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Cosmological Implications of Diffusion: The Hubble Tension

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Abstract

The paper introduces an "ab initio" model to calculate the timeline of the temperature field of the Big Bang radiation field in the universe and its connection with the Hubble law. The theoretical approach is rooted in the concept of quantum uncertainty and has a quantum character. The aim is to emphasize that the big bang energy diffusion throughout the expanding universe is enough to account also for the Hubble tension.

Keywords

Quantum Theory, Relativity, Universe Timeline, Universe Age, Hubble Tension

1. Introduction

A recent paper [1] has shown how to find via dimensional analysis an equation having the form

$$\nabla\cdot\left(D\xi\hbar\nabla\chi\right)+energy'=energy'', \tag{1.1}$$

where χ is in general a dimensionless scalar field, ξ an arbitrary proportionality factor and D a function of space and time coordinates having physical dimensions $length^2/time$. In this paper $\nabla\cdot\left(D\xi\hbar\nabla\chi\right)$ is then regarded with specific reference to the well known Fourier heat conduction equation, in which case D is the heat diffusion coefficient in agreement with its physical dimensions, whereas $\chi=T/T_0$ describes the temperature field T normalized by the constant temperature T_0. The solution of (1.1) is sought specifying that $energy'=energy'\left(\boldsymbol{r}_0,t\right)$ represents the source term of a local thermal field $T=T\left(\boldsymbol{r},t\right)$; so the starting equation of diffusion model reads

$$\nabla\cdot\left(D\hbar\nabla\chi\right)+\varepsilon_s=\varepsilon \tag{1.2}$$

with appropriate boundary conditions. The reference system defines the coordinate

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$\boldsymbol{r}_0$ of the energy source term ε_s. It is necessary now to specify also the physical context where applies the diffusion model. Usually (1.2) describes solid state problems, where heat diffuses from a given source coordinate $\boldsymbol{r}_0$ to the neighbor space region where is calculated $T(\boldsymbol{r},t)$ as a function of time at any $\boldsymbol{r}-\boldsymbol{r}_0$. In this paper (1.2) has instead cosmological meaning: the diffusion medium is the universe, T is the temperature field from the initial condition represented by the Big Bang energy. The aim of the paper is to explain how to calculate the thermal field throughout the universe expanding after the Big Bang from an arbitrarily small initial size to today's size. The contribution of heat convection is deliberately omitted, to show that the expansion of the universe can explain itself the time dependence of $T(t)$ consistent with that observed throughout the universe by defining appropriately ε_s. Section 2 aims to solve (1.2) in order to stimulate general considerations of cosmological character; section 3 extends the results to the further cosmological problem known in the literature as "Hubble tension".

2. Cosmological Concept of Diffusion

Consider first a body of matter of mass m and volume V that define its density ρ. According to (1.1), a typical example of diffusion model concerns the thermal field induced in the body by the presence of a source of energy, around which heat diffuses by temperature gradient driven conduction mechanism. If the body also contains concentration gradients of local impurities, then the final result of energy and mass diffusion mechanisms is to bring matter towards a uniform equilibrium T and uniform composition. Calculate now $\delta\rho$ when change both V, e.g. when increases the temperature of the body, and m, e.g. because of chemical reaction of the body with the environment. Write then in principle

$$\frac{\delta\rho}{\rho}=\left(\frac{\delta m}{V}-m\frac{\delta V}{V^2}\right)\Big/\left(m/V\right)=\frac{\delta m}{m}-\frac{\delta V}{V}=\delta\log(\rho)=\log\left(\frac{\rho}{\rho_0}\right) \tag{2.1}$$

whence

$$\frac{\delta\rho}{\rho_0}=\frac{\delta\rho}{\rho}\frac{\rho}{\rho_0}=\frac{\rho}{\rho_0}\left(\frac{\delta m}{m}-\frac{\delta V}{V}\right)=\frac{\rho}{\rho_0}\log\left(\frac{\rho}{\rho_0}\right);$$

at the left hand side appears the relative change of ρ with respect to the initial ρ_0. Clearly results $\delta\rho/\rho_0 \lesseqgtr 0$ depend on whether $\delta m/m \lesseqgtr \delta V/V$. Is relevant in particular the minus sign: it means that the relative change of mass $\delta m/m$ of the body is smaller than the relative change $\delta V/V$ of its volume. Consider now a system of several bodies each one of which is characterized by its own ρ_j. If all of the j-th bodies are subjected to such changes, then it is sensible to assess the total change of the system

$$\frac{\Delta\rho}{\rho_0}=-\sum_j\frac{\rho_j}{\rho_0}\log\left(\frac{\rho_j}{\rho_0}\right). \tag{2.2}$$

The same holds of course for all V_j-th elementary volumes of a unique and isolated body of matter. The tendency to the equilibrium means that all ρ_j become equal, so that this equation turns into

$$\left.\frac{\Delta\rho}{\rho_0}\right|_{eq} = -\frac{\rho_{tot}}{\rho_0}\log W \quad W = \left(\frac{\rho_{eq}}{\rho_0}\right)^N \quad \rho_j \equiv \rho_{eq} \quad 1 \le j \le N. \tag{2.3}$$

In fact, the equilibrium state is possible thanks to the concepts of dissipation of a local excess thermal energy and local displacement of matter, e.g. due to possible concentration gradients of impurities, both implied by the maximum global entropy with all $\rho_j = \rho_{eq}$.

This conclusion is extensible to the cosmology because it is rooted in the universal concept of entropy. Examine thus the idea of extending these well known concepts to the radiation field of the whole universe.

In principle nothing hinders to refer the index *j* to all bodies of the universe having volume $V = \sum_j V_j$ and local masses $m_j = \rho_j V_j$. However now it is necessary to specify the meaning of δm and δV. The latter can be identified with the observed universe expansion, the former with the progressive creation of mass in the available volume; the initial Big Bang energy fulfills in principle this requirement.

To guess the physical context implied by an expanding universe, regarded as the general framework within which are allowed physical events, consider the following three points.

(i) The expansion implies size growth of space time and thus, in particular, stretching of light wavelengths propagating in the universe *i.e.* energy loss of photons, which in turn means cooling of the e.m. radiation energy field.

(ii) As ρ_j decrease because $\delta m/m < \delta V/V$, then differentiating (2.2) one finds

$$\delta\left(\frac{\Delta\rho}{\rho_0}\right) = -\sum_j \frac{\delta\rho_j}{\rho_0}\left(\log\left(\frac{\rho_j}{\rho_0}\right)+1\right) = -\sum_j \frac{\delta\rho_j}{\rho_0}\log\left(\frac{\rho_j}{\rho_0}\right)$$
$$\sum_j \rho_j = const \quad \sum_j \delta\rho_j = 0.$$

Let ρ_0 be the initial Planck density at the time $t = t_{Pl}$, whereas ρ_j is that at the time $t > t_{Pl}$; then each term of the sum reads

$$\delta\left(\frac{\Delta\rho}{\rho_0}\right) = -\sum_j \frac{\rho_j(t+\delta t) - \rho_j(t)}{\rho_0}\log\left(\frac{\rho_j}{\rho_0}\right)$$

because the change of ρ_j is reasonably due to its time dependence; as indeed $\rho_j(t+\delta t) < \rho_j(t)$ because of the universe expansion during δt, one infers

$$\delta\left(\frac{\Delta\rho}{\rho_0}\right) \ge 0,$$

which actually is nothing else but the second law of thermodynamics.

(iii) Redistribution of the initial Big Bang energy energy in an increasing volume of universe means decreasing the global energy density. Consider indeed $T\delta S$ during growth: if $\delta S(t)$ increases, then expectedly $T(t)$ decreases. In this model, heat diffusion accounts for the dynamic energy balance in a growing environment; in other words, heat diffusion is the leading mechanism to transfer energy from bulk previously formed to a freshly formed volume of universe.

These hints highlight what has to do with the classical concept of diffusion coefficient with the cosmology: it concerns the residual energy/radiation field that pervades the universe after the Big Bang.

Regard the big bang as initial event characterized by an excess of energy that pushes outwards the current space time boundary, which thus starts growing to dissipate energy into an increasing V. Let this amount of energy fill all space time available during growth; heat diffusion is the simplest hypothesis about a possible mechanism to transfer energy throughout the new space time volumes allowed by the growth process. In other words, the thermal field in V is due to heat flux diffusing throughout the space time: this is a time process towards the thermal equilibrium of the universe. Let the matter be actually at rest, it is displaced outwards by the drag effect of the expanding space time itself; δm is not related to diffusion driven displacement of matter gradients throughout the universe, rather it concerns the formation of new matter along with new space. If so, then neglect matter and energy convection, whereas it is enough to implement a classical differential equation valid for a continuous and homogeneous heat flux progressively distributed throughout an increasing region of available space time; nevertheless this still implies decreasing ρ because $\delta m/m < \delta V/V$ means insufficient local mass growth into a larger space volume. Even with this meaning of δm and δV the Equations (2.2) and (2.3) keep their conceptual validity. Thus (1.1) can be solved to find the T time profile of the radiation thermal field during universe growth. This means expressing $\chi = \chi(x,y,z,t)$ via the dimensionless temperature T/T_0 defining the energy $k_B T$: the constant T_0 has been introduced for dimensional reasons.

The first problem is to guess the diffusion coefficient $D = D(\chi)$ and the source term $\mathcal{S} = \mathcal{S}(\chi)$ to solve the heat Equation (1.1), which reads now in dimensionless temperature form

$$\begin{gathered} \nabla\cdot(D\nabla\chi) + \mathcal{S} = \dot{\chi} \quad \frac{energy'}{\hbar} = \mathcal{S}(\chi) \quad D = D(\chi) \\ \frac{energy''}{\hbar} = \dot{\chi} \quad \chi(x,y,z,t) = \frac{T}{T_0}. \end{gathered} \tag{2.4}$$

A first reasonable hint is the source term $\mathcal{S} \propto T$: *i.e.* the higher T, the stronger the energy released by $\mathcal{S}$ in its own V_j and available for the neighbor elementary volumes V_j' of universe. Moreover let be $D \propto T^2$: the square T dependence of D means that the bulk energy supplied by each V_j is efficiently exchanged with the elementary volume V_j' facing V_j to dissipate energy towards regions further away from the source. In other words, the ability to diffuse from V_j more energy than the incoming one prevents local energy increase and thus local T gradients. In this way the higher T, the higher D and thus the ability of V_j' to exchange energy with another newly created V_j''. This is the specificity of a system, the universe, able to regenerate itself; such D is reasonably guessed having the form $D = D_0 \chi^2$, with D_0 dimensional constant. Since $\chi^2 \nabla \chi = \nabla\left(\chi^3\right)/3$, the first addend of (2.4) reads $\left(D_0/3\right)\nabla\cdot\left(\nabla\chi^3\right)$; write then (2.4) as

$$\frac{D_0}{3}\nabla\cdot\left(\nabla\left(\chi^3\right)\right)+\xi\chi=\dot{\chi} \quad D=D_0\chi^2 \quad \mathcal{S}=\xi\chi. \tag{2.5}$$

Next with the position $\dot{\chi}=\xi\chi-q\chi^3$, being q a proportionality dimensional factor to be determined, (2.5) yields

$$\nabla^2\left(\chi^3\right)=-\frac{3q}{D_0}\chi^3 \quad \dot{\chi}=\xi\chi-q\chi^3 \quad q=q\left(x,y,z\right) \quad \xi=\xi\left(t\right). \tag{2.6}$$

In principle the source term coefficient ξ is expected to be a function of time; if the initial Big Bang energy source is unique and located in a unique place, wherever this place in the new born universe might be, then $\xi=\xi\left(t\right)$ means that the energy of source term changes at increasing times to account for the energy converted into the expansion rate. The factor q accounts for the fact that the source term determining $\dot{\chi}$ can be located somewhere in the universe. Although (2.5) and (2.6) do not imply any specific information about the actual locations of ξ and q, these mere definitions allow finding a simple and reasonable solution of (2.5); it is immediate to verify that the general solution of the second (2.6) is

$$\begin{gathered}\chi=\sqrt{\frac{g_1}{f'+2qg_2}} \quad f'=f'\left(x,y,z\right)\\ g_1=g_1\left(t\right)=\exp\left(2\int_{bb}^{t}\xi\mathrm{d}t\right) \quad g_2=g_2\left(t\right)=\int_{bb}^{t}g_1\left(t\right)\mathrm{d}t,\end{gathered} \tag{2.7}$$

where f' is an arbitrary function of space coordinates only, which takes the meaning of arbitrary integration constant with respect to the time integration. The notation indicates that the time integrals of ξ span from the Big Bang time bb to an arbitrary later time, e.g. today's time. It is worth noticing that putting $\xi=0$ means neglecting the source term $\mathcal{S}$ in the starting Equation (2.4), as this appears in (2.5); however even in this case χ remains a sensible solution, as it must be. Indeed $g_1=1$ and $g_2=t+const_0$ yield

$$\chi_0=\sqrt{\frac{1}{f'+2q\left(t+const_0\right)}}, \tag{2.8}$$

with notation emphasizing a mere heat diffusion equation in an adiabatic system tending to its equilibrium state; the time evolution of the universe should follow the $T\propto t^{-1/2}$ law without divergence problems, whatever f' and q might be. It also appears that $\xi=\pm\xi_0\neq 0$ would imply $g_1=\exp\left(\pm 2\xi_0 t\right)$ and $\xi_2=\pm\left(2\xi_0\right)^{-1}\exp\left(\pm 2\xi_0 t\right)$; *i.e.* even with a constant source term χ_0 does not diverge because

$$\chi_{\xi=const}=\sqrt{\frac{\exp\left(\pm 2\xi_0 t\right)}{f'\pm\left(q/2\xi_0\right)\exp\left(\pm 2\xi_0 t\right)}} \quad f'\neq q/2\xi_0$$

with finite limit for $t\rightarrow\in 3.45$. This ensures that the assumptions (2.5) are sensible.

Returning to the general case $\xi(t)\neq 0$, take advantage of the arbitrariness of f' to put for simplicity $f'=2q$ and rewrite then the solution (2.7) of the second (2.6) as

$$\chi=\frac{1}{\sqrt{q}}\frac{1}{2}\sqrt{\frac{1}{(1+g_2)/g_1}}. \tag{2.9}$$

In this particular case χ results factorized by a space function q and a time function of ξ. In other words, the simplifying assumption $f'=2q$ is useful to split (2.7) into the space and time functions of (2.9), which are assessed separately via straightforward considerations and compared easily with experimental data.

-As concerns the space function q plug now (2.9) in the first (2.6), which takes thus the form

$$\nabla^2\left(\frac{1}{q^{3/2}}\right)=-\frac{3}{D_0}\frac{1}{q^{1/2}}\ \Rightarrow\ \nabla^2Q^3=-\zeta Q\quad Q=q^{-1/2}\quad \zeta=\frac{3}{D_0},$$

whose general solution reads in integral form

$$Q=C_2-(x+y+z)\pm\int_{a_0}^{Q}\frac{6a^2}{\sqrt{6C_1-6a^4\zeta}}\mathrm{d}a \tag{2.10}$$

being C_1 and C_2 the integration constants; a is a dummy integration parameter in turn dependent on D_0. In practice (2.10) correlates space coordinates x,y,z and $q^{-1/2}$; once guessing appropriately the boundary conditions via the integration constants, the numerical integration yields $q^{-1/2}$ at any x,y,z. The lack of information about the source location does not prevent interesting information via (2.7). Two examples of such information are shortly exemplified here considering (2.9).

-For example, to emphasize the first kind of information consider in particular

$$\chi=\frac{Q}{2}\sqrt{\frac{1}{(1+g_2)/g_1}}\qquad Q\sim\langle Q\rangle\sin\left(Q_1(x+y+z)\right) \tag{2.11}$$

i.e. Q has an oscillating space trend around an average value $\langle Q\rangle$ with x,y,z extended throughout the universe volume; this hint admits that the main time line described by $\sqrt{g_1/(1+2g_2)}$ is subjected to space ripples of T described by the space function Q with amplitude modulated by $\langle Q\rangle$ and space frequency modulated by Q_1. In effect it is known that the CMB is not completely smooth and uniform, rather, it shows faint anisotropy and T oscillations.

-Consider the time factor of (2.9) and note that although $\xi(t)$ is not known, it is certainly possible to expand in series the function $(1+g_2)/g_1$ around an appropriate $t=t_0$. Trivial considerations on the coefficients of series expansion

$$\xi(t)=c_1/(t+\tau)+c_2/(t+\tau)^2+\cdots$$

show that $(1+g_2)/g_1\approx C_0+C_1(t-t_0)+C_2(t-t_0)^2+\cdots$; the constant τ has been included to avoid divergence for $t\to t_{bb}$. Plugging into (2.9) and collecting the terms C_i' that multiply t separately from the constant terms, (2.11) up to the second order reads

$$\frac{T}{T_0\langle Q\rangle}\approx\sqrt{\frac{1}{C'_0+C'_1t+C'_2t^2+\cdots}}\qquad C'_i=C'_i\left(t_0,\tau,T_0,c_1,c_2,C_0,C_1,C_2\right),\tag{2.12}$$

This result must be assessed with the condition that for $t=0$ must hold $T_{Pl}\sqrt{C'_0}/T_0\langle Q\rangle=1$, *i.e.* $\sqrt{C'_0}=T_0\langle Q\rangle/T_{Pl}$, being T_{Pl} the Planck temperature. Replacing in (2.12) it yields $T=T_{Pl}\left(1+C''_1t+C''_2t^2\right)^{-1/2}$ with $C''_1=C'_1/C'_0$, $C''_2=C'_2/C'_0$; hence, introducing for shortness best fit coefficients C''_1 and C''_2, the result reads

$$T\approx\frac{T_0\langle Q\rangle/\sqrt{C'_0}}{\sqrt{1+C''_1t+C''_2t^2+\cdots}}\Rightarrow T\approx\frac{T_{Pl}}{\sqrt{1+1.8\times10^{43}t+10^{27}t^2+\cdots}}$$

$$T_{Pl}=\frac{T_0\langle Q\rangle}{\sqrt{C'_0}}=1.4\times10^{32}\ \text{K};$$

the coefficients, calculated with best fit methods e.g. requiring that today's T is 2.72 K, show that the series converges. The plot of **Figure 1** is compliant with the literature results.

Further considerations that deserve specific attention should concern χ of (2.7), especially to calculate "ab initio" the series coefficients (2.12); these considerations however are omitted for brevity because out of the purposes of this paper. Nevertheless, it is easy to guess the values of these coefficients even in the particular case (2.11). Let $C''_1=1/t_{Pl}$ in order to refer T to the Planck time t_{Pl}, as suggested by the best fit calculation. Write thus

$$T\approx\frac{T_{Pl}}{\sqrt{1+\left(t/t_{Pl}\right)+\left(t/t_{cr}\right)^2+\cdots}}\tag{2.13}$$

As concerns C''_2, note that exists in fact a further link as concerns the relationship between T and t. Calculate the radiation field momentum p as $pc=hc/\lambda$ and equate the energy at the left hand side to k_BT; it yields an additional condition to link T to t because it is possible to write

$$pc=\frac{hc}{\lambda}=k_BT\ \Rightarrow\ \frac{1}{t_{cr}}=\frac{c}{\lambda_{cr}}=\frac{k_B}{h}T_{cr},\tag{2.14}$$

being T_{cr} the critical temperature corresponding to the critical time t_{cr}. These quantities are identified in **Figure 2**, which shows a changed profile at increasing times when the square term is no longer negligible with respect to the linear term. Whatever the actual physical reason of this time dependence might be, let us assign to t_{cr} arbitrary values to observe how changes T_{cr}. The **Figure 1** is calculated implementing four trial values of C''_2. As expected, when $\left(t/t_{cr}\right)^2\ll t/t_{Pl}$ the plots overlap, whereas at times $t>t_{cr}$ appear four different profiles emphasizing the respective times at which begin the changes of T profiles. Compare then T_{cr} corresponding to the different t_{cr} requiring their consistency with (2.14). **Figure 2** shows in greater detail the region of the plot around the possible t_{cr}: it appears that only the solid line curve fulfills this condition, which therefore identifies uniquely the coefficient of (2.13).

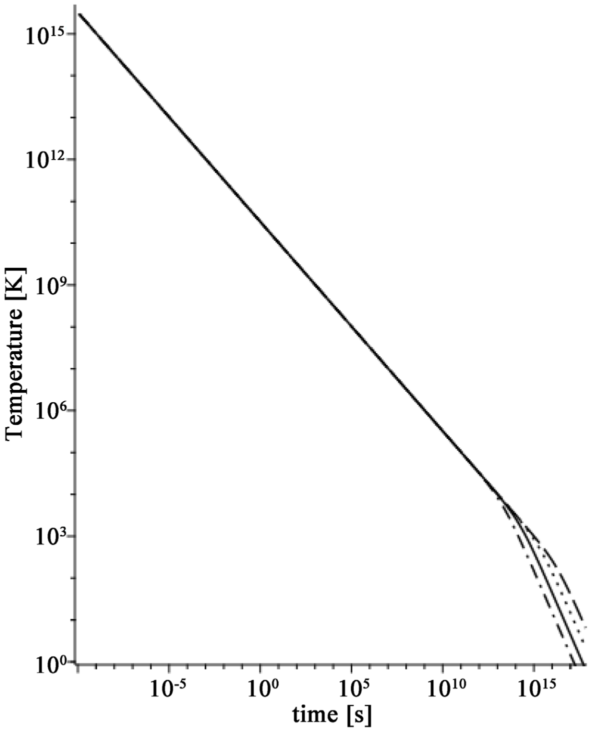

Figure 1. LogLogPlot of temperature (Kelvin) vs time (seconds).

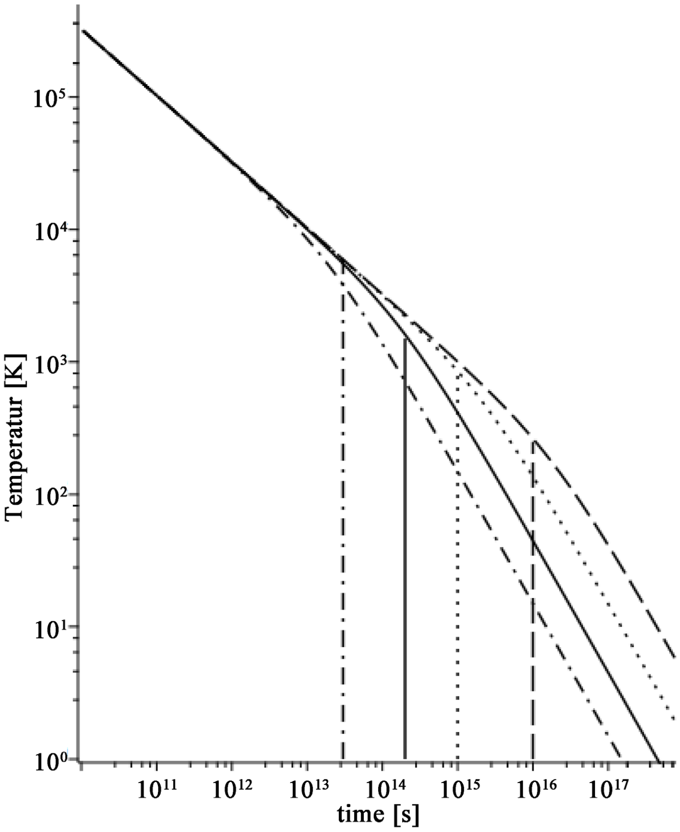

Figure 2. Detail of **Figure 1** to evidence the T profiles at times around t_{cr}. The vertical lines evidence the links of the respective T_{cr} with T.

Holds thus the conclusion: the big bang energy diffusion only, without convection mechanisms, is enough and adequate to explain decently the time profile of the various ages of the universe.

Note at this point that the physical dimension of D/t is square velocity; so write formally $D/t = v^2$ as $D = sv$, being in general $s = vt$ an arbitrary length. Dividing this latter by s^2 one finds $D/s = \mathcal{H} = v/s$, being H a function of t^{-1}. Specify then v in order to give this result a physical sense. For example define $v \equiv v_s$ *i.e.* $v_s = \delta s/\delta t$, in which case one concerns the time change δs of the current length $s = s(t)$ during the time range δt. So

$$\mathcal{H} = \frac{1}{s}\frac{\delta s}{\delta t}$$

Next specify further what does s represent. If s is the radius of a sphere, v_s/s represents the relative change of size of the sphere. This is not at all trivial according to (2.1): e.g. calculating from this result $\delta\rho/\rho$ one could infer information about the entropy of the material of which the sphere is made. Also, since $\hbar\nabla\cdot\boldsymbol{v} = \varepsilon$ represents an energy, one could calculate also the kinetic energy of the sphere if is known the current mass m generated by the big bang and $\delta(\delta v_s/\delta t)/\delta t'$ by integrating $v'\delta(\delta v_s/\delta t)/(v'\delta t')$ times $m\hbar$. So even an abstract dimensional approach like that proposed here can in fact provide concrete physical information once specifying the pertinent conceptual frame. If the sphere represents the universe, this still holds regarding H with reference to the Hubble law, as it is done in the next section.

3. Cosmological Implications: The Hubble Tension

This section makes explicit reference to the uncertainty equation, whose implications are introduced in a previous paper [1]:

$$\delta x\delta p_x = n\hbar = \delta\varepsilon\delta t \qquad \frac{\hbar G}{c^2} = \frac{length^3}{time}. \tag{3.1}$$

With reference to (3.1) define a range δs as follows

$$\delta s_{\pm} = \frac{n\hbar}{\delta p_x} \pm \delta x = \frac{n\hbar}{\delta\varepsilon\delta t'/\delta x'} \pm \delta x \qquad \delta p_x = \frac{\delta\varepsilon\delta t'}{\delta x'}, \tag{3.2}$$

which reads

$$\delta s_{\pm} = c\delta t \pm \delta x \qquad c = \frac{\delta x'}{\delta t'} \qquad \delta t = \frac{n\hbar}{\delta\varepsilon};$$

thus merging both chances,

$$\delta s_+\delta s_- = \delta s^2 = c^2\delta t^2 - \delta x^2 = inv \Rightarrow \frac{\delta s^2}{\delta t^2} = c^2 - v_x^2 \qquad v_x = \frac{\delta x}{\delta t} \tag{3.3}$$

yields

$$\frac{\delta s^2}{\delta t^2} = \frac{\delta x^2}{\delta t^2} - v_x^2 \Rightarrow \frac{\delta s^2}{\delta t^2} = c^2 - v_x^2 = c^2\beta_{sr}^2 \qquad c = \frac{\delta x}{\delta t} \qquad \beta_{sr}^2 = 1 - \frac{v_x^2}{c^2}. \tag{3.4}$$

Moreover the second (3.2) yields $\delta(\varepsilon \pm const) = \delta(p_x c)$, which reads

$\varepsilon \pm const = p_{x\pm}c$; as before, multiplying the chances $\varepsilon + const = p_{x+}c$ and $\varepsilon - const = p_{x-}c$ side by side one finds $\varepsilon^2 - const^2 = (p_{x+}c)(p_{x-}c)$ *i.e.*

$$\varepsilon^2 = (pc)^2 + const^2. \tag{3.5}$$

No further comments are needed for (3.3) and (3.5), which have been mentioned to ensure that the positions (3.2) and the subsequent steps are correct. Eventually (3.4) implies

$$\frac{1}{s^2}\frac{\delta s^2}{\delta t^2} = H_{sr}^2 = c^2\frac{\beta_{sr}^2}{s^2}, \tag{3.6}$$

being s an arbitrary length defined in the same reference system of the ranges δs and δt.

Equation (3.6) is an equation of special relativity, its physical meaning is related to the quoted invariant.

Now (3.6) is rewritten by adding v'^2/s^2 at both sides, with v' arbitrary velocity, as

$$\left(\frac{1}{s}\frac{\delta s}{\delta t}\right)^2 + \frac{v'^2}{s^2} = \left(\frac{c^2}{s^2} - \frac{v_x^2}{s^2}\right) + \frac{v'^2}{s^2} \Rightarrow H_u^2 = \left(\frac{1}{s}\frac{\delta s}{\delta t}\right)^2 + \frac{v'^2}{s^2}, \tag{3.7}$$

which reads also

$$H_u^2 = \frac{c^2\beta^2}{s^2} = \frac{v_u^2}{s^2} \qquad \beta^2 = 1 - \frac{v_x^2 - v'^2}{c^2} \qquad v_u = \beta c: \tag{3.8}$$

the resulting v_u^2 merges $v_x^2 - v'^2$ to fulfill the condition $v_u = \beta c \le c$. Although the form of (3.8) is analogous to (3.6), H_u is defined after having introduced the arbitrary v' in (3.7). The new β, different from β_{sr} defined by v_x only, changes in principle the physical meaning of the initial (3.6) and thus the conceptual frame of the results.

As v_x and v' of (3.7) are independent each other and both arbitrary, owing to $H_u \ne H_{sr}$ regard H_u in the conceptual frame of a curved space time; this in fact comes from the additional $v'^2/s^2 \ne 0$ of (3.7) with $v_u \le c$ owing to the condition $\beta c \le c$ only. Dividing both sides of (3.8) by a constant H_0^2 one finds

$$\frac{H_u^2}{H_0^2} = \frac{c^2}{s^2 H_0^2} - \frac{(v_u/H_0)^2}{s^2} \qquad v_u^2 = v_x^2 - v'^2 \qquad 0 \le v_u \le c. \tag{3.9}$$

As H_u^2 has physical dimensions $time^{-2}$ like ρG, it is possible to write

$$\frac{c^2}{s^2} = \zeta\rho G, \tag{3.10}$$

where the arbitrary s fits any given ρ via the dimensionless proportionality factor ζ. Thus the parameters ζ, v, ρ in (3.9) and (3.10) allow writing

$$\frac{H_u^2}{H_0^2} = \frac{\rho}{\rho_0} - k\frac{(c/H_0)^2}{s^2} \qquad H_0 = \sqrt{\zeta\rho_0 G} \qquad k = \frac{v_u^2}{c^2} \qquad 0 \le k \le 1. \tag{3.11}$$

If in particular $v' = const$, then this result calculated for $k = 0, 1$ is compliant with the rescaled form of the first Friedman equation, whereas $k = const/s^2 \ne 0$

is related to the space time curvature. Note however that H_u takes a minimum value for $k=1$ and a maximum value for $k=0$; so $H_u = H_u(k)$ fulfills

$$H_{u_{min}}(k \to 1) < H_u < H_{u_{max}}(k \to 0). \tag{3.12}$$

Anyway it follows from (3.7)

$$H_u = \frac{c\beta}{s} \quad hH_u = h\nu\beta \quad \nu = \frac{c}{s} \Rightarrow h\nu = \frac{\epsilon_0}{\beta} = \frac{hH_u}{\beta}: \tag{3.13}$$

as H_u has physical dimensions $time^{-1}$, is in principle guessable the corresponding energy $\epsilon = hH_u$, which in turn implies $\delta\epsilon = h\delta H_u$ and thus a range of possible values of H_u and ϵ. Also, since a time range δt must be related to $\delta\epsilon$, it follows that through $\delta\epsilon = n\hbar/\delta t$ is appropriate to define

$$\frac{\delta H_u}{H_u} = \frac{\delta\epsilon}{\epsilon} = \frac{n_u\hbar/\delta t}{\epsilon}. \tag{3.14}$$

Differentiating now the first (3.8) write

$$\delta\frac{H_u}{c} = \frac{\delta\beta}{s} - \frac{\beta\delta s}{s^2} = -\frac{\delta\left(v_u^2\right)}{2\beta sc^2} - \frac{\beta\delta s}{s^2} = -\frac{\delta\left(v_u^2\right)/c}{2H_u s^2} - \frac{H_u}{c}\frac{\delta s}{s};$$

hence, as in turn $\delta s/s = H_u\delta t/c$, one finds

$$-\frac{\delta H_u}{c} = \frac{\delta\left(v_u^2\right)/2cs^2}{H_u} + H_u^2\frac{\delta t}{c}. \tag{3.15}$$

This result reads by definition

$$-\left(H_u' - H_u''\right) = \left(\frac{v'^2}{s^2} - \frac{v''^2}{s^2}\right)\frac{1}{2H_u} + H_u^2\left(t' - t''\right), \tag{3.16}$$

which suggests

$$-H_u' - \frac{H'^2}{2H_u} - H_u^2 t' = -H_u'' - \frac{H''^2}{2H_u} - H_u^2 t'' \quad H'^2 = \frac{v'^2}{s^2} \quad H''^2 = \frac{v''^2}{s^2}; \tag{3.17}$$

if the primed and double primed terms are regarded in different reference systems, (3.16) implies

$$H_u^2 t + H_u + \frac{H^2}{2H_u} = \mathcal{T} = invariant \quad \mathcal{T} = time^{-1}. \tag{3.18}$$

Regard thus H and t in (3.18) as fixed parameters: then $2H_u t + 1 - H^2/2H_u^2 = 0$ solved with respect to $H_u = H_{min}$ reads $2H_{min}t + 1 - H^2/2H_{min}^2 = 0$, which requires in turn for any H the minimum condition

$$H^2 = 4tH_{u_{min}}^3 + 2H_{u_{min}}^2. \tag{3.19}$$

Guessing then

$$H \approx \frac{\xi}{t} \quad \xi = const \;\Rightarrow\; \xi^2 \approx 4\left(H_{u_{min}}t\right)^3 + 2\left(H_{u_{min}}t\right)^2, \tag{3.20}$$

one finds that H^2 of (3.18) is uniquely identified by its own chance of fulfilling this boundary condition of minimum of H_u at any given *t*; *i.e.* replacing

(3.19) into (3.18) yields

$$\mathcal{T} = H_u^2 t + H_u + \frac{2tH_{u_{min}}^3 + H_{u_{min}}^2}{H_u} \quad \mathcal{T}_{min} = \mathcal{T}\left(H_u = H_{u_{min}}\right) = 3H_{u_{min}}^2 t + 2H_{u_{min}}. \tag{3.21}$$

The equations to be calculated are therefore (3.21) and (3.14), here rewritten via (3.6) replacing $\delta t = \delta s/\left(sH_u\right)$ as

$$h\delta H_u = \delta\epsilon = \frac{n\hbar}{\delta t} = \frac{n\hbar}{\delta s/sH_u} \Rightarrow \frac{\delta H_u}{H_u} = \frac{s}{\delta s}\frac{n_u}{2\pi}. \tag{3.22}$$

Note that are useful for the next calculations the approximate definitions deductible from (3.8)

$$H_u = \frac{1}{s}\frac{\delta s}{\delta t} \approx \frac{1}{t} \text{ for } \delta s \approx s \text{ and } \delta t \approx t \Rightarrow \frac{\delta H_u}{H_u} \approx \frac{n_u}{2\pi}: \tag{3.23}$$

these positions implement $\delta s = s - s_0$ and $\delta t = t - t_0$ regarded with $s \gg s_0$ and $t \gg t_0$. In principle this is possible because the range boundaries are arbitrary; if in particular s and t define cosmological length and time, in practice these approximations are a sensible boundary condition extrapolated at the cosmological scale of today's universe age and size with respect to the big bang conditions with the same t_0 and s_0.

Enentually rewrite (3.23) as There is a direct way to assess and compare $\delta H_u/H_u$ of (GZL) and (HSH) with experimental data (3.31). Consider (3.23)

$$\delta H_u = n_u H \Rightarrow \delta\epsilon = n_u hH \quad H = \frac{2\pi}{t_u}: \tag{3.24}$$

owing to the physical dimensions oh H and H_u, this last result is the cosmological equivalent of the quantum $\delta\varepsilon = \varepsilon_2 - \varepsilon_1 = nh\nu$, which means that ε_2 differs from ε_1 bu an integer number $1 \le n_i \le n$ of quanta $h\nu$ such that $\varepsilon_1 \le n_i h\nu \le \varepsilon_2$; in (3.24) each quantum is hH, which in fact completes the basic guess of (3.18) and (3.21) about the physical meaning of H: as in (3.13) H_u is related to $\epsilon = hH_u$, by analogy here H is reasonably related to hH.

First of all this definition of H justifies why it can be regarded as a fixed parameter to calculate the minimum $H_{u_{min}}$ of the values allowed for H_u; indeed t is a variable parameter to carry out calculations at any given age t and size r of the universe; t_u and r_u are today's respective values. Accordingly quantized $\delta H_u = H_u'' - H_u'$ means that upper range boundary H_u'' differs from lower range boundary H_u' by an integer number n of discrete steps H_{u_i}, as in fact suggested by the early (3.12); now it is reasonable to assume these intermediate values as $H_{u_i} = n_i \nu_{u_i}$ around $H_{u_{min}}$. Therefore the plot of $\mathcal{T}$ vs H_u can be presented as a function of $H_{u_{min}} \pm n_i \nu_{u_i}$ for various quantum numbers n_i.

These ideas are now checked: the next part of text aims to show that the idea of regarding hH as quantum of energy is strictly linked to the Hubble tension.

At this point to carry out calculations of cosmological interest, introduce order of magnitude estimates of some key universe parameters [2].

The data of interest are the Hubble parameter H_u and Einstein cosmological parameter Λ, the universe radius r_u and age t_u, whose numerical estimates shown in **Table 1** are reported for convenience also here:

Table 1. Estimated cosmological data reported in [2].

	Today's universe in Planck units	
Property of present-day observable universe	Approximate number of Planck units	Equivalents
Age	8.08×10^{60} tp	4.35×10^{17} s or 1.38×10^{10} years
Diameter	5.4×10^{61} Ip	8.7×10^{26} m or 9.2×10^{10} light-years
Mass	approx. 10^{60} mp	3×10^{52} kg or 1.5×10^{22} solar masses (only counting stars) 10^{80} protons (sometimes known as the Eddington number)
Density	1.8×10^{-123} mp·lp^{-3}	9.9×10^{-27} kg·m^{-3}
Temperature	1.9×10^{-32} Tp	2.725 K temperature of the cosmic microwave background radiation
Cosmological constant	10^{-122} lp^{-2}	10^{-52} m^{-2}
Hubble constant	10^{-61} tp^{-1}	10^{-18} s^{-1} 10^{2} (km/s)/Mpc

$$r_u^* \sim 4.35\times10^{26}\text{ m} \quad t_u^* \sim 4.35\times10^{17}\text{ s} \quad \Lambda^* \sim 10^{-52}\text{ m}^{-2} \quad H_u^* \sim 10^{-18}\text{ s}^{-1}: \tag{3.25}$$

the starred notation reminds that these values are mere estimates. The mass M_u^* of the universe is not quoted because it amounts to 3 × 10^{52} kg counting the stars only; this value is close the one 2 × 10^{52} kg calculated in [3] under the same approximation.

The literature value $\Lambda_t^* = c^2\Lambda^*$, which directly compares with the physical dimensions of the square Hubble factor, is

$$\frac{\Lambda_t^*}{H_u^{*2}} \sim 2 \quad \Lambda_t^* = c^2\Lambda^* \sim 10^{-36}\text{ s}^{-2} \sim H_u^{*2}. \tag{3.26}$$

Deserve attention also the estimates of critical density ρ_{cr}^* and mass M_u^* of the universe

$$\rho_{cr}^* \sim 9.9\times10^{-27}\frac{\text{kg}}{\text{m}^3} \quad M_u^* \sim 3.52\times10^{52}\text{ kg}, \tag{3.27}$$

where ρ_{cr}^* has been calculated via the Friedmann equation,

$$\rho_{cr} = \frac{3H_0^{*2}}{8\pi G} = 9.4\times10^{-27}\frac{\text{kg}}{\text{m}^3} \quad H_0^* = 2.2\times10^{-18}\text{ s}^{-1}, \tag{3.28}$$

whereas M_u^* concerns reductively the visible mass accessible to the observation counting the visible stars only. Eventually quote here also the quantum vacuum mass density reported in [4] from cosmological data:

$$\rho_{vac} = (60.3\pm1.3)\times10^{-31}\frac{\text{g}}{\text{cm}^3}. \tag{3.29}$$

The values of quantum vacuum density and energy density calculated in [5] implementing (3.1)

$$\rho'_{vac} = 58\times10^{-31}\,\frac{\text{g}}{\text{cm}^3} \quad \eta'_{vac} = \rho'_{vac}c^2 = 5.2\times10^{-9}\,\frac{\text{erg}}{\text{cm}^3} \tag{3.30}$$

agree reasonably with (3.29). These estimates rise two questions.

The first one is: why is Λ^* of the order of $\left(r_u^*\right)^{-2}$ and Λ_t^* of the order of H_u^{*2} ?

In principle this question is legitimate, because Einstein introduced the constant Λ to counterbalance the effect of gravity and obtain a static universe, whereas in the first (3.26) Λ^* seems proportional to the parameter H_u^* describing the universe expansion. As concerns the Hubble factor, accurate values have been measured in the frame of the Planck and SHOE collaborations and calculated with various methods [6]:

$$H'' = 73.5\pm1.4 \quad H' = 67.8\pm1.4 \quad \langle H\rangle = 70.7\,(\text{km/s})/\text{Mpc} \tag{3.31}$$

and

$$H'' = 73.30\pm1.04 \quad H' = 67.4\pm0.5\,(\text{km/s})/\text{Mpc}. \tag{3.32}$$

The conversion factor to s^{-1} is

$$1\,(\text{km/s})/\text{Mpc} = 3.24\times10^{-20}\ \text{s}^{-1} \ \Rightarrow\ \langle H\rangle = 2.29\times10^{-18}\ \text{s}^{-1}. \tag{3.33}$$

The discrepancy between the values of H' and H'', known as "Hubble tension", is currently explained postulating tentatively that the Hubble parameter is a function of time; so the observed results should depend on the distances of light emitting sources from the observer, which include far or close objects and thus different ages of universe expansion with correspondingly different time dependent values of H_u. Moreover, the experimental difficulties of determining red shifts and distances to be correlated with the pertinent recession rates certainly affect accurate estimates of the observed values of H_u.

However this way of explaining both experimental observations and theoretical models to justify the Hubble tension is doubtful:

the mere time dependence of H_u should reveal a continuity of values, spreading from the oldest to the youngest light source, to account for the respective age dependent $H_u(t)$. The same holds also for the inaccuracy of the distance estimates. The crucial problem is that in fact the data reported in (3.31) reveal a gap between the experimental error bars: in other words, even considering the upper and lower error boundaries in the most restrictive way, the values of H' and H'' in either range $74.9 \leftrightarrow 72.1$ and $69.2 \leftrightarrow 56.4$ do not overlap, suggesting instead that two distinct classes of H_u should actually exist. The same holds for (3.32).

In fact, the preliminary considerations of this section prospect the Hubble tension as a natural corollary of quantum premises: (3.22) and (3.24) have emphasized that the gap δH_u of (3.31) is an actual energy gap, as it concerns $\epsilon = \hbar H = energy$. This appears instead natural in the theoretical frame where energy and time ranges are related to (3.1).

-On the one hand the uncertainty ranges of dynamical variables have been preliminarily introduced in this paper without hypotheses about their specific meaning, but merely as a general quantum basis in (3.1); so it is not surprising that $\delta H_u/H_u$ of (3.7) is contextually compatible with two values H' and H''.

-On the other hand the present way to regard (3.31) rises the second question: if δH_u splits into two separate error bars $\delta H'$ and $\delta H''$, *i.e.* two separate uncertainty ranges within which fall the central values H' and H'', which one of the two ranges do the Friedman equations refer to?

The purpose of this subsection is to propose a possible answer to the aforesaid points, to emphasize their actual physical meaning and to show that values reasonably related to (3.27) and (3.29) are calculable in the frame of the present theoretical model. As a matter of fact, (3.33) prospect a preliminary hint in this respect: the literature value to calculate ρ_{cr}^{*} in (3.27) fits surprisingly the average value (3.33).

Note that regarding separately H' and H'' does not prevent the uniqueness of recession rate, because in fact the Hubble law implements $\dot{s}$ and s: even at the boundary of the universe, δH_u concerns an interface layer between an internal environment where hold the physical laws we know and an external nothingness. Whatever the recession rate $\dot{s}$ of the universe boundary might be, the Hubble law allows writing in principle

$$\delta\epsilon_u = h\left(H_u'' - H_u'\right) = h\left(\frac{\dot{s}}{s''} - \frac{\dot{s}}{s'}\right) \Rightarrow \delta\epsilon_u = h\frac{\dot{s}\delta s}{s's''} \qquad \delta s = s' - s'', \qquad (3.34)$$

i.e. different values of H_u with a uniquely defined recession rate of a boundary layer of finite thickness δs, in which case s' and s'' are the distances of the observer from the inner and outer layer. Note that (3.34) automatically implies that δs uniquely defined at a given time introduces a universe with Gaussian curvature proportional to $\left(s''s'\right)^{-1}$. A first corollary of (3.34) is then

$$\frac{\delta\epsilon_u}{\delta s} = h\frac{\dot{s}}{s's''}: \qquad (3.35)$$

moreover

$$\frac{\delta\epsilon_u}{\delta\dot{s}} = h\frac{\delta s}{s's''} \Rightarrow \frac{\delta}{\delta t}\frac{\delta\epsilon_u}{\delta\dot{s}} = h\frac{\delta}{\delta t}\frac{\delta s}{s's''}.$$

If $s's'' = const$ and $\delta s = s$, i.e in particular $\delta s = s - 0$, then the right hand side reads

$$\frac{h}{s's''}\dot{s} \Rightarrow \frac{\delta}{\delta t}\frac{\delta\epsilon_u}{\delta\dot{s}} = \frac{\delta\epsilon_u}{\delta s}$$

A second corollary follows owing to the physical dimensions of $\delta\epsilon_u/\delta s = force$, and regarding $\left(s's''\right)^{-1}$ as Gauss curvature. One infers that $force \propto Gauss\ curvature$, being $h\dot{s}$ the proportionality factor introducing the principal curvatures $\left(const'/s'\right)\left(const''/s''\right)$ of the boundary of the universe. Noting that $h\dot{s}$ has physical dimensions $mass \times length^3 \times time^{-2}$, nothing hinders to define $h\dot{s} = Gm'm''$, even without any proportionality constant as m'

and m'' are arbitrary, in which case (3.35) is precisely the Newton law. Strictly speaking the standard form of the Newton formula could be acknowledged directly compliant itself with this conclusion via the $r^{-2}=\left(r'r''\right)^{-1}$ dependence formally guessable for the gravitational interaction range likewise $\left(s's''\right)^{-1}$ of (3.35): however this reasoning, hidden in the Newton law although identifiable in principle, appears instead clear in (3.34) concerning the curved boundary shell of expanding universe. Note that $h\dot{s}$ can be rewritten as

$$\frac{h\dot{s}}{m''c^2}=\frac{m'G}{c^2} \Rightarrow \frac{\dot{s}}{r_m}=\nu \quad \nu=\frac{m''c^2}{h} \quad r_m=\frac{m'G}{c^2}. \tag{3.36}$$

Eventually it is also worth noticing that this reasoning holds even for the Coulomb law, because e^2 has the same physical dimensions of m^2G, so that it is still possible to write $h\dot{s}\leftrightarrow e^2$ in agreement with (3.35). In other words, replacing $h\dot{s}\rightarrow e^2$ and $m'G/c^2\rightarrow r''$, (3.35) reads

$$\frac{e^2}{m''c^2}=r'':$$

regarding m'' as electron mass and relating r'' to e^2/r'' the result still has an identifiable physical meaning, i brings to the classical radius of the electron.

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Since the cosmological parameters of Figure 3 are estimates, establish some initial relationships that suggest how to calculate more rational values t_u and Λ: the strategy to follow is to find acknowledged results and sensible correlations from the available estimates reported in the literature, even looking for new values as close as possible to these latter.

-According to (3.23) $H_u^*\approx t_u^{*-1}\approx 10^{-18}\ \mathrm{s}^{-1}$ implies $H_u^*/\sqrt{\Lambda^*}\approx 10^8\ \mathrm{m/s}$ and suggests that a reasonable and reliable expectation value could be *c*; assume thus

$$\frac{H_u}{\sqrt{\Lambda}}=c \quad H_u=H_u\left(t\right) \quad \Lambda=\Lambda\left(t\right), \tag{3.37}$$

which does not require that both parameters at the left hand side are constant, rather they possibly evolve as a function of time in order to fulfill coherently this condition. The unstarred notation indicates reviewed values to be implemented and then assessed in the next calculations. In effect H_u and Λ are time and space parameters; describing the universe through them means describing the curved space time itself.

-Take r_u directly from Figure 3, it is certainly an allowed value for the universe radius; rather it is necessary to check the corresponding time t_u at which this actual space size is in fact attained. In other words, it is guessable an actual t_u more accurately representative of the age of the universe than the estimate t_u^*.

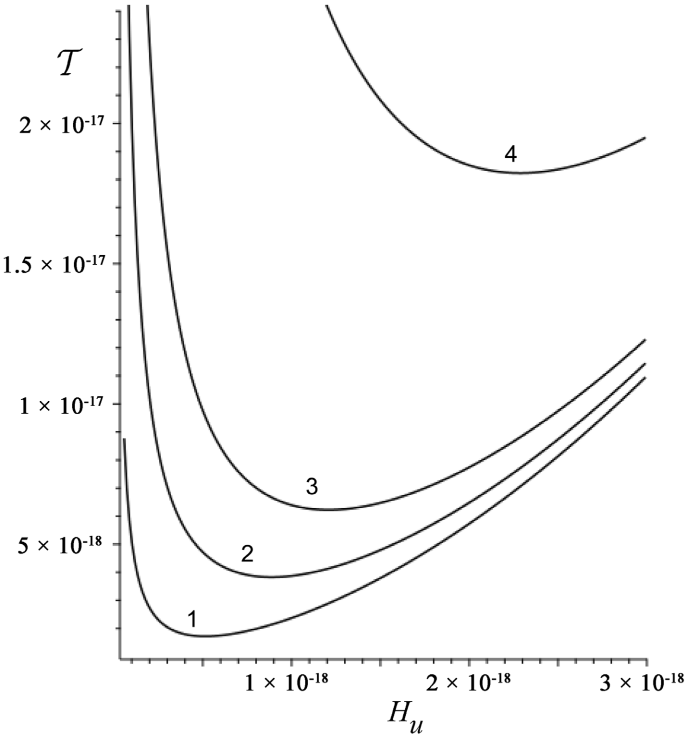

Figure 3. Plot of $\mathcal{T}$ vs H_u for various values of H: the curves are calculated for $H = 1\times10^{-18}\ \text{s}^{-1}$, $H = 2\times10^{-18}\ \text{s}^{-1}$, $H = 3\times10^{-18}\ \text{s}^{-1}$ and $H = 7.2\times10^{-18}\ \text{s}^{-1}$ respectively.

Of course one could have also taken for granted t_u^* and look for a consequent r_u; however, to implement in the following path just outlined, we assume $r_u \equiv r_u^*$ and look for a reliable t_u in turn consistent with new Λ and H_u as close as possible to the quoted Λ^* and H_u^*.

-The strategy of implementing and next assessing the input data of Figure 3, suggests the chance of calculating M_u and ρ_u via the gravitational radius (3.36) with $r_m = r_u$; the result is

$$\frac{M_u G}{c^2} \approx r_u \quad \Rightarrow \quad M_u \approx 5.9\times10^{53}\ \text{kg}. \tag{3.38}$$

It is preliminarily acceptable that this mass M_u of universe results in (3.38) about twenty times greater than that of Table 1, estimated counting the stars only; actually the content of our universe is much more complex than its mere visible matter, so that even this result can be tentatively accepted. This is more than mere hypothesis: actually (3.36) establishes a correlation between mass m and length r_m via the constant dimensional coefficient G/c^2, which reads $mass \leftrightarrow space\ length$. The physical meaning of this correlation appears introducing the respective differentials of δm and δr_m, which in turn can be nothing else but $\delta r_m = \left(G/c^2\right)\delta m$. This correlation is interpreted in turn according to (3.1) writing

$$m \leftrightarrow r_m \quad \delta m \leftrightarrow \delta r_m \quad \Rightarrow \quad \delta r_m = \frac{\delta\epsilon\delta t}{\delta p_m} \quad \delta m \leftrightarrow \frac{\delta\left(energy\right)}{force}, \tag{3.39}$$

whose physical meaning is related to $\delta m = const\delta r_m$: the change of m implies the force $\delta p_m/\delta t$ because δp_m is in turn defined by $n\hbar/\delta r_m$. As the initial $m \leftrightarrow r_m$ implies $\delta m \leftrightarrow \delta r_m$, the last (3.39) reads in fact

$$\delta r_m \leftrightarrow \frac{\delta(energy)}{force}.$$

This result replicates in fact (3.35) and (3.36), thus confirming that: (i) *force* is related to the mass m driven space deformation δr_m and that (ii) *force* does not involve directly the particle masses themselves but is mediated by their energy and space deformation fields. Quantum and relativistic concepts merge in (3.38).

To explain what δm does mean itself, plug δr_m into (3.1) to emphasize the resulting connection between mass change and space range deformation. As δr_m defines formally $\delta p_m = n\hbar/\delta\left(mG/c^2\right)$, one finds $\delta p_m = nh/\delta\left(2\pi r_m\right)$ *i.e.* both $2\pi\delta r_m = \delta\lambda$ and $\delta p_m = h/\delta\lambda$. To link Λ_t and H_u remind first (3.26), $\Lambda_t^* \approx 2H_u^{*2}$, and note that $\int H_u^* \mathrm{d}H_u^* = H_u^{*2}/2 + const$. Write thus

$$\Lambda_t^* = \int H_u^* \mathrm{d}H_u^* = \frac{H_u^{*2}}{2} \equiv 2\left(\frac{H_u^*}{2}\right)^2 + const. \tag{3.40}$$

To replace the similarity between the estimate $\Lambda_t^* \sim 2H^{*2}$ of (3.26) with the equality between more reliable values $\Lambda_t = 2H_u^2$, note that (3.40) implies $H_u = H_u^*/2$ and thus via (3.23)

$$\frac{H_u^*}{2} = H_u \Rightarrow \frac{2}{t_u} = \frac{1}{t_u^*} \quad t_u = 2t_u^* \tag{3.41}$$

simply postulating Λ_t related to H_u as $\Lambda_t = 2H_u^2$ in agreement with (3.26). It follows thus from (3.37)

$$\begin{gathered} \Lambda = \left(\frac{H_u}{c}\right)^2 = 1.6\times10^{-53}\ \mathrm{m}^{-2} \\ H_u = 1.2\times10^{-18}\ \mathrm{s}^{-1} = 37\ (\mathrm{km/s})/\mathrm{Mpc} \quad t_u = 8.7\times10^{17}\ \mathrm{s}, \end{gathered} \tag{3.42}$$

whence

$$\Lambda_t = \Lambda c^2 = 1.4\times10^{-36}\ \mathrm{s}^{-2} \quad \Lambda_t = 1.2\times10^{-18}\ \mathrm{s}^{-1}. \tag{3.43}$$

In this respect note that owing to (3.7) $H_u < c/s$; putting $s = r_u$, one finds $t_u < r_u/c$ *i.e.* $t_u < 1.5\times10^{18}$ s; so the last (3.42) is sensible. However, the universe results are older. However further checks are needed to support the validity of (3.41) and exclude its character of "ad hoc" hypothesis. An immediate check of (3.23) is possible right now according to some straightforward considerations.

-Put in (3.23) H_u just calculated in (3.42) and $n_u = 1$, fundamental energy state $\delta\left(hH_u\right)$ of (3.22); the comparison with the observed values (3.31) reads

$$\delta H_u = 73.5 - 67.8 = 5.7\ (\mathrm{km/s})/\mathrm{Mpc} \Leftrightarrow \frac{37}{2\pi} = 5.89\ (\mathrm{km/s})/\mathrm{Mpc}. \tag{3.44}$$

The agreement is enough to conclude that H_u' and H_u'' of (3.34) are central values in the respective ranges; this point is further concerned below.

-Calculate the vacuum energy density η_u and related mass density ρ_u

starting from

$$V_{\Lambda} = \frac{1}{\Lambda^{1.5}} = \left(\frac{c}{H_u}\right)^3 = 1.6\times10^{79}\ \mathrm{m}^3 \quad V_t = \left(t_u c\right)^3 = 1.8\times10^{79}\ \mathrm{m}^3,$$

which implies owing to (3.38) and (3.23)

$$\rho_u = \frac{M_u}{4\pi V_{\Lambda}/3} = 8.8\times10^{-27}\ \frac{\mathrm{kg}}{\mathrm{m}^3} \quad \eta_u = \rho_u c^2 = 7.9\times10^{-10}\ \frac{\mathrm{J}}{\mathrm{m}^3}; \tag{3.45}$$

significantly η_u can be calculated via t_u and r_u, it supports the idea that t_u of (3.41) corresponds to r_u. These values, consistent with (3.29) and (3.30), agree reasonably also with

$$\frac{c^2}{r_u^2 G} = 7.1\times10^{-27}\ \frac{\mathrm{kg}}{\mathrm{m}^3} \quad \frac{c^4}{r_u^2 G} = 6.4\times10^{-10}\ \frac{\mathrm{J}}{\mathrm{m}^3}. \tag{3.46}$$

The consistency of η_u and η_{vac} suggests that matter and vacuum in the universe are at the equilibrium.

-Eventually the second result is reasonably acknowledged as the critical density (3.27) of the Friedman equation, in agreement with (3.30). The crucial comparison between actual density of the universe and Friedman critical density turns into the correlation between H_0 related to the expansion of the universe and the quantum vacuum density.

Is encouraging the fact that two different ways to calculate ρ_u coincide, in particular because (NVH) does not involve explicitly M_u but the correlation between r_u and t_u.

-Calculate preliminarily (3.19) with some trial values of H^2, which yields the respective values of $H_{u_{min}}$; with these values calculate $\mathcal{T}_{min}$ and then $\mathcal{T} = \mathcal{T}\left(H_{u_{min}}, H_u\right)$ of (3.21) as a function of H_u. The plots of **Figure 3** are calculated with three arbitrary values of *H* in (3.18) and of course the true $H = 2\pi/t_u$ of (3.24) too; the purpose is to exemplify how the *H* affects the outcomes of the present model. Note in this respect that both $H_{u_{min}}$ and $\mathcal{T}_{min}$ move to higher values in the plot at increasing *H*. The specific value of *H* of interest is

$$H = \frac{2\pi}{t_u} = 7.2\times10^{-18}\ \mathrm{s}^{-1}; \tag{3.47}$$

also this crucial value is calculated with (3.42). With this *H* the plot of $\mathcal{T}$ vs H_u shown in **Figure 4** evidences the corresponding value

$$H_{u_{min}} \equiv \left\langle H \right\rangle = 2.28\times10^{-18}\ \mathrm{s}^{-1}, \tag{3.48}$$

i.e. $H_{u_{min}}$ coincides with $\left\langle H_u \right\rangle$ of (3.33) inferred from experimental data, and thus with (3.44) too. It clarifies the actual physical meaning of Friedmann's equation implementing a mere average value; indeed

$$\rho_{cr} = 9.4\times10^{-27} = \frac{3H_{u_{min}}^2}{8\pi G}\ \frac{\mathrm{kg}}{\mathrm{m}^3}.$$

in reasonable agreement with the Friedman critical density highlighted in **Figure 3**. This result can also be obtained via the cosmological parameters only

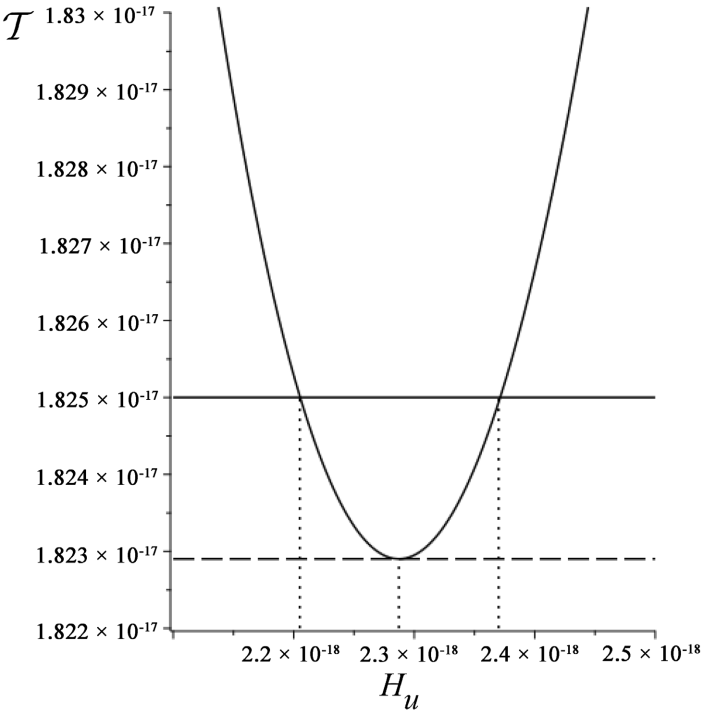

Figure 4. Plot of $\mathcal{T}$ vs H_u for $H = 7.2\times10^{-18}\ \text{s}^{-1}$, Equation (3.18). The horizontal dashed line emphasizes the values of $\mathcal{T}_{min} = \mathcal{T}_{min}(H)$ calculated by (3.21), the solid line the next energy level of δH_u; the vertical dots identify the respective $H_{u_{min}} = H_{u_{min}}(H) \pm \nu_u$ crossing the higher energy level.

with the help of (3.21) and (FCD)

$$\frac{H_{u_{min}}^2}{G}\frac{H_{u_{min}}}{\mathcal{T}_{min}} = 9.8\times10^{-27}\ \frac{\text{kg}}{\text{m}^3} \quad \mathcal{T}_{min} = 1.8\times10^{-17}\ \text{s}^{-1}. \tag{3.49}$$

Eventually the definition $r_m \equiv r_u$ and its corollary $G \equiv c^2 r_u / M_u$ calculated with (3.48) imply, again with t_u of (3.42),

$$c^2 r_u = 3.92\times10^{43} \leftrightarrow \frac{H_{u_{min}}}{\Lambda^{1.5} t_u} = 3.95\times10^{43}\ \frac{\text{m}^3}{\text{s}^2} \quad G = \frac{H_{u_{min}}}{\Lambda^{1.5} M_u t_u} = 6.69\times10^{-11}. \tag{3.50}$$

Let us conclude now this section returning to the Hubble tension considering (3.24), (3.33) and (3.48). The plot of fig 4 shows that the Hubble tension at $H_{u_{min}} \pm \nu$ is nothing else but the recession rate driven excited energy level splitting $h(\mathcal{T}_{min} \pm \nu)$ of $hH_{u_{min}}$; once having found $H_{u_{min}}$, $h\delta H_u = \left(hH_{u_{min}} + \nu\right) - \left(hH_{u_{min}} - \nu\right) = 2\nu$; then via (3.23) $2\nu = \sqrt{\Lambda_t}/2\pi$ yields $\nu = 10^{-19}\ \text{s}^{-1}$. Eventually it is enough to calculate

$$H' = \langle H \rangle - \nu = 2.17\times10^{-18}\ \text{s} = 67.0\ (\text{km/s})/\text{Mpc}$$
$$H'' = \langle H \rangle + \nu = 2.39\times10^{-18}\ \text{s} = 73.8\ (\text{km/s})/\text{Mpc}$$

to find

$$H' = 2.17\times10^{-18}\ \text{s} = 67.0\ (\text{km/s})/\text{Mpc}$$
$$H'' = 2.39\times10^{-18}\ \text{s} = 73.8\ (\text{km/s})/\text{Mpc}$$

as already obtained in (3.44) in agreement with (3.31) and (3.32).

4. Conclusion

The paper shows that relevant cosmological problems are successfully addressed via the formalism of the quantum uncertainty. The evolution of the universe is more than a mere list of events characterizing the various ages that succeeded each other during its lifetime. Instead of first concentrating on the details of the various eras, as a function of which should be inferred the respective temporal domains, this model indicates the chance of providing the general background as a function of which is deductible the possible time steps of evolution. In other words, the whole frame determines the succession of specific events, not vice-versa. Considering the individual pieces of a puzzle without a preliminary overview of the whole physical scenario is reductive and complicates any attempt at a theoretical approach. The concept of uncertainty concerns, in principle, the delocalization of a quantum particle in a region of space time during a time lapse, which implies allowed ranges of conjugate energy and momentum of the particle. So, it is not obvious "a priori" its extension to cosmological problems. Yet the results of this model show that the uncertainty is a reliable conceptual basis useful to problems even beyond the typical nano/micro scale of the standard quantum mechanics.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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