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A Blind Spot in the Reframing of a Universe of Possibles: Towards a Suitable Model for Decision-Making Theory and A.I.

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Abstract

Bayesian inference model is an optimal processing of incomplete information that, more than other models, better captures the way in which any decision-maker learns and updates his degree of rational beliefs about possible states of nature, in order to make a better judgment while taking new evidence into account. Such a scientific model proposed for the general theory of decision-making, like all others in general, whether in statistics, economics, operations research, A.I., data science or applied mathematics, regardless of whether they are time-dependent, have in common a theoretical basis that is axiomatized by relying on related concepts of a universe of possibles, especially the so-called *universe* (or the *world*), the *state of nature* (or the *state of the world*), when formulated explicitly. The issue of where to stand as an observer or a decision-maker to reframe such a universe of possibles together with a partition structure of knowledge (*i.e.* semantic formalisms), including a copy of itself as it was initially while generalizing it, is not addressed. Memory being the substratum, whether human or artificial, wherein everything stands, to date, even the theoretical possibility of such an operation of self-inclusion is prohibited by pure mathematics. We make this blind spot come to light through a counter-example (namely Archimedes' *Eureka* experiment) and explore novel theoretical foundations, fitting better with a *quantum* form than with fuzzy modeling, to deal with more than a reference universe of possibles. This could open up a new path of investigation for the general theory of decision-making, as well as for Artificial Intelligence, often considered as the science of the imitation of human abilities, while being also the science of knowledge representation and the science of concept formation and reasoning.

Keywords

Decision-Making, Innovation, Universe of Possibles, A.I., Quantum Form, Fuzzy Modeling

1. Introduction

Bayesian inference model is identified because—as an optimal processing of incomplete information [1]—more than other models, it better captures the way in which any decision-maker learns and updates his degree of rational beliefs about a possible state of nature θ among all enumerated ones whether in human or artificial memory (a theory, a hypothesis, an event, an observation, or an occurrence), in order to make a better judgment while taking into account new evidence E (new knowledge, new measurement, new sampling data, or other standpoints) [2]. Such framework operates within a universe of possibles [all possible states of nature] given beforehand. And in general, it is common practice to rely on scientific models working so, exploring and exploiting, in whole or in part, a specific pre-defined research space that is a universe of possibles.

This is true for dealing with a decision-making theory: "(…) there exists a considerable area of design practice where standards of rigor in inference are as high as one could wish. I refer to the domain of so-called "optimization methods" (…) The optimization problem is to find an admissible set of values of the command variables, compatible with the constraints, that maximize the (expected value of the) utility function for the given values of the environmental parameters" ([3], p. 116). This is true especially when complex situations are being analyzed and decision makers are called upon to make optimized choices (e.g. minimal loss, maximum gain, etc.), notably those emanating from operations research, data science, knowledge discovery in databases, Artificial Intelligence and adaptive learning (such as metaheuristics, hybrid-metaheuristics and hyper-heuristics, etc), with underlying deterministic models or stochastic ones, taking an empirical approach, with or without prior knowledge. On a cyclical basis, they collect information, and often stochastically, with a view to enhancing understanding of the problem (based on different phases that can be classified in exploration phases or diversification phases), storing it in a myriad of possible forms, whether collectively (considering the problem as a whole) or inter individually (considering one solution in relation to another), then sorting through it so as to reduce dispersion (in the phase called exploitation or intensification).

This is true also for pure or applied mathematics, economics, statistics, or design science. What these scientific models have in common, regardless of whether they are time-dependent, is a theoretical basis that is axiomatized by relying on related concepts of "the universe" (or "the world"), "the state of nature" (or "state of the world"), or the "development of a state of nature" (or a true "state of the world") where knowledge is partitioned as if it were always there, somewhere, in one part or another of the "universe of possibles" [3] [4] [5] [6]. "The world (is) the object about which the person is concerned. A state (of the world) (is) a description of the world, leaving no relevant aspect undescribed. The true state (of the world) (is) the state that does in fact obtain, *i.e.*, the true description of the world" ([4], p. 9). This means that irrelevant aspects of a description of the world may be undescribed, provided irrelevance/relevance is not time-dependent, in accordance with scientific knowledge.

A change in representation thus results in a convenient redistribution of probabilities attached to existing possible states of nature θ (hypothesis or theories, etc.) after the addition of previously unknown or uncertain observations E. Such a redistribution is obtained by Bayesian conditioning of probabilities attached to θ and E, while ensuring that the total always stands at 1 according to the theory of probabilities [4]. In other words, the decision-maker's[1] scientific knowledge space is expanding whenever an event obtains at a state of the world (*i.e.* of the universe of possibles) including such an information set: "For [a player] to know [an event] is itself an event" ([5], p. 264). This is to say: "Semantic formalism consists of a partition structure [of a space of states of the world]" (Ibid)[2]. But it turns out that a counter-example shows a change in representation which involves a reframing of the universe of possibles [all possible states of nature θ_j], relying on the extension [7] to a novel partition structure of a novel space of states of nature θ_j^* (novel hypothesis or theories, etc.) based on an ancient partition structure of a prior space of states of nature θ_j (hypothesis or theories, etc.). The counter-example in question is namely Archimedes' thought experiment, which led to his famous "Eureka" moment[3] (that word being at the root of "heuristics"), by using a reproducible method described in the Palimpsest[4]. Hence, as a consequence, scientific model foundations must shift and evolve. There is a blind spot in such a reframing of a universe of possibles [all possible states of nature] as it raises the issue of how but also wherein—in which fuzzy multiverse of possibles or quantum universe of possibles—it is supposed to take place.

The article is structured as follows: In the first section we shall present the Bayesian inference modeling proposed for general decision-making theory or Artificial Intelligence. In the second section, we shall consider the emblematic counter-example of Archimedes' Eureka, which involves a reframing of the universe of possibles [all possible states of nature] unlike the Bayesian inference modelling, while revealing a blind spot with a moment of undecidability. In the third section, we shall discuss new theoretical foundations of reframing the un-

[1]Also called the player.

[2]"a space Ω of states of the world, together with a partition of Ω for each player, whose atoms represent information sets of that player; Ω is called the universe. Like in probability theory, events are subsets of Ω; intuitively, an event is identified with the set of all those states of the world at which the event obtains. Thus, an event E obtains at a state ω if and only if ω ∈ Ω, and a player *i* 'knows' E at Ω if and only if E includes his information set at ω. For *i* to know E is itself an event denoted KÆ: it obtains at some states ω and at others does not" (Ibid).

[3]An interjection taken from the Greek (ερηκα) translated as "I found it".

[4]An interjection taken from the Greek (*ερηκα*) translated as "I found it".

iverse of possibles [all possible states of nature], that it would be tempting to formalize with fuzzy modeling, but nonetheless fits better with a quantic form.

2. Bayesian Inference Modeling for General Decision-Making Theory or A.I.

Bayesian inference modeling is based on interpreting p-value as a degree of rational belief (in hypothesis θ, or theory, etc.) and on conditioning that probability on knowledge of new data E (*i.e.*, evidence, observation, etc.) denoted as $p(\theta|E)$ or expressed as "the probability p of hypothesis θ given E (after getting relevant evidence)…". For the purposes of this theorem, $p(\theta)$ and $p(\theta|E)$ are, respectively, a priori and a posteriori probabilities of θ. They are brought together under the Bayes Theorem: $p(\theta|E)\cdot p(E)=p(E|\theta)\cdot p(\theta)$. In other words, when considering hypothesis or theory θ, comparing the probability that is assigned to it, before and after evidence E is obtained—$p(\theta)$ and $p(\theta|E)$ respectively –, would be equivalent to comparing in the same relationship, the probability assigned to evidence $p(E)$ and the probability assigned to the likelihood $p(E|\theta)$. By reference to probabilities, the decision-maker weighs his trust in his own choice and his perceived credibility in information received from others, before he/she begins updating his judgment.

Jaynes [1] proposed a general decision-making theory which posits an objectively derived model of Bayesian inference, including in situations where information is incomplete, requiring reliance on probabilistic inductive reasoning. "By 'inference' we mean simply: deductive reasoning whenever enough information is at hand to permit it; inductive or plausible reasoning when—as is almost invariably the case in real problems—the necessary information is not available. But if a problem can be solved by deductive reasoning, probability theory is not needed for it; thus, our topic is the optimal processing of incomplete information" [1]. The latter is intended to reflect "the phenomenon of a person who tells the truth and is not believed, even though the disbelievers are reasoning consistently. The theory explains why and under what circumstances this will happen (…) New data that we insist on analyzing in terms of old ideas (that is, old models which are not questioned) cannot lead us out of the old ideas (…) Old data, when seen in the light of new ideas, can give us an entirely new insight into a phenomenon" [1][5].

The rules to solve the problem of inference are as follows [1]: 1) enumerate the possible states of nature θ_j, discrete or continuous, as the case might be; 2) assign prior probabilities $p(\theta_j|I)$ which represent whatever prior information I you have about them, before any measurement; 3) assign sampling probabilities $p(E_i|\theta_j)$, which represent the likelihood of the measurements, that is, prior knowledge about the mechanism of measurement process yielding the possible, observable data sets E_i; 4) Digest any additional evidence $E = E_1 E_2$… (Sampling data) and, by application of Bayes' Theorem, obtain the posterior probabilities

[5]For instance, the visual perception, the discovery of Neptune, etc.

$p(\theta_j|E\ I)$, which means taking into account new data feedback as it comes in. That is the end of the inference problem and probabilities $p(\theta_j|E\ I)$ yield all information regarding the possible states of nature θ_j that can be known a posteriori. In other words, the set of available information can be factored into the calculation of probabilities $p(\theta_j|E\ I)$ pertaining to the states of nature. All of these probabilities are interrelated through Bayes' theorem:

$$p(\theta_j / E I) \cdot p(E I) = p(\theta_j / I) \cdot p(E / \theta_j I)$$

As for the final steps of the decision-making process, they are: 5) enumerate the possible decisions D_i; 6) express what was sought to be accomplished, as a function of preferences (minimize the expected loss/maximize the expected gains) by associating possible decisions with states of nature $L(D_i|\theta_j)$; and 7) make the decision D_i that leads to the most preferred expected outcome (minimized expected loss/maximum expected gain) for θ_j.

It should be noted, however, that this general theory of decision-making based on a Bayesian inference modeling does not expressly refer to the universe of possibles [all possible states of nature] (or the "world" as Savage puts it) in its equations. Insofar as the universe has been posited once and for all, there is no need to do so. Moreover, once the states of nature θ_j are formulated, information I on these states of nature, which is assumed to be held as a priori and as introduced into the decision process at step (2) through the equations, does not (but should) serve the same function. If that had been the case, information I should have been included either in every step of the process or at least at steps (3) and (6). But anyway, as will be developed later, once the universe of possibles [all possible states of nature] is reframed—relying on the extension to a novel partition structure of a novel space of states of nature θ_j^* (novel hypothesis or theory, etc.) based on an ancient partition structure of a prior space of states of nature θ_j (hypothesis or theories, etc.) –, the step (7) becomes undecidable, although a decision leading to the most preferred expected outcome for θ_j is expected to be made. Indeed, a function of preferences L associating possible decisions D_i with states of nature θ_j or rather θ_j^* (depending on the reference universe of possibles), as any function, is a binary relation which is one-one [$L(D_i|\theta_j)$] or many-one [$L(D_i; D_k|\theta_j)$], not one-many [$L(D_i|\theta_j; \theta_k; \theta_j^*)$] just as it is in the blind spot of reframing—by extension—the universe of possibles [all possible states of nature] [8]. Besides, as we can note, that is why no novel state of nature θ' (novel hypothesis or theories, etc.) is introduced in the general theory of decision-making based on a Bayesian inference modeling.

In light of the foregoing, we shall consider the emblematic counter-example of Archimedes' Eureka moment, that precisely involves a reframing—by extension – of the universe of possibles [all possible states of nature]. Forgetting then the meaning of a universe of possibles [all possible states of nature] (or the "world") previously set forth—as everything that exists and the assumption that it is possible to possess perfect knowledge about it [5]—we must turn to a consideration of what is held to be universal in accordance with a current state of scientific knowledge, in the sense of a "universality of reference". This means that, while irrelevant aspects of a description of the world may be wrongly undescribed because irrelevance/relevance is time-dependent, the description of the world is prone to be reframed by extension. Such a change in representation allows us to bear out the truth of Cédric Villani's remark, which we might paraphrase by saying that, like mathematics, an innovative idea "can change the world"[6].

3. Counter-Example: Archimedes' Eureka Moment

Before discussing Archimedes' thought experiment and explaining its non-conformity with the general decision-making theory based on Bayesian inference modeling, let us first consider how it acquired its status as a counterexample.

3.1. Status as a Counter-Example

According to Larousse (French dictionary), a theory is an organized set of principles, rules, and scientific laws used to describe and explain observed phenomena. In more formal language, a theory is a consistent set of statements containing all of its consequences. In science, a theory cannot demonstrate its own consistency. To be universally valid, it must be provable. Its experimental verification [9] would assume an infinite set of favorable outcomes (*i.e.*, outcomes of interest), which is unsustainable. As a next best alternative, a theory is therefore viewed as being constructed from incomplete information and accepted as true until it is contradicted, notably by a counter-example [10] [11]. A counter-example is an indefinitely reproducible experiment that contradicts a theory and may suffice to refute it or refine it, wholly or at least in part, making it more efficient [12]. The thought experiment that brought Archimedes to the Eureka moment, occupies the status of counter-example for the general theory of decision-making based on Bayesian inference modeling. Archimedes constructed a new representation for understanding a class of phenomena (floating bodies) and solved a decision-making situation once thought insoluble. This accomplishment was by no means fortuitous but was, rather, derived from a reproducible method that he described in a 100-page letter written to Eratosthenes[7] (276 BC to 194 BC) [13]. The letter, which was lost for nearly 2000 years, reappeared in 1906, only to be lost again until 1998 (the Archimedes Palimpsest Project). The "method" consists in having a strategically situated observer who compares an unknown object to a known one (based on a specially designed artificial model whose behavior is known). To sum up, he had created a method for modeling and simulation of an unknown object. The thought experiment was ingenious in that he had to find a way to monitor the characteristic properties of an unknown object potentially belonging to a new overarching reference universe, while reasoning within a long-standing reference universe where a known object was taken as the

[6]From statements made by 2010 Fields Medal winner at TedxParis conference held in 2012.

[7]Eratosthenes is famous for having calculated the circumference of the Earth with great accuracy (39,375 km), within 10% of the actual figure.

model. Hence, the need for the observer to be appropriately "situated". "Indeed, I assume that someone among the investigators of to-day or in the future will discover by the Method here set forth still other propositions which have not yet occurred to us" [14]. Archimedes was so proud of having used this method to determine that the ratio of the volume of a sphere to the volume of the circumscribed cylinder worked out to be two-thirds that he asked for the figure of a sphere and cylinder to be modeled in stone on top of his grave. He is also generally considered to be one of the greatest scientists of all time. "It is just possible that Archimedes, could he come to life long enough to take a post-graduate course in mathematics and physics, would understand Einstein, Bohr, Heisenberg and Dirac better than they would understand themselves" [15].

The reproducibility of Archimedes' Method stems from its scientific formulation. Like the Law of Floating Bodies, it can be discussed separately from Archimedes. Only a science can build itself on a body of knowledge that can be discussed separately from its formulators and the class of objects and phenomena to which it is applied.

3.2. A Decision-Making Situation Once Thought Insoluble

Hieron II, king of Syracuse, had chosen Archimedes to supervise an engineering project of unprecedented scale: a sailing vessel 50 times bigger than a standard ancient warship, named the Syracusia after his city. He wanted to construct the largest ship ever, which was destined to be given as a present to Egypt's ruler, Ptolemy. In Archimedes's day, no one had attempted anything like this. The Syracusia was successfully completed and arrived in Alexandria on its first and only voyage. At the core of the Syracusia story is a keel[8] (korone in Greek, corona in Latin, crown in English). This sounds like another story: Hieron II would have asked a goldsmith to craft a solid-gold crown, in tribute to the immortal gods. He had reason to suspect that some of the gold had been replaced by silver, and asked Archimedes to find out if he had been cheated. No known solution was available to solve the problem as presented. That is, the solution set was empty. At that stage, even operations research or Artificial Intelligence would have been unsuccessful. To the great benefit of science, Archimedes, upon entering his bath, noticed the increase in the water level, due to the volume of water his body had displaced, and cried "I found it! [Eurêka]" [16]. He had two lumps of pure gold and pure silver brought to him, each weighing the exact same as the crown. He immersed the bar of silver into a large vessel, with water filled to the brim, and measured the volume of displaced water. Then he repeated the experiment under exactly the same conditions, using a bar of gold, and observed that a smaller volume of water had been displaced. Next, he conducted the experiment with the solid-gold crown and noted that the volume of displaced water was greater than was the case with the gold bar. He had found out the fraud and his cry of Eureka, upon realizing that he had solved the vexing problem, has

[8]A long piece of wood or metal along the bottom of a boat that forms part of its structure and helps to keep the boat balanced in the water.

since become the emblematic cry of discovery or comprehension. It is of little real importance that the experiment probably did not transpire exactly as reported by Roman architect Marcus Vitruvius Pollio (1st century BC). What is important is the inference modeling employed by Archimedes – a line of reasoning that does not conform to the Bayesian inference modeling proposed for general decision-making theory or Artificial Intelligence, even during the experimentation stage.

3.3. Reframing the Reference Universe of Possibles

It was not simply a matter of comparing the behavior of objects in the context of a given reference universe of possibles [all possible states of nature in the air], but also of noting differences in the apparent behavior of a single object while considering the reference universes of possibles [all possible states of nature in the air and underwater] which, until then, had been deemed to be independent. Hence, the need to reason independently of the reference universe as it was originally perceived [all possible states of nature in the air] and consider a reference universe broadened to encompass all fluids [all possible states of nature in all fluids], in order to conceive a more general relationship linking the apparent weight of objects to the displaced volumes of fluid and, in this way, identify a new property, that is, volumetric weight or density: "anybody completely or partially submerged in a fluid (gas or liquid) at rest is acted upon by an upward, or buoyant, force the magnitude of which is equal to the weight of the fluid displaced by the body." The property thought to account for the expected behavior of an object is thus conditioned[9] by the reference universe of possibles in which its use is imagined. It would thus appear more logical to talk about embedding or situational properties [17] rather than intrinsic properties (independent of embedding in a referent) to account for the interpretation of observable phenomena or behaviors: the weight of an object in a fluid, its color in a luminous atmosphere, its price on a market, etc. And if certain properties are deemed to be intrinsic, it is because they are dependent on a reference universe[10] of possibles [2]. Over time, a commonsense knowledge base has been built up and ultimately come to prevail, through the force of collective representations. Imagining that things might be otherwise has proved to be a challenging reflective exercise [18].

The processual pattern can be applied to any immaterial objects (software, etc.) whose properties (weight, volume, value, etc.) and behaviors change in keeping with the reference universe of possibles in which they are embedded (situations of use, markets, etc.). Weighing choices, evaluating possibilities, gains, losses or risks, are all part of the underlying process that fuels reasoning during decision-making. The decision maker (whether an individual or an organization) need merely change the name of the reference universes of possibles,

[9]That depends on the interaction with the other substances comprising the reference universe (molecules of air, water, etc.).

[10]Set theory in mathematics (where a provable property, respectively a refutable one, depends on the system of axioms, whereas a property that is neither provable nor refutable-in other words, which is undecidable-is independent of the system of axioms).

observables and comparison tools, reconsider the possible states of nature, to identify with Archimedes' thought experiment.

3.4. A Novel Partition Structure of an Extended Space of States of Nature: Reference Universes of Possibles and Decidability

Let us consider the following statement—the two scales of a balance have to be in equilibrium to account for two objects of the same weight. According to the partition structure of the space of states of nature θ_j in the air, a decision maker would be led to accept this proposition as necessarily true. But according the partition structure of the space of states of nature θ_j underwater, a decision maker wouldn't be led to accept this proposition as necessarily true. Hence, considering both partition structures, the statement becomes undecidable (neither provable nor refutable). It is impossible to assign it a truth value (as true or false) as long as a novel partition structure of a novel space of states of nature θ_j^* in all fluids is not designed—this is to say, as long as the fluid in which the weighing was carried out is not specified. Accordingly, two scales of a balance have to be in equilibrium to account for two objects of the same volume weight or density. A way to define a variation of volume is thus to consider the weight of the object and the same weight of the fluid in which the object is immersed. Conversely, the weight of the volume of the object may differ from the weight of the same volume of the fluid in which the object is immersed. Density is not expressed in units, but as a ratio and that of water is equal to 1 as a reference. This is why the decision-maker's semantic formalism (*i.e.* the partition structure) must change together with the novel space of states of nature θ_j^*—comparison of density in all fluids). The outcome of the methodology is significant (see **Table 1**): decidability is linked to a partition structure of the space of states of nature θ_j or rather to that of the space of states of nature θ_j^* as well as to the meaning ascribed to universal properties that are respectively weight or rather volume weight (*i.e.* density). In physics, such a revision of constants related to physical properties to bring experimental data into line with theory is a so-called renormalization.

For the same observation, such as that obtained from the weighing of objects where the two scales of a balance are in equilibrium, explanatory hypothesis θ in the initial reference universe of possibles [all possible states of nature in the air] is necessarily the equality of weight; whereas explanatory hypothesis θ in the other reference universe of possibles [all possible states of nature underwater] may be not necessarily equality of weight. Rather, explanatory hypothesis θ^* in the final reference universe of possibles [all possible states of nature in all fluids], is the equality of density—a property unknown in the initial reference universe of possibles (in the air). As a corollary to this, the observation of a balanced weighing in the initial reference universe of possibles [all possible states of nature in the air] is consistent with hypothesis θ of the equality of weight as well as the observation of an unbalanced (immersed) weighing in the other reference universe of possibles [all possible states of nature underwater]. An observation

Table 1. Constructing a more generalizing representation, reference universe of possibles and decidability.

Observations	Reference Universe of possibles $U \rightarrow U_k \supset U^*$ [all possible states of nature $\Theta_j \rightarrow \Theta_j^*$]	Explanatory Hypotheses: Partition structure $P(\Theta_0/\Theta_1) \rightarrow$ Partition structure $P^*\left(\Theta_0^* / \Theta_1^*\right)$
Balanced weighting E_1	Initial U_1 (air)	Θ_0: equality of weights
	New and independent U_2 (water)	Θ_1: not necessarily equality of weights
	Unknown	undecidable (knowing that the existence of U_2 is possible)
	U^* broadened to encompass all fluids	Θ_0^*: equality of densities (volumetric weights)
Unbalanced weighing E_2	Initial U_1 (air)	Θ_0: inequality of weights
	New and independent U_2 (water)	Θ_1: not necessarily an inequality of weights
	Unknown	undecidable (knowing that the existence of U_2 is possible)
	U^* broadened to encompass all fluids	Θ_1^*: inequality of densities (volumetric weights)

such as an unbalanced weighing, considered as a priori unfavorable to explanatory hypothesis θ such as the equality of weight in the initial reference universe of possibles [all possible states of nature in the air] may be favorable to it in the other reference universe of possibles [all possible states of nature underwater]. Only hypothesis θ' of equality of density in the final reference universe of possibles [all possible states of nature in all fluids] can dispel the obvious contradiction.

It is not only a matter of reasoning rationally and consistently, by reference to an interpretable universe of possibles, while striving to estimate the plausibility of various explanatory hypotheses θ_j, based on observations, but also a question of bringing to bear ever more generalizing representations and coherences. In this way, our theoretical constructions are continually put to the test, under experimental conditions that are reproducible, every time that the referential universe of possibles changes, which involves a (temporary) absence of decidability (*i.e.*, undecidability). "Embracing contradictory forces can inspire learning, discovery, and creativity" [19].

According to the Bayesian inference model, the decision-maker's semantic formalism would consist of a prior partition structure P_i of an initial space of the states of nature (*i.e.* a universe of possibles in the air) and of another partition structure P_f of another space of the states of nature (*i.e.* a universe of possibles underwater), in such a way that P_i and P_f cannot coexist without being contradictory, until a novel partition structure P^* of an extended space of the states of nature (*i.e.* a universe of possibles including all fluids) is designed. Hence, the issue of how and wherein—in which fuzzy multiverse or quantum universe of possibles—P^* can be designed.

4. Implications under the Bayesian Inference Modeling Proposed for General Decision-Making Theory or A.I.: Fuzzy Model or Quantic Form?

First, we will discuss the implications for the Bayesian inference modeling proposed for general decision-making theory or Artificial Intelligence. Then we shall examine the foundations of a theory that is intended to be more generalizing.

4.1. Reformulating the System of Rules

Given that all comparisons between hypotheses or observations must be relativized in relation to the reference universes U_k of possibles [all possible states of nature], rules (3) and (4) of the general theory of decision-making should be reformulated, as follows: (3) assign sampling probabilities $p(E_i|\theta_j\ U_k)$ which represent the likelihood of the measurements, reflecting prior knowledge about the mechanism of measurement process yielding observable data sets E_i in a reference universe U_k of possibles [all possible states of nature]; (4) digest any additional evidence $E = E_1\ E_2\ldots$ (Sampling data) and, by application of Bayes' Theorem, obtain the posterior probabilities $p(\theta_j|E\ I\ U_k)$. The probabilities $p(\theta_j|E\ I\ U_k)$ yield all information about possible states of nature θ_j that can be known a posteriori in reference universe U_k of possibles [all possible states of nature in all fluids]. The reformulation of steps (3) and (4) has a notable impact on those that ensue. Assuming that the possible decisions D_i set out in step (5) are not affected (if only two decisions Yes/No or 0/1 were ever proposed), it is obvious that steps (6) and (7) are bound to be affected. Indeed, the expression of what is sought to be accomplished, as a function of preferences (minimize the expected losses/maximize the expected gains) linking possible decisions with states of nature $L(D_i|\theta_j)$ will be transformed into $L(D_i|\theta_j\ U_k)$. A concomitant transformation will take place, in step (7), regarding decision D_i which leads to the most preferred expected outcome for θ_j (minimized expected loss/maximum expected gain) in reference universe U_k of possibles [all possible states of nature], given that the novel hypotheses θ_j^* must be formulated pursuant to the reframing of reference universe U^* of possibles (encompassing U_k). Otherwise, the problem becomes undecidable (see **Table 1**). Technically, that would proceed by imposing a forced *coupling* [20] on probabilities $p(\theta_j|E\ I\ U_k)$ within U^* (see **Figure 1**).

The theory of probabilities [4] was not intended to apply to several reference universes Uk (in-sofar as the probability of any reference universe of possibles cannot exceed 1). It is important to note that, as already explained, it is not a matter of adding any previously unknown or un-certain observations Ei [21] [22] to a reference universe Uk of all possible states of nature θj (hypotheses or

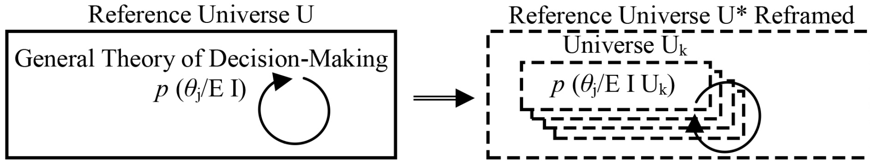

Figure 1. Decision, inference and reframing of the reference universe of possibles.

theories, etc.) by conveniently redistributing the probabilities so as to ensure that the total always stands at 1.

Now inspired by the sciences of nature, some scientists [20] [23] have begun to formalize the *coupling* of two or more previously separated universes of possibles (such as two separated probability spaces, etc.) in each of which an operator (a function, differentiation, transformation, map, etc.) is defined, knowing that it is difficult to describe a deformation of one operator on the other. However, it is possible to extend both of them to the overall universe of possibles (*i.e.* overall space of the states of nature) assigning value 0 outside its universe of possibles. This can be made by multiplying each of them by a function that has value 1 in one universe of possibles and 0 outside. If we also normalize the two functions in such a way that their sum is identically 1, the couple may then be called partition of unit. Thanks to this extension, the two operators act on the same set of functions, allowing a *coupling* (or heterogeneous assemblage), which is a deformation of an operator on the other. This process of *coupling* can be formally expressed as a linear combination of the two extended operators. Since the first function takes value 1 in the first sub-universe of possibles (that was originally the first universe of possibles) and 0 outside, the *coupling* coincides with the first operator in the first sub-universe of possibles. Analogously, since the second function takes value 1 in the second sub-universe (that was originally the second universe of possibles), the *coupling* coincides with second operator in the second sub-universe of possibles. The resulting operator is then a smooth transformation of the first operator into the second one. Note that this is just one of the many *coupling* possibilities, especially the quantum translation.

4.2. Discussion of the Theoretical Foundations of Reframing the Reference Universe of Possibles: Quantic Form Rather Than Fuzzy Modeling?

While raising the issue of a Quantic Form rather than Fuzzy Modeling, the representation that we have constructed and the assessment that we make regarding any space of all possible states of nature and the impact of our decisions (emanating from both our behavior and our actions) thus depend on: 1) the reference universe of possibles together with the partition structure of such a space of the states of nature that makes decidability, 2) the potential for identifying other universes of possibles (in line with the problem to be solved) and partitioning structures with such spaces of the states of nature which makes undecidability, 3) the potential for reframing by extension the reference universe of possibles so as to encompass all others, together with a novel partition structure of such a mega-space of all spaces of the states of nature.

4.3. A Partition Structure of a Space of the States of the World to Make Decidability

As stated earlier, the belief in an absolute and eternal universality can be posited as an axiom. The implications of this can be seen in the Bayesian inference modeling proposed for general decision-making theory or Artificial Intelligence. Theoreticians (in economics, epistemic modal logic, etc.) have formulated such a conception, as so-called semantic knowledge, in a unifying axiomatic principle[11] [7], modeled in terms of partitions on a comprehensive space of "possible states of the world" (*i.e.* universe of possibles). A partition structure of the space of the world is thus of use to give symbols of the language a meaning and a truth value (semantic) to validate a formal (logical/mathematical) theory [10]. The basic principle is to consider a theory as consistent (*i.e.* mathematically correct), if it is possible to define a world where this theory is true. Semantics (truth) and syntax (proof) are independent and equivalent [24]. To illustrate, according to Newton's theory of gravitation and its principle of equivalence, gravity acts equally on all bodies—this is the proof. Now the demonstration was carried out by D. Scott during the Apollo 15 mission in 1971, when a hammer (1.32 kg) and a feather (0.03 kg) released at the same moment hit the ground at the same time – this is for truth.

4.4. Partitioning Structures with Many Spaces of the States of the World Makes Undecidability

Another conception is to posit as an axiom that universality depends on the (time-dependent) state of knowledge. Under this somewhat constructivist concept—an idea seized upon by a mathematical theory dubbed intuitionism [25] – every decision maker must know which reference universe of possibles serves as the lens through which he/she can look in order to reframe, decide and act in an informed manner. The acquisition of a novel understanding of the world depends on his ability to foresee the broadening of his knowledge beyond the bounds of his rationality [3] and ultimately place into perspective the reasons behind his previous viewpoints and stances. This may happen in the space of all possible parts wherein partition structures are designed. Such space is always wider than the space of the world itself (*i.e.* all possible states of nature), as it includes all possible sub-spaces, the entire space of the world as a whole and even the empty space. Indeed, let us remember and note that mathematical structures and their relations are designed according to set theory [26]. The latter is regarded as a fundamental theory of all arithmetic and analysis, axiomatized by Zermelo E. & Fraenkel A. (Ibid) and known as ZFC system, according to which it is possible to "assign to an arbitrary logically definable notion a *set* (as one), or *class* (as many), as its *extension*" [27]. So, the validation of formal human construction and the notion of mathematical truth are based on set theory and its

[11]The axioms of Conscience, Omniscience, Knowledge, Transparency, and Prudence.

metatheoric language. Being conveniently "situated" in such a space of all possible parts, it is interesting to note that the observer (*i.e.* the decision-maker) necessarily works somewhere outside the space of the world (*i.e.* the universe of possibles), as Archimedes put it in the Palimpsest. Thus, the observer also necessarily made and stored, somewhere in a human or artificial memory, a copy of the space of the world. "(…) since it includes a version of the entire mathematical world, and one might expect it to be difficult to construct explicitly. In fact, Gödel's Second Incompleteness Theorem forbids even the theoretical possibility of such a construction, since it would entail the non-contradiction of the ZFC system, which the theorem asserts, is impossible to show from ZFC, *i.e.* within the framework of set theory" [7]. From outside the space of the world, the observer can reframe it while exploring the existence of others. In addition, assuming he/she finds some in the space of all possible parts of all possible spaces of the world, he/she has to cross the barrier of undecidability as to what encompassing all these possible worlds implies. To achieve this, he/she cannot rely on the Bayesian inference modeling, but rather on intuition, elaborating and nuancing the forms of rationality and corresponding codified languages [28], to design novel partition structures. 'The intuition–rationality paradoxical tension will be present in any strategic decision-manipulate making process (…) Understanding better how rationality and intuition interact during decision-making has, however, remained a major challenge (…) Some researchers suggest that intuition is the main mechanism through which choices are made, and the role of rational thinking is to evaluate the product of intuitive processing [29] [30].

Note also that, from the space of all possible parts of all possible spaces of the world, to devise a novel partition structure together with a mega-space of the world encompassing all possible spaces of the world, generates new dimension: "the term *dimension* can be defined as the unique mega-space that is built by infinite general-spaces, subspaces and micro-spaces that are systematically interconnected (…) any dimension needs to be studied using the ideas of spaces/sets or sub-spaces/sub-sets or partitions/cuts" ([31], p. 340).

4.5. Reframing by Extension the Space of the World Together with a Novel Partition Structure: Quantic Form Rather Than Fuzzy Modeling

The main theories applied in fuzzy modeling are fuzzy logic and the fuzzy set theory. Such fuzzy modeling theories, including fuzzy inference reasoning, enable us to work with imprecise information [often available in the form of sentences of natural language only [32] [33]. They have yielded impressive outcomes in decision-making when dealing with classes of objects or things with a continuum of grades of membership ranging between 0 and 1 [34]. Fuzzy models require partitioning the universe into parts, with the specificity that they need not be precisely formed and can overlap. Thus, such fuzzy modeling theories clearly assume to work and explore within a given reference universe U of possibles. They do not assume to work within a so-called *Class* [27] [35], neither between many universes of possibles U_k (*i.e.* spaces of the world) nor between a reference universe of possibles U (*i.e.* a space of the world) and a reference extended universe of possibles U^* (*i.e.* mega-space of the world) including U. Moreover, fuzzy sets are especially characterized by membership (characteristic) functions, which are relations of the type one-one or many-one and not of the type one-many (*i.e.* $U_k \supset U^*$), as previously explained.

The process of reframing a reference universe of possibles U (*i.e.* a space of the world) requires the observer or the designer to deal with a quantum translation: 1) quantum coherence/decoherence, 2) superposition of states (quantum notation in a Hilbert space: $|U\rangle + |U_k\rangle$, 3) non-commutability of observations or observers (*i.e.* irreversibility) [36]. Indeed, reference universes of possibles U and U_k (*i.e.* spaces of the world) have independent states until the existence of a reference universe of possibles U^* encompassing them (*i.e.* a mega-space of the world) gets its own translation through a superposition of these states. É. Galois [37] opened the way by studying the extension of structures. It is necessary to understand by extension of structures, the generation of a more general structure such as U^* generalizing U while including it (*i.e.* a mega-space of the world including any pre-existing space of the world). Yet, a fundamental issue, still opened, is where to travel searching for reference universes of possibles U_k with the aim of extending them, if no universe of possibles U^* is preexisting. According to formal logic, working within a reference universe U means that it has to remain unique. This is a basic rule (called contraction): "U and U" must be reduced to "U". Therefore, U and U^* cannot be active simultaneously. This is also true for U and U_k. Thus, we must switch to another more suited logic form, namely a quantum one, to consider U, U_k, U^* in quantum states respectively. Also, since the process of reframing a reference universe of possible U is time-dependent, a quantum form seems to be well suited to formalize it. To put it another way, memory being the substratum, whether human or artificial, wherein everything stands [3] [38], especially wherein the universe of possibles U is stored, then, where to stand as an observer or work as a designer and where to store a novel universe of possibles U^* including a copy of itself (as it was initially) while generalizing it, if not in a quantum universe? To date, even the theoretical possibility of such an operation of self-inclusion is prohibited by pure mathematics [7]. Such approach remains to be explored and could open up a new path of investigation.

4.6. Application of the Quantic Form to Loyds' Problem

The continuous tracing problem [39] consists in connecting points (9) with lines (4) without lifting the pencil. Thinking Outside the Box is another name for an original, creative and clever way of thinking. We chose this problem because it is deemed practically impossible to solve (you have to think about getting out of the mental framework) and makes us manipulate elementary objects which are the points, the links between these points, the links between these links. Neither

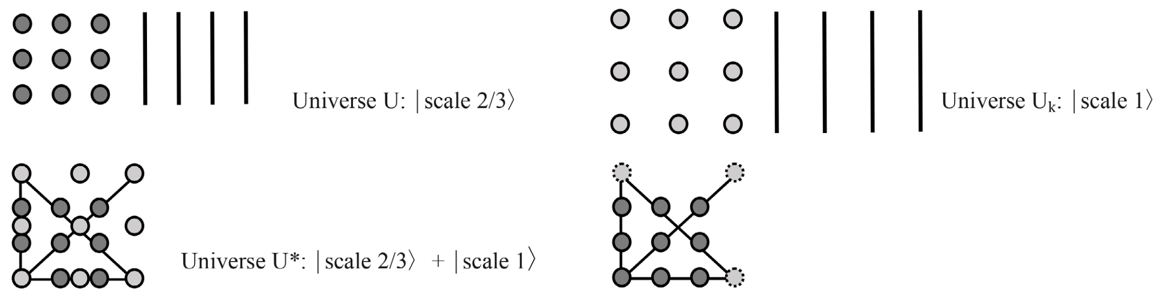

Figure 2. Quantic Form applied to the Loyds' problem.

Operations Research nor A.I. nor even adaptive learning will enable us to overcome Loyds' problem. Instead, applying the Quantic Form provides a solution (Cf Figure 2).

5. Conclusion

Bayesian inference model [1] is an optimal processing of incomplete information that, more than other models, better captures the way in which any decision-maker learns and updates his degree of rational beliefs about possible states of nature, in order to make a better judgment while taking new evidence into account. Such a scientific model proposed for the general theory of decision-making, like all others in general, whether in statistics, economics, operations research, A.I., data science or applied mathematics, regardless of whether they are time-dependent, have in common a theoretical basis that is axiomatized by relying on related concepts of a universe of possibles, especially the so-called *universe* (or the *world*), the *state of nature* (or the *state of the world*), when formulated explicitly. The issue of where to stand as an observer or a decision-maker to reframe such a universe of possibles together with partition structures of knowledge (*i.e.* semantic formalisms), including a copy of itself (as it was initially) while generalizing it, is not addressed. Memory being the substratum, whether human or artificial, wherein everything stands, to date, even the theoretical possibility of such an operation of self-inclusion is prohibited by pure mathematics. We have made this blind spot come to light through a counter-example (namely Archimedes' thought experiment, which led to his famous "Eureka" moment) and have explored novel theoretical foundations, fitting better with a *quantum* form than with fuzzy modeling, to deal with more than a reference universe of possibles (*i.e.* space of the world). Indeed, the last ones have independent states (*i.e.* decoherence) until the existence of a mega-space of the world encompassing them, together with a novel meta-partition structure of knowledge, gets its own translation through the superposition of these states (*i.e.* coherence). This could open up a new path of investigation for the general theory of decision-making, as well as for Artificial Intelligence, often considered as the science of the imitation of human abilities [40], while being also the science of knowledge representation and the science of concept formation and reasoning [2] [41].

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Jaynes, E.T. (2003) Probability Theory: The Logic of Science. Cambridge University Press, 19

[2] Seongmin, A.P., Goïame, S., O'Connor, D.A. and Dreher, J.-C. (2017) Integration of Individual and Social Information for Decision-Making in Groups of Different Size. *PLOS Biology*, **15**, e2001958.

[3] Simon, H.A. (1996) The Science of the Artificial. MIT Press, 116.

[4] Savage, L. (1954 [1972]) The Foundations of Statistics. John Wiley and Sons.

[5] Aumann, R.J. (1999) Interactive Epistemology I: Knowledge. *International Journal of Game Theory*, **28**, 263-300. https://doi.org/10.1007/s001820050111

[6] Samuelson, L. (2004) Modeling Knowledge in Economic Analysis. *Journal of Economic Literature*, **42**, 367-403. https://doi.org/10.1257/0022051041409057

[7] Dehornoy, P. (2007) Au-delà du forcing: La notion de vérité essentielle en théorie des ensembles. In: Joinet, J.-B., Ed., *Logique*, *dynamique et cognition*, Editions de la Sorbonne, Logique, Langage, Sciences, Philosophie (Coll.), 147-169.

[8] Codd, E.F. (1990) The Relational Model for Database Management, Version 2. Addison-Wesley Publishing Company, Inc.

[9] Popper, K.R., de Launay, M.I. and de Launay, M.B. (1985) Conjectures et réfutations: La croissance du savoir scientifique. Payot.

[10] Tarski, A. (1969) Truth and Proof. *Scientific American*, **220**, 63-77. https://doi.org/10.1038/scientificamerican0669-63

[11] Taleb, N.N. (2010) The Black Swan: The Impact of the Highly Improbable. 2nd Edition, Penguin.

[12] Séguy-Duclot, A. (2011) Recherches sur le langage. Librairie philosophique. J. Vrin.

[13] Heath, T.R. (2007) The Method of Archimedes. Cosimo Classics.

[14] Heiberg, J.L. (1909) A Newly Discovered Treatise of Archimedes: Heiberg' Translation from the Greek. *The Monist*, **19**, 202-224. https://doi.org/10.5840/monist190919234

[15] Bell, E.T. (1986) Men of Mathematics. Touchstone, 19.

[16] de Chaufepié, J.G. (1750) Nouveau dictionnaire historique et critique. Biblioteca Naz. Vittorio Emanuale.

[17] Lautman, A. (2006) Les mathématiques, les idées et le réel physique. Librairie Philosophique Vrin.

[18] Carlile, P.R. (2004) Transferring, Translating, and Transforming: An Integrative Framework for Managing Knowledge across Boundaries. *Organization Science*, **15**, 555-568. https://doi.org/10.1287/orsc.1040.0094

[19] Lewis, M.W. and Smith, W.K. (2014) Paradox as a Metatheoretical Perspective: Sharpening the Focus and Widening the Scope. *The Journal of Applied Behavioral Science*, **50**, 127-149. https://doi.org/10.1177/0021886314522322

[20] Longo, G. and Montevil, M. (2014) Perspectives on Organisms—Biological Time, Symmetries and Singularities. Springer.

[21] Knight, F.H. (2006) Risk, Uncertainty, and Profit. Cosimo Classics.

[22] Shackle, G. (1961) Decision, Order and Time. Cambridge University Press.

[23] Sarti, A., Citti, G. and Piotrowski, D. (2019) Differential Heterogenesis and the Emergence of Semiotic Function. *Semiotica*, **2019**, 1-34. https://doi.org/10.1515/sem-2018-0109

[24] Nagel, E. and Newman, J.R. (2005) Gödel's Proof. Routledge Classics.

[25] Brouwer, L.E.J. (1913) Intuitionism and Formalism. *Bulletin of the American Mathematical Society*, **20**, 81-96. https://doi.org/10.1090/s0002-9904-1913-02440-6

[26] Jech, T. (1978) Set Theory. Academic Press.

[27] Zermelo, E. (1908) Untersuchungen über die Grundlagen der Mengenlehre. I. *Mathematische Annalen*, **65**, 261-281. https://doi.org/10.1007/bf01449999

[28] Gonseth, F. (1926, 1974) Les fondements des mathématiques, de la géométrie d'Euclide à la relativité générale et à l'intuitionnisme. Librairie scientifique A. Blanchard.

[29] Kahneman, D., Slovic, P. and Tversky, A. (1982) Judgment under Uncertainty: Heuristics and Biases. Cambridge University Press.

[30] Calabretta, G., Gemser, G. and Wijnberg, N.M. (2016) The Interplay between Intuition and Rationality in Strategic Decision Making: A Paradox Perspective. *Organization Studies*, **38**, 365-401. https://doi.org/10.1177/0170840616655483

[31] Estrada, M.A.R. (2011) Multi-Dimensional Coordinate Spaces. *International Journal of the Physical Sciences*, **6**, 340-357.

[32] Melin, P. and Castillo, O. (2008) Fuzzy Modeling Fundamentals. Wiley Online Library.

[33] Cohen, K., Nicholas, E., Barnabas, B. and Kreinovich, V. (2023) Fuzzy Information Processing. Springer.

[34] Zadeh, L.A. (1965) Fuzzy Sets. *Information and Control*, **8**, 338-353. https://doi.org/10.1016/s0019-9958(65)90241-x

[35] Cantor, G. (1883) Grundlagen einer allgemeinen Mannigfaltigkeitslehre. Teubner.

[36] Connes, A. (2019) La géométrie et le quantique. Les grandes voix de la recherche (Coll.). CNRS.

[37] Liouville, J. (1846) Œuvres mathématiques d'Evariste Galois, suivies d'un avertissement de Liouville. *Journal de Mathématiques Pures et Appliquées*, **11**, 1-62.

[38] Laird, J.E. (2008) Extending the Soar Cognitive Architecture. *Frontiers in Artificial Intelligence and Applications*, **171**, 224-235.

[39] Loyd, S. (1914) Ŗoutside the box thinkingŗ [archive], Cyclopedia of Puzzles. The Lamb Publishing Company, Random House.

[40] Pomerol, J. (1997) Artificial Intelligence and Human Decision Making. *European Journal of Operational Research*, **99**, 3-25. https://doi.org/10.1016/s0377-2217(96)00378-5

[41] Newell, A. and Simon, H.A. (1972) Human Problem Solving. Prentice-Hall.