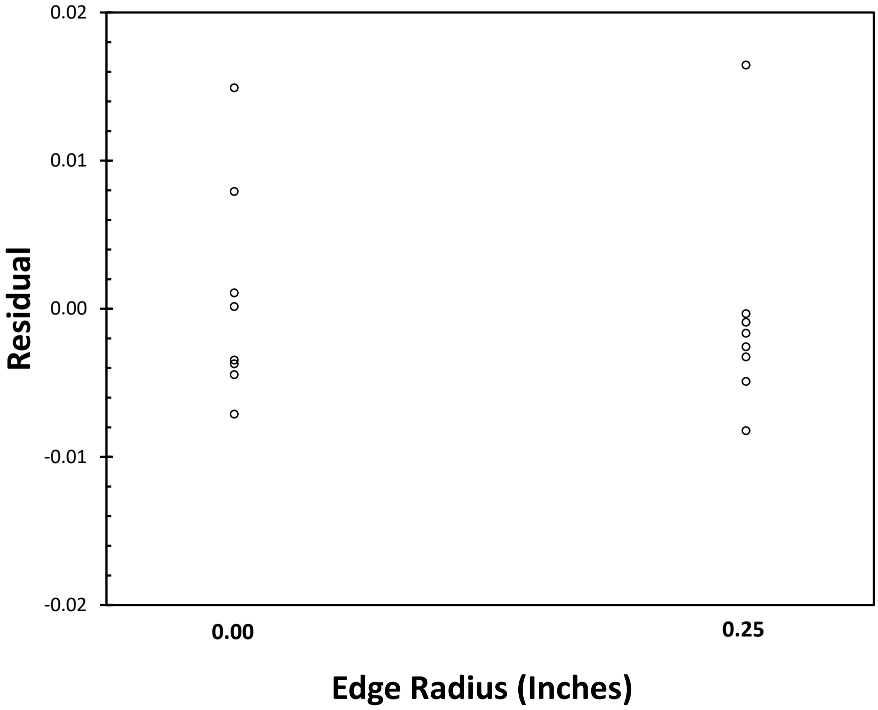

Figure 6. Edge radius vs. residual.

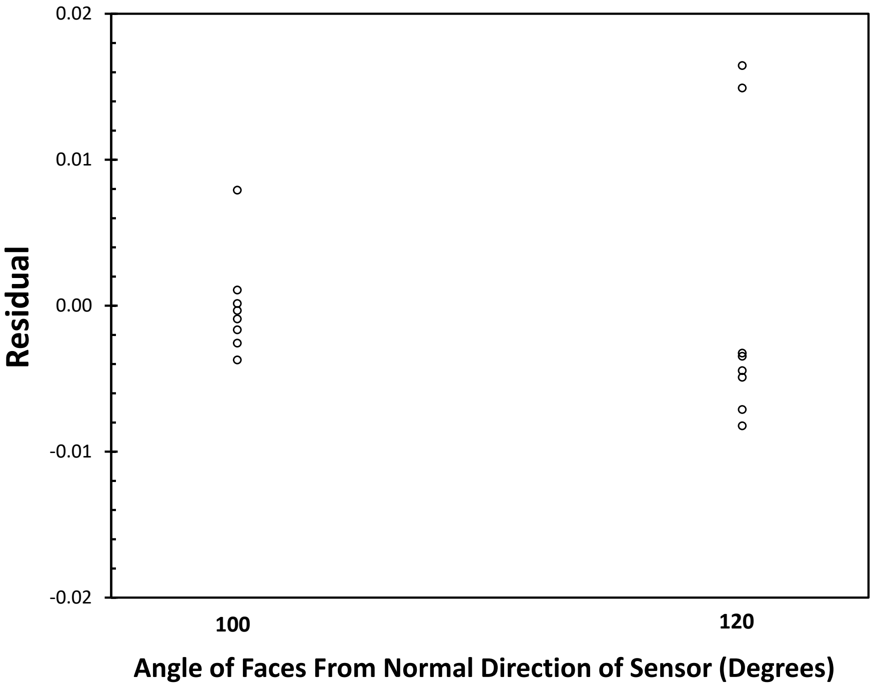

Figure 7. Angle of faces from normal direction of sensor vs. residual.

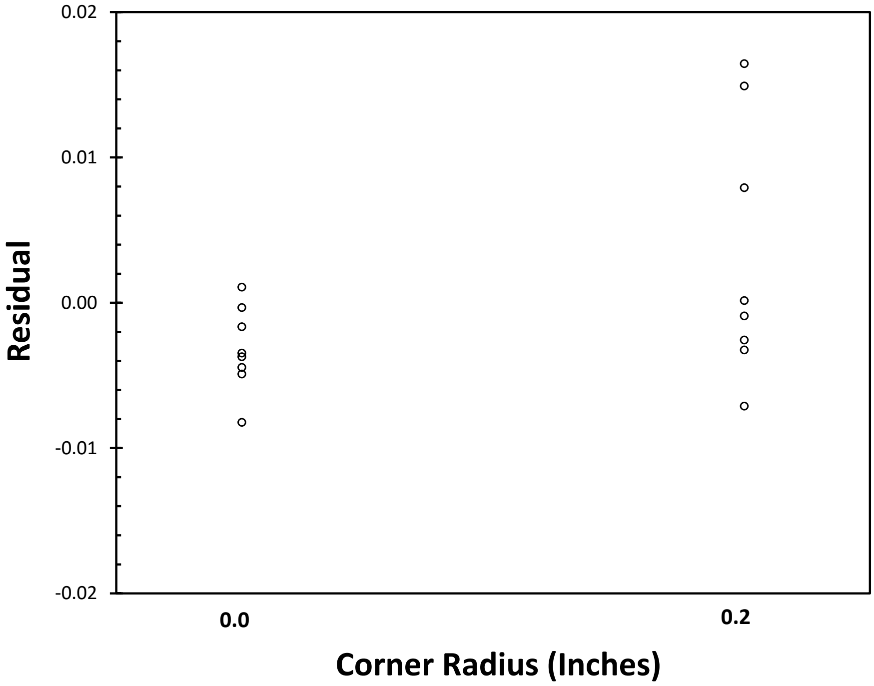

Figure 8. Corner radius vs. residual.

Through analysis of **Figures 5-9**, it visually appears as if there may be a difference in the variance of the residuals for all 5 figures. To confirm this, a separate F-test was conducted on the variance of the residuals at each level for all five control variables, which concluded that there was a statistically significant difference in the variance of the residuals for all the control variables besides the distance from the radar. Interestingly, all the variables that showed a changing variance with a change in their level were variables that concerned the geometric shape of the target itself, meaning that because the levels were changed, the return signal was scattered in more erratic ways leading to a less predictable RCS. Specifically interesting, in the case of the angle between the faces and the normal direction of the sensor, which was the only geometric value that had a significant impact on the RCS, there was a much more variable residual at the higher level. This increased variance indicates that specimens with a larger angle of incidence (the angle between the signal and the faces of the specimen) are more reactive to slight changes in position/orientation that may have occurred between different runs of the experiment. As priorly discussed, the angle of reflection is equal to the angle of incidence, meaning that specimens with steeper angles would amplify the effect of any small positional errors that may have become present, scattering radar signals in further directions, resulting in increased measurement error of specimens with a higher incidence angle. This helps to draw the conclusion that the measured RCS of an object with a large angle of incidence may be more difficult to predict.

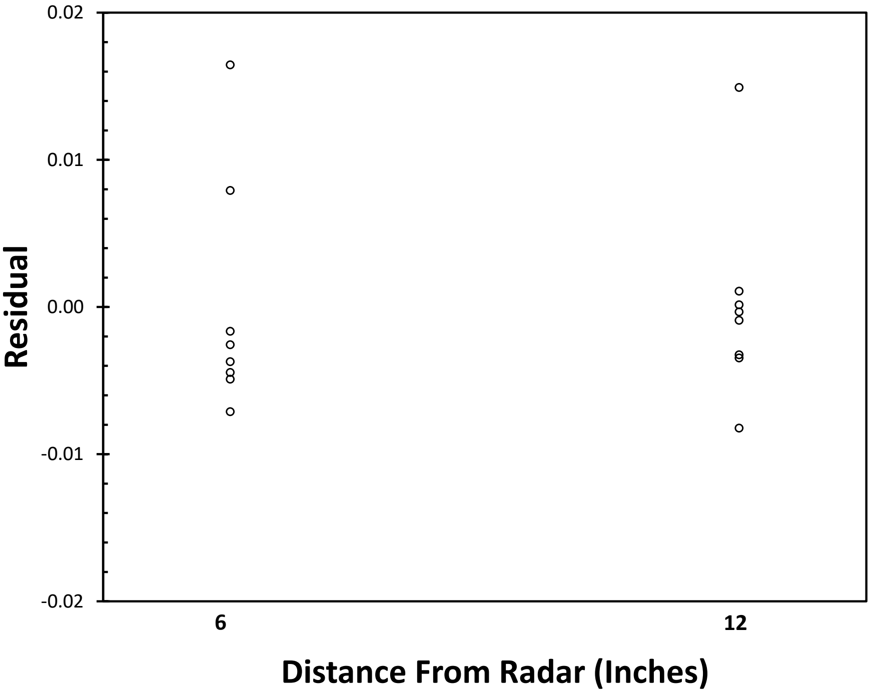

Figure 9. Distance from radar vs. residual.

Ultimately, because the only control variables and interactions that had a statistically significant impact on RCS reduction were the angle between the faces and the normal direction of the sensor, the distance between the sensor and the test specimen, and the interaction between the two variables just mentioned, it can be concluded that the scale of the other variables may have been the limiting factor to their RCS reduction capabilities, as it is generally known in radiology science that the control variables used in this experiment should have all had a statistical impact on the data. Despite this short coming, the F-test conducted on the variance of the residuals showed that these control variables had an impact on the radar scattering capabilities of the targets, and targets with control levels that would have been expected to reduce RCS (such as an increased number of faces which scatters the radar in more directions, curved edges which scatters the radar more effectively, an increased angle between the faces and the normal direction, and curved corners which reduces the surface area of the target) had a more variable RCS, meaning that the radar deflected off of these targets was deflected in a more sporadic way than others, making the RCS less predictable and maybe more difficult to track in military application.

Lastly, the application of the above research has many possibilities for not just military application, but application in anything where RCS must be maximized and minimized. Despite radar technology being present in modern science and engineering for nearly 90 years, it is still a constantly developing field with frequent breakthroughs. Specifically, two recent studies published in the Open Journal of Microphysics found similar results to one another while studying the first born double differential cross section for ionization of hydrogen at various electron ejection angles [12] [13]. Breakthroughs like this can lead to more advanced radar and radar stealth technology, and thus it is important to continue studying these phenomena. The research discussed in this paper may be used as guidance to RCS centered design, leading to a more efficient design, specifically in stealth fighters. Primarily, by ensuring that the surfaces of a given craft are aligned to maximize the angles between the faces and any oncoming radar signals, the craft can be designed in a thoughtful way to minimize the RCS. Furthermore, equations 4 may be utilized by engineers to minimize the RCS of a design within given engineering restraints and requirements. It may also allow them to gain a quantitative estimation of the magnitude of RCS for a given product based on certain given geometries. The findings concluded in this paper serve as a tool for engineers and scientists, which may be used to create more stealthy designs based upon a multitude of given geometries.

Acknowledgements

Thanks are extended to Dr. Murat for his generously shared expertise in the field of statistical analysis. Without the guidance of Dr. Murat, the conducted experiment would not have been inspired or conducted.

Furthermore, thanks are extended to Professor Bohn, whose guidance and help in the Jacksonville University Maker Space was an invaluable asset.

Moreover, thanks are extended to the Jacksonville University engineering department for its funding through the form of material suppliance and access to necessary parts for the experimentation procedure.

Lastly, thanks are extended to the classmates of the author, for their competitive spirit and will to learn, which pushed the scientist to dive deeper into a detail-oriented approach to this experiment.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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