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A Comprehensive Study of Soliton Pulse Propagation Using Modified Differential Transform Methods

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Abstract

In this article, a modified version of the Differential Transform Method (DTM) is employed to examine soliton pulse propagation in a weakly non-local parabolic law medium and wave propagation in optical fibers. This semi-analytic method has the advantage of overcoming the obstacle of the hardest nonlinear terms and is used to explain the origin of the bright and dark soliton solutions through the Schrödinger equation in its non-local form and the Radhakrishnan-Kundu-Laksmannan (RKL) equation. Numerical examples demonstrate the effectiveness of this method.

Keywords

Soliton Pulse, Parabolic Law Medium, Differential Transform Method, Bright Solution, Dark Solution, Schrödinger Equation, Radhakrishnan-Kundu-Laksmannan

1. Introduction

Complex physical phenomena that occur in a variety of scientific and engineering fields can be portrayed by utilizing Nonlinear Partial Differential Equations (NLPDEs), which are commonly utilized in nonlinear optics, plasma physics, solid-state physics, chemical kinematics, biology, fluid mechanics, chemistry, and many other fields, There are several numerical methods to solve NLPDEs such as DTM [1]-[8], Adomian Decomposition Method (ADM) [9]-[14], Variational Iteration

Method (VIM) [15], Homotopy Perturbation Method (HPM) [16], Homotopy Analysis Method (HAM) [17] [18] and various other methods can be utilized.

One of the fastest-growing research areas in nonlinear optics and quantum optics is optical soliton perturbation. Analytical research results are widely visible, but numerical studies have received less attention [19]-[26]. Recent research on optical solutions has been widely covered, and the nonlinear Schrödinger equation highlights the importance of understanding the Second-Order Nonlinear Schrödinger Equation (SONLSE) [19] [20] to build on previous studies regarding the existence of SONLSE optical solitons. Therefore, we investigate SONLSE with weakly non-local and parabolic laws.

One significant contribution to the understanding of optical fibers is the Radhakrishnan-Kundu-Laksmannan (RKL) equation. The RKL equation is a nonlinear partial differential equation that describes the evolution of light pulses in optical fibers, accounting for various physical phenomena such as dispersion, nonlinearity, and attenuation.

Several authors have investigated the RKL equation using both analytical and semi-analytical methods. In [27], the author found a single soliton solution to the RKL equation. In [28], the authors examined the bifurcations of exact traveling wave solutions for the generalized RKL problem. In [29], the author applied Lie group analysis to obtain bright and dark soliton solutions for the RKL equation. In [30], the authors employed the extended trial function integration scheme to find optical solitons of the RKL equation. In [31], the authors discovered chirp-free optical bright soliton solutions for the RKL problem. In [32], the author derived periodic and solitary waveforms as exact general solutions to the RKL problem. In [33], the authors used the Laplace-Adomian decomposition method to find approximate solutions for optical solitons of the RKL equation.

The RKL equation and the non-local Schrödinger equation provide bright and dark solitons, which have different characteristics. Dark solitons are intensity dips that are adapted to defocusing medium, whereas bright solitons are high-intensity, localized peaks. Long-range interaction is introduced by the non-local Schrödinger equation, which can broaden solitons and affect their stability. The characteristics of these solitons are further altered by medium-specific parameters and interactions in other media, such as nonlinear optics, BECs, and plasma.

In this article, we use a modified version of DTM to solve the SONLSE with weakly non-local and parabolic laws, as well as the RKL equations. In this method, the nonlinear function is replaced by its accelerated Adomian polynomial, and the dependent variable components are replaced by their corresponding differential transform components of the same index [6]. The modified version of DTM is a highly effective tool for numerically analyzing these solitons. Two types of solutions are studied numerically, and the details are discussed in the following sections.

2. Differential Transform Method (DTM)

We considered $u(x,t)$ to be analytic and differentiated continuously in the domain of interest, then let,

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=t_0}, \tag{1}$$

where $U_k(x)$ is transformed function and the differential inverse transform of $U_k(x)$ is defined as follows:

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)(t-t_0)^k, \tag{2}$$

in real applications, the function $u(x,t)$ in Equation (2) can be written by a finite series as follows:

$$u(x,t) = \sum_{k=0}^{n} U_k(x)t^k, \tag{3}$$

where $t_0 = 0$, and the value of n is decided by the convergence of the series coefficients.

Some basic properties of the DTM for solving PDE. Let $u(x,t)$, $r(x,t)$, and $b(x,t)$ be the original functions and $U_k(x)$, $R_k(x)$ and $B_k(x)$ are their corresponding transformed functions. Then for constant α the following hold:

1) If $u(x,t) = r(x,t) \pm b(x,t)$, then $U_k(x) = R_k(x) \pm B_k(x)$.
2) If $u(x,t) = \alpha r(x,t)$, then $U_k(x) = \alpha R_k(x)$.
3) If $u(x,t) = \partial r(x,t)/\partial t$, then $U_k(x) = (k+1)R_{k+1}(x)$.
4) If $u(x,t) = \partial^k r(x,t)/\partial t^k$, then $U_k(x) = (k+1)(k+2)\cdots(k+m)R_{k+m}(x)$.
5) If $u(x,t) = \partial^k r(x,t)/\partial x^k$, then $U_k(x) = \partial^k R_k(x)/\partial x^k$.

In the modified version of DTM, the nonlinear function $N(u)$ is replaced by its accelerated Adomian polynomial of index k as follows:

$$N(u) = \sum_{k=0}^{\infty} A_k, \tag{4}$$

where A_k is the accelerated Adomian polynomial generated by El-Kalla [9]-[14]

$$A_k = f(s_k) - \sum_{i=0}^{k-1} A_i, \tag{5}$$

where $s_k = u_0 + u_1 + u_2 + u_3 + \cdots + u_k$ is the partial sum.

3. Convergence Analysis for Modified Version of DTM

Our methodology is based on the following theorem.

Theorem 3.1. Given a known analytic function $f(u)$ with u given by a convergent series as $u(x,t) = \sum_{k=0}^{\infty} c_k(x)t^k$, a convenient evaluation of $f(u)$ can be made by

$$f(u) = f\left(\sum_{k=0}^{\infty} c_k(x)t^k\right) = \sum_{k=0}^{\infty} A_k(c_0, c_1, c_2, \cdots, c_k)t^k, \tag{6}$$

where A_k s are the accelerated Adomian polynomials into which the function $f(u)$ is decomposed. That is, $f(u) = \sum_{k=0}^{\infty} A_k(u_0, u_1, u_2, \cdots, u_k)$.

The main idea of the suggested approach is expressed in the lemma that follows.

Lemma 3.1. $U_k(x) = DT\{u(x,t)\}$ and consequently $u(x,t) = DT^{-1}\{U_k(x)\}$. Assume A_ks represent the accelerated Adomian polynomials relating to nonlinear operator *N*. It holds that

$$DT\{N(u)\} = A_k(U_0, U_1, \cdots, U_k) \quad (7)$$

Proof. By Theorem 3.1 and recalling Equation (1) and Equation (3), we can deduce

$$N(u) = N\left(\sum_{k=0}^{\infty} U_k(x) t^k\right) = \sum_{k=0}^{\infty} A_k(U_0, U_1, \cdots, U_k) t^k. \quad (8)$$

Talking the differential transform from both sides of Equation (8) gives

$$DT\{N(u)\} = DT\left\{\sum_{k=0}^{\infty} A_k(U_0, U_1, \cdots, U_k) t^k\right\}. \quad (9)$$

As $A_k(U_0, U_1, \cdots, U_k)$ is independent of *x*, the transform passes through to yield

$$DT\{N(u)\} = \sum_{k=0}^{\infty} A_k(U_0, U_1, \cdots, U_k) DT\{t^k\} = \sum_{k=0}^{\infty} A_k(U_0, U_1, \cdots, U_k) \delta(n-k). \quad (10)$$

The definition of the Kronecker delta function forces $n = k$. Hence,

$$DT\{N(u)\} = A_k(U_0, U_1, \cdots, U_k). \quad (11)$$

Remarkably, **Lemma 3.1** provides a straightforward and practical method for assessing the differential transforms of any specified nonlinear expression.

4. Governing Model (1) and Mathematical Analysis

We study SONLSE with weakly non-local and parabolic laws,

$$iq_t + c_1 q_{xx} + \left(c_2 |q|_{xx}^2 + c_3 |q|^2 + c_4 |q|^4\right) q = 0, \quad (12)$$

where, q_t is the temporal evolution, c_1 is the coefficient of the second-order dispersion, c_2 is the coefficient of the weakly non-local nonlinearity, c_3 and c_4 are coefficients of the parabolic law nonlinearity [20].

Analysis of the Modified Version of DTM

In this section, we present the modified version of DTM for solving the SONLSE with weakly non-local and parabolic laws. Let us split the solution of Equation (12) as follows:

$$q(x,t) = y(x,t) + iu(x,t), \quad (13)$$

where $y(x,t)$ and $u(x,t)$ are the real and imaginary values of $q(x,t)$. Substituting Equation (13) into Equation (12) to get,

$$\begin{aligned} &i(y+iu)_t + c_1(y+iu)_{xx} + c_2\left((y+iu)(y-iu)\right)_{xx} + c_3(y+iu)(y-iu) \\ &+ c_4\left((y+iu)(y-iu)\right)^2 (y+iu) = 0. \end{aligned} \quad (14)$$

After simplifying Equation (14), we can get the real part,

$$-u_t + c_1 y_{xx} + c_2 y\left(y^2\right)_{xx} + c_2 y\left(u^2\right)_{xx} + c_3 y^3 + c_3 u^2 y + c_4 y^5 + 2c_4 u^2 y^3 + c_4 u^4 y = 0. \quad (15)$$

Imaginary part,

$$y_t + c_1 u_{xx} + c_2 u\left(y^2\right)_{xx} + c_2 u\left(u^2\right)_{xx} + c_3 y^2 u + c_3 u^3 + c_4 y^4 u + 2c_4 y^2 u^3 + c_4 u^5 = 0. \quad (16)$$

We can write Equation (15), (16) in the form,

$$-\frac{\partial u(x,t)}{\partial t} + c_1 \frac{\partial^2 y(x,t)}{\partial x^2} + \mathbb{N}_1(y,u) = 0, \quad (17)$$

$$\frac{\partial y(x,t)}{\partial t} + c_1 \frac{\partial^2 u(x,t)}{\partial x^2} + \mathbb{N}_2(y,u) = 0. \quad (18)$$

where $\mathbb{N}_1(y,u) = \sum_{n=0}^{\infty} A_n$ and $\mathbb{N}_2(y,u) = \sum_{n=0}^{\infty} B_n$ are nonlinear terms for Equations (15), (16) and A_n, B_n are the accelerated Adomian polynomials. Applying the modified version of DTM for Equation (17) and Equation (18), we can get the recurrence relation:

$$U_{k+1}(x) = \frac{1}{k+1}\left(c_1 \frac{\partial^2 Y_k(x)}{\partial x^2} + \mathbb{N}_1(y,u)\right), \quad (19)$$

$$Y_{k+1}(x) = \frac{-1}{k+1}\left(c_1 \frac{\partial^2 U_k(x)}{\partial x^2} + \mathbb{N}_2(y,u)\right). \quad (20)$$

Then the inverse of a modified version of DTM takes the form:

$$u(x,t) = \sum_{k=0}^{n} U_k(x) t^k, \quad (21)$$

$$y(x,t) = \sum_{k=0}^{n} Y_k(x) t^k. \quad (22)$$

With the initial conditions,

$$U_0(x) = u(x,0), \quad Y_0(x) = y(x,0). \quad (23)$$

5. Governing Model (2) and Mathematical Analysis

The RKL equation is given by [32],

$$iq_t + aq_{xx} + b|q|^{2n} q + i\beta q_{xxx} + i\alpha\left(|q|^{2n} q\right)_x = 0, \quad n \neq 0. \quad (24)$$

where, $q = q(x,t)$ is a complex value, that represents the wave profile, and the variables x and t represent the temporal and spatial coordinates respectively. n is the arbitrary index, $i^2 = -1$, a, b, β, and α are parameters of Equation (24).

Analysis of the Modified Version of DTM

In this section, we present the modified version of DTM for solving the RKL equation. Let us split the solution of Equation (24) as follows:

$$q(x,t) = y(x,t) + iu(x,t), \quad (25)$$

substituting Equation (25) into Equation (24) to get,

$$i\left(y+iu\right)_t+a\left(y+iu\right)_{xx}+b\left(\sqrt{y^2+u^2}\right)^n\left(y+iu\right)+i\beta\left(y+iu\right)_{xxx}+i\alpha\left(\left(\sqrt{y^2+u^2}\right)^{2n}\left(y+iu\right)\right)_x=0. \tag{26}$$

After simplifying Equation (26), we can get to the real part,

$$-u_t+ay_{xx}-\beta u_{xxx}+by\left(y^2+u^2\right)^n-2n\alpha u^2\left(y^2+u^2\right)^{n-1}u_x-\alpha\left(y^2+u^2\right)^n u_x-2n\alpha uy\left(y^2+u^2\right)^{n-1}y_x=0. \tag{27}$$

Imaginary part,

$$y_t+au_{xx}+\beta y_{xxx}+bu\left(y^2+u^2\right)^n+2n\alpha y^2\left(y^2+u^2\right)^{n-1}y_x+\alpha\left(y^2+u^2\right)^n y_x+2n\alpha uy\left(y^2+u^2\right)^{n-1}u_x=0. \tag{28}$$

We can write Equation (27), (28) in the form,

$$-\frac{\partial u(x,t)}{\partial t}+a\frac{\partial^2 y(x,t)}{\partial x^2}-\beta\frac{\partial^3 u(x,t)}{\partial x^3}+\mathbb{N}_1(y,u)=0, \tag{29}$$

$$\frac{\partial y(x,t)}{\partial t}+a\frac{\partial^2 u(x,t)}{\partial x^2}+\beta\frac{\partial^3 y(x,t)}{\partial x^3}+\mathbb{N}_2(y,u)=0. \tag{30}$$

where, $\mathbb{N}_1(y,u)=\sum_{n=0}^{\infty}A_n$ and $\mathbb{N}_2(y,u)=\sum_{n=0}^{\infty}B_n$ are nonlinear terms for Equations (27), (28) and A_n, B_n are the accelerated Adomian polynomials. Applying the modified version of DTM for Equations (29), (30), we can get the recurrence relation:

$$U_{k+1}(x)=\frac{1}{k+1}\left(a\frac{\partial^2 Y_k(x)}{\partial x^2}-\beta\frac{\partial^3 U_k(x)}{\partial x^3}+\mathbb{N}_1(y,u)\right), \tag{31}$$

$$Y_{k+1}(x)=\frac{-1}{k+1}\left(a\frac{\partial^2 U_k(x)}{\partial x^2}+\beta\frac{\partial^3 Y_k(x)}{\partial x^3}+\mathbb{N}_2(y,u)\right). \tag{32}$$

Then the inverse of the modified version of DTM takes the form:

$$u(x,t)=\sum_{k=0}^{n}U_k(x)t^k, \tag{33}$$

$$y(x,t)=\sum_{k=0}^{n}Y_k(x)t^k. \tag{34}$$

With the initial conditions,

$$U_0(x)=u(x,0),\quad Y_0(x)=y(x,0). \tag{35}$$

6. Applications to the Models

In this section, the modified version of DTM will be used to solve the models for Equation (12) *and* Equation (24). Two solutions will be studied in this section. The details will be shown in the next two subsections.

6.1. Bright Solutions

Case (1): the solution of Equation (12) is given by [19],

$$q(x,t) = \frac{a_1 A\left(\sinh(x-2kc_1t)+\cosh(x-2kc_1t)\right)}{2A^2\sinh(x-2kc_1t)\cosh(x-2kc_1t)+2A^2\left(\cosh(x-2kc_1t)\right)^2-A^2+1}\times e^{i(kx+\mu t)}, \quad (36)$$

where,

$$\mu = \frac{-1}{48}k^2a_1^4c_4 - \frac{1}{8}k^2a_1^2c_3 + \frac{1}{48}a_1^4c_4 + \frac{1}{8}a_1^2c_3, \quad (37)$$

$$c_1 = \frac{1}{48}a_1^4c_4 + \frac{1}{8}a_1^2c_3, \quad (38)$$

$$c_2 = \frac{1}{24}c_4a_1^2. \quad (39)$$

Case (2): the solution of Equation (24) is given by,

$$q(x,t) = 3^{\frac{1}{2n}}\times\left(A\operatorname{sech}\left(B(x-vt)\right)\right)^{\frac{1}{n}}\times e^{i(kx+\mu t)}, \quad (40)$$

where,

$$A = \sqrt{\frac{(n+1)\left(3\beta v-a^2\right)}{\alpha\beta}}, \quad (41)$$

$$B = \sqrt{\frac{(n+1)\left(3\beta v-a^2\right)}{\alpha\beta}}. \quad (42)$$

This solution represents a bright soliton with $3\beta v-a^2>0$.

6.2. Dark Solution

The solution of Equation (12) is given by [19],

$$q(x,t) = \frac{a_2\left(A^4+4A^2\sinh(x-2kc_1t)\cosh(x-2kc_1t)-1\right)}{A^4+4A^2\left(\cosh(x-2kc_1t)\right)^2-2A^2+1}\times e^{i(kx+\mu t)}, \quad (43)$$

where,

$$\mu = a_2^4c_4 + \frac{2}{3}k^2a_2^4c_4 + \frac{1}{2}k^2a_2^2c_3 + a_2^2c_3, \quad (44)$$

$$c_1 = \frac{-1}{6}a_2^2\left(4a_2^4c_4+3c_3\right), \quad (45)$$

$$c_2 = \frac{-1}{6}c_4a_2^2. \quad (46)$$

7. Numerical Simulation

We represent numerical simulation for Bright and Dark solutions.

7.1. Applications for Bright Solutions

Case 1: We take the values of parameters for Equation (36) as follows:

$$c_1 = \frac{-5}{96},\ c_2 = \frac{1}{48},\ c_3 = \frac{-1}{2},\ c_4 = \frac{1}{2},\ a_1 = -1,\ k = 1,\ \mu = 0 \text{ and } A = 1.$$

Case 2: We take the values of parameters for Equation (40) as follows:

$$n = \frac{1}{2},\ \beta = 1.34,\ \alpha = 2.88,\ v = 0.66,\ a = 0.7,\ b = 2.4.$$

The cases for the Bright solutions are discussed in **Table 1**, **Table 2**, and **Figures 1-6**.

Table 1. The Absolute Error (AE) between the exact and the approximate solution for Equation (36), case 1.

x	t		
	0.1	0.5	1
−60	4.304×10^{-36}	2.713×10^{-33}	4.386×10^{-32}
−50	9.481×10^{-32}	5.975×10^{-29}	9.662×10^{-28}
−25	6.827×10^{-21}	4.302×10^{-18}	6.957×10^{-17}
−10	2.231×10^{-14}	1.406×10^{-11}	2.274×10^{-10}
−8	1.649×10^{-13}	1.039×10^{-10}	1.680×10^{-9}
−6	1.217×10^{-12}	7.676×10^{-10}	1.241×10^{-8}
−2	1.946×10^{-11}	1.305×10^{-8}	2.276×10^{-7}
0	1.226×10^{-9}	7.656×10^{-7}	1.221×10^{-5}
2	1.877×10^{-11}	1.089×10^{-8}	1.584×10^{-7}
6	1.212×10^{-12}	7.517×10^{-10}	1.190×10^{-8}
8	1.642×10^{-13}	1.017×10^{-10}	1.611×10^{-9}
10	2.222×10^{-14}	1.377×10^{-11}	2.181×10^{-10}
25	6.798×10^{-21}	4.214×10^{-18}	6.673×10^{-17}
50	9.442×10^{-32}	5.852×10^{-29}	9.268×10^{-28}
60	4.286×10^{-36}	2.657×10^{-33}	4.207×10^{-32}

Table 2. The AE between the exact and the approximate solutions for Equation (40), case 2.

x	t		
	0.1	0.3	0.5
−60	4.27×10^{-22}	8.94×10^{-22}	1.31×10^{-21}
−50	2.41×10^{-19}	5.05×10^{-19}	7.44×10^{-19}
−25	1.82×10^{-12}	3.82×10^{-12}	5.63×10^{-12}
−10	2.45×10^{-8}	5.18×10^{-8}	6.61×10^{-8}
−8	8.86×10^{-8}	1.91×10^{-7}	1.59×10^{-7}
−6	3.47×10^{-7}	8.11×10^{-7}	5.60×10^{-7}
−2	8.91×10^{-6}	2.63×10^{-5}	5.03×10^{-5}

Continued

0	1.54×10^{-5}	4.54×10^{-5}	7.47×10^{-5}
2	9.21×10^{-6}	2.94×10^{-5}	4.77×10^{-5}
6	4.03×10^{-7}	1.52×10^{-6}	3.49×10^{-6}
8	9.91×10^{-8}	3.81×10^{-7}	1.07×10^{-6}
10	2.72×10^{-8}	1.07×10^{-7}	3.10×10^{-7}
25	2.02×10^{-12}	7.96×10^{-12}	2.34×10^{-11}
50	2.67×10^{-19}	1.05×10^{-18}	3.09×10^{-18}
60	4.72×10^{-22}	1.86×10^{-21}	5.48×10^{-21}

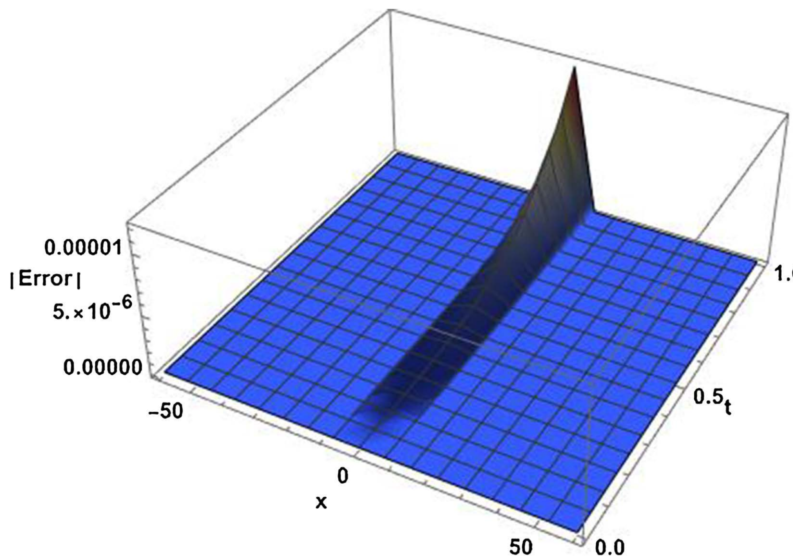

Figure 1. The AE between the exact solution and the approximate solution in (3-D) for Equation (36), case (1).

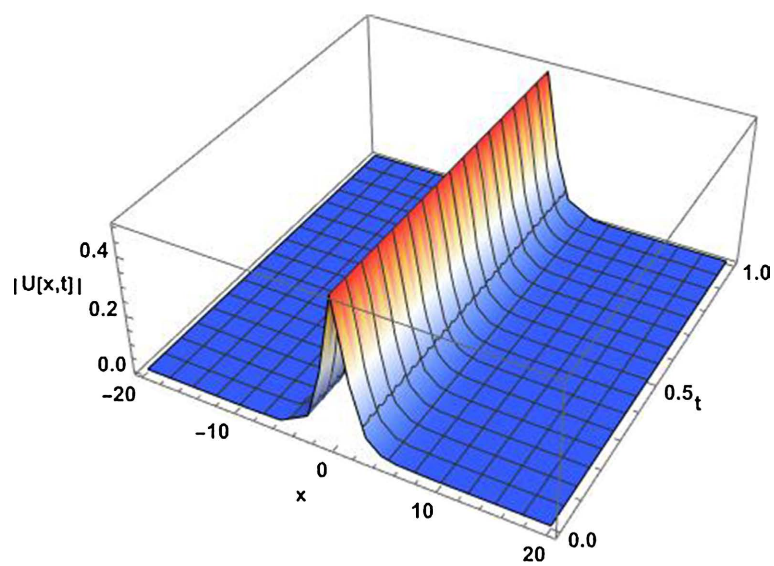

Figure 2. The approximate solution $U(x,t)$ in (3-D) for Equation (36), case (1).

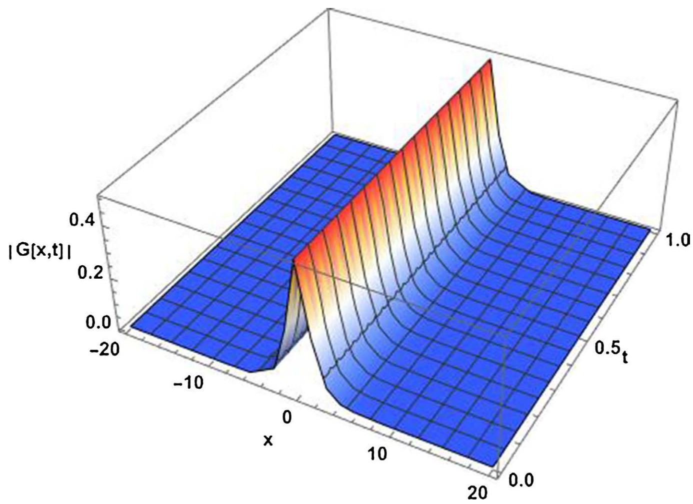

Figure 3. The exact solution $G(x,t)$ in (3-D) for Equation (36), case 1.

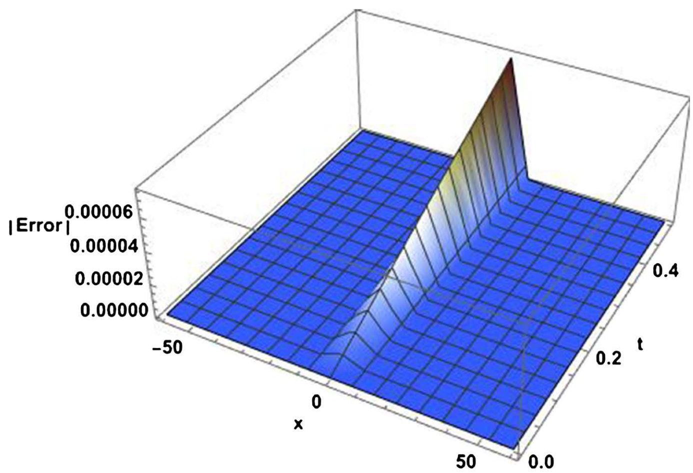

Figure 4. The AE between the exact solution and the approximate solution in (3-D) for Equation (40), case (2).

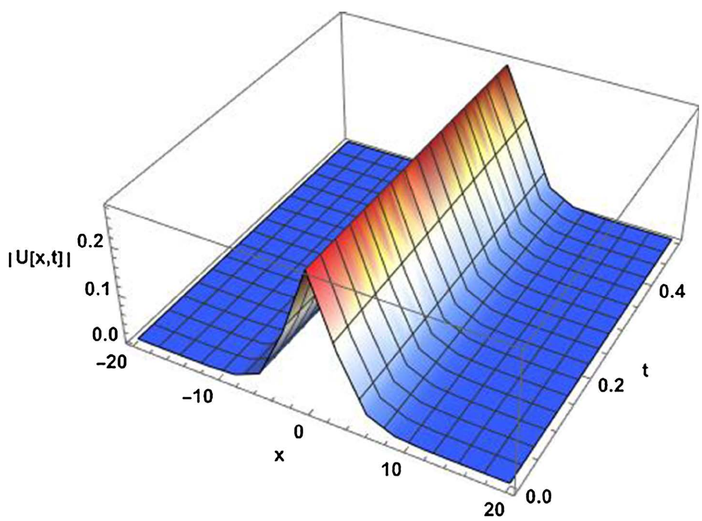

Figure 5. The approximate solution $U(x,t)$ in (3-D) for Equation (40), case (2).

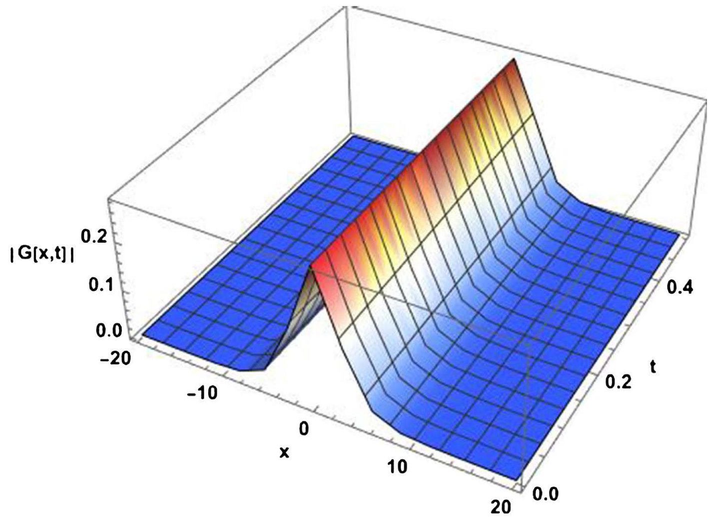

Figure 6. The exact solution $G(x,t)$ in (3-D) for Equation (40), case (2).

7.2. Applications for Dark Solution

We take the values of parameters for Equation (43) as follows:

$$c_1 = \frac{-1}{12},\ c_2 = \frac{-1}{12},\ c_3 = \frac{-1}{2},\ c_4 = \frac{1}{2},\ a_2 = -1,\ k = 1,\ \mu = \frac{1}{12} \text{ and } A = 1.$$

The case for the Dark solution is discussed in **Table 3**, and **Figures 7-9**.

Table 3. The AE between the exact and the approximate solutions for Equation (43).

x	t		
	0.1	0.5	1
−60	2.009×10^{-10}	1.255×10^{-7}	2.009×10^{-6}
−50	2.009×10^{-10}	1.255×10^{-7}	2.009×10^{-6}
−25	2.009×10^{-10}	1.255×10^{-7}	2.009×10^{-6}
−10	2.009×10^{-10}	1.255×10^{-7}	2.009×10^{-6}
−8	2.009×10^{-10}	1.255×10^{-7}	2.009×10^{-6}
−6	2.005×10^{-10}	1.253×10^{-7}	2.004×10^{-6}
−2	1.609×10^{-9}	1.028×10^{-6}	1.690×10^{-5}
0	1.446×10^{-8}	9.045×10^{-6}	1.448×10^{-4}
2	1.592×10^{-9}	9.739×10^{-7}	1.516×10^{-5}
6	2.005×10^{-10}	1.253×10^{-7}	2.005×10^{-6}
8	2.009×10^{-10}	1.255×10^{-7}	2.009×10^{-6}
10	2.009×10^{-10}	1.255×10^{-7}	2.009×10^{-6}
25	2.009×10^{-10}	1.255×10^{-7}	2.009×10^{-6}
50	2.009×10^{-10}	1.255×10^{-7}	2.009×10^{-6}
60	2.009×10^{-10}	1.255×10^{-7}	2.009×10^{-6}

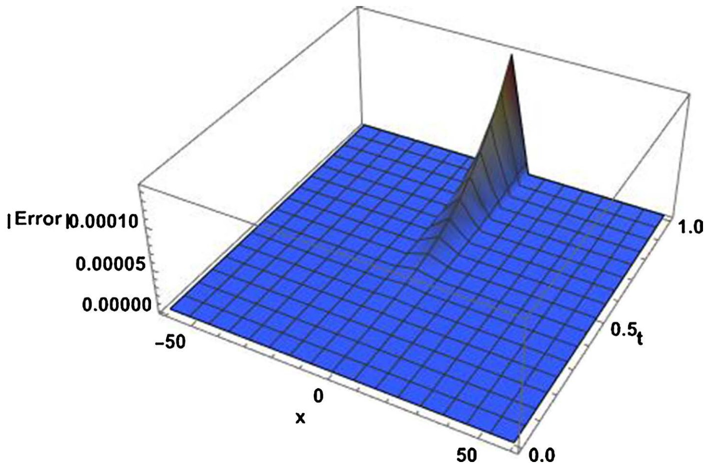

Figure 7. The AE between the exact solution and the approximate solution in (3-D) for Equation (43).

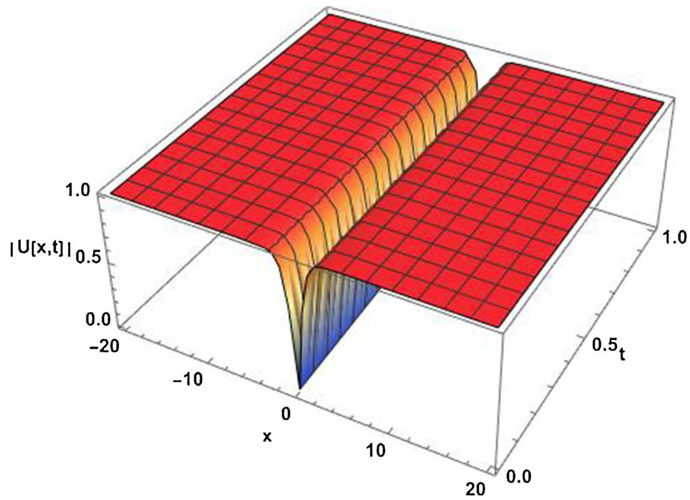

Figure 8. The approximate solution $U(x,t)$ in (3-D) for Equation (43).

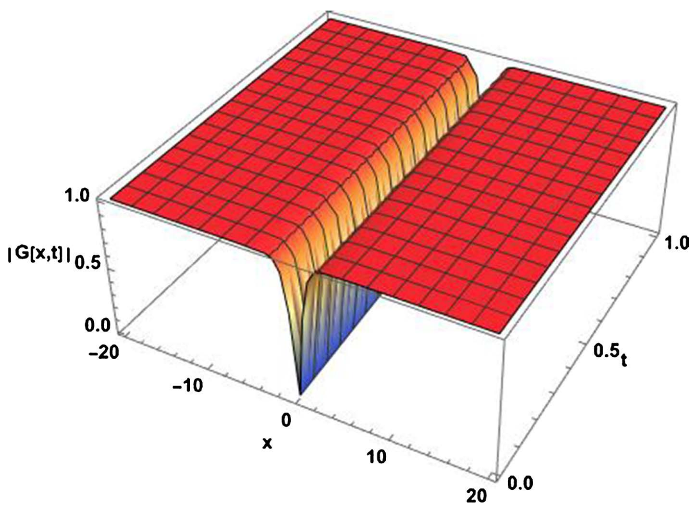

Figure 9. The exact solution $G(x,t)$ in (3-D) for Equation (43).

8. Conclusion

This article investigates soliton pulse propagation in a weakly non-local parabolic law medium and wave propagation in optical fibers using a modified version of DTM. This modified version of the DTM used a new polynomial called El-Kalla polynomial. It has the advantage of overcoming the obstacle of the hard nonlinear terms which is difficult to handle with using DTM and this technique is a powerful tool that requires minimal time to quantitatively analyze the origin of bright and dark soliton solutions through the Schrödinger equation in its non-local form and the RKL equation. The solution obtained using this modified technique shows excellent agreement with the exact solution using a few iterations.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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