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Utilizing ACP Alpha Beta ($\alpha\beta$) Nonlinear Mathematics for Analyzing Astrophysics and Electrostatic Separation Data (Applications 3 and 4)

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Abstract

Analyses of astrophysics and electrostatic separation data were illustrated with the Asymptotic Curve Based and Proportionality Oriented (ACP) nonlinear math for relating two physical variables. The fundamental physical law asserts that the nonlinear change of continuous variable Y is proportional to the nonlinear change in continuous variable X. Mathematically, this is expressed as $\mathrm{d}\alpha\{\mathrm{Y}, \mathrm{Yu}, \mathrm{Yb}\} = -\mathrm{K}\mathrm{d}\beta\{\mathrm{X}, \mathrm{Xu}, \mathrm{Xb}\}$, with Yu, Yb, Xu, and Xb representing the upper and baseline asymptotes of Y and X. Y is the continuous cumulative numbers of the elementary y and X is the continuous cumulative numbers of elementary x. K is the proportionality constant or equally is the rate constant.

Keywords

Alpha Beta ($\alpha\beta$) Nonlinear Math, Asymptotic Concave and Convex Curve, Upper and Baseline Asymptote, Demulative Numbers (Opposite to Cumulative Numbers), Coefficient of Determination (COD), Proportionality and Position Constant, Skewed Bell and Sigmoid Curve

1. Introduction

In a previous publication, we used the relationship between "1" and "0.99999…" to prove the nonlinear numbers are associated with a unique upper asymptote and the change of nonlinear numbers need to be measured relative to asymptotes [1] [2]. In this article, we start with a short review of ACP nonlinear math, followed by introducing the required basic equations for use in analyses.

In Part I, we use first order nonlinear equation (with 1q in Y dependent variable) to describe Kepler's Third Law on expressing the relationship between the semimajor axis to the sidereal period of planets; and in Part II, we use second order nonlinear equation (with 2q in Y dependent variable) to demonstrate the analysis of the separation of tribo-charged particles along a uniform electrical conducting copper plate.

The following gives the key features of the ALPHA BETA ($\alpha\beta$) NONLINEAR MATH.

- Continuous numbers can be categorized as linear and nonlinear based on the presence or absence of asymptotes. Asymptotes are never part of the nonlinear numbers.
- Two types of zero are linear zero and nonlinear zero: a linear zero is sandwiched between positive and negative numbers; a nonlinear zero is a baseline asymptote of nonlinear numbers that can decrease toward nonlinear zero but will never reach or touch the nonlinear zero.
- Two mathematical axioms govern nonlinear math: Axiom I focuses on continuity, stating that continuous numbers are dynamic, non-terminating, monotonically increasing or decreasing, and always maintain continuity. Axiom II asserts that asymptotes are never part of nonlinear numbers; they are approachable but cannot be touched or crossed.
- The standard scale for nonlinear numbers is a 10 based logarithmic scale; its characteristic is the existence of a nonlinear zero, which is approachable but cannot be touched or plotted on the rectilinear coordinate graph, e.g., the baseline nonlinear zero $Yb = (0) = \phi$, and baseline nonlinear zero $Xb = (0) = \phi$ cannot be plotted in the rectilinear graph.
- When either the linear numbers or the cluster of nonlinear numbers are assigned or plotted on the axes of graphs, these numbers are called face values of the numbers. For linear numbers, face values are equivalent to true values. For nonlinear numbers, the nonlinear face values are the measurement of nonlinear variables relative to their asymptotes, such as $(Yu - Y)$, $(Xu - X)$, and $(qYu - qY)$; the true values of nonlinear numbers are obtained by nonlinear logarithmic transformation of the nonlinear face values, such as $q(Yu - Y)$, $q(Xu - X)$, and $q(qYu - qY)$.
- "d" is designated as "change" or "variation" in equations. Linear cases are expressed as $dY = KdX$, indicating that the change in linear Y is proportional to the change in linear X. where K is the proportionality constant. For reading differential equations involving nonlinear numbers, we read $d(q(Yu - Y)) = KdX$ as the <u>change</u> of nonlinear true values $q(Yu - Y)$ is proportional to the <u>change</u> of linear numbers X, or the <u>nonlinear change</u> of face values $(Yu - Y)$ is proportional to the <u>linear change</u> of linear numbers X, etc.
- Four types of graphs exist: primitive elementary, primary, leading, and proportionality graphs. Primitive elementary graphs depict vertical elementary y against various horizontal X. Primary graphs illustrate cumulative Y or demulative Y

against cumulative X. Leading graphs feature an asymptotic curve with a continuously changing slope. Proportionality graphs are characterized by a straight line expressible as a two-parameter proportionality equation.

2. Importance of Concave and Convex Curves and Its Association with Asymptotes

Physical phenomena are generally expressible with an asymptotic concave or convex curve. These concave and convex curves can be in an ordinary or secondary order of nonlinearity and can be used to derive various differential equations based on their inherited proportionality.

3. Basic Equations and Their Graphical Expressions [1] [2]

The following differential equations are characterized by a single proportionality constant K, while the integral equations have an additional integral constant C for dictating the position of the straight line in the graph; thus, we also call the C a position constant. We can classify equations based on the order of nonlinearity, as shown in **Table 1(a)**, or based on the comparison of the linearity of two variables, as shown in **Table 1(b)**.

Table 1. (a) basic equations I (based on the order of Y variable); (b) basic equations II (based on the comparison between linearity and nonlinearity).

(a)

Order	Eq. ( )	Differential Equation	Eq. ( )	Integral Equation	Yb or Yu
Zero	1	$dY = -KdX$	1a	$Y = -KX + C$	0
First	2	$d(q(Y - Yb)) = -KdX$	2a	$q(Y - Yb) = -KX + qC$	1 Yb
	3	$d(q(Yu - Y)) = -KdX$	3a	$q(Yu - Y) = -KX + qC$	1 Yu, 1 Yb (hiding)
	4	$d(q(Y - Yb)) = -Kd(q(X - Xb))$	4a	$q(Y - Yb) = -K(q(X - Xb)) + qC$	1 Yb
	5	$d(q(Yu - Y)) = -Kd(q(X - Xb))$	5a	$q(Yu - Y) = -Kq(X - Xb) + qC$	1 Yu, 1 Yb (hiding)
Second	6	$d(q(qYu - qY)) = -KdX$	6a	$q(qYu - qY) = -KX + qC$	1 Yu, 1 Yb (hiding)
	7	$d(q(qYu - qY)) = -Kd(q(X - Xb))$	7a	$q(qYu - qY) = -K(q(X - Xb)) + qC$	1 Yb, 1 Yb (hiding)

*above equations have two parameters K and C; when $C = 1$, $qC = q1 = 0$; Independent variable be either dX or $d(q(X - Xb))$.

(b)

group	Eq. ( )	Differential Equation	Eq. ( )	Integral Equation	Asymptote in Y
A	1	$dY = -KdX$	1a	$Y = -KX + C$	0
B1(Yb)	2	$d(q(Y - Yb)) = -KdX$	2a	$q(Y - Yb) = -KX + qC$	1 (single Yb)
B2(Yu)	3	$d(q(Yu - Y)) = -KdX$	3a	$q(Yu - Y) = -KX + qC$	2 (one Yb hiding)
	6	$d(q(qYu - qY)) = -KdX$	6a	$q(qYu - qY) = -KX + qC$	2 (one Yb hiding)
C	4	$d(q(Y - Yb)) = -Kd(q(X - Xb))$	4a	$q(Y - Yb) = -K(q(X - Xb)) + qC$	1 (single Yb & Xb)
	5	$d(q(Yu - Y)) = -Kd(q(X - Xb))$	5a	$q(Yu - Y) = -Kq(X - Xb) + qC$	2 (one Yb hiding, 1 Xb)
	7	$d(q(qYu - qY)) = -Kd(q(X - Xb))$	7a	$q(qYu - qY) = -K(q(X - Xb)) + qC$	2 (one Yb hiding, 1 Xb)

Table 1(a) is based on the order of nonlinearity: the zero-order equation has no asymptote and no logarithmic transformation in Y, such as Equations (1) and (1a); the first order equations have one logarithmic transformation and has 1q, such as Equations (2) to (5); the second order equations have two logarithmic transformations and has 2q, such as Equations (6) and (7). In comparing two variables, the zero-order equation states the change of linear Y is proportional to the change of linear X, Equation (1); the first-order equations state the nonlinear change of Y is either proportional to the change of linear X, Equations (2) and (3), or proportional to the nonlinear change of X, Equations (4) and (5). Equations (6) and (7) state the change of nonlinear Y in a second order of nonlinearity is proportional to the linear change of X or proportional to the change of nonlinear X. Table 1(b) is based on the comparison of linearity/nonlinearity: we classify equations into linear-by-linear, Equation (1); nonlinear-by-linear, Equations (2), (3), and (6); and nonlinear-by-nonlinear, Equations (4), (5), and (7).

3.1. Expression of Linear-by-Linear Equations (Equation (1), (1a)) [1] [2]

Figure 1 uses three straight lines to demonstrate the change of Y is proportional to the change of X in Equations (1) and (1a). The straight line means the existence of proportionality [3]-[6]. The equations have two parameters K and C. The three K (−1.5, −3, and 3) give the directions and slopes of the straight lines, and the three C (64, 0, and 30) give the position of the straight lines. All three lines can extend continuously forever in two directions and all pass through linear zero.

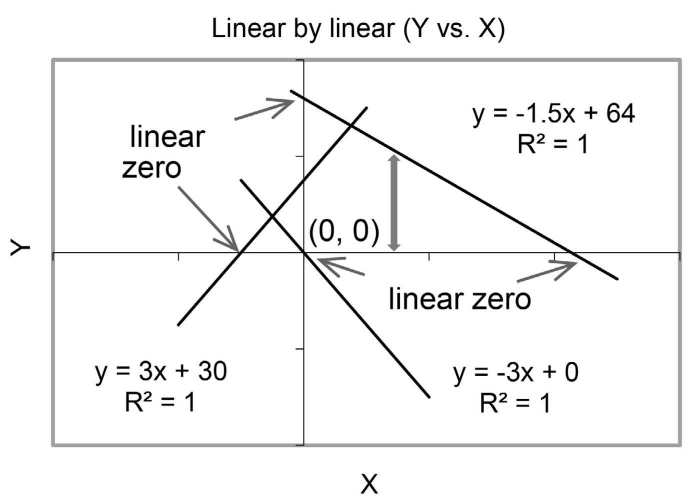


Figure 1. Linear by linear phenomenon.

3.2. Expression of Nonlinear Equations (Equations (2, 2a), (3, 3a), (4, 4a), and (6, 6a)). Solid Double-Arrows Are for Nonlinear Face Values, Dashed Double-Arrow Are for Linear Values

Figure 2 gives four asymptotic curves, and Figure 3 gives their corresponding proportionality plot for Equations (2), (3), (4), and (6). [1] [2]

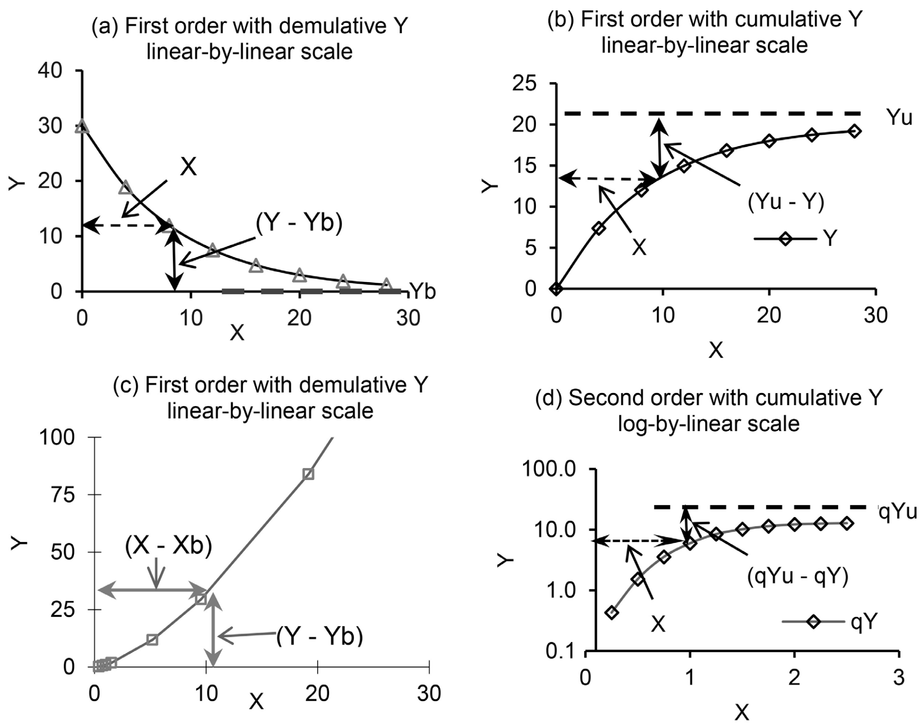


Figure 2. Asymptotic curves: (a) first order with demulative *Y*; (b) first order with cumulative *Y*; (c) first order with demulative *Y*; (d) second order with cumulative *Y*.

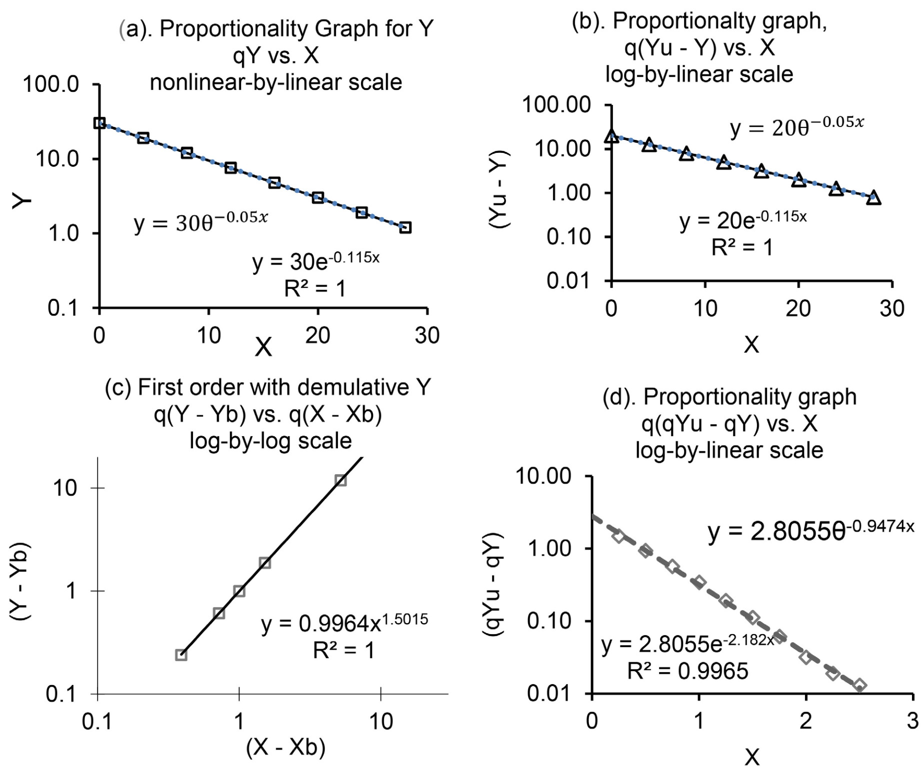


Figure 3. Proportionality plots: (a) derived from **Figure 2(a)** with demulative *Y*; (b) derived from **Figure 2(b)** with cumulative *Y*; (c) derived from **Figure 2(c)** with demulative *Y* and demulative *X*; (d) derived from **Figure 2(d)** with second order nonlinearity in cumulative *Y*.

In Figure 2, the solid double arrows are a measurement of face values relative to the upper and baseline asymptotes Yu or Yb. The dashed double arrows are a measurement of face values X relative to zero. The relationship between the solid and dashed arrows is that as the solid arrow increases, the dashed arrow decreases, or *vice versa*. For example, in Figure 2(a) it is the change of ($Y - Yb$) is negatively proportional to the change of X; in Figure 2(b) the change of ($Yu - Y$) is negatively proportional to the change of X; in Figure 2(c) the change of ($Y - Yb$) is negatively proportional to the change of ($X - Xb$); and in Figure 2(d) the change of ($qYu - qY$) is negatively proportional to the change of X.

Figure 3 gives proportionality plots for corresponding figures in Figure 2. That is: Figure 3(a) is derived from Figure 2(a); Figure 3(b) is derived from Figure 2(b); Figure 3(c) is derived from Figure 2(c); and Figure 3(d) is derived from Figure 2(d). The straight-line in Figure 3 means the existence of proportionality for qY vs. X; $q(Yu - Y)$ vs. X; $q(Yu - Yb)$ vs. $q(Xu - Xb)$ and $q(qYu - qY)$ vs. X, [3]-[6].

4. The Semimajor Axis versus Sidereal Period Relationship of Planets

Johannes Kepler, the renowned astronomer, unveiled his celebrated third law, elucidating the dynamic interplay between two celestial objects as they orbit each other under the influence of mutual gravitational attraction in 1619. [7]. His law posits that the squares of the sidereal periods of planets Y are equivalent to the cubes of their semimajor axes X. In other words, if a planet's sidereal period (the time it takes to orbit the Sun) is measured in years, and the semimajor axis length is measured in astronomical units, then Kepler's third law is succinctly expressed as:

$$Y^2 = X^3 \tag{8}$$

The correlation between the sidereal period and the semimajor axis can be aptly described using the proportionality equations in Equation (4) and Equation (4a). The nonlinear change in face values ($Y - Yb$) is proportional to the nonlinear change in face values ($X - Xb$). The baseline asymptote of "X" is $Xb = \phi$, and the baseline asymptote of Y is $Yb = \phi$. Consequently, the equation takes the form:

$$d\left(q\left(Y - Yb\right)\right) = Kd\left(q\left(X - Xb\right)\right) \tag{9}$$

Integration of the above equation yields Equation (10).

$$\log\left(Y - Yb\right) = K\log\left(X - Xb\right) + \log C \tag{10a}$$

$$\text{with} \quad Yb = \phi \quad \text{and} \quad Xb = \phi \tag{10a-1}$$

$$\log Y = K\log X + \log C \quad \text{with the removal of the notion of } Yb \text{ and } Xb \tag{10b}$$

Table 2 gives the observed data by Kepler. The direct plot of the sidereal period versus the semimajor axis is given in Figure 4(a), indicating the XY line is a nonlinear concave line with both X and Y decreasing toward their baseline asymptote $Yb = \phi$ and $Xb = \phi$. These baseline asymptotes are nonlinear zeros that cannot be plotted in the rectilinear graph; thus, the "0" in the linear rectilinear graph is only a surrogate zero but not a real nonlinear zero. For a true representation, it is essential to plot the nonlinear numbers on the nonlinear logarithmic scale.

Table 2. Observed data by Kepler [7].

Planet	Sidereal period P (Y) (in years)	Semimajor axis a (X) (in AU)
Mercury	0.24	0.39
Venus	0.61	0.72
Earth	1.00	1.00
Mars	1.88	1.52
Jupiter	11.86	5.20
Saturn	29.46	9.54
Uranus	84.01	19.18
Neptune	164.79	30.06
Pluto	247.70	39.44

Figure 4(b) is a re-plot of Figure 4(a), with the sidereal period Y plotted on a logarithmic scale. The curve is approaching the baseline asymptote Yb, $Yb = \phi$, but will never reach the nonlinear zero. Nonlinear zero is the asymptote of Y but is never part of the curve. Similarly, Figure 4(c) is a re-plot of Figure 4(a), with the semimajor axis plotted on a logarithmic scale. The curve is approaching the baseline asymptote Xb, $Xb = \phi$ is a nonlinear zero that will never be reached. Ultimately, by plotting Y versus X in a log-by-log scale, we obtain a proportionality graph of qY versus qX, having a regression equation of $y = 0.9964x^{1.5015}$.

From Table 2, one notices that $Y = 1$ and $X = 1$ for the Earth. Thus, using Earth for initial condition values, the C value in Equation (4a) is resolved to be $C = 1$. Consequently, $\log C = \log 1 = 0$, and Equation (10a) is reduced to Equations (11a) and (11b).

$$\log Y = K \log X \tag{11a}$$

$$\log Y = \log X^{K} \tag{11b}$$

In graphical expressions of Kepler's third law: Figure 4(a) gives a direct plot of Y vs. X in a linear-linear scale; Figure 4(b) gives a plot of qY vs. X in a log-linear scale; Figure 4(c) gives a plot of Y vs. qX in a linear-log scale; Figure 4(d) gives a plot of qY vs. qX in a log-log scale. The straight line in the log-by-log scale means the existence of proportionality between qY vs. qX, [3]-[6]. The plot of Equation (10a) is shown in Figure 4(d), which gives a straight line in a log-log graph with the regression equation $y = 0.9964X^{1.5015}$, it is $Y = CX^{K}$. With the equation K, $K = 1.5015 \cong 3/2$, and position constant C, $C = 0.9964 \cong 1$, we notice the straight line passes through $Y = 1$ and $X = 1$ for the Earth. Thus, we write Equation (11b) as Equation (12a). We can also write them as Equations (12b) and (12c). These are Kepler's third law. It is like the Six Tenth Law in plant engineering.

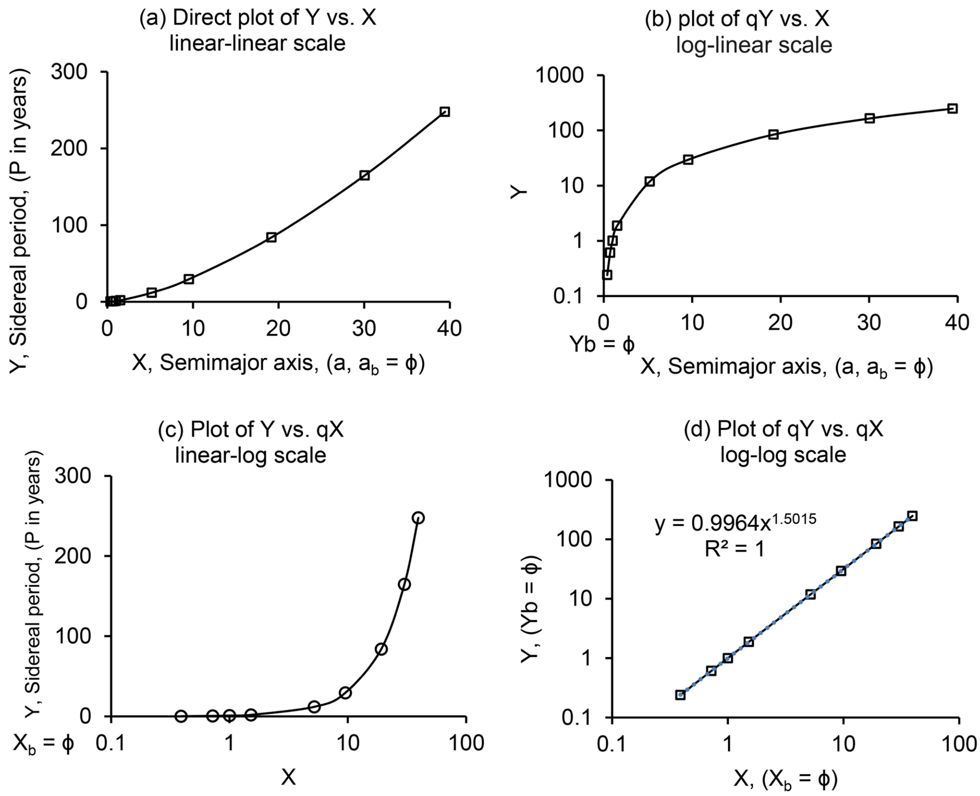


Figure 4. Graphical expression of Kepler's third law.

$$\log Y = \log X^{3/2} \tag{12a}$$

$$Y = X^{3/2} \tag{12b}$$

$$Y^2 = X^3 \tag{12c}$$

The derivation of the above equations from the general ACP equation confirms that it is consistent with the modeling of the existing empirical evidence. In terms of ACP nonlinear math, it states that the nonlinear change of sidereal periods is proportional to the nonlinear change of semimajor axes, with a proportionality constant at 1.5 or 3/2, *i.e.*, the proportionality constant is $K = 3/2$. We can also state that the rate of nonlinear change of sidereal periods with respect to the nonlinear change of semimajor axes is a constant with the rate constant as $K = 3/2$. In this context, the proportionality constant and the rate constant refer to the same concept but are expressed using different terms. Note: Kepler's law was derived empirically, but Newton was able to derive it mathematically [7].

5. Electrostatic Separation of Fine Particles (Example with Variation in Asymptotes)

In this section, let's explore the use of a second order nonlinear equation to analyze a more complex experiment of the tribo-electrostatic separation of fine particles. The technology of tribo-electrostatic separation of particles is a well-known old technology that dates back to the time of Thomas Edison.

5.1. A general Case in Injecting and Deposition of Tribo-Charged Fine Particles

The separation and deposition of tribo-charged particles along a uniform copper plate is a nonlinear-by-nonlinear phenomenon. In the separation experiment, the tribo-charged particles are carried by an air stream and injected horizontally from one end into a rectangular box of 10 cm in height, 30 cm in width, and 122 cm long. The top and bottom plates are copper conductors connecting to a DC source, with one of each assigned as the positive plate and the negative plate. The electric potential between the plates is maintained at 50 Kilo Volt. The positively charged particles are deposited on the negative plate, and the negatively charged particles are deposited on the positive plate. Figure 5 shows the profiles for the deposition of particles [8].

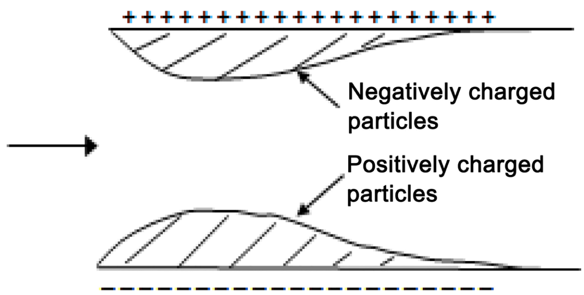


Figure 5. Profiles for deposition of particles.

Table 3. Data for deposition of particle. (Image of Excel Table)

B14 | =B13+B11

	A	B	C	D	E	F	G	H	I	J
1	Position cm		Range	Mass g	Cum. Mass. g	qYu - qY	q(qYu - qY)			
2	X	qX		y	Y					
3	3	0.4771	0 -3	0.13	0.13			Active Yu =		
4	6	0.7782	3 - 6'	1.79	1.92			R^2 =		
5	12	1.0792	6 - 12'	5.19	7.11					
6	24	1.3802	12 - 24'	5.40	12.51					
7	48	1.6812	24 - 48	2.50	15.01					
8	96	1.9823	48 - 96	1.10	16.11					
9	120	2.0792	96 - 120	0.24	16.35					
10										
11	ΔYu	0.04		0.08		0.16		0.32		
12			R^2 for 0.16		R^2 for 0.08		R^2 for 0.16		R^2 for 0.32	
13	Yu0 =	16.35		16.35		16.35		16.35		
14	Yu1 =	16.39		16.43		16.51		16.67		
15	Yu2 =	16.43		16.51		16.67		16.99		
16	Yu3 =	16.47		16.59		16.83		17.31		
17	Yu4 =	16.51		16.67		16.99		17.63		
18	Yu5 =	16.55		16.75		17.15		17.95		
19	Yu6 =	16.59		16.83		17.31		18.27		
20	Yu7 =	16.63		16.91		17.47		18.59		
21	Yu8 =	16.67		16.99		17.63		18.91		

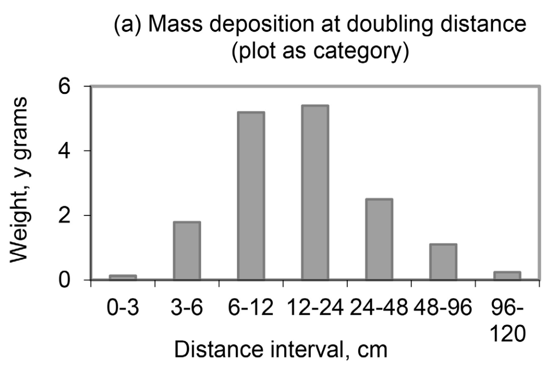
In one experiment, the mass of the negatively charged particles was collected at intervals between 0 - 3, 3 - 6, 6 - 12, 12 - 24, 24 - 48, 48 - 96, and 96 - 120 cm. These intervals are increasing in a nonlinear fashion. The span of these distances is doubled each time in sequence, except the last one. In a nonlinear phenomenon, the collection of data in a nonlinear fashion is extremely important.

In Table 3, sample collection position X in cm, sample mass "y" in gram, and cumulative mass Y are given in Columns A, D, and E. Column F is for the calculation of face-values ($qYu - qY$), and Column G is for the calculation of true value $q(qYu - qY)$. The lower part of the table, Raw 11 to 21, is used to search for the optimal upper asymptote Yu.

Raw 14 to Raw 21 is for the generation of Estimated Yu from $Yu1$ to $Yu8$. In the guided estimation of Yu, we first select the last Y (the largest acquired) data in Column E (*i.e.*, Cell E9) as the base, *i.e.*, $Yu0 = 16.35$ and input it into Raw 13 of Cells B13, D13, F13, and H13. We also select four incremental Yu (ΔYu) at about 0.25%. 0.5%, 1% and 2% of $Yu0$, *i.e.*, 0,04, 0.08, 0.16, and 0.32, and input into Raw 11 of Cells B11, D11, F11, and H11. Then, we generate $Yu1$ to $Yu8$ for each incremental ΔYu. The formula for $Yu1$ in Cell B14 is "B13 + B11", as shown in the Formula bar on top of the table, it is 16.39. We can copy Cell B14 into Cell B15 through B21 to complete Column B. Likewise, we calculate Cell D14 as "D13 + D11", then copy Cell D14 into Cell D15 through D 21 to complete Column D. We do the same for Column F and Column H.

By plotting Column D(D3:D9) vs. Column C(C3:C9) for y versus "Range", we obtain column-plot Figure 6(a); by plotting Column D vs. Column A for y versus X, we obtain line-form Figure 6(b).

Figure 6 gives primitive elementary graphs without giving a clue about the existence of the proportionality relationship between mass (y) and distance X. However, we can uncover the true physical meaning behind their data by calculating the cumulative weight and plotting the cumulative mass Y along the distance X. A cumulative curve is an integration of the area under the curve (AUC) [9] [10].



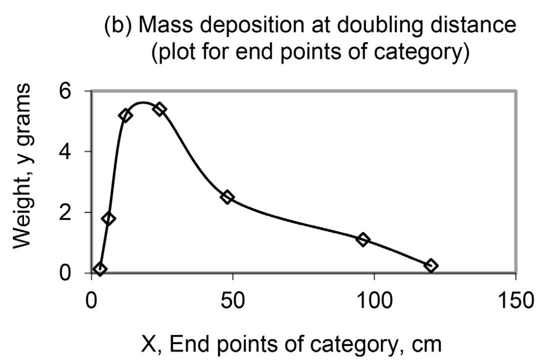


Figure 6. Primitive elementary graph of particle deposition: (a) plot of y versus X in column form; (b) plot of y versus X in line form.

Figure 7(a) is a plot of cumulative mass Y versus cumulative distance X in a linear-linear graph (column A versus column E), showing a sigmoid curve with a very small initial tail. This is a primary graph with its sigmoidal curve representing the integration of area under the curve (AUC) in Figure 6(b). Figure 7(b) is a semi-log plot of Figure 7(a), obtained by converting the y-axis from a linear into a nonlinear logarithmic scale. This plot gives an asymptotic convex curve with continuous change in the slope of the line, like Figure 2(d). It is a leading graph because it will lead us to a proportional graph. In this leading graph, the continuous change in the slope of the curve reveals that as the vertical distance measured from the upper asymptote ($qYu - qY$) increases, the horizontal distance decreases, or *vice versa*.

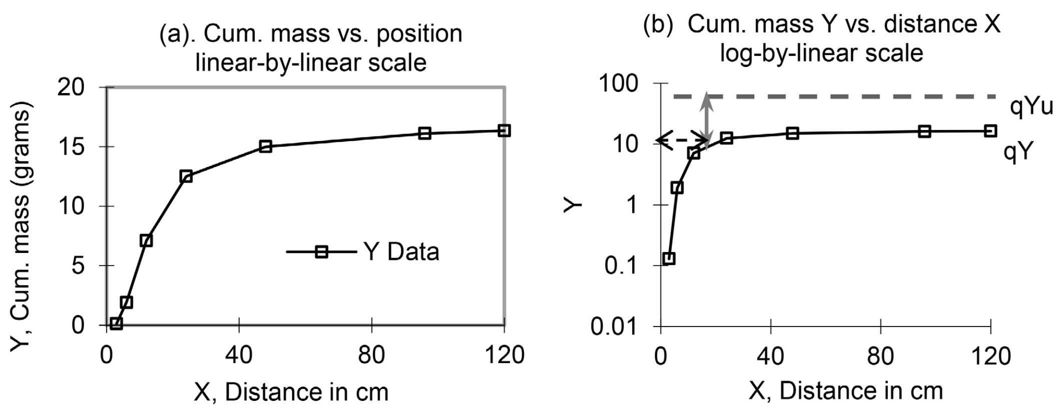


Figure 7. Cumulative mass versus distance: (a) Y vs. X in linear-linear scale; (b) Y vs. X in log-linear scale.

Table 4. Calc. of Columns F, G, and Cell J4 using Active Yu in Cell J3; Also copy Cell J4 to Cell C14.

J4 | =CORREL(B3:B9,G3:G9)^2

	A	B	C	D	E	F	G	H	I	J
1	Position cm		Range	Mass g	Cum. Mass. g	qYu - qY	q(qYu - qY)			
2	X	qX		y	Y					
3	3	0.4771	0 -3	0.13	0.13	2.1006	0.32235	Active Yu =		16.39
4	6	0.7782	3 - 6'	1.79	1.92	0.9313	-0.03092	R^2 =		0.94063
5	12	1.0792	6 - 12'	5.19	7.11	0.3627	-0.44044			
6	24	1.3802	12 - 24'	5.40	12.51	0.1173	-0.93062			
7	48	1.6812	24 - 48	2.50	15.01	0.0382	-1.41796			
8	96	1.9823	48 - 96	1.10	16.11	0.0075	-2.12590			
9	120	2.0792	96 - 120	0.24	16.35	0.0011	-2.97420			
10										
11	ΔYu	0.04		0.08		0.16		0.32		
12			R^2 for 0.16		R^2 for 0.08		R^2 for 0.16		R^2 for 0.32	
13	Yu0 =	16.35		16.35		16.35		16.35		
14	Yu1 =	16.39	0.94063	16.43		16.51		16.67		
15	Yu2 =	16.43		16.51		16.67		16.99		
16	Yu3 =	16.47		16.59		16.83		17.31		
17	Yu4 =	16.51		16.67		16.99		17.63		
18	Yu5 =	16.55		16.75		17.15		17.95		
19	Yu6 =	16.59		16.83		17.31		18.27		
20	Yu7 =	16.63		16.91		17.47		18.59		
21	Yu8 =	16.67		16.99		17.63		18.91		

At this point, let us mention how to locate the unique (optimal) upper asymptote *Yu* for the straight line. There are several ways to locate optimal upper asymptote *Yu*, such as (A) Template digital method; (B) Analog graphical resolution method; and (C) Use of a Fortran program or a Python-coded program. In the Template digital method, we make use of Excel tables such as Tables 3-6. In Analog graphical resolution, we search for a straight line with maximum COD (Coefficient of Determination) by stepwise elimination of concave and convex date-lines.

In the Template digital method, let us continue with Table 3 and start filling Table 4. Referring to Table 4, we need to assign an active *Yu* for the calculation of Column F(F3:F9), Column G(G3:G9), and Cell J4 for COD. Let us assign the estimated *Yu*, *Yu*1, in Cell B14 as the first Active *Yu* and input it into Cell J3. Column F (F3:F9) is used to calculate nonlinear face value (q*Yu* – qY) using the Active *Yu* 16.39. The formula for Cell F3 is "=log(J3) – log(E3)", *i.e.*, 2.1006, we copy Cell F3 to F4 through F9 to complete the column, as shown in the table. Column G (G3:G9) is used to calculate the logarithmic transformation of the face value by taking the log of Column F, *i.e.*, Cell G3 = log(E3), etc.

Table 5. "Input" active *Yu* to Cell J3 and "copy" Cell J4 to Cell C 14, etc., using estimated *Yu* in Columns B(B14:B21), Column E(E14:E21), Column G(G14:G21), and Column I(I14:I21). (Image of Excel Table)

J4 =CORREL(B3:B9,G3:G9)^2

	A	B	C	D	E	F	G	H	I	J
1	Position cm		Range	Mass g	Cum. Mass. g	qYu - qY	q(qYu - qY)			
2	X	qX		y	Y					
3	3	0.4771	0 -3	0.13	0.13	2.1006	0.32235	Active Yu =		16.39
4	6	0.7782	3 - 6'	1.79	1.92	0.9313	-0.03092	R^2 =		0.94063
5	12	1.0792	6 - 12'	5.19	7.11	0.3627	-0.44044			
6	24	1.3802	12 - 24'	5.40	12.51	0.1173	-0.93062			
7	48	1.6812	24 - 48	2.50	15.01	0.0382	-1.41796			
8	96	1.9823	48 - 96	1.10	16.11	0.0075	-2.12590			
9	120	2.0792	96 - 120	0.24	16.35	0.0011	-2.97420			
10										
11	ΔYu	0.04		0.08		0.16		0.32		
12			R^2 for 0.04		R^2 for 0.08		R^2 for 0.16		R^2 for 0.32	
13	Yu0 =	16.35		16.35		16.35		16.35		
14	Yu1 =	16.39	0.94063	16.43	0.96491	16.51	0.98357	16.67	0.99537	
15	Yu2 =	16.43	0.96491	16.51	0.98357	16.67	0.99537	16.99	0.99915	
16	Yu3 =	16.47	0.97659	16.59	0.99137	16.83	0.99864	17.31	0.99716	
17	Yu4 =	16.51	0.98357	16.67	0.99537	16.99	0.99915	17.63	0.99378	
18	Yu5 =	16.55	0.98817	16.75	0.99752	17.15	0.99846	17.95	0.99003	
19	Yu6 =	16.59	0.99137	16.83	0.99864	17.31	0.99716	18.27	0.99627	
20	Yu7 =	16.63	0.99368	16.91	0.99911	17.47	0.99555	18.59	0.99262	
21	Yu8 =	16.67	0.99537	16.99	0.99915	17.63	0.99378	18.91	0.97912	

Then, we calculate Active COD in Cell J4. The formula for Cell J4 is "=CORREL(B3:B9, G3:G9)^2", as shown in the formula bar on the top line of Table 5.

Cell J4 gives active COD at 0.94063. We then copy the obtained COD in Cell J4 into the Cell next to $Yu1$ in Column C, *i.e.*, we copy 0.94063 in Cell J4 into Cell C14. We then sequentially input the estimated Yu into Active Yu cell in J3, whence the values in Column F, G, and Cell J4 all change. We sequentially input estimated Yu in Column B (B14:B21), Column D (D14:D21), Column F (F14:F21), and Column H (H14:H21) and sequentially copy the COD value in Cell J4 into Cell next to the estimated Yu, we obtain **Table 5**.

When using 4 levels of ΔYu, at 8 estimated Yu for each level, ($4 \times 8 = 32$), it is like casting a net to catch fish. We are certain to catch all the fish, small and big. From there, we can pick what we want. That is, we are certain to locate all the COD, and from there, we can identify the largest COD, as shown in the Green Cells in the table.

In Column I, the COD increases from 0.99537 to reach a maximum of 0.99915, then decreases. In Column C, the COD increases continuously. In Column G, the COD increases to reach the maximum and then decreases. By plotting Column F (F14:F21) versus Column G (G14:G21), we obtain **Figure 8**, showing the COD increase From $Yu1$ to $Yu2$ to reach $Yu4$ at maximum and then decrease toward $Yu8$.

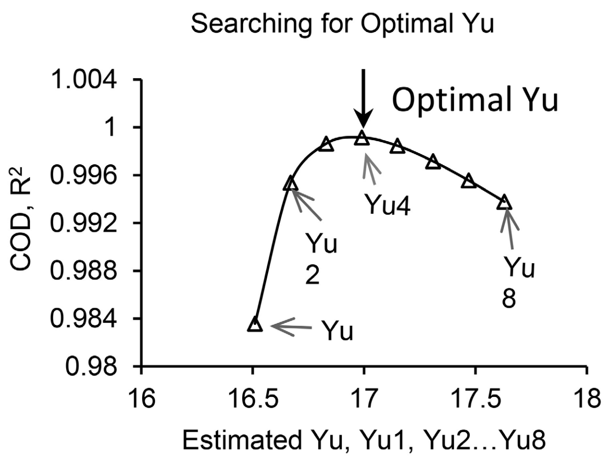


Figure 8. Searching for optimal Yu.

By inserting the maximum (optimal) COD 16.99 into the Active Cell in Cell J3, we obtain **Table 6**. Then, by plotting Column B (B3:B9) vs. Column G (G3:G9) for qX vs. $q(qYu - qY)$, we obtain **Figure 9(a)** with a straight-line and a trendline equation $y = -1.3284x + 0.9833$, the proportionality constant K is 1.3284, and the trendline gives $R^2 = 0.9991$. Meanwhile, by first plotting Column A versus Column F in a liner-by-linear scale and then converting both axes into a log-by-log scale, we obtain a straight line in **Figure 9(b)**. The trendline equation gives $y = 9.6227x^{1.328}$ with $R^2 = 0.9974$. The proportionality constant K is $K = 1.328$. As expected, K is the same in a liner-by-linear plot of **Figure 9(a)** and in a log-by-log plot of **Figure 9(b)**. The straight line in **Figure 9(a)** in a linear-by-linear scale means the existence of proportionality between $q(qYu - qY)$ vs. qX. The straight line in **Figure 9(b)** in a log-by-log scale means the existence of proportionality between ($qYu - qY$) vs. qX, [3]-[6].

Table 6. Calculation with active $Yu = 16.99$. (Image of Excel Table)

J4 | f_x =CORREL(B3:B9,G3:G9)^2

	A	B	C	D	E	F	G	H	I	J
1	Position cm		Range	Mass g	Cum. Mass. g	qYu - qY	q(qYu - qY)			
2	X	qX		y	Y					
3	3	0.4771	0 -3	0.13	0.13	2.1163	0.32557	Active Yu =		16.99
4	6	0.7782	3 - 6'	1.79	1.92	0.9469	-0.02370	R^2 =		0.99915
5	12	1.0792	6 - 12'	5.19	7.11	0.3783	-0.42214			
6	24	1.3802	12 - 24'	5.40	12.51	0.1329	-0.87636			
7	48	1.6812	24 - 48	2.50	15.01	0.0538	-1.26912			
8	96	1.9823	48 - 96	1.10	16.11	0.0231	-1.63643			
9	120	2.0792	96 - 120	0.24	16.35	0.0167	-1.77792			
10										
11	ΔYu	0.04		0.08		0.16		0.32		
12			R^2 for 0.04		R^2 for 0.08		R^2 for 0.16		R^2 for 0.32	
13	Yu0 =	16.35		16.35		16.35		16.35		
14	Yu1 =	16.39	0.94063	16.43	0.96491	16.51	0.98357	16.67	0.99537	
15	Yu2 =	16.43	0.96491	16.51	0.98357	16.67	0.99537	16.99	0.99915	
16	Yu3 =	16.47	0.97659	16.59	0.99137	16.83	0.99864	17.31	0.99716	
17	Yu4 =	16.51	0.98357	16.67	0.99537	16.99	0.99915	17.63	0.99378	
18	Yu5 =	16.55	0.98817	16.75	0.99752	17.15	0.99846	17.95	0.99003	
19	Yu6 =	16.59	0.99137	16.83	0.99864	17.31	0.99716	18.27	0.99627	
20	Yu7 =	16.63	0.99368	16.91	0.99911	17.47	0.99555	18.59	0.99262	
21	Yu8 =	16.67	0.99537	16.99	0.99915	17.63	0.99378	18.91	0.97912	

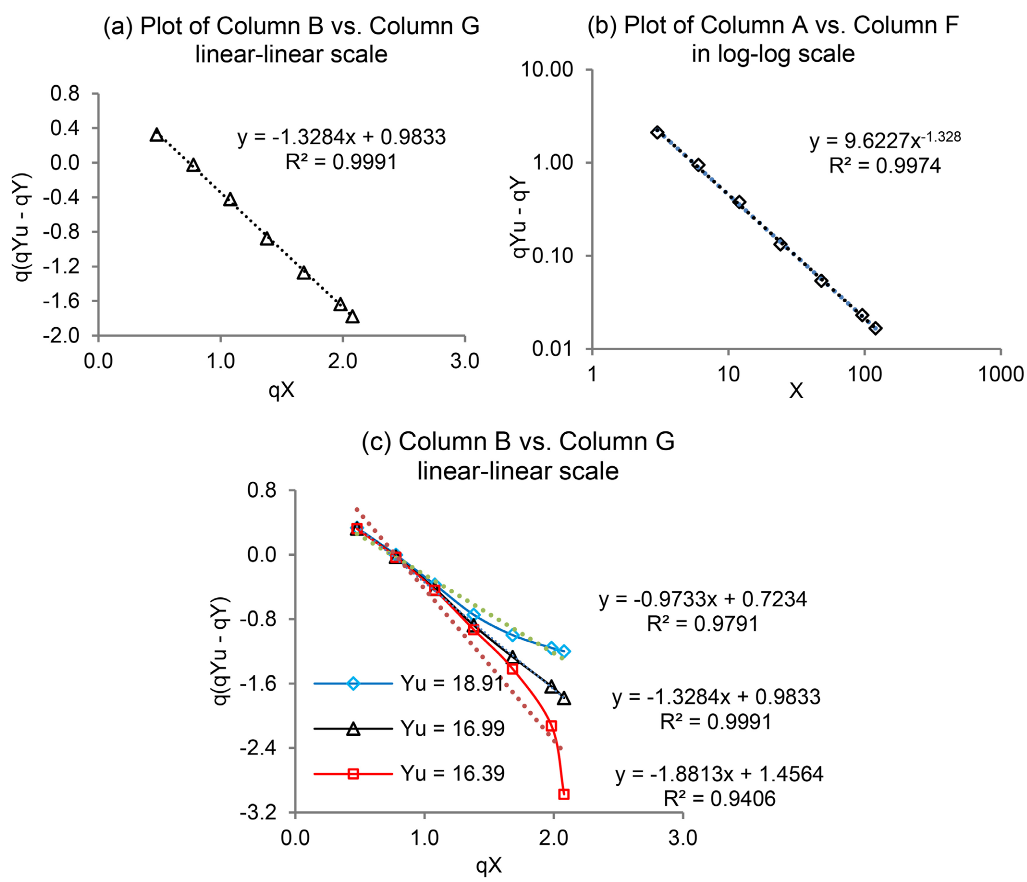


Figure 9. (a) Column B vs. Column G, in linear-linear scale; (b) Column A vs. Column F, in log-log scale. (c) three lines for three Yu.

In the analog graphical resolution method, we first copy **Figure 9(a)** into **Figure 9(c)**. Then, we insert other lines with different Active *Yu*. For example, values in Column B vs Column G of Table 5 are used to get the equation line along with the equation and COD. It shows that at the Active *Yu* of 16.39, the *K* increases from 1.3284 (in **Figure 9(c)**) to 1.8813, the COD decreases from 0.9991 to 0.9406, and the data shows a convex downward line. Similarly, we can generate a separate worksheet with the new Active *Yu* at 18.91 and inset the curve and COD into the same graph. At this new series, the *K* decreases to 0.9733, and the data shows a concave bend-up line, with the COD decrease to 0.9791. The above method of identifying straight lines in **Figure 9(c)** is an analog graphical resolution method that has been used extensively in the past three decades [3] [11]-[13].

To approximate the optimal *Yu* quickly, we can use a Python program. This high-level programming language allows us to determine the optimal *Yu* of any data in a very short period. In addition, we can ensure that the approximation is much more precise. The Python program is listed in **Appendix A**, with 37 lines of coding and 6 comments. It determines the optimal *Yu* of the same data set used previously. The user must input the *X* and *Y* values of their data set. Note that the number of *X* values and *Y* values must be equal. The user must also ensure that the input *Y* is a cumulative value that always increases. Finally, the user must set the maximum *Yu* value they want to test and the increment between each test. If a sufficient *Yu* value is not found, consider increasing the maximum or using a smaller increment. The listed coding example uses a maximum of 18 estimated *Yu* and an increment of 0.01. Running the above code on the same data as shown returns an optimal *Yu* of 16.96 with an R^2 of 0.99917. Thus, the program found a *Yu* with a slightly higher correlation than we had previously found.

This code operates on the same mathematical basis as previous approximations of *Yu* by testing many possible *Yu* values and determining which of them creates the highest correlation in logarithmic form. However, the code can use much smaller increments and find an optimal *Yu* much faster. The program specifically tests every incremented *Yu* between the highest *Y* value and the input maximum *Yu* value. It returns the *Yu* that had the highest R^2 value, along with the R^2 of that *Yu*. If the user wishes to see the trend of R^2 values as the *Yu* changes, they can uncomment the print statements near the end of the code.

We can also use the Fortran program to search for *Yu*. However, it is more tedious and more time-consuming and thus not recommended for general use. For example, it used up to 12 pages of programming in the book to give the results [14].

5.2. Effect of Silica on the Profiles of Particle Deposition—Simplicity out of Complex Case

Like the last example of general fine particle deposition, let us use a set of data from four experiments. The four experiments included one experiment with raw coal powder without an addition of silica (0% silica); the other three experiments were with a mixture of 25% silica, 50% silica, and 75% silica. Raw coal powder generally contains some impurity, which, like silica, serves as a negative charge acceptor, while pure coal is positively charged during a tribo-charging process.

Table 7 lists the weight of clean coal collected at various interval positions in Columns J through M. The first sample is collected from 0 to 3 cm, the second sample from 3 to 6 cm, etc. The table also lists the cumulative weight and calculated nonlinear face values ($qYu - qY$) using unique values of Yu: 70, 86.5, 81, and 74.8. Figure 10 gives a three-dimensional plot for mass collected at seven nonlinear distance intervals for four levels of silica mixture. The profile for each level of silica in the 3-D graph is the equivalent of a plot of the primitive graph. For original coal (0% silica), the most mass was collected in the 12 - 24 cm interval. At 25% by weight addition of fine sand, the mass peak shifted toward the inlet of the separator (6 - 12 cm interval).

Table 7. Effect of silica on particle deposition. (Image of Excel Table)

F5 =(LOG(F2)-LOG(B5))

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		0% silica	25% silica	50% silica	75% silica	0% silica	25% silica	50% silica	75% silica	0% silica	25% silica	50% silica	75% silica
2	Yu					70	86.5	81	74.8				
3	posit, X	cum. Y	cum.Y	cum.Y	cum.Y	(qYu-qY)	(qYu-qY)	(qYu-qY)	(qYu-qY)	wt. y	wt. y	wt. y	wt. y
4	cm									g	g	g	g
5	3	0.06	6.3	13.8	24.3	3.0669	1.1377	0.7686	0.4883	0.06	6.3	13.8	24.3
6	6	3.36	28.7	41.8	54.9	1.3188	0.4791	0.2873	0.1343	3.3	22.4	28.0	30.6
7	12	19.97	59.8	66.6	68.2	0.5447	0.1603	0.0850	0.0401	16.61	31.1	24.8	13.3
8	24	44.01	78.3	75.9	72.0	0.2015	0.0433	0.0282	0.0166	24.04	18.5	9.3	3.8
9	48	60.47	83.7	78.6	73.5	0.0636	0.0143	0.0131	0.0076	16.46	5.4	2.7	1.5
10	96	65.62	85.4	79.9	74.2	0.0281	0.0056	0.0059	0.0035	5.15	1.7	1.3	0.7
11	120	66.36	85.7	80.4	74.5	0.0232	0.0040	0.0032	0.0017	0.74	0.3	0.5	0.3

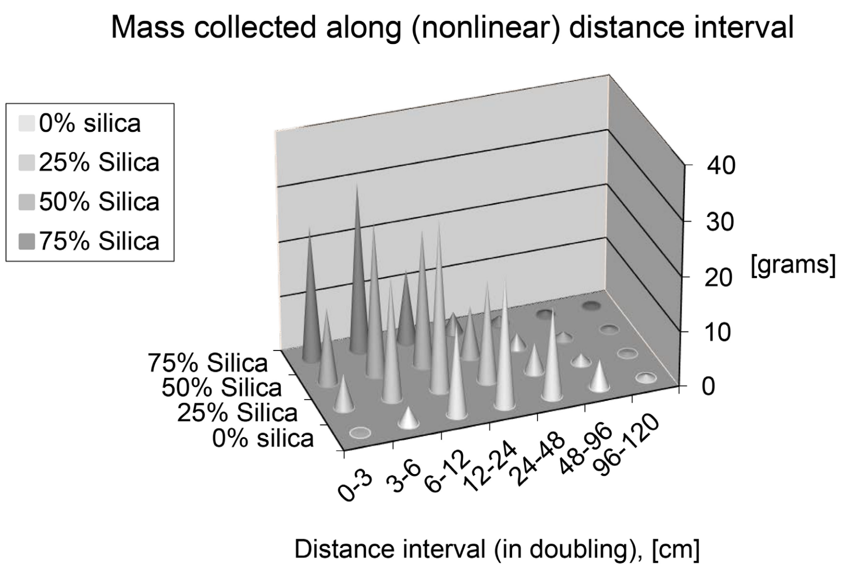


Figure 10. Mass collected versus distance.

Further addition of sand resulted in more of a shift toward the separator's inlet. When plotting the above 3-D information in two-dimensional graphs, we obtain **Figure 11**, showing various bell-shaped primitive graphs when connecting the tips of data points for various levels of silica mixture (not shown in the graph). Primitive graphs are not used much in data analysis. Fortunately, we can resort to nonlinear analysis using cumulative data and primary graphs. To do this, the first task is to generate a series of cumulative data Y and calculate nonlinear face values ($qYu - qY$), as indicated in **Table 7** and the primary graph **Figure 12**.

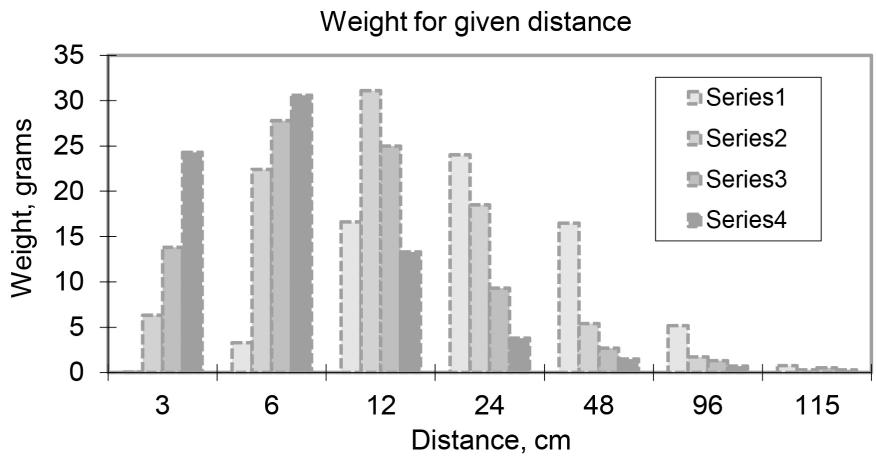


Figure 11. Weight versus distance.

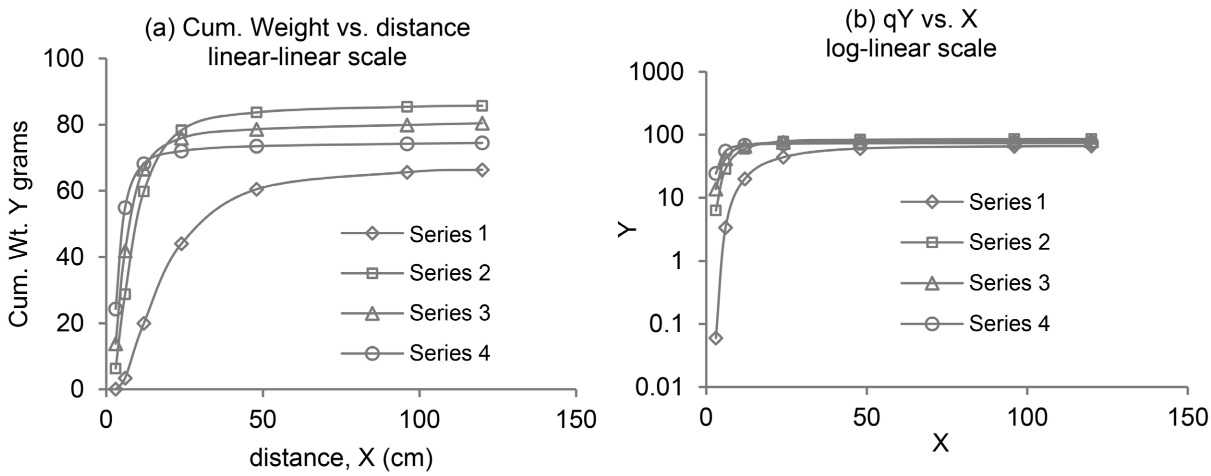


Figure 12. Weight vs. distance, (a) linear-linear scale; (b) log-linear scale.

Subsequently, by plotting cumulative weight Y versus cumulative distance X, we obtain **Figure 12(a)**. This is the primary graph where the curve of series 1 (0% silica) is shown as a sigmoid curve. The other three curves are also expected to show a similar sigmoid curve with small initial tails if the sample collection is available at smaller X ranges and with sufficient sample collection resolution in the short distance range.

By converting the y-axis in **Figure 12(a)** from a linear into a nonlinear logarithmic scale, we obtain Figure 12(b). This is a leading graph where the slopes of the curves are changing continuously, implying that the nonlinear change of nonlinear face-values, $qYu - qY$, is negatively proportional to the change of X or to the nonlinear change of X, as the indication in Figure 2(d).

After locating Yu for each curve like the last general case, we can proceed to plot $(qYu - qY)$ versus X on a log-log graph, as shown in Figure 13, where the equations are $(qYu - qY) = CX^K$. The equation parameters are summarized in Table 8.

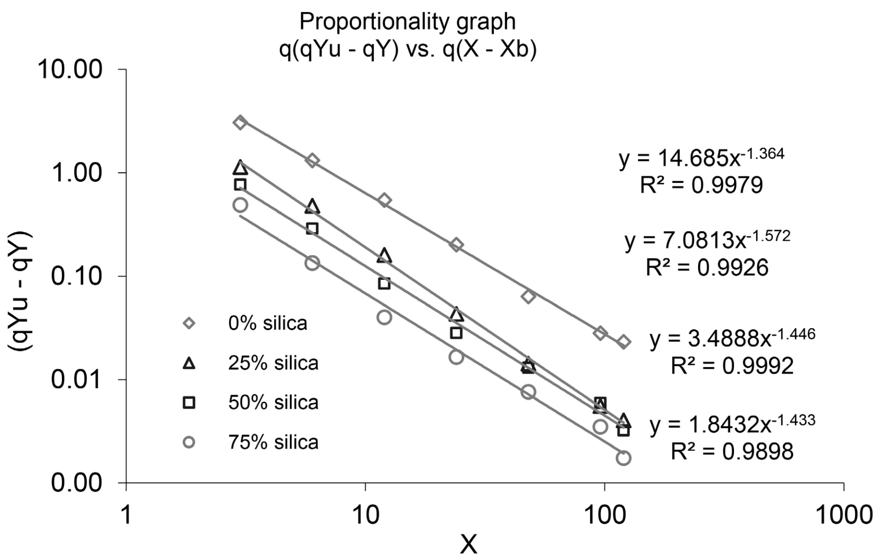


Figure 13. Proportionality plot for $(qYu - qY)$ vs. X.

Table 8. Equation parameters Yu, K, and C.

% Silica	Parameters		
	Yu	K	C
0	70.0	1.3645	14.685
25	86.5	1.5721	7.0813
50	81.0	1.4457	3.4888
75	74.8	1.4330	0.9898

The four straight lines in Figure 13 have a slope (K) between 1.3645 and 1.5721. These similar slopes reflect the fact that the experimental parameters, *i.e.*, the gas carrying rate, the physical dimension of the box, and the DC polarities, are fixed. The asymptote (Yu) of the original coal is 70.0. As fine silica is added to the coal, electrons on the coal surface transfer to the silica surface during the tribo-charging stage and thus make the coal surface more positive.

Consequently, the asymptote of the mixture with 25% silica dramatically increases to 86.5. Subsequent addition of silica causes a decrease in the asymptote from 86.5 to 81.0 for 50% silica addition and to 74.8 for 75% silica addition. The decreases in the asymptotes for these two later cases are due to a dilution of the coal in the feed as the total feed mass is maintained constant. The position constant (integral constant C) of the equations reflects the collection of the coal near the inlet. The position constants decrease from 14.685 to 7.0813, 3.4888, and to 3.4888 as the silica content increases, as shown in **Table 8**.

In essence, if we pay attention to the asymptotes and measure the nonlinear change relative to the asymptotes, we can get the ACP nonlinear model shown in the proportionality graph **Figure 13**. This graph, along with ACP nonlinear equations, has significantly enhanced the description of the data in **Figures 10-12**.

6. Discussions

We present the analyses of astrophysics and electrostatic separation data using a simple nonlinear concept and using multiple tables and graphs for illustrations. In the future, the practitioners can simplify the presentation by using a single worksheet, a single proportionality graph, and a single summary table.

In our analysis, we compare the monotonically increasing nonlinear number Y with the other monotonically increasing nonlinear number X. Specifically, we compare their nonlinear face value with each other. The analysis is so simple and easy that every high school student can do it.

Traditional XY math is insufficient to describe the nonlinear phenomena; we need to extend the XY math into the $\alpha\beta$ Math to account for the existence of asymptotes, *i.e.*, we need to extend $XY = \{(X), (Y)\}$ into $\alpha\beta = \{\alpha(Y, Yu, Yb), \beta(X, Xu, Xb)\}$. The $\alpha\beta$ Nonlinear Math classifies continuous monotonically increasing numbers into linear and nonlinear numbers. Nonlinear numbers are associated with asymptotes, and their measurement of difference is the face value of the nonlinear numbers. The nonlinear face value can be a difference, a ratio of difference, or with logarithmic transformation, such as $(Yu - Y)$, $(Y - Yb)$, $(Yu - Y)/(Y - Yb)$, and $(qYu - qY)$. The application of $(Yu - Y)/(Y - Yb)$, which behaves like a logistic equation, will be demonstrated in a separate article.

The Alpha Beta ($\alpha\beta$) Math is a science for connecting a straight line to concave and convex asymptotic curves, sigmoid and various bell curves in physical science, engineering, and life and biomedical sciences [1]-[6] [9]-[13]. We provide illustrations for building Excel Templates along with a Python high-level programming language to solve for upper asymptotes and build a straight-line proportionality equation.

7. Conclusions

- We can describe nonlinear phenomena with a simple proportionality equation and four types of graphs: primitive, primary, leading, and proportionality graphs. It is important to learn how to distinguish between a primitive elementary graph and a primary graph. We cannot use a primitive elementary graph alone to build a mathematical relationship because each elementary "y" has no mathematical connectivity, and one elementary number cannot mathematically relate to the other cumulative numbers. Instead, we must resort to relating one cumulative number with the other cumulative numbers and using the primary graph for mathematical analysis, where we can have both continuous numbers Y and continuous numbers X exist as cumulative Y and cumulative X. Cumulative numbers mean the existence of connectivity.

- Data in astrophysics and particle separation physics must obey the law of nature. The experimental law dictates that the dependent variable be either proportional to or negatively proportional to the independent variable. We can build a straight-line proportionality relationship for the dependent-independent data in log-linear and log-log graphs. This article demonstrates a methodology for solving the key upper asymptotes for the proportionality equation using Microsoft Excel along with Python program via determining the "coefficient of determination". Our example includes a systematic demonstration of Excel data manipulation and extensive graphing.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Symbols

q = Log; $\alpha\beta$ (extension of XY); ϕ = (0) (nonlinear zero); x = elementary variable, y or (y) = elementary variable or y = equation y (inside the graph); X = cumulative of x, Y = cumulative of y or (y).

Appendix A. Python Programming for Searching the Upper Asymptote Yu

```
import math
def q(n):
    return math.log10(n)
def avg(l):
    return sum(l) / len(l)
# USER INPUTTED VALUES
X = [3, 6, 12, 24, 48, 96, 120] # X and Y MUST be the same length
Y = [.13, 1.92, 7.11, 12.51, 15.01, 16.11, 16.35] #cumulative y (should always be increasing)
max_pred = 18 #input the highest value Yu that should be tested
deltaYu = .01 #input the change in Yu for each test, smaller will generally give a more accurate approximation
# CALCULATED VALUES
qX = list(map(q, X))
qY = list(map(q, Y))
Yu = Y[len(Y) - 1] + deltaYu #start with the last value of Y + deltaYu
maxCorr = 0
bestYu = -1
while Yu <= max_pred:
    qYu = q(Yu)
    qDif = qY.copy()
    for i in range(len(qY)):
        qDif[i] = qYu - qY[i]
    qqDif = list(map(q, qDif))
    aqX = avg(qX)
    aqqDif = avg(qqDif)
    numList = qqDif.copy()
    denX = qX.copy()
    denY = qqDif.copy()
    for i in range(len(qqDif)):
        numList[i] = (qX[i] - aqX) * (qqDif[i] - aqqDif)
        denX[i] = (qX[i] - aqX)**2
        denY[i] = (qqDif[i] - aqqDif)**2
    corrNum = sum(numList)
    corrDen = math.sqrt(sum(denX) * sum(denY))
    corrSquare = (corrNum / corrDen)**2
    if corrSquare > maxCorr:
        maxCorr = corrSquare
        bestYu = Yu
    #UNCOMMENT FOR EXACT CORRELATION FOR EACH YU
    #print("Current Yu: ", Yu)
    #print("Correlation: ", corrSquare)
    Yu += deltaYu
#FINAL APPROXIMATION
print("Approximate Yu: ", bestYu)
print("Correlation: ", maxCorr)
```