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# Application of ACP Nonlinear Math in Analyzing Arithmetic and Radiation Transmission Data (Application 1 & 2) [4-21-2024, 820P] (V)

**Ralph W. Lai**[1], **Melisa W. Lai-Becker**[2], **Grace Cheng-Dodge**[3]

[1]28 Cornerstone Ct., Doylestown, PA, USA
[2]Harvard Medical School, Cambridge, MA, USA
[3]Independent Education Consultants, West Hartford, CT, USA
Email: ba70inpa@yahoo.com, mlaibecker@partners.org, gscheng1@gmail.com

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## Abstract

In this study, we explore the application of ACP (asymptotic curve based and proportionality oriented) Alpha Beta ($\alpha\beta$) Nonlinear Math to analyze arithmetic and radiation transmission data. Specifically, we investigate the relationship between two variables. The novel approach involves collecting elementary "$y$" data and subsequently analyzing the asymptotic cumulative or demulative (opposite of cumulative) $Y$ data. In part I, we examine the connection between the common linear numbers and ideal nonlinear numbers. In part II, we delve into the relationship between X-ray energy and the radiation transmission for various thin film materials. The fundamental physical law asserts that the nonlinear change in continuous variable $Y$ is negatively proportional to the nonlinear change in continuous variable $X$, expressed mathematically as $d\alpha = -Kd\beta$. Here: $d\alpha$ {$Y$, $Yu$, $Yb$} represents the change in $Y$, with $Yu$ and $Yb$ denoting the upper and baseline asymptote of $Y$. $d\beta$ {$X$, $Xu$, $Xb$} represents the change in $X$, with $Xu$ and $Xb$ denoting the upper and baseline asymptote of $X$. $K$ represents the proportionality constant or rate constant, which varies based on equation arrangement. $K$ is the key inferential factor for describing physical phenomena.

## Keywords

Asymptotic Concave and Convex Curve, Upper and Baseline Asymptote, Demulative vs. Cumulative, Coefficient of Determination, Proportionality and Position Constant, Skewed Bell and Sigmoid Curve

## 1. Introduction

In the realm of Newtonian calculus, the slope and tangent of curves hold paramount importance. Their utility is widely recognized in general mathematics. However, when dealing with continuous concave or convex curves, the conventional approach falls short. Specifically, it inadequately addresses the existence of asymptotes related to continuous cumulative numbers.

In this study, we embark on a journey through ACP (Asymptotic Curve-based and Proportionality-oriented) Nonlinear Math. Our exploration begins with a concise review of its principles. Subsequently, we introduce essential equations for analyzing various data. Part I focuses on the interplay between common linear numbers and ideal nonlinear numbers, while part II delves into the intricate relationship between X ray energy and radiation transmission across various thin film materials.

Let's delve into the key feature of Alpha Beta ($\alpha\beta$) Nonlinear Math:

### 1.1. Categorization of Continuous Numbers

- Continuous numbers can be classified as either linear or nonlinear based on the presence or absence of asymptotes.
- Notably, asymptotes are never part of the nonlinear numbers.

### 1.2. Two Types of Zero

- Linear Zero: Sandwiched between positive and negative numbers.
- Nonlinear Zero: Serves as the baseline asymptote for nonlinear numbers. Nonlinear numbers can approach nonlinear zero but never quite reach or touch it.

### 1.3. Mathematical Axioms Governing Nonlinear Math

- Axiom I: Emphasizes continuity, asserting that continuous numbers are dynamic, non-terminating, and perpetually maintain their continuity.
- Axiom II: Declares that asymptotes are never part of nonlinear numbers; they remain approachable but unattainable.

### 1.4. The 10-Based Logarithmic Scale

- Nonlinear numbers adhere to this scale.
- Its characteristic is the existence of a nonlinear zero, which is approachable but cannot be touched or plotted on the rectilinear coordinate graph, e.g., the baseline nonlinear zero $Yb = (0)$ and baseline nonlinear zero $Xb = (0)$ cannot be plotted in the rectilinear graph.

## 2. We Need New Nonlinear Math That Can Address Asymptotes

Many physical phenomena are associated with nonlinear variables where the variables have continuity and are associated with asymptotes such that the two variables can be expressed with an asymptotic concave or convex curve. These concave and convex curves can be in an ordinary or secondary nonlinearity.

Based on these asymptotic curves and their inherited proportionality, we can derive various differential and integral equations to express/interpret the physical phenomena.

## 3. Graphical Expressions of Basic Equations

Let's explore the graphical expression of basic equations, focusing on nonlinear variables and their asymptotic behavior, but first let's familiarize ourselves with the following terms and definitions specific to the ACP nonlinear math.

1) Face Values and True Values:

- When plotting numbers on a graph, we distinguish between face values and true values.
- for linear numbers, face values are equivalent to true values.
- for nonlinear numbers, the nonlinear face values represent measurements relative to their asymptotes. Examples include $(Yu - Y)$, $(Y - Yb)$, and $(qYu - qY)$.
- True values of nonlinear numbers are obtained by applying nonlinear logarithmic transformation to the face values, such as $q(Yu - Y)$, $q(Y - Yb)$, and $q(qYu - qY)$.

2) Linear vs. Nonlinear Equations:

- Linear cases involve equations like $(dY = KdX)$, where the change in linear $(Y)$ is proportional to the change in linear $(X)$, with $(K)$ as the proportionality constant.
- When dealing with nonlinear numbers, we encounter differential equations like $(d(q(Yu - Y)) = KdX))$. Here, the <u>change</u> in nonlinear true values $(q(Yu - Y))$ is proportional to the <u>change</u> in linear true value $(X)$, or the <u>nonlinear change</u> in face values $(Yu - Y)$ is proportional to the <u>linear change</u> in $(X)$.

3) Types of Graphs:

- Primitive Elementary Graphs: These depict vertical elementary $(y)$ against horizontal $(X)$.
- Primary Graphs: They illustrate cumulative $(Y)$ or demulative $(Y)$ against cumulative $(X)$.
- Leading Graphs: These feature asymptotic curves with continuously changing slopes.
- Proportionality Graphs: Characterized by straight lines expressible as two-parameter proportionality equations.

4) Comparing Variables: We can express comparisons between variables in three groups shown in **Table 1** for addressing dependent variable $Y$:

- Group A: The change in linear $(Y)$ is proportional to the change in linear $(X)$ (Equation. (1)).
- Group B: The change in nonlinear $(Y)$ is proportional to the change in linear $(X)$ (Equation (2) has a baseline asymptote $Yb$, Equations (3) and (4) have an upper asymptote $Yu$).
- Group C: The change in nonlinear $(Y)$ is proportional to the change in nonlinear $(X)$ (Equations (5), (6), and (7)).

Table 1. Basic differential and integral equations.

| group | Eq. ( ) | Differential Equation | Eq. ( ) | Integral Equation | Asymptote in Y |
|---|---|---|---|---|---|
| | | | | | |
| A | 1 | dY = -KdX | 1a | Y = -KX + C | 0 |
| | | | | | |
| B1 | 2 | d(q(Y – Yb)) = -KdX | 2a | q(Y – Yb) = -KX + qC | 1 (single Yb) |
| | | | | | |
| B2 | 3 | d(q(Yu – Y)) = -KdX | 3a | q(Yu – Y) = -KX + qC | 2 (one Yb hiding) |
| | 4 | d(q(qYu – qY)) = -KdX | 4a | q(qYu – qY) = -KX + qC | 2 (one Yb hiding) |
| | | | | | |
| C | 5 | d(q(Y – Yb)) = -Kd(q(X – Xb)) | 5a | q(Y – Yb) = -K(q(X – Xb)) + qC | 1 (single Yb & Xb) |
| | 6 | d(q(Yu – Y)) = -Kd(q(X – Xb)) | 6a | q(Yu – Y) = -Kq(X – Xb) + qC | 2 (one Yb hiding, 1 Xb) |
| | 7 | d(q(qYu – qY)) = -Kd(q(X – Xb)) | 7a | q(qYu – qY) = -K(q(X – Xb)) + qC | 2 (one Yb hiding, 1 Xb) |

Note: the above equations have two parameters K and C; when C = 1, qC = q1 = 0; independent variable be either dX or d(q(X – Xb)).

Table 1 gives the basic differential and integral equations of the ACP nonlinear math, where $K$ is a proportionality constant and $C$ is an integral constant for dictating the position of the straight-line in the graph. We also call the $C$ as a position constant. In Part I of this article, we discuss Equation (3) for ideal arithmetic data, and in Part II discuss Equation (7) for the relationship between the X ray energy and the radiation transmission of thin films.

### 3.1. Graphs and Equations in Linear-By-Linear Cases (Equation (1), (1a))

Equations (1) and (1a) describe the general linear-by-linear phenomena, where the change of linear $Y$ is proportional to the change of linear $X$, as shown in Figure 1. The three straight lines indicate the change of $Y$ is proportional to the change of $X$ (*i.e.*, Equation (1)) and have two parameters $K$ and $C$ in its integral Equation (1a). The three $K$ (−1.5, −3, and 3) give the directions and slopes of the straight lines, and the three $C$ (64, 0, and 30) give the position of the straight lines. All three lines can extend continuously forever in two directions and all pass through linear zero.

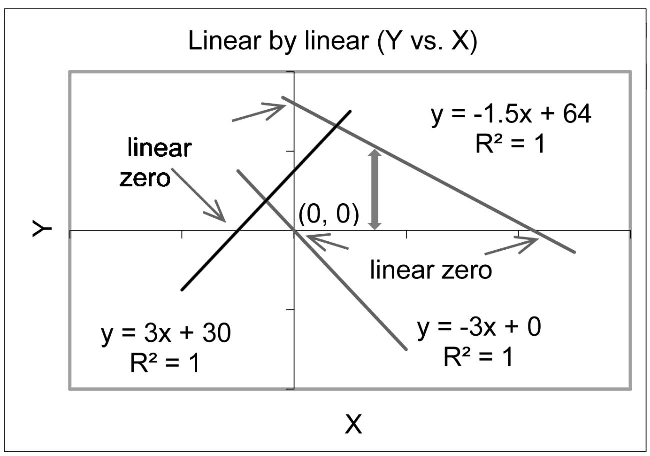



Figure 1. Linear by linear phenomenon.

### 3.2. Graphs and Equations in Nonlinear Cases

**Figure 2** gives four asymptotic curves, where **Figures 2(a)-(c)** are for ordinary order of nonlinear variable $Y$, with equations corresponding to Equation (2), Equation (3), and Equation (5). **Figure 2(d)** is for the second order of nonlinear variable $Y$ with equation corresponding to Equation (4). Solid double-arrows are for nonlinear face values. Dashed double-arrows are for linear values. In **Figure 2(d)**, the horizontal $X$ can either be a linear $X$ or be a nonlinear $X$. In a nonlinear case, its nonlinear face value is $(X - Xb)$.

In **Figure 2**, the solid double arrows are a measurement of face values relative to the upper and baseline asymptotes $Yu$ and $Yb$. The dashed double arrows are measurement of face values $X$ relative to zero. The relationship between the solid double arrows and dashed double arrows is that as the solid arrow increases the dashed arrow decreases, or vice versa. For example, in differential equation forms, in Equation (2), it is the change of $(Y - Yb)$ is negatively proportional to the change of $X$; or in Equation (3) the change of $(Yu - Y)$ is negatively proportional to the change of $X$; or in Equation (4) the change of $(qYu - qY)$ is negatively proportional to the change of $X$.

**Figure 3** gives their corresponding proportionality plot for Equations (2)-(5). In **Table 1**, based on dependent variable $Y$, we classify Equations (2), (3), (5) and (6) as first order nonlinear differential equations, where we have a single "$q$" in $Y$ variable; and classify Equations (4) and (7) as second order nonlinear differential equations, where we have 2 "$q$" in $Y$ variable.

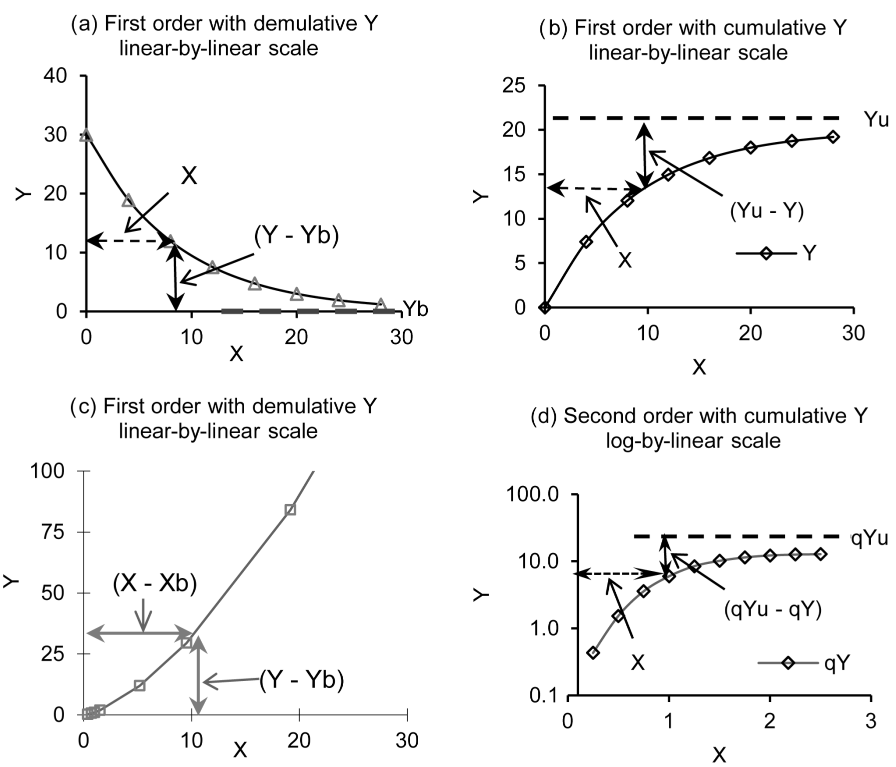



**Figure 2.** Asymptotic curves: (a) is first order with demulative $Y$; (b) is first order with cumulative $Y$; (c) is first order with demulative $Y$; (d) is second order with cumulative $Y$.

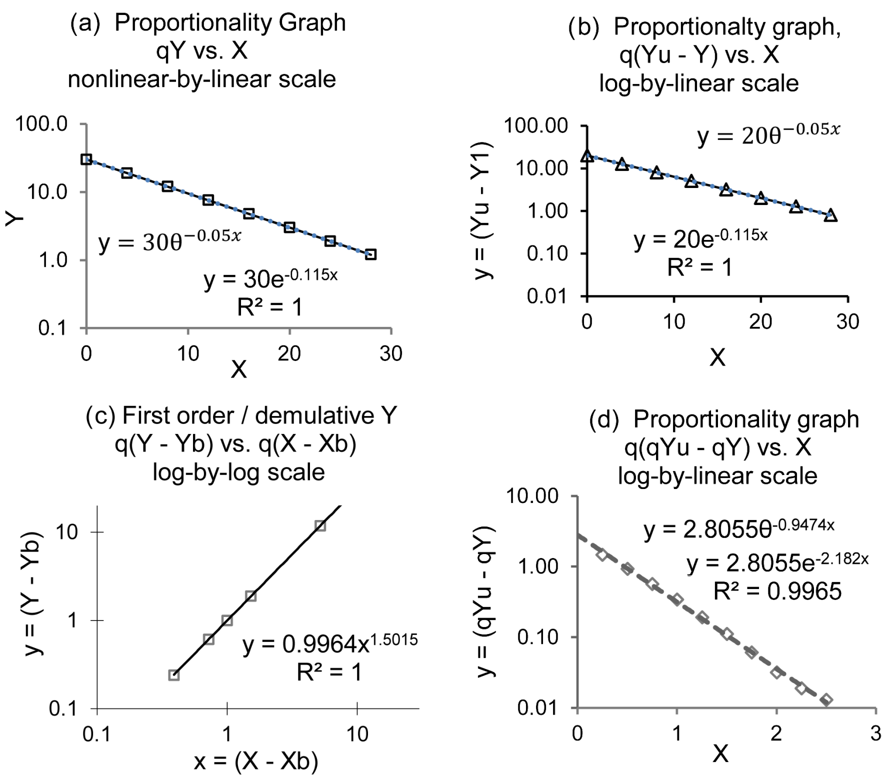



**Figure 3.** Proportionality plots: (a) is derived from **Figure 2(a)** with demulative *Y*; (b) is derived from **Figure 2(b)** with cumulative *Y*; (c) is derived from **Figure 2(c)** with demulative *Y* and demulative *X*; (d) is derived from **Figure 2(d)** with second order nonlinearity in cumulative *Y*.

## 4. Principle for Identification of Unique or Optimal Upper Asymptote *Yu*

A convex asymptotic curve is always associated with a unique upper asymptote. In analyses we may not necessary get that unique asymptote, but we can use guided estimation to identify an optimal asymptote based on the last value of *Y* data. In principle, we use the last *Y* (the largest acquired number) value as base to generate 7 - 8 estimated *Yu*, as shown in **Figure 4(a)**, where the COD (coefficient of determination) for comparing of two variable in a given estimated *Yu*

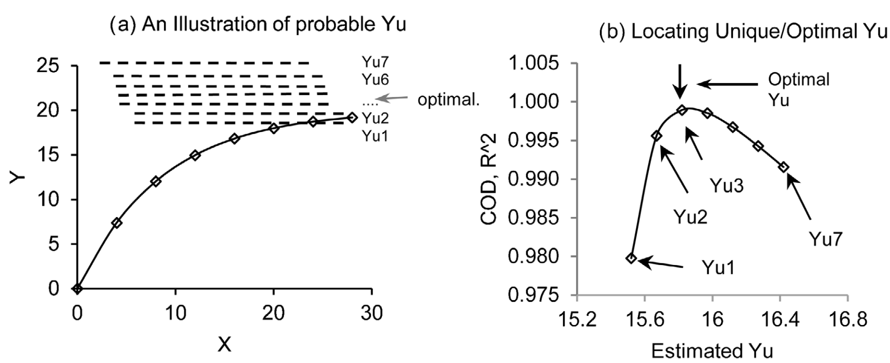



**Figure 4.** (a) An illustration for assigning estimated *Yu*, and (b) Locating unique *Yu* with estimated *Yu* vs. COD ($R^2$).

increases from the base to reach an optimal then decrease, as shown in Figure 4(b), e.g., the COD increase from *Yu*0 to *Yu*1 and *Yu*2 to reach a maximum at *Yu*3, then decrease to *Yu*4 toward *Yu*7. COD is a measure of the goodness of fit for a straight-line to relating two variables.

## 5. An Illustration with Ideal Nonlinear Numbers 0.9, 0.99, 0.999, 0.9999, 0.99999…

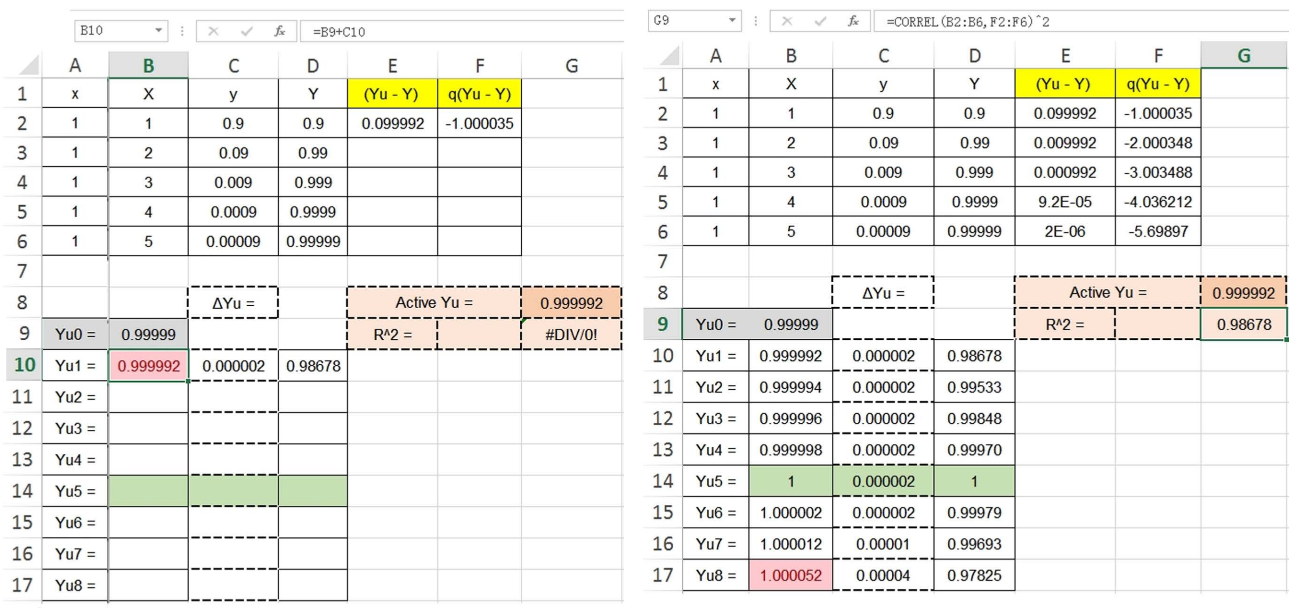
In the traditional math classes, students are taught that 0.999999… or 0.9, 0.99, 0.9999… = $0.\dot{9}$ = 1. This expression is erroneous because 0.9, 0.99, 0.999, 0.9999… are dynamic non-terminating and forever continuously moving nonlinear numbers, they cannot equate to 1, because the "1" is static, never moves and never being a part of the nonlinear numbers. What is wrong is the use of an equal sign "=". The correct expression should be 0.9, 0.99, 0.999, 0.9999, 0.99999…… → 1. We can use "→" or "~" but not "=". A dynamic moving number cannot equate to a static number, Newton's law cannot be violated.

In this analysis, we aim to demonstrate that the number "1" serves as the unique asymptote for nonlinear numbers. To illustrate this, we'll refer to Figure 5 (Left) (Image of Excel table).

In the table:

1) Data Columns:

- Column A: Contains elementary numbers denoted as "*x*".
- Column B: Corresponds to the cumulative number "*X*", which cumulates as we progress through the values of "*x*".
- Column C: Represents elementary numbers "*y*" (e.g., 0.9, 0.09, 0.009, etc.).
- Column D: Displays the cumulative numbers "*Y*" for succession of *X*.



Left (B10: =B9+C10)

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | x | X | y | Y | (Yu - Y) | q(Yu - Y) | |
| 2 | 1 | 1 | 0.9 | 0.9 | 0.099992 | -1.000035 | |
| 3 | 1 | 2 | 0.09 | 0.99 | | | |
| 4 | 1 | 3 | 0.009 | 0.999 | | | |
| 5 | 1 | 4 | 0.0009 | 0.9999 | | | |
| 6 | 1 | 5 | 0.00009 | 0.99999 | | | |
| 7 | | | | | | | |
| 8 | | | ΔYu = | | Active Yu = | | 0.999992 |
| 9 | Yu0 = | 0.99999 | | | R^2 = | | #DIV/0! |
| 10 | Yu1 = | 0.999992 | 0.000002 | 0.98678 | | | |
| 11 | Yu2 = | | | | | | |
| 12 | Yu3 = | | | | | | |
| 13 | Yu4 = | | | | | | |
| 14 | Yu5 = | | | | | | |
| 15 | Yu6 = | | | | | | |
| 16 | Yu7 = | | | | | | |
| 17 | Yu8 = | | | | | | |

Right (G9: =CORREL(B2:B6,F2:F6)^2)

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | x | X | y | Y | (Yu - Y) | q(Yu - Y) | |
| 2 | 1 | 1 | 0.9 | 0.9 | 0.099992 | -1.000035 | |
| 3 | 1 | 2 | 0.09 | 0.99 | 0.009992 | -2.000348 | |
| 4 | 1 | 3 | 0.009 | 0.999 | 0.000992 | -3.003488 | |
| 5 | 1 | 4 | 0.0009 | 0.9999 | 9.2E-05 | -4.036212 | |
| 6 | 1 | 5 | 0.00009 | 0.99999 | 2E-06 | -5.69897 | |
| 7 | | | | | | | |
| 8 | | | ΔYu = | | Active Yu = | | 0.999992 |
| 9 | Yu0 = | 0.99999 | | | R^2 = | | 0.98678 |
| 10 | Yu1 = | 0.999992 | 0.000002 | 0.98678 | | | |
| 11 | Yu2 = | 0.999994 | 0.000002 | 0.99533 | | | |
| 12 | Yu3 = | 0.999996 | 0.000002 | 0.99848 | | | |
| 13 | Yu4 = | 0.999998 | 0.000002 | 0.99970 | | | |
| 14 | Yu5 = | 1 | 0.000002 | 1 | | | |
| 15 | Yu6 = | 1.000002 | 0.000002 | 0.99979 | | | |
| 16 | Yu7 = | 1.000012 | 0.00001 | 0.99693 | | | |
| 17 | Yu8 = | 1.000052 | 0.00004 | 0.97825 | | | |

Figure 5. (Image of Excel table) Left: initial input; Right: partially filled with active *Yu* 0.999992.

2) Connectivity:

- Cumulative numbers exhibit connectivity, whereas elementary numbers do not.
- When comparing two numbers, it is essential to compare cumulative numbers against other cumulative numbers, rather than elementary numbers.

3) Continuous Cumulative Numbers:

- We focus on Continuous cumulative numbers, which is monotonically increasing and ensure both continuity and incremental behavior.
- The last value of "$Y$" in Column D is 0.99999 (represented by five nines in Cell D6).

4) Base of Estimated $Yu$ ($Yu0$):

- We take this value (0.99999) as the base for our estimated $Yu$.
- It is input into Cell B9.

5) Incremental $Yu$ ($\Delta Yu$):

- We enlist a small incremental value, $\Delta Yu$, in Column C (C10:C17).
- These increments will help calculate eight estimated $Yu$ values, ranging from $Yu1$ to $Yu8$.
- The first $\Delta Yu$ is 0.000002 (located in Cell C10).

6) Generating Estimated $Yu$:

- Using the formula "B10 = B9 + C10", we compute the first estimated $Yu$ in Cell B10.
- We repeat this process for the remaining seven $Yu$ values (from $Yu2$ to $Yu8$), copying the formula down the column.

Next, we reserve Cell G8 for inputting an active upper asymptote, Active $Yu$. We input $Yu1$ value in Cell B10, *i.e.*, 0.999992, into Cell G8, as shown in **Figure 5** (Right) (Image of Excel table). We shall use this active $Yu$ to calculate ($Yu - Y$) in Column E. Formula for Cell E2 is "=$G$8-D2", e.g., 0.999992 − 0.9 = 0.09992. We then copy Cell E2 into Cell E3 through Cell E6 to complete the column, shown in the right-hand side of table. Column F is for the calculation of log of column E, e.g., log(0.999992) = -1.0000347, as shown in the right-hand side table.

To obtain COD for $Yu1$, $Yu2$, $Yu3$….to $Yu8$ in Column D (D10:D17), we first input $Yu1$ value 0.999992 into Cell G8, then use formula to get COD in Cell G9. Formula for Cell G9 is "= CORREL (B2:B6, F2:F6)^2". **Figure 5** (Right) (Image of Excel table) has complete data in Column E and F; thus we get the value 0.98678. We record this value into Cell D10 parallel to $Yu1$, as 0.98678.

Let us continue to use **Figure 5** and generate **Figure 6** (Image of Excel table). First, selecting $\Delta Yu$:

- Our goal is to choose 4 $\Delta Yu$ values that generate numbers between 0.99999 (five nines) and 0.999999 (six nines) and select 3 $\Delta Yu$ slightly above 1 for Column B (B10:17), as shown in Column C (10: 17) of **Figure 6**.

Then, by inputting $Yu5$ value "1" into Active Cell G8, whence all the ($Yu - Y$), $q(Yu - Y)$, and the COD value in Cell G9 all changed. The Formula bar for COD is shown on the top line of the table. We record the COD value in Cell G9 into Cell D14 parallel to $Yu5$ line. We sequentially input the rest value of $Yu2$, $Yu3$, *Yu*4, etc. value into Cell G8 and sequentially record the obtained COD in Cell G9 into Colum D (D10:D17).

What we have in mind is that when the estimated *Yu* is between the last *Y* (0.99999) and the unique *Yu*, the COD for comparing two equation variables, e.g., *q* (*Yu* − *Y*) versus *X* and *q* (*qYu* − *qY*) versus *X*, is less than 1 but will increase toward 1. After reaching 1, the COD will decrease as the estimated *Yu* increases. By plotting Column B (B10:B17) vs. Column D (D10:D17) in **Figure 6** (Image of Excel table), we obtain **Figure 7** for Estimated *Yu* vs. $R^2$ in a linear scale, where the data line initially increases to reach the maximum of unique asymptote at $R^2 = 1$ and then decreases.

By plotting Column F (F2:F6) versus Column B (B2:B6) for *q* (*Yu* − *Y*) versus *X* in linear-linear scale, we obtain **Figure 8(a)** showing the data line on a

G9 | =CORREL(B2:B6,F2:F6)^2

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | x | X | y | Y | (Yu - Y) | q(Yu - Y) | |
| 2 | 1 | 1 | 0.9 | 0.9 | 0.1 | -1 | |
| 3 | 1 | 2 | 0.09 | 0.99 | 0.01 | -2 | |
| 4 | 1 | 3 | 0.009 | 0.999 | 0.001 | -3 | |
| 5 | 1 | 4 | 0.0009 | 0.9999 | 1E-04 | -4 | |
| 6 | 1 | 5 | 0.00009 | 0.99999 | 1E-05 | -5 | |
| 7 | | | | | | | |
| 8 | | | ΔYu = | | Active Yu = | | 1 |
| 9 | Yu0 = | 0.99999 | | | R^2 = | | 1.00000 |
| 10 | Yu1 = | 0.999992 | 0.000002 | 0.98678 | | | |
| 11 | Yu2 = | 0.999994 | 0.000002 | 0.99533 | | | |
| 12 | Yu3 = | 0.999996 | 0.000002 | 0.99848 | | | |
| 13 | Yu4 = | 0.999998 | 0.000002 | 0.99970 | | | |
| 14 | Yu5 = | 1 | 0.000002 | 1 | | | |
| 15 | Yu6 = | 1.000002 | 0.000002 | 0.99979 | | | |
| 16 | Yu7 = | 1.000012 | 0.00001 | 0.99693 | | | |
| 17 | Yu8 = | 1.000052 | 0.00004 | 0.97825 | | | |

**Figure 6.** (Image of Excel table) Data calculation with *Yu*5 = 1.

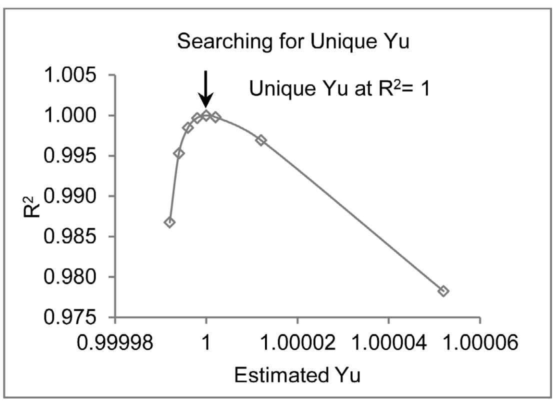



**Figure 7.** Searching for Unique *Yu*.

straight-line and a regression equation $y = -1x + 8E-13$ or $q(Yu - Y) = -1X + 8E-13$, where the proportionality constant $K$ is 1 ($KX = 1X$) and equation COD is at $R^2 = 1$. Alternatively, by plotting Column E (E2:E6) versus Column B (B2:B6) for $(Yu - Y)$ versus $X$ in log-linear scale, we obtain **Figure 8(b)** showing a straight-line and a regression equation $y = 1\theta^{-x}$ or $1e^{-2.303x}$. ($\theta = 10$).

Next, let us copy the worksheet of **Figure 6** (Image of Excel table) (with active $Yu$ in Cell G8 at 1) into four new worksheets in Excel, and with each one worksheet change the active $Yu$ in Cell G8 into 1.00052, 1.000052, 0.999992, and 0.9991. Subsequently, let us copy **Figure 8(a)** into **Figure 9**, and then click inside **Figure 9** to get Excel manual of "select data", as shown in **Figure 10**. After sequentially adding the four data sets, we obtain **Figure 9**, indicating we get the straight-line only at $Yu = 1$ with $R^2 = 1$, other values of estimated Yu give either concave bend up curve or convex bend down curves.

Overall, we use four depositions in **Figures 6-9** to confirm that the nonlinear numbers 0.9, 0.99, 0.999, 0.9999…have a unique upper asymptote 1. (A): Green Cells of Cell B14 and Cell D14 in **Figure 6** (EI template) have the highest estimated $Yu$ of 1 and the largest COD of 1; (B): **Figure 7** for Estimated $Yu$ vs. $R^2$

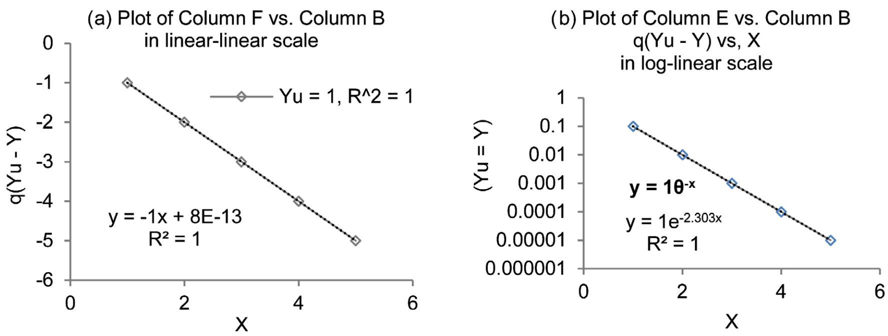



**Figure 8.** (a) Plot of Column F vs. Column B, (b) Plot of Column E vs. Column B.

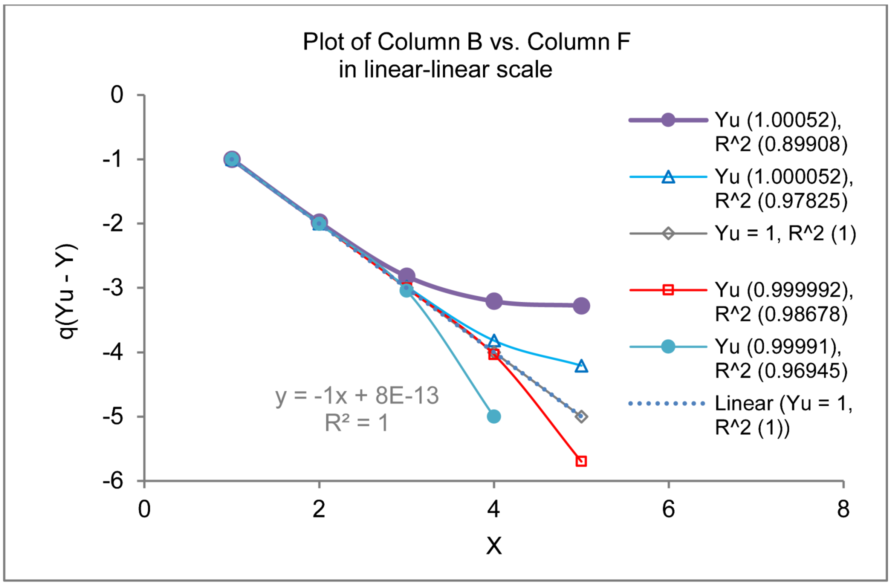



**Figure 9.** Plot of Column B vs. Column F.

shows the data line initially increases to reach the maximum of unique asymptote at $R^2 = 1$ and then decreases; (C): Figure 8 shows the data line on a straight-line with a regression equation having the proportionality constant $K$ at 1 ($KX = 1X$) and equation COD is at $R^2 = 1$; (D): Figure 9 indicates that we get a straight-line only at $Yu = 1$ with $R^2 = 1$, other values of estimated $Yu$ give either concave bend up curve or convex bend down curves.

Overall, to view the nonlinear number 0.9, 0.99, 0.9999…in graph form, we can summarize them in sequence as: primary graph, leading graph, pre- proportionality graph, and proportionality graphs. Referring to Figure 6 (Image of Excel table), by plotting Column D (D2:D6) vs. Column B (B2:B6) for $Y$ vs. $X$, we obtain a primary graph, as shown in Figure 11. It is also a leading graph due to its continuous change in the slope of the line. By placing an upper asymptote $Yu$ line as reference line, we consider the graph as a leading graph for deriving the differential Equation (3). Where it indicates that the nonlinear change in face value ($Yu - Y$) is negatively proportional to the linear change of $X$, as shown in the graph, *i.e.*, we see that as the solid double arrow increases the shaded double arrow decreases, or vice versa. In this case, the primary graph is also a leading graph.

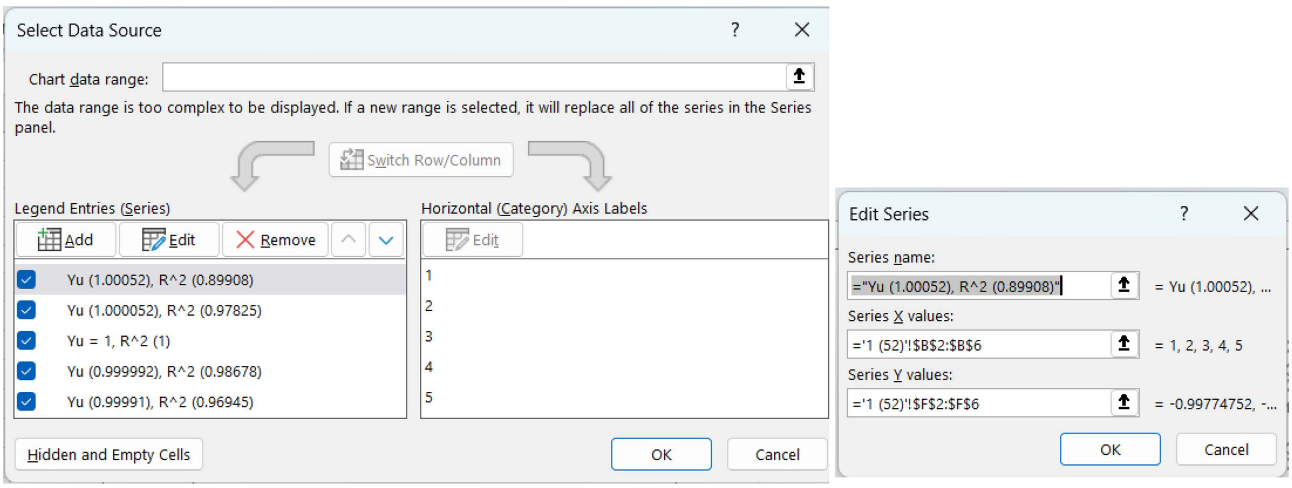



**Figure 10.** Excel "Select Data" manual and series input.

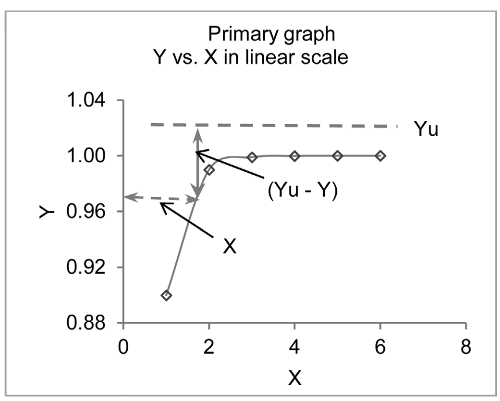



**Figure 11.** Primary graph for nonlinear number 0.9, 0.99, 0.999…

By plotting Column E (E2:E6) vs. Column B (B2:B6) for ($Yu - Y$) vs, $X$, we obtain **Figure 12** as a pre-proportionality graph. Subsequently, by converting its y-axis into logarithmic scale, we obtain the proportionality graph **Figure 8(b)** in log-by-linear scale. Alternately, when we plot Column F (F2:F6) vs. Column B (B2:B6), we obtain proportionality graph **Figure 8(a)** in linear-by-linear scale.

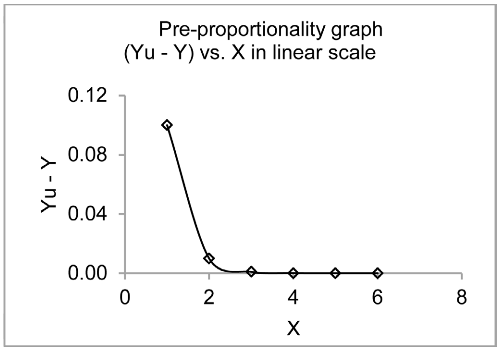



**Figure 12.** Pre-proportionality graph for nonlinear number 0.9, 0.99, 0.999…

## 6. Relationship between X-Ray Energy and X-Ray Transmission of Diamond, Be, and Kapton Films

In the last section we have discussed the simple case of nonlinear-by-linear phenomenon, where the nonlinear numbers have a single nonlinearity and a single asymptote. In this section we expand the nonlinear phenomena to include a higher order of nonlinearity and a possibility of nonlinear numbers having two asymptotes by using an example to relate X-ray transmission versus x-ray energy.

The relationship between the transmissions of x-rays through beam line windows and x-ray energy in the soft x-ray energy range is a nonlinear-by-nonlinear phenomenon. As part of their research at the Brookhaven National Laboratory, Monowitz and Gordon compared the differences in transmission of soft x-rays through windows comprised of different materials, such as beryllium (Be), Kapton, and diamond [1]. **Figure 13** gives the transmissions ($T = I/I_0$) of diamond film, Be, Kapton, and a combination of Be plus Kapton versus the x-ray energy. This graph is plotted on a semi-log (log-linear) scale, where the slope of the curves is changing continuously, indicating that we can describe the physical phenomenon with a simple equation. This graph is called a leading graph because it can lead us to a proportionality graph where the straight line in the graph can directly provide desired equation for describing physical phenomenon.

Now, let us convert the scale of y-axis in **Figure 13** into a linear scale. We obtain **Figure 14** showing a series of sigmoid curves. This graph is a plot of cumulative numbers of $Y$ versus cumulative numbers of $X$ on a Cartesian coordinate graph and is called a primary graph. This type of graph is a familiar graph that one may encounter in many science and engineering data presentations. **Figure 13** and **Figure 14** indicate that $Y$ numbers are nonlinear numbers and that $Y$ numbers have an upper asymptote $Yu = 1$ and a baseline asymptote $Yb = (0)$ (Note: nonlinear logarithmic scale always has nonlinear zero as its bottom asymptote, which

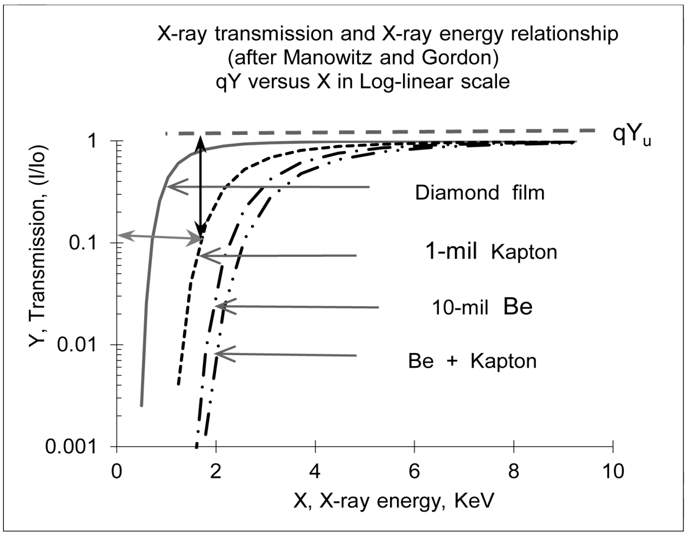



**Figure 13.** X-ray transmission vs. X-ray energy, log-by-linear scale.

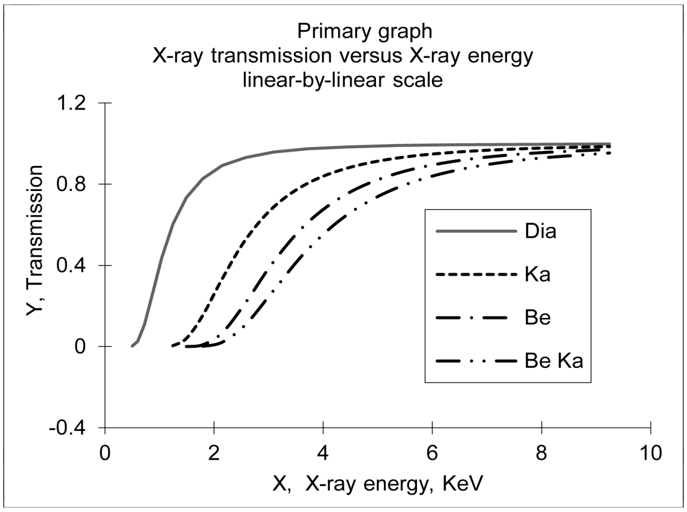



**Figure 14.** X-ray transmission vs. X-ray energy, linear-by-linear scale.

cannot be reached or touched, and cannot be shown on the scale).

Next, let us examine two double arrows along the curve in Figure 13. Since the slope of the curve changes continuously, it reveals that as the vertical distance of solid double arrow from the upper asymptote, ($qYu - qY$), increases, the horizontal distance of double arrow measured from the bottom asymptote, ($X - Xb$), decreases, or vice versa. By plotting these two arrow quantities as nonlinear face-values on a log-log graph, we obtain four straight lines, as shown in Figure 15.

Figure 16 is a graph of $dY/dX$ vs. $X$, where a series of skewed bell-shaped curves are shown. This graph is called a primitive graph. A primitive graph (e.g., Figure 16) is different from a primary graph (e.g., Figure 14). The primary graph is a plot of cumulative $Y$ numbers vs. cumulative $X$ number with monotonic increasing of both $Y$ and $X$, while the primitive graph has data up and down.

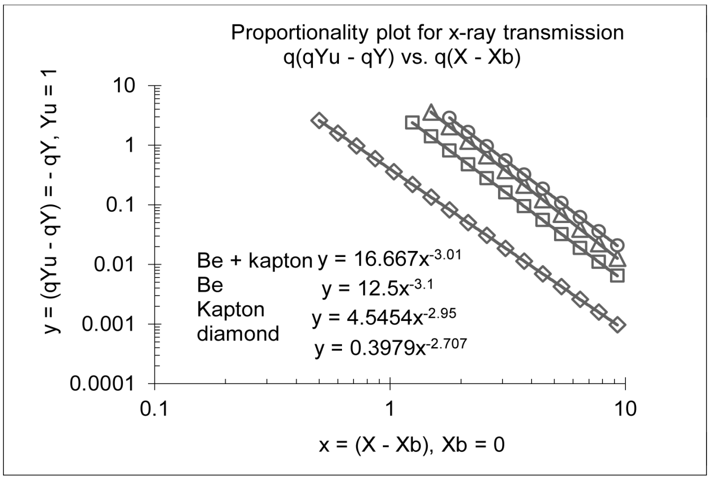



**Figure 15.** Proportionality plot for X-ray transmission.

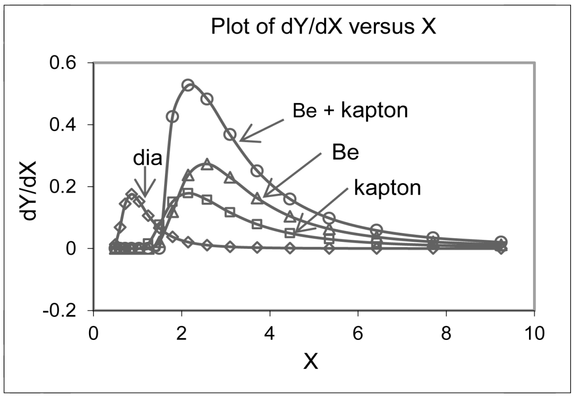



**Figure 16.** Plot of $dY/dX$ vs. $X$.

The equation of the straight line in **Figure 15** is Equation (7a) in **Table 1**, where $K$ is the proportionality constant, and $C$ is a position constant or integral constant. The differential form of the equation is Equation (7) in **Table 1**.

It states that the change of true-values $q(qYu - qY)$ is negatively proportional to the change of true-values $(X - Xb)$, or the nonlinear change of face-values $(qYu - qY)$ is negatively proportional to the nonlinear change of face-values $(X - Xb)$. Nonlinear face-values are to be calculated and plotted on logarithmic nonlinear scale, as shown in **Figure 15**, while we can also plot true-values vs. true-values on linear-linear scale to show the 4 straight lines (not shown).

Since $Xb = (0)$, $Yu = 1$ and $\log 1 = 0$, Equation (6) simplifies to $-q(-qY) = -KqX + qC$. The four straight lines for diamond film, Be, Kapton, and Be plus Kapton have $K$ values given as 2.71, 2.95, 3.10, 3.01 and $C$ values are 0.3979, 4.5454, 12.5, and 16.6667, for diamond film, Kapton, Be, and Kapton + Be, respectively. These straight lines suggest that the proportionality exists and that there is a simple physical law governing the relationship between x-ray transmission and x-ray energy.

In the above phenomenon, we simply take the $Y$ numbers and calculate its logarithm $qY$ and plot - $qY$ versus $qX$ on a log-log nonlinear scale to obtain the final proportionality graph. There is no need to search for asymptotes because one (bottom) asymptote is nonlinear zero, $Xb = 0$ and the other (upper) asymptote is $Yu = 1$, and $\log 1 = 0$.

In the data collection, Figure 13 shows only the asymptotic convex curve without showing the nonlinear nature of the curve, e.g., it did not show the nonlinear additive nature of Be plus Kapton from the individual Be and Kapton. However, by advancing the analysis to include Figures 14-16, we have more understanding of the nonlinear nature of the nonlinear phenomena. For example, it should be interesting to notice, from examining Figure 13, that the algebraic additive property of logarithms is applicable to the combination of films, e.g., for a given x-ray energy, transmission of Kapton ($Y1$) times transmission of Be ($Y2$) equals transmission of Kapton plus Be ($Y12$), *i.e.*,

$$Y1 \times Y2 = Y12 \quad \text{or}$$

$$\log Y1 + \log Y2 = \log Y12$$

The numerical examples are:

at $X = 4$ KeV, $Y1 = 0.83$, $Y2 = 0.68$, and $Y12 = 0.56$, which gives $0.83 \times 0.68 = 0.56$;

at $X = 3$ KeV, $Y1 = 0.64$, $Y2 = 0.38$, and $Y12 = 0.24$, which gives $0.64 \times 0.38 = 0.24$.

The above knowledge may help researchers in advancing their design of experiments.

A nonlinear phenomenon of higher order of nonlinearity, in general, can have multiple graphs including primitive graph, primary graph, leading graph and proportionality graph, as demonstrated in the above example. In this type of nonlinear phenomenon, we will obtain a straight-line on a log-log graph when plotting one nonlinear face-value on a logarithmic scale versus the other nonlinear face-values on the other logarithmic scale. The data will yield a convex asymptotic curve on a semi-log graph as leading graph and a sigmoid curve on a linear-by-linear graph as primary graph.

In summary, this physical example is the simplest nonlinear phenomenon that happened to be without any need of math skill to solve or search for asymptotes. In other examples we may encounter a more complex situation and require intense search for asymptote to understanding the real nonlinear phenomena.

## 7. Discussions

Discussion on Nonlinear Numbers and Asymptotic Curves

We explore the use of nonlinear numbers, specifically 0.9, 0.99, 0.999, 0.9999, and 0.99999, to illustrate the concept of convex asymptotic curves. Our goal is to derive a differential equation expressed as: [$d(q(Yu - Y)) = -KdX$]. Here:

- ($Y$) represents a variable that exhibits nonlinear behavior.
- ($X$) represents a linear variable.
- ($Yu$) denotes the upper asymptote.
- ($Yu - Y$) represents the nonlinear face value, and ($q(Yu - Y)$) represents the

nonlinear true value.

- We systematically input data to an Excel table, using the last acquired ($Y$) value as the base to estimate upper asymptotes.
- By assigning an active upper asymptote, we calculate the nonlinear face and true value.
- We then determine the coefficient of determination (COD) for all estimated upper asymptotes.
- The resulting table and graph confirm the existence of a unique upper asymptote.
- We conclude that the nonlinear numbers (0.9, 0.99, 0.999, 0.9999, 0.99999) should be expressed as approaching 1, rather than using an equal sign (e.g., $0.\dot{9} = 1$). The latter is incorrect.

In a previous publication, we extended this analysis to include 9 nonlinear numbers up to 0.999999999, yielding consistent results [2]. The true is that the nonlinear numbers 0.9, 0.99, 0.9999… are an ideal nonlinear number having a unique upper asymptote $Yu$, $Yu = 1$. It is a one-sided nonlinear number without a bottom asymptote. In general cases, a nonlinear number has both baseline asymptote and an upper asymptote, with baseline asymptote assumes nonlinear zero.

Expression of X-Ray Energy and Radiation Transmission

The second part of presentation for the expression of the relationship between X-ray energy and the radiation transmission of various thin film materials is a good illustration to demonstrate the connectivity among asymptotic convex curve, sigmoidal curve, straight-line proportionality graph, and various skewed bell primitive graph, all arise from the second order nonlinear equation. This example is the simplest case of the second order nonlinear phenomenon that assumes an upper asymptote of $Yu = 1$ and baseline asymptote of $Yb = (0)$. In general cases all physical phenomena need to identify their upper asymptote $Yu$, as will be illustrated in Part 3 and 4 of this series [3].

In our analysis, we compare the monotonic increasing nonlinear numbers $Y$ with the other monotonic increasing nonlinear numbers $X$. Specifically; we compare their nonlinear face value or true value with each other. The analysis is simple and easy such that every high school student can do it. In contrast, the traditional analysis may involve tedious calculations or may need help with commercial software or help from statistician. Moreover, we need minimal numbers of sampling around 7 to 10.

Traditional $XY$ math is insufficient to describe the nonlinear phenomena; we need to extend the $XY$ math into the $\alpha\beta$ Math to account for the existence of asymptotes, *i.e.*, we need to extend $XY = \{(X),(Y)\}$ into $\alpha\beta = \{\alpha(Y, Yu, Yb), \beta(X, Xu, Xb)\}$. The $\alpha\beta$ Math classifies continuous monotonic numbers into linear and nonlinear numbers. Nonlinear numbers are associated with asymptotes, and their measurement of difference is the face value of the nonlinear numbers. The nonlinear face value can be a difference, a ratio of difference, or with logarithmic transformation (a logarithmic transformation is also called the nonlinear transformation), such as $(Yu - Y)$, $(Y - Yb)$, $(Yu - Y)/(Y - Yb)$, and $(qYu - qY)$.

The Alpha Beta ($\alpha\beta$) Math is a science for connecting a straight line to concave and convex asymptotic curves, sigmoid, and various bell curves such as in biomedical and physical sciences [4]. We provide illustration for building Excel Templates to solve for upper asymptotes and building a straight-line proportionality equation.

## 8. Conclusions

We use ACP (asymptotic curves based and proportionality oriented) nonlinear math for mathematical analysis and graphical interpretation. In the realm of nonlinear mathematics, we encounter continuous variables that exhibit asymptotic behavior. These variables necessitate careful consideration in their differential equation and graphical representations. Specific items are:

1) We use the ACP nonlinear math to confirm that a continuous nonlinear variable needs to account for its asymptote in their differential equation and graphical expression for their changes. The variation in arithmetic data can be described by the first order nonlinear equation, while the variation in radiation transmission data can be described by the second order nonlinear equation. We can describe the second order nonlinear phenomena with second order proportionality equation and four types of graphs: primitive, primary, leading, and proportionality graphs.

2) It is important to learn how to distinguish between a primitive elementary graph and a primary graph. We cannot use a primitive elementary graph alone to build a mathematical relationship, because each elementary "$y$" has no mathematical connectivity, and one elementary number cannot mathematically relate to the other cumulative numbers. Instead, we must resort to relating one cumulative number with the other cumulative numbers and using the primary graph for mathematical analysis, where we can have both continuous numbers $Y$, and continuous numbers $X$ exist as cumulative $Y$ and cumulative $X$. Cumulative numbers are monotonically increasing numbers that guarantee continuity and the increment of number.

3) We can build a straight-line proportionality relationship for the dependent-independent variables in a log-linear and log-log graphs. This article demonstrates a methodology for solving the key upper asymptotes for the proportionality equation using Microsoft Excel via determining the "coefficient of determination". Our example includes systematic demonstration of Excel data manipulation and extensive graphing.

The $\alpha\beta$ Math classifies continuous monotonic numbers into linear and nonlinear numbers. Nonlinear numbers are associated with asymptotes, and their measurement of difference is the face value of the nonlinear numbers. The nonlinear face value can be a difference, a ratio of difference, or with logarithmic transformation (a logarithmic transformation is also called the nonlinear transformation).

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

## References

[1] Manowitz, B. and Gordon, B. (1992) Data from X-19A Beam Line. Brookhaven National Laboratory, Upton, New York.

[2] Lai, R.W., Lai-Becker, M.W. and Agathokleous, E. (2023) Zeno's Paradoxes and Lie Tzu's Dichotomic Wisdom Explained with Alpha Beta ($\alpha\beta$) Asymptotic Nonlinear Math. *JAMP*, **11**, 1209-1249. https://doi.org/10.4236/jamp.2023.115080

[3] Lai, R.W., Lai-Becker, M.W. and Cheng-Dodge, G. (2024) Application of ACP Nonlinear Math in Analyzing Astrophysics and Electrostatic Separation Data. (Ready for Publication).

[4] Lai, R.W. (2015) The Wonderful Mathematical Connectivity of Alpha Beta ($\alpha\beta$) Math—The Science of Connecting a Straight Line to Asymptotic, Sigmoid, and Various Bell Curves in Biomedical and Physical Sciences. http://www.researchgate.net/

## Symbols

$\theta = 10$

$q$ = Log (nonlinear logarithmic)

$\alpha\beta$ (extension of $XY$)

$\phi$ = (0) (nonlinear zero)

$x$ = elementary independent variable,

$y$ or ($y$) = elementary dependent variable or $y$ = equation $y$ (inside the graph)

$X$ = cumulative of $x$, $Y$ = cumulative of $y$ or ($y$)