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Reciprocity Relation between Alternative Gravity Formulas

Hans Hermann Otto

Department of Materials Science and Crystallography, Clausthal University of Technology, Clausthal-Zellerfeld, Germany
Email: hhermann.otto@web.de

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Abstract

We compare *Newton*'s force law of universal gravitation with a corrected simple approach based on *Bhandari*'s recently presented work, where the gravitation constant G is maintained. A reciprocity relation exists between both alternative gravity formulas with respect to the distances between mass centers. We conclude a one-to-one mapping of the two gravitational formulas. We don't need *Einstein*'s construct of spacetime bending by matter.

Keywords

Gravity Formulas, Reciprocity Distance Relation, Newton, Einstein, Bhandari, Higgs Field, Dirac's Large Number

1. Introduction

Newton's well-known law of universal gravitation, acting along the line intersecting two "point" bodies of masses m_1 respectively m_2 with a distance of R, takes the form of [1]:

$$F_N = G \cdot \frac{m_1 \cdot m_2}{R^2} \tag{1}$$

where $G = 6.67430(15) \times 10^{-11} \left(\frac{\mathrm{m}^3}{\mathrm{kg} \cdot \mathrm{s}^2} \right)$ is the gravitation constant [2].

Later, *Einstein* postulated in his general theory of relativity that gravitation is curvature of spacetime caused by massive bodies [3]. With methods of differential geometry, he derived in 1915 his field equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\tilde{R}g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} = \kappa \cdot T_{\mu\nu} \tag{2}$$

where $G_{\mu\nu}$ represents the *Einstein* tensor, $R_{\mu\nu}$ is the *Ricci* tensor, $T_{\mu\nu}$ is the

energy-momentum tensor, $g_{\mu\nu}$ is a metric tensor, $\tilde{R}$ is the curvature scalar (don't confuse with the distance R), c is the speed of light, and Λ is the cosmological constant that is attributed to large scale dynamics of the cosmos.

Already in 1953, *Sciama* presented a reliable theory about the origin of gravity (inertia) as inductive effect of distant matter. The gravitational field of a moving universe was calculated by means of *Maxwell*-type field equations [4]. *Newtonian* Equation (1) arises from the gravitational effect of a rotating universe in agreement with *Mach*'s principle. He suggested for general motions that inertial forces have to be derived from a tensor potential.

Recently, *Bhandari* and *Bhandari* set out to demonstrate that an external energy source may power our universe and that gravity may be due to cancellation of energy lines in the shadow regions of mass objects, creating an energy vacuum that causes the gravitational force between the masses [5]. The reader should study this new approach in the original publication.

The reader may also compare this effect with the *Casimir* effect as the dominant interaction between nano-scale objects, where the omnipresent quantum electromagnetic vacuum energy in parts is displaced between, for instance, two perfectly conducting parallel plates [6]. The inverse quartic dependence with distance is different from the inverse quadratic dependence of gravity in *Newton*'s approach respectively electromagnetic forces. Also, forces between interacting electric dipoles respectively magnetic dipoles show an inverse quartic dependence with distance.

The many attempts to expand *Einstein*'s approach are beyond the intention of this contribution. For instance, a post-quantum theory of classical gravity discussed the need of quantization of gravity [7].

2. An Alternative Gravity Formula

The present author has just commented on the *Bhandari* approach, explained in the Introduction Chapter, and developed a simple formula that describes the gravitational force quite accurately by maintaining the well-known G constant [8]. With the aid of **Figure 1** and **Figure 2**, we can estimate the shadow regions of massive objects verifying the red volume of the shadow region as (see **Appendix**):

$$V_2 = \frac{2}{3}\pi r_2^3\left(1-\sqrt{1-\left(\frac{r_1}{R}\right)^2}\right) \tag{3}$$

$$V_1 = \frac{2}{3}\pi r_1^3\left(1-\sqrt{1-\left(\frac{r_2}{R}\right)^2}\right) \tag{4}$$

Dividing by the volume of each sphere gives us simpler relative volume expression V_i':

$$V_2' = \frac{1}{2}\left(1-\sqrt{1-\left(\frac{r_1}{R}\right)^2}\right) = \frac{1}{2}\left(1-\cos\left(\delta_1\right)\right) \tag{5}$$

$$V_1' = \frac{1}{2}\left(1-\sqrt{1-\left(\frac{r_2}{R}\right)^2}\right) = \frac{1}{2}\left(1-\cos\left(\delta_2\right)\right) \tag{6}$$

where δ_i are the respective half-cone angles.

The shadow is not a cone but a spherical cutout, a cone capped by a spherical section. The cone base radius a is equal for both bodies, because:

$$a = \frac{r_1 \cdot r_2}{R} \tag{7}$$

The diameter d of the cut circle, where the two shadow cones meet (**Figure 1**), can be approximated by:

$$d \approx 2 \cdot \frac{r_1 \cdot r_2}{r_1 + r_2} \tag{8}$$

giving a circular area of:

$$A \approx \pi \cdot \left(\frac{r_1 \cdot r_2}{r_1 + r_2}\right)^2 = \pi \cdot \left(\frac{a \cdot R}{r_1 + r_2}\right)^2 \tag{9}$$

We can also write down the interesting reciprocity relation:

$$\frac{2}{d} = r^{-1} \approx r_1^{-1} + r_2^{-1} \tag{10}$$

which resembles the relation $R_{12}^{-1} = R_1^{-1} + R_2^{-1}$ for parallel connection of *Ohmic* resistors.

Indeed, when replacing charge by mass, we jump from electrodynamics to gravity.

The gravitational force law then results in:

$$F = \frac{4 \cdot G \cdot M_1 \cdot M_2}{R^2\left(1-\sqrt{1-\left(\frac{r_2}{R}\right)^2}\right)\left(1-\sqrt{1-\left(\frac{r_1}{R}\right)^2}\right)} \tag{11}$$

For $r_i \ll R$,

$$\sqrt{1-\left(\frac{r_i}{R}\right)^2} \approx 1-\frac{1}{2}\left(\frac{r_i}{R}\right)^2 \tag{12}$$

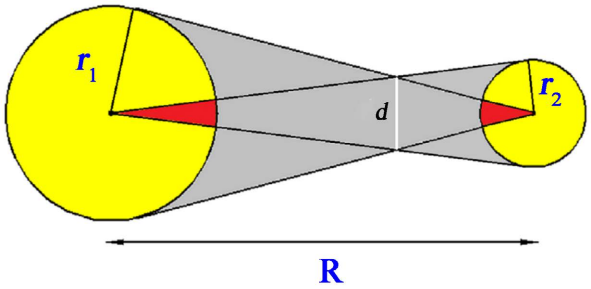

Figure 1. Illustration of the spherical cutoff shadow between two spherical bodies of radii r_1 respectively r_2 with a separation of R. Red: Projected shadow volumes as spherical cutouts. In the grey region, energy lines are devoid.

$$F \approx \frac{4 \cdot G \cdot M_1 \cdot M_2}{\frac{r_1^2 r_2^2}{4R^2}} = \frac{16 \cdot G \cdot M_1 \cdot M_2}{a^2} \tag{13}$$

The case $R \to 2r_i + \delta$ in Equation (12) will be considered in a separate contribution.

When finally denote the force by *ExS* (means external energy source) and obtain the following formula, where M_1 respectively M_2 are now masses of the shadow spherical cutouts, and r_1 respectively r_2 are the radii of these spherical bodies (Figure 1 and Figure 2):

$$F_{ExS} = 16 \cdot G \cdot M_1 \cdot M_2 \cdot \frac{R^2}{r_1^2 \cdot r_2^2} \tag{14a}$$

$$F_{ExS} = 16 \cdot G \cdot M_1 \cdot M_2 \cdot a^{-2} \tag{14b}$$

Remarkably, the squared distance between the centers of the masses is reciprocal in relation (14) compared to the *Newtonian* gravity formula:

$$F_N = G \cdot \frac{m_1 \cdot m_2}{R^2}, \quad F_{ExS} = 16 \cdot G \cdot M_1 \cdot M_2 \cdot \frac{R^2}{r_1^2 \cdot r_2^2} \tag{15}$$

In the *Newtonian* formula, the gravitational force F_N is inversely proportional to the second power of distance of the mass centers, whereas for F_{ExS} we estimated the second power of distance between the mass centers in the nominator.

Reciprocity relations are frequently observed in physics. Most important is the reciprocal duality between particles and waves [9]. The reciprocity relation between *Sommerfeld*'s structure constant [10] and *Guynn*'s galactic velocity is another prominent example [11] [12] [13]. In mathematics, the golden mean is a famous example: $\varphi = \frac{1}{\Phi}$, where $\varphi = \frac{\sqrt{5}-1}{2}$, $\Phi = \frac{\sqrt{5}+1}{2}$.

Examples demonstrating the accuracy of our gravity Formula (14) are given in Ref. [8] and below. For sake of compatibility, we used successfully weighted radii in case of eccentric planets such as Jupiter, which should be also included in *Wikipedia*.

$$r_{Jup} = \left(\frac{3m}{4\pi D}\right)^{\frac{1}{3}} = 6.99125 \times 10^7 \,(\mathrm{m}) \tag{16}$$

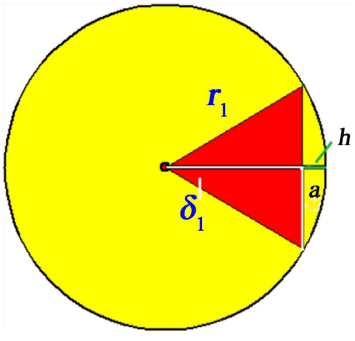

Figure 2. Explanation of the variables used. Red: The projected cone with a base radius *a*, completed by a spherical cap of height *h* to give the spherical cutout.

In the same way, we get for the Sun $r_{Sun} = 6.96025 \times 10^{8}\,(\mathrm{m})$.

We calculated gravitational forces between the Sun and the following planets:

Earth $F_{ExS} = 3.5448 \times 10^{22}\,(\mathrm{N}),\ F_N = 3.5424 \times 10^{22}\,(\mathrm{N}),\ \frac{F_{ExS}}{F_N} = 1.0007$ (17a)

Mars $F_{ExS} = 1.7953 \times 10^{21}\,(\mathrm{N}),\ F_N = 1.7934 \times 10^{21}\,(\mathrm{N}),\ \frac{F_{ExS}}{F_N} = 1.0011$ (17b)

Jupiter $F_{ExS} = 4.5719 \times 10^{23}\,(\mathrm{N}),\ F_N = 4.5659 \times 10^{23}\,(\mathrm{N}),\ \frac{F_{ExS}}{F_N} = 1.0013$ (17c)

The gravitational force between Earth and Moon is calculated to be:

$$F_{ExS} = 1.979 \times 10^{20}\,(\mathrm{N}),\ F_N = 1.981 \times 10^{20}\,(\mathrm{N}),\ \frac{F_{ExS}}{F_N} = 0.9990 \tag{18}$$

As a resume, we conclude a one-to-one mapping of the two gravitational formulas. However, relation (14) is only an approximation [8], and if we consider the external energy source approach as true, then vice versa the *Newtonian* gravity relation must be considered as a quite good approximation.

We can in Equation (14b) transform the masses in rest energies and get the following equation:

$$F_{ExS} = 16\frac{G}{c^4}\left(\mathrm{N}^{-1}\right)\cdot\frac{M_1c^2}{a}(\mathrm{N})\cdot\frac{M_2c^2}{a}(\mathrm{N}) \tag{19}$$

where $\frac{4G}{c^4} = \kappa' = 3.305088 \times 10^{-44}\left(\mathrm{N}^{-1}\right)$. Using the maximum force being [14]:

$$F_{\max} = \frac{c^4}{4G} = \frac{1}{\kappa'} \tag{20}$$

we finally get:

$$F_{ExS} = \frac{4}{F_{\max}}\left(\mathrm{N}^{-1}\right)\cdot\frac{M_1c^2}{a}(\mathrm{N})\cdot\frac{M_2c^2}{a}(\mathrm{N}) \tag{21}$$

The reader may compare the factor $\kappa = \frac{8\pi G}{c^4}$ in *Einstein*'s field equations with the similar factor $\kappa' = \frac{4G}{c^4}$ in our relation (19):

$$\kappa' = \frac{\kappa}{2\pi} \tag{22}$$

The new approach is of course a simplified one and does not consider variations in the G "constant". For instance, periodic variations of the gravitational constant and the length of day (*LOD*) were recently attributed by *Guynn* to the influence of Jupiter's orbit and alignment relative to the galaxy [15], based on measurements of *Anderson et al.* [16]. Jupiter is by far the most massive planet in the solar system. It is always engineers who come up with such excellent ideas. The late astronomer *Johannes Kepler* (*1571, †1630) would be very pleased about *Gynn*'s finding.

3. Outlook

We should all work hard now to bring together the ideas of *Guynn* [11] [15], *Bhandari* and *Bhandari* [5], *Suleiman* [17], *El Naschie* [18], Pellis [19], *Markoulakis* [20] and some others, including ideas of the present author [12] [13] [21], into a common picture of new physics and reality. We will understand, why obscure dark matter, which is strongly coupled to moving baryonic matter, may be explained by the speed dependent "viscous" drag exerted on moving objects by the repressed otherwise invisible (superluminal) construct of energy lines from an external energy source, similar to the recently successfully verified effect of gravitomagnetism as kinetic effect caused by mass "currents" (charge is replaced by mass) on gravity [22] [23].

The energy lines or an energy field penetrating our world may be similar to the *Higgs* field. Similarities between the *Higgs* boson as composite particle with no spin and superconductivity may pave the way to understand dark constituents of our universe [24]. A simple formula for the mass m_H of the *Higgs* boson was recently derived by the present author [24]:

$$m_H \approx 133.3959128\left(m_p + m_e\right) = 125.2298\ \text{GeV}/c^2 \tag{23}$$

where

$$133.3959128 = \frac{\alpha_1}{\varphi^2} \tag{24}$$

and

$$\alpha_1 = \varphi^2 \frac{m_{Hi}}{m_p + m_e} \tag{25}$$

$\alpha_1 = 50.9527^\circ$ is a new magic angle [25] and $\varphi = \frac{\sqrt{5}-1}{2}$ is the golden mean, m_p is the proton mass respectively m_e the electron mass. The number in relation (16) is related to *Dirac*'s large number (*DLN*) [26] [27]:

$$\sqrt[20]{\frac{10^{43}}{\pi}} = \sqrt[20]{DLN} = 133.3959128 \tag{26}$$

A "thought-provoking concept" as potential explanation for dark matter rooted in information physics was recently suggested by *Menin* [28].

4. Conclusion

According to *Sciama*, the *Newtonian* gravity equation arises from the gravitational effect of a rotating universe in agreement with *Mach*'s principle. However, based on the concept of *Bhandari*, gravity may arise from displacement of an energy field in the shadow of massive objects. An alternative gravity formula maps one-to-one *Newton*'s results, but indicates reciprocity to *Newton*'s formula with respect to the squared distance between the mass centers. *Einstein*'s construct of spacetime bending is unnecessary.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Newton, I. (1687) Philosophiä Naturalis Principia Mathematica. Imprimatur S. Pepys, London. https://doi.org/10.5479/sil.52126.39088015628399

[2] The NIST Reference of Constants, Units and Uncertainty (2018) *NIST* Gaitherburg, MD 20899, USA. https://physics.nist.gov/cuu/Constants/

[3] Einstein, A. (1915) Die Feldgleichungen der Gravitation. *Sitzung der Physikalisch-Mathematischen Klasse*, **25**, 844-847.

[4] Sciama, D.W. (1953) On the Origin of Inertia. *Monthly Notices of the Royal Astronomical Society*, **113**, 34-42. https://doi.org/10.1093/mnras/113.1.34

[5] Bhandari, P.N. and Bhandari, N.M. (2023) Fundamental Forces Are Not Fundamental as Our 3-D Universe Is Driven by an External Energy Source. *International Journal of Physics Research and Applications*, **6**, 167-179. https://doi.org/10.29328/journal.ijpra.1001068

[6] Casimir, H.B.G. (1948) On the Attraction between Two Perfectly Conducting Plates. *Proceedings of the Koningklijke Nederlandse Akademie van Wetenschappen*, **51**, 793-795.

[7] Oppenheim, J. (2023) A Postquantum Theory of Classical Gravity? *Physical Review* **13**, Article ID: 041040. https://doi.org/10.1103/PhysRevX.13.041040

[8] Otto, H.H. (2023) Comment to Bhandari and Bhandari: Fundamental Forces Are Not Fundamental as Our 3-D Universe Is Driven by an External Energy Source. https://vixra.org/abs/2306.0109

[9] Otto, H.H. (2020) Reciprocity as an Ever-Present Dual Property of Everything. *Journal of Modern Physics*, **11**, 98-121. https://doi.org/10.4236/jmp.2020.111007

[10] Sommerfeld, A. (1919) Atombau und Spektrallinien. Friedrich Vieweg & Sohn, Braunschweig.

[11] Guynn. P. (2018) Thomas Precession Is the Basis for the Structure of Matter and Space. viXra: 1810.0456.

[12] Otto, H.H. (2022) Comment to *Guynn*'s Fine-Structure Constant Approach. *Journal of Applied Mathematics and Physics*, **10**, 2796-2804. https://doi.org/10.4236/jamp.2022.109186

[13] Otto, H.H. (2022) Galactic Route to the Strong Coupling Constant $\alpha_s(m_z)$ and Its Implication on the Mass Constituents of the Universe. *Journal of Applied Mathematics and Physics*, **10**, 3572-3585. https://doi.org/10.4236/jamp.2022.1012237

[14] Kenath, A., Schiller, C. and Sivaram, C. (2022) From Maximum Force to the Field Equation of General Relativity and Implications. *International Journal of Modern Physics D*, **31**, Article ID: 2242019. https://doi.org/10.1142/S0218271822420196

[15] Guynn, P. (2021) Jupiter's Orbit and Alignment Relative to the Galaxy Is the Physical Basis of Periodic Variations of Gravitational Constant G and the Length of Day. viXra.org: 2111.0073v1.

[16] Anderson, J.D. *et al.* (2015) Measurement of Newton's Gravitational Constant and the Length of Day. arXiv: 1504.06604.

[17] Suleiman, R. (2019) Relativizing Newton. Nova Scientific Publisher, New York, 1-207.

[18] El Naschie, M.S. (2017) Elements of a New Set Theory Based Quantum Mechanics with

Application in High Quantum Physics and Cosmology. *International Journal of High Energy Physics*, **4**, 65-74. https://doi.org/10.11648/j.ijhep.20170406.11

[19] Pellis, S. (2022) Dimensionless Unification of the Fundamental Interactions. *SSRN Electronic Journal*, **2**, 1-49. https://doi.org/10.2139/ssrn.4201780

[20] Markoulakis, E. (2023) The Superluminous Vacuum. *International Journal of Physical Research*, **11**, 18-25. https://doi.org/10.14419/ijpr.v11i1.32269

[21] Otto, H.H. (2022) Does a Relationship between Superconductivity and Dark Energy Exist? https://www.researchgate.net/publication/369370988_Does_a_Relationship_Between_Superconductivity_and_Dark_Energy_Exist

[22] Ludwig, G.O. (2021) Galactic Rotation Curve and Dark Matter According to Gravitomagnetism. *The European Physical Journal C*, **81**, Article No. 186. https://doi.org/10.1140/epjc/s10052-021-08967-3

[23] Broeils, A.H. (1992) The Mass Distribution of the Dwarf Spiral NGC 1560. *Astronomy & Astrophysics*, **256**, 19-32.

[24] Otto, H.H. (2023) Higgs Boson Mass Relations and Hole Superconductivity. https://vixra.org/abs/2401.0023

[25] Otto, H.H. (2023) Creation of Paired Entities Is Ever Governed by the Golden Mean: About the Nested Repeatability of Living and Cosmic Processes and the Origin of the Universe. https://vixra.org/abs/2401.0036

[26] Dirac, P.A.M. (1974) Cosmological Models and the Large Number Hypothesis. *Proceedings of the Royal Society* A, **338**, 439-446. https://doi.org/10.1098/rspa.1974.0095

[27] Kosinov, M. (2023) Fractal Theory of Proton Mass: Fractal Proton. The Origin of Constant m_p/m_e=1836.1526… The Law of Baryogenesis. Fractal Mechanism of Baryonic Asymmetry. https://www.researchgate.net/publication/372108587_FRACTAL_THEORY_OF_PROTON_MASS_Fractal_Proton_The_origin_of_the_constant_mpme_18361526_The_law_of_baryogenesis_Fractal_mechanism_of_baryonic_asymmetry

[28] Menin, B. (2023) From Black Holes to Information Erasure: Uniting Bekenstein's Bound and Landau's Principle. *Journal of Applied Mathematics and Physics*, **11**, 2185-2194. https://doi.org/10.4236/jamp.2023.118140

Appendix

Volume of a cone

$$V_{cone} = \frac{1}{3}\pi a^2 (r - h) \tag{27}$$

Volume of a sphere section

$$V_{sec} = \frac{\pi}{6} \cdot h\left(3a^2 + h^2\right) \tag{28}$$

Volume of a sphere cutout

$$V_{cutout} = \frac{2}{3}\pi r^2 h \tag{29}$$

$$h = r - \sqrt{r^2 - a^2} \tag{30}$$

$$V_2 = \frac{2}{3}\pi r_2^2 \left(r_2 - \sqrt{r_2^2 - \frac{r_1^2}{R^2} r_2^2} \right) \tag{31}$$

$$V_2 = \frac{2}{3}\pi r_2^3 \left(1 - \sqrt{1 - \left(\frac{r_1}{R}\right)^2} \right) = \frac{2}{3}\pi r_2^3 \left(1 - \cos\left(\delta_1\right)\right) \tag{32}$$