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Atomic Quantum Mechanics Based on Atomic Functions

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Abstract

Based on theorems, the Atomic AString Functions theory, evolving since the 1970s, is introduced into Quantum Mechanics to represent a wave function via the shifts and stretches of smooth finite Atomic Function pulses/solitonic atoms. It leads to a novel 'atomic interpretation' where wave functions become the superpositions of localized Atomic Wave Functions, which can also describe collapsed wave functions, represent Gaussians, uphold Heisenberg's uncertainly principle, and a more generic concept of Atomic Harmonic Oscillator. Atomic Functions can solve the boundary wave function discontinuity problem for particle-in-a-box and other solutions by introducing atomic wave packets. It highlights some limitations of the Schrödinger equation, yielding harmonic representations that may not be flexible enough to satisfy complex boundary conditions. The theory follows more generic research on Atomic Spacetime, quantum gravity, and field theories to derive common mathematical blocks of unified fields similar to loop quantum gravity and strings theories.

Keywords

Quantum Mechanics, Atomic Function, Atomic Wave Function, Atomic Oscillator, Atomic Spacetime, Unified Theories

1. Introduction and Main Ideas

This paper continues the research [1]-[9] started in 2017 on the expansion of the theory of Atomic AString Functions [1]-[19] known since the 1970s to new domains of theoretical physics, including General Relativity, spacetime and field theories [20]-[31] soliton theories [4] [6] and now towards Quantum Mechanics [20]-[28] [32]-[35]. The theory of Atomic Functions has been evolving since the

1970s after pioneering works by Academician of NAS of Ukraine V. L. Rvachev[1] (Ukrainian V. L. Rvachov) and V. A. Rvachev [10]-[12], who discovered and systematically researched the smooth finite pulse function for which derivatives are conveniently expressed via the function itself:

$$up'(x) = 2up(2x+1) - 2up(2x-1). \tag{1.1}$$

Being a smooth finite compactly-supported function like widely-used splines, this function provides a unique ability to exactly compose polynomials of any order $P_n(x)$, hence represents the wide class of analytic functions $f(x)$ and their superpositions representable by polynomials via Taylor's series [1]-[19]:

$$P_n(x) \equiv \sum_k C_k up\left(\frac{x-ka}{a}\right);\ \ f(x) = \sum_{m=-\infty}^{\infty} c_m up\left(\frac{x-b_m}{a_m}\right). \tag{1.2}$$

Because of their universality, Atomic Functions have been often called 'mathematical atoms' [10]-[15] due to their ability—like from 'atoms'—to compose polynomials, complex analytic functions, and solutions of differential equations of mathematical physics, including General Relativity (GR) and Quantum Mechanics (QM) [20]-[31].

Following the research [1]-[9] started in 2017 on the theorem-based theory expansion to mathematical physics, field, and spacetime theories, the ability to represent spacetime and fields via superposition of localized Atomic AString Functions provides the unique mathematical apparatus to introduce into continuous theories the finite objects like quanta/elementary distortions/metriants [1]-[9] leading to the fields unification ideas based on a common 'mathematical ancestor' like in string and Loop Quantum Gravity theories [23]-[30]. For example, in Quantum Mechanics focused in this work, continuous wave functions can be composed of localized Atomic Wave Functions, which also replace probabilistic Gaussians and collapsed wave functions, leading to atomic interpretation of QM and fixing some boundary discontinuity problems of Schrödinger equation. Importantly, the same Atomic Functions can be used in other fields and Atomic Spacetime theories [1] [2], leading to unification theories based on the common 'mathematical ancestor' [1]-[9], including Quantum Gravity [22]-[31]. Atomic interpretation of fields also realizes A. Einstein's "perfectly thinkable", "atomic theory" 1933 ideas [1] [21] where all fields are just "intricate distortions of spacetime" promoted by S. Hawking [29]. The background, challenges, and contributions to Atomic Functions theory are described in the following historical review.

2. Brief History of Atomic and AString Functions

The theory of Atomic Functions (AF) [10]-[19] has been evolving since 1967-1971 when Academician of NAS of Ukraine V. L. Rvachev and V. A. Rvachev [10] had envisaged a finite pulse function $up(x)$ for which derivatives (also pulses) would

[1]Volodymyr Logvinovich Rvachov (1926-2005), Academician of the National Academy of Sciences of Ukraine, author of 600 papers, 18 books, mentor of 80 PhDs, 20 Doctors and Professors including the author. https://en.wikipedia.org/wiki/Volodymyr_Rvachov

conveniently be similar to the original pulse shifted and stretched by the factor of 2:

$$up'(x) = 2up(2x+1) - 2up(2x-1), |x| \le 1: up(x) = 0, |x| > 1. \quad (2.1)$$

This and other similar functions possess unique properties of infinite differentiability, smoothness, nonlinearity, nonanalyticity, finiteness, and compact support and can serve as widely used splines. The most significant is that other functions like polynomials, trigonometric, exponential, and other analytic functions can be represented either exactly (like polynomials) or via a converging series of shifts and stretches of AFs. So, like from 'mathematical atoms' [10]-[19], smooth functions can be composed of the AF superpositions, and because of that, those 'atoms' have been called Atomic Functions in the 1970s. The foundation of AF theory [10]-[14] has been developed in Ukraine and enriched by many followers from different countries, notably by schools of V.F. Kravchenko [13]-[14] [33] B. Gotovac, H. Gotovac [15] [16] and the author [1]-[9] with the number of papers and books observed in [14] has grown to a few hundred.

3. History of Atomic AString Functions in Atomic Spacetime and Fields Research

In 2017, the author noted that AF $up(x)$ (1.1) is a composite pulse consisting of two kink functions called AStrings [1]-[9], making them more generic:

$$up(x) = AString(2x+1) - AString(2x-1) = AString'(x). \quad (3.1)$$

AString is related to the shifted and stretched Fabius function [31] known since 1966 but has been obtained within AF theory as a 'composing branch' and integral of $up(x)$. Mutual relationships (2.1) and (3.1) imply that theories and approximating advantages involving AFs can be reformulated via AStrings too. Composing AF pulse (2.1) via kink-antikink pair (3.1) of nonlinear AStrings resembles topological 'solitonic atoms' (or bions) from the theory of soliton dislocations leading to the theory of Atomic Solitons (2018) [4] [6] where AString (3.1) becomes a solitonic kink while $up(x)$ is a 'solitonic atom' made of AStrings. This 'solitonic atoms' analogy also enforces the naming of AF as 'mathematical atoms' in the 1970s. The ability of finite AFs to compose smooth polynomials, analytic functions, and solutions of differential equations, including GR and QM, leads to novel interpretations of atomic spacetime and field composition from "solitonic atoms"/atomic solitons described in [1]-[9].

The mathematical foundation of the 'atomization' of physical fields is based on the sequence of Atomization Theorems [1] [2] [3] applied hereafter for QM. Starting from the core theorems from the 1970s and enriched in the 2020s for recently introduced AStrings with the so-called Atomic Series [1]-[3], it was extended in [2] [3] to new theorems for complex analytic functions, nonlinear theories, and finally nonlinear General Relativity equations [1]-[9] [18] leading to the theory of Atomic Spacetime where Atomic AString Functions can be not only introduced into GR but also deduced from GR surprisingly finding support from A. Einstein

[21] who envisaged in 1933 the "atomic theory" for finite "regions of space" with "discrete energies" resembling finite Atomic Functions. The Atomization Theorems and AF theory can be applied to many physical theories, including electromagnetism, elasticity, heat conductivity, soliton theories, and field theories [1]-[30], including QM [33] focused in this work. A unified representation of fields composed of superpositions of finite Atomic AString Functions may offer some novel variants of unified theory under research now [1]-[9] [22] where, like in string theory, fields become interconnected, having a common mathematical ancestor.

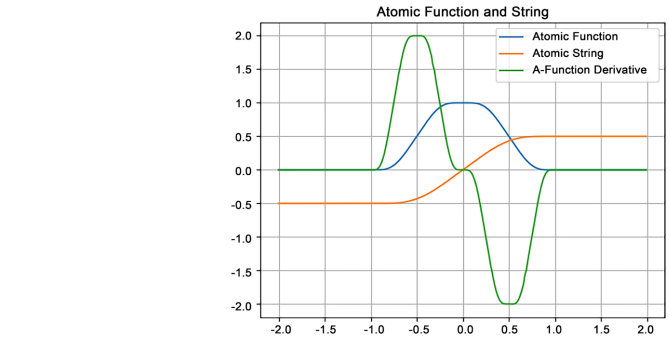
4. Atomic and AString Functions

Let's describe Atomic [10]-[19] and AString [1]-[9] Functions in more detail.

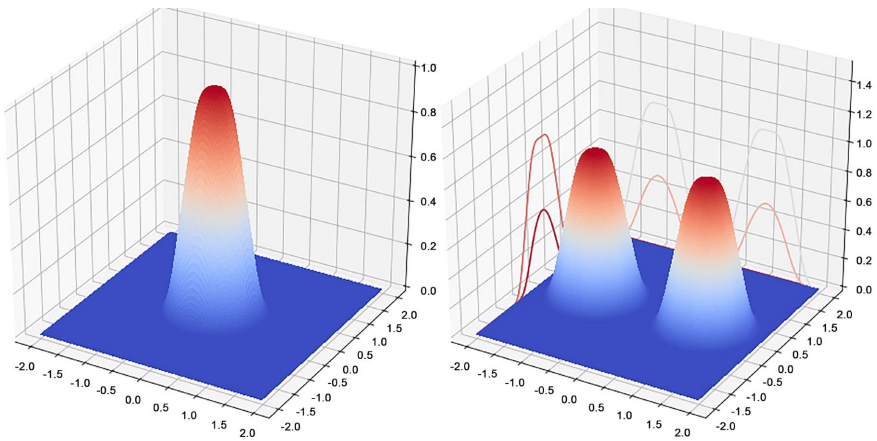
4.1. Atomic Function

Atomic Function (AF) (V. L. Rvachev, V. A. Rvachev, [10]) $up(x)$ was introduced in 1967-1971 as a finite compactly supported non-analytic infinitely differentiable function (**Figure 1**) with the first derivative conveniently expressible via the function itself shifted and stretched by the factor of 2:

$$up'(x) = 2up(2x+1) - 2up(2x-1), |x| \le 1;\ up(x) = 0, |x| > 1. \tag{4.1}$$



(a)



(b) (c)

Figure 1. (a) Atomic function pulse with its derivative and integral (AString); (b) Atomic function pulse ('solitonic atom') in 2D; (c) Two atomic function pulses ('solitonic atoms' or 'atomic solitons').

With exact Fourier series representation[1]-[9] [10]-[13],

$$up(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{itx}\prod_{k=1}^{\infty}\frac{\sin\left(t2^{-k}\right)}{t2^{-k}}dt,\ \int_{-1}^{1}up(x)dx=1, \tag{4.2}$$

the values of $up(x)$ can be calculated with computer scripts [2] [4] [19].

Higher derivatives $up^{(n)}$ and integrals I_m can also be expressed via $up(x)$ [1]-[13] [15]

$$up^{(n)}(x)=2^{\frac{n(n+1)}{2}}\sum_{k=1}^{2^n}\delta_k up\left(2^n x+2^n+1-2k\right),$$

$$\delta_{2k}=-\delta_k,\ \delta_{2k-1}=\delta_k,\ \delta_1=1;$$

$$I_m(x)=2^{C_m^2}up\left(2^{-m}x-1+2^{-m}\right),\ x\le 1;$$

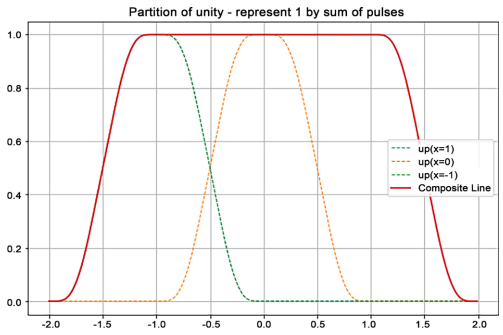
$$I_m(x)=2^{C_m^2}up\left(2^{-m+1}-1\right)+\frac{(x-1)^{m-1}}{(m-1)!},\ x>1;$$

$$I_1(x)=up\left(2^{-1}x-2^{-1}\right);\ I_1'(x)=up(x). \tag{4.3}$$

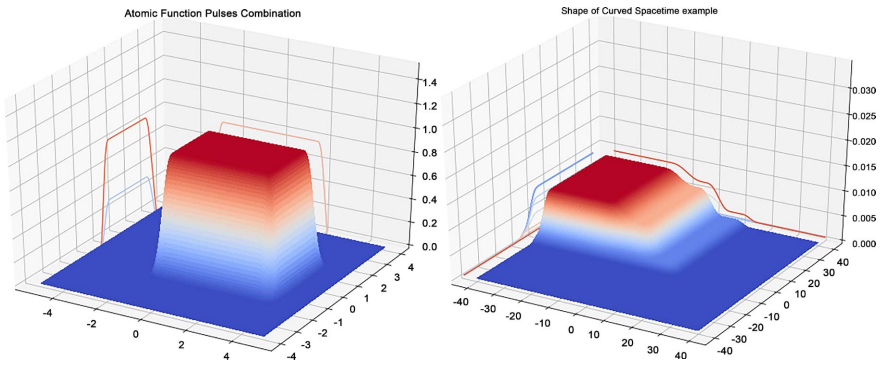
Due to special double symmetry, AF provides partition of unity [10]-[19] to exactly represent the number 1 by summing up individual overlapping pulses set at regular points... −2, −1, 0, 1, 2… (**Figure 2(a)**):

$$\cdots up(x-2)+up(x-1)+up(x)+up(x+1)+up(x+2)+\cdots\equiv 1.$$

$$up(x)=up(-x),\ x\in[-1,1];\ up(x)+up(1-x)=1,\ x\in[0,1]. \tag{4.4}$$



(a)



(b) (c)

Figure 2. (a) Partition of unity with Atomic Functions; (b) Representation of flat surface via summation of Afs; (c) Curved surface as a superposition of 'solitonic atoms'.

Generic AF pulse of width $2a$, height c, and center positions b, d is

$$up(x,a,b,c,d=0)=d+c*up\left((x-b)/a\right)\cdot\int_{-a}^{a}cup(x/a)\mathrm{d}x=ca. \tag{4.5}$$

Multi-dimensional atomic functions [1]-[14] (**Figure 1**, **Figure 2**) can be constructed as either multiplications or radial atomic functions:

$$up(x,y,z)=up(x)up(y)up(z),$$

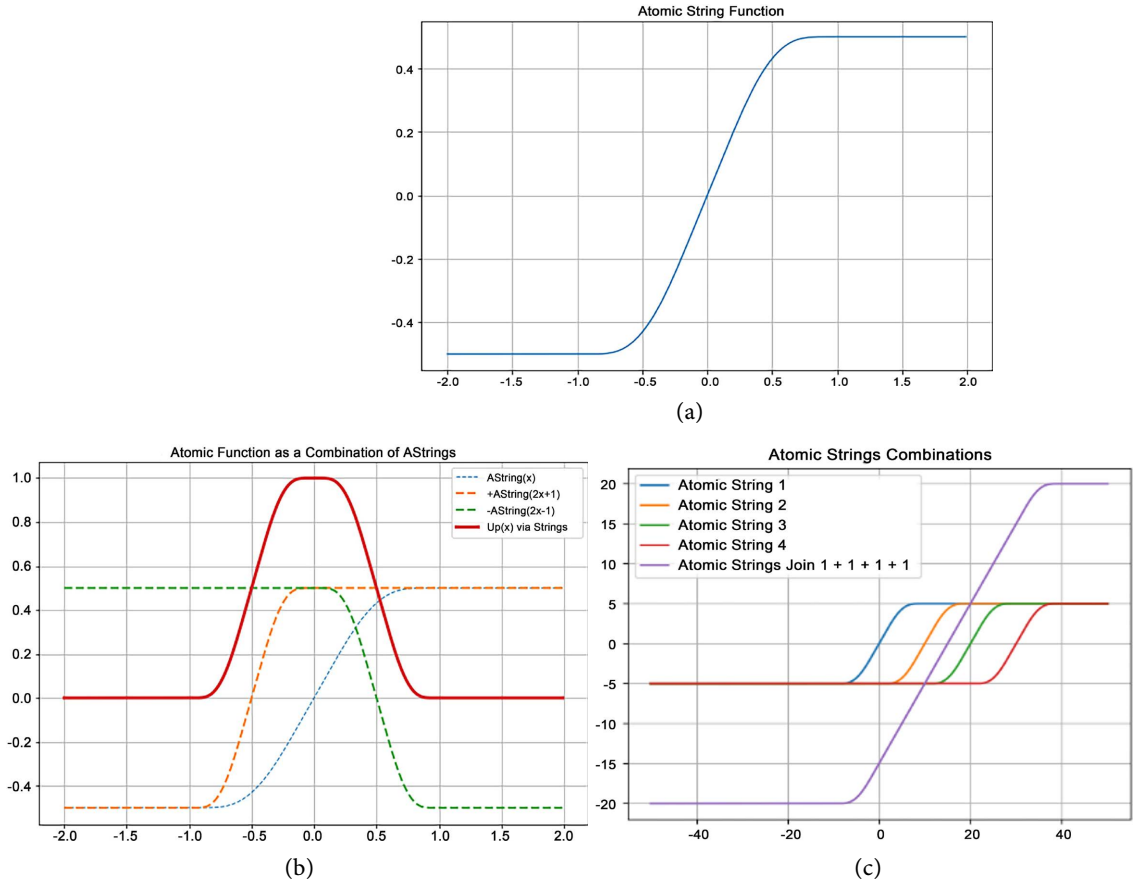
$$up(r)=up\left(\sqrt{x^2+y^2+z^2}\right),\ \iiint cup\left(\frac{x}{a},\frac{y}{a},\frac{z}{a}\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z=ca^3. \tag{4.6}$$

As a normalized finite function, atomic function possesses statistical properties [10]-[14] of being a weighted density distribution of the random variable ξ composed from independent equally distributed variables ξ_k:

$$\xi=\sum_{k=1}^{n}\xi_k 2^{-k},\ \xi_k\in[-1,1];\ up(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{e}^{itx}\prod_{k=1}^{\infty}\frac{\sin\left(t2^{-k}\right)}{t2^{-k}}\mathrm{d}t. \tag{4.7}$$

4.2. AString Function

AString function (**Figure 3**) proposed in 2018 by the author [1]-[9] as a separate function as the integral (4.3) and 'composing branch' of $up(x)$:



(a) (b) (c)

Figure 3. (a) Atomic string function (AString); (b) Atomic function as a combination of two AStrings; (c) Representation of a straight line segment by summing of AStrings.

$$AString'(x) = AString(2x+1) - AString(2x-1) = up(x).$$

$$AString(x) = up(x/2 - 1/2) - 1/2. \tag{4.8}$$

AString has a form of a solitary kink (**Figure 3(a)**), which can compose a straight line $y = x$ as a translation of finite AString kinks (**Figure 3(c)**):

$$x \equiv AString\left(x - \frac{1}{2}\right) + AString\left(x + \frac{1}{2}\right), \ x \in \left[-\frac{1}{2}, \frac{1}{2}\right];$$

$$\begin{aligned} x \equiv \cdots & AString(x-2) + AString(x-1) + AString(x) \\ & + AString(x+1) + AString(x+2) \cdots \end{aligned} \tag{4.9}$$

The shifted and stretched AString kink function can be generalized as

$$AString(x, a, b, c, d = 0) = d + c * AString((x-b)/a). \tag{4.10}$$

Importantly, the Atomic Function pulse (4.6) can be presented as a sum of two opposite AString kinks (**Figure 3(b)**), making AStrings and AFs deeply related to each other:

$$up(x, a, b, c) = AString\left(x, \frac{a}{2}, b - \frac{a}{2}, c\right) + AString\left(x, \frac{a}{2}, b + \frac{a}{2}, -c\right). \tag{4.11}$$

Introduction in 2018 of AString functions capable of composing straight and curved geodesics and fields via superposition of solitonic kinks inspired the spacetime 'atomization'/quantization research [1]-[9] leading to Atomic Spacetime theory published in 2022-2024 [1] [2].

4.3. Atomic Series, Atomic Splines, and "Mathematical Atoms" as Building Blocks of Analytic Functions

Atomic and AString Functions (or briefly *Atomics*) become useful due to their unique approximation properties. Like from 'mathematical atoms' [10]-[14] as founders coined them, analytic functions and smooth surfaces (**Figure 2**, **Figure 3**) can be composed of a superposition of 'atoms' via Atomics via Atomic Series [1]-[18] with an *exact* representation of polynomials of any order

$$\frac{1}{4}\sum_{k=-\infty}^{k=+\infty} kup\left(x - \frac{k}{2}\right) \equiv \sum_{k=-\infty}^{k=+\infty} AString(x-k) \equiv x;$$

$$\sum_{k=-\infty}^{k=+\infty}\left(\frac{k^2}{64} - \frac{1}{36}\right) up\left(x - \frac{k}{4}\right) \equiv x^2,$$

$$\begin{aligned} x^n &\equiv \sum_{k=-\infty}^{k=+\infty} C_k up\left(x - k2^{-n}\right) \\ &= \sum_{k=-\infty}^{k=+\infty} C_k \left(AString\left(2\left(x - k2^{-n}\right) + 1\right) - AString\left(2\left(x - k2^{-n}\right) - 1\right)\right). \end{aligned} \tag{4.12}$$

Importantly for physics, only a limited number of neighboring finite 'atoms' are required to calculate a polynomial value at a given point. It means Atomics can also represent—"atomize"—any analytic function [33] (a function representable by converging Taylor's series) with calculable coefficients:

$$y(x)=\sum_{m=0}^{\infty}\frac{y^{(m)}(0)}{m!}x^m=\sum_{m=0}^{\infty}B_m x^m=\sum_{m=0}^{\infty}\sum_{k=-\infty}^{k=+\infty}B_m C_k up\left(x-k2^{-m}\right)$$
$$=\sum_{mk=-\infty}^{\infty}c_{mk}up\left(\frac{x-b_{mk}}{a_{mk}}\right)=\sum_{l=-\infty}^{l=+\infty}AString\left(x,a_l,b_l,c_l\right). \tag{4.13}$$

Analytic functions not only represent a wide range of polynomial, trigonometric, exponential, hyperbolic, and other functions but also their sums, derivatives, integrals, reciprocals, multiplications, and superpositions [33]. They all can be 'atomized' with any predefined degree of precision, underscoring the most important property of Atomics. Instead of sums (4.12) and (4.13), we will be using short notation with localized basis atomic functions $A_k(x)$ and function values $y^{(k)}$ at node *k*, assuming summation over repeated indices *k*:

$$y(x)=A_k(x)y^{(k)};\quad f(x,y,z)=A_k(x,y,z)f^{(k)}. \tag{4.14}$$

Composing functions from finite pieces can also be achieved with widely-used polynomial splines but with the limitations that n-order splines can exactly reproduce only n-order polynomials (eq cubic parabola cannot be exactly composed of quadratic splines) as well as only with limited smoothness of connections between splines. Atomic Splines are more generic and provide a smooth connection between splines, leading to spacetime and field quantization ideas where Atomics pretend to be the 'building blocks' of fields [1]-[9].

4.4. Atomic Solitons

Being solutions of special kinds of nonlinear differential equations with shifted arguments (4.1), (4.8), AStrings, and Atomic Functions possess some mathematical properties of lattice solitons called Atomic Solitons [4] [6]. AString is a solitonic kink whose particle-like properties exhibit themselves in the composition of a line (4.9) and kink-antikink 'atoms' (4.8) (**Figure 3**). Being a composite object (4.8) made of two AStrings, AF $up(x)$ is not a true mathematical soliton but rather a solitonic atom, like 'bions' or 'dislocation atoms' [4] [6].

5. Atomization Theorems

Unique properties of Atomic and AString Functions allow the formulating of the *Atomization Theorems* [1]-[4], stating how scalar, vector, tensor functions, and solutions of linear and nonlinear differential equations can be represented via a series of Atomics/Atomic Splines/Atomic Solitons leading to field quantization/atomization ideas. The proof of the theorems formulated below is provided in a detailed paper [2].

Theorem 1 (*Polynomial atomization theorem*). Polynomials of any order can be exactly represented/atomized via the Atomic Series of Atomic and AString Functions, with a limited number of pulses at any given point:

$$\left(x^n\right)^{(n)}=c\sum_{k=-\infty}^{k=+\infty}up(x-k)\equiv c,$$
$$x^n=I_n\left(c\sum_{k=-\infty}^{k=+\infty}up(x-k)\right)=\sum_{k=-\infty}^{k=+\infty}C_k up\left(x-k2^{-n}\right). \tag{5.1}$$

$$P_n(x) = x^n + a_1 x^{n-1} + \cdots + a_n \equiv \sum_k C_k up\left(\frac{x-ka}{a}\right) = \sum_k AString\left(x, a_k, b_k, c_k\right) = A_k(x) P_n^{(k)}. \tag{5.2}$$

Based on (4.12) and finiteness, it states that only a few neighboring Atomics are required to calculate a polynomial value at a given point (**Figure 4(a)**).

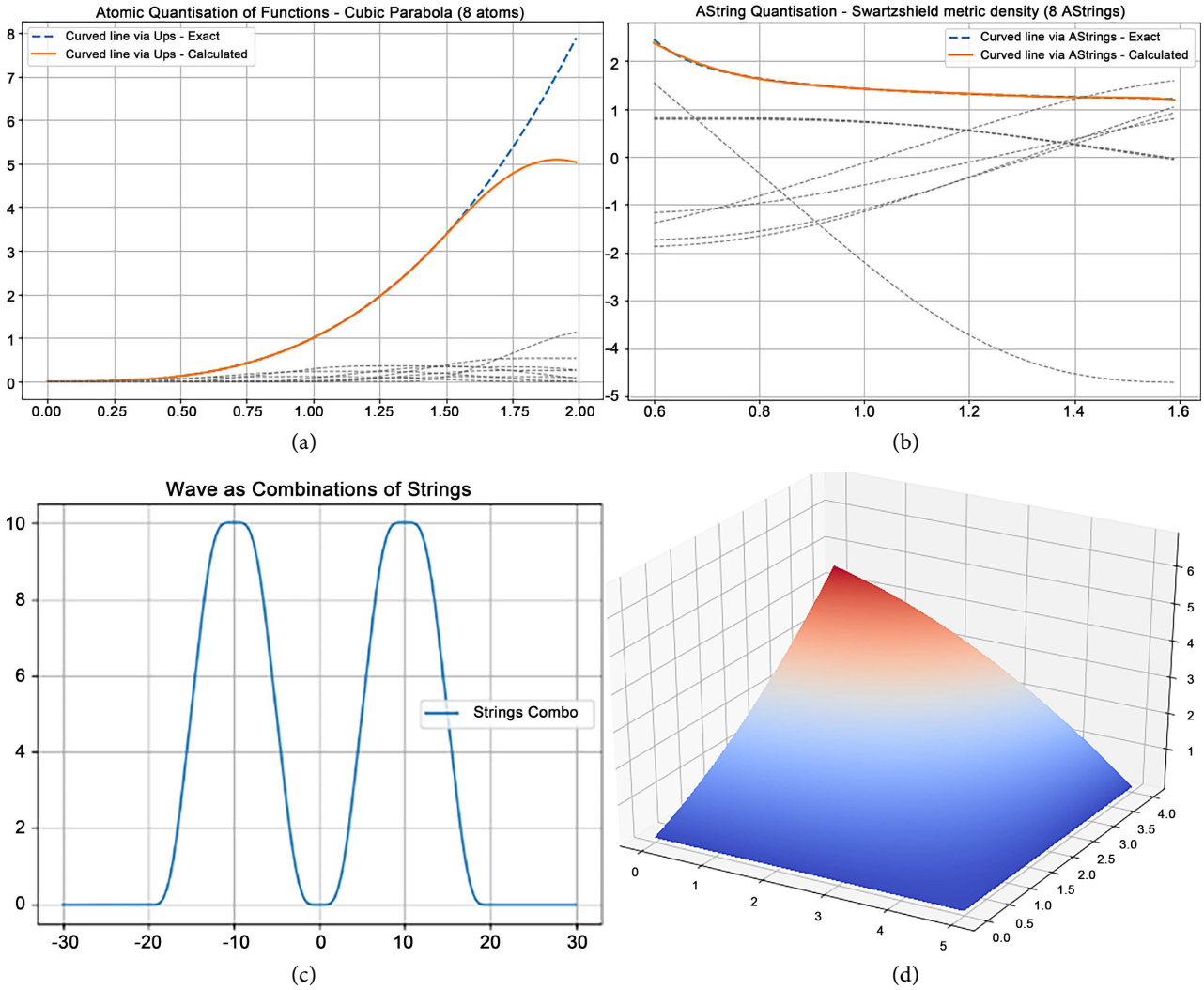


Figure 4. Representing sections of polynomials and analytic functions with AStrings and atomic functions. (a) Cubic parabola via 8 Atomic Functions; (b) Schwarzschild metric function; (c) Wave-like formation; (d) 2d surface.

Theorem 2 (*Analytic atomization theorem*). Analytic functions representable by converging Taylor's series via polynomials can be represented/atomized via converging Atomic Series of localized Atomic and AString Functions:

$$y(x) = \sum_{m=0}^{\infty} \frac{y^{(m)}(0)}{m!} x^m = \sum_{m=0}^{\infty} B_m x^m = \sum_{m=0}^{\infty} \sum_{k=-\infty}^{k=+\infty} B_m C_k up\left(x - k2^{-m}\right) = \sum_{mk=-\infty}^{\infty} c_{mk} up\left(\frac{x-b_{mk}}{a_{mk}}\right) = \sum_{l=-\infty}^{l=+\infty} AString\left(x, a_l, b_l, c_l\right) = A_k(x) y^{(k)}. \tag{5.3}$$

By definition, an analytic function [33] is representable via conversing Taylor's series by polynomials, which in turn can be expressed via Atomics (5.1). It means physical fields described by exponential, trigonometric, hyperbolic, and other analytic functions are also atomizable. This theorem proven in [2] can be generalized to various combinations of analytic functions [1]-[4] [33] with the following.

Theorem 3 (*Complex analytic atomization theorem*). Complex functions $y(x)$ that are sums $y=y_1+y_2$, products $y=y_1y_2$, reciprocals $y=1/y_1$ ($y_1\neq 0$), inverse $y(y_1)=x$, derivatives $y=y_1'$, integrals $I(y)$, and superposition $y=y_1(y_2)$ of analytic functions $y_1(x), y_2(x)$ can be represented/atomized by Atomic Series over Atomic and AString Functions.

Polynomic, trigonometric, exponential, and other analytic functions are the solutions of some linear differential equations (LDE), implying that Atomization Theorems can be extended to differential equations [1]-[8] [10]-[15].

Theorem 4 (*LDE atomization theorem*). Solutions of linear differential equations with constant coefficients can be represented/atomized via series over Atomic and AString Functions:

$$L(y)=y^{(n)}(x)+a_1y^{(n-1)}(x)+\cdots+a_{n-1}y'(x)+a_ny(x)=0;$$
$$y(x)=A_k(x)y^{(k)}. \tag{5.4}$$

This theorem can be generalized [1]-[3] to equations with variable analytic coefficients frequently appearing in mathematical physics. Atomization of composite functions like $\arctan(\exp(x)), \operatorname{sech}(x)$ satisfying nonlinear soliton differential equations implies that the atomization procedure is also applicable to some nonlinear differential equations [1]-[9] [20]-[24], including GR.

Atomization Theorems can be extended to many dimensions, noting that multidimensional *n*-order polynomials in *m*-dimensions $P_{mn}=P_n(x_1,\cdots,x_m)$, which are some multiplications of 1D polynomials exactly representable by Atomics (Theorem 1), are also exactly representable by multiplications of Atomic Functions (multidimensional atomic functions (4.9) $\boldsymbol{UP}(\boldsymbol{a}_k,\boldsymbol{b}_k,\boldsymbol{c}_k)$ which in turn are AStrings combinations (4.8).

Theorem 4 (3*D atomization theorem*). Representable by converging Taylor's series, multidimensional analytic functions with their sums, multiplications, reciprocals, derivatives, integrals, and superpositions can be represented/atomized via Atomic Series over localized multidimensional Atomic and AString Functions:

$$\begin{aligned} P_{mn}&=P_n(x_1,\cdots,x_m)=\prod_{i=1}^m P_n(x_i)=\sum_k\prod_{i=1}^m up_i(x_i,a_{ik},b_{ik},c_{ik})\\ &=\sum_k \boldsymbol{UP}(\boldsymbol{a}_k,\boldsymbol{b}_k,\boldsymbol{c}_k)=\sum_l \boldsymbol{AString}(\boldsymbol{a}_l,\boldsymbol{b}_l,\boldsymbol{c}_l)=A_k(x_1,\cdots,x_m)P_{mn}^{(k)}. \end{aligned} \tag{5.5}$$

Similar to the 1D case, the atomization procedure can be extended to multidimensional differential equations containing linear differential operators like Laplacian and Poisson operators widely used in mathematical physics:

$$L(y_1,\cdots,y_m)(x_1,\cdots,x_m)=a_{ijmn}\frac{\partial^m y_i}{\partial x_j^n}=0;\ \nabla_i=\frac{\partial}{\partial x_i};\ \Delta=\sum\frac{\partial^2}{\partial x_i^2};\ \Delta+k;\ \Delta\Delta.$$

$$y_i\left(x_j\right) \equiv \sum_{ijjkl} up_i\left(x_j, a_{ijl}, b_{ijl}, c_{ijl}\right) = \sum_{jjkl} AString_i\left(x_j, a_{ijk}, b_{ijk}, c_{ijk}\right). \quad (5.6)$$

In summary, formulated Atomization Theorems proven in [1] [2] demonstrate how polynomials, complex multi-dimensional analytical functions, and solutions of linear and nonlinear differential equations can be represented/atomized via superpositions of localized Atomic and AString Functions.

6. Atomization Theorems and Atomic Functions in Quantum Mechanics

With linear Schrödinger operator

$$L_S\left(\psi\left(x_i,t\right)\right) = i\hbar\frac{\partial\psi\left(x_i,t\right)}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial^2\psi\left(x_i,t\right)}{\partial x_i^2}, \quad (6.1)$$

the Schrodinger equation as the main equation of QM for a quantum wave function $\psi\left(x_i,t\right)$ influenced by the potential $V\left(x_i,t\right)$ has the form [20]

$$L_S\left(\psi\left(x_i,t\right)\right) = V\left(x_i,t\right)\psi\left(x_i,t\right). \quad (6.2)$$

Linearity of the Schrödinger operator allows splitting the wave function into spatial and temporal parts:

$$\psi\left(x_i,t\right) = \psi\left(x_i\right)\psi\left(t\right);\ \ \psi\left(x_1,x_2,x_3,t\right) = \psi\left(x_1,x_2,x_3\right)\psi\left(t\right) \quad (6.3)$$

resulting in stationary Schrödinger equation solvable for discrete eigenvalues E, which are the manifestation of the main quantum mechanics idea

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi\left(x_i\right)}{\partial x_i^2} + V\left(x_i\right)\psi\left(x_i\right) = E\psi\left(x_i\right). \quad (6.4)$$

Originally, the Schrödinger Equation (6.2) was obtained [20] based on the hypothesis of L. de Broglie that a 1D wave function can be represented as a harmonic wave matching that k and w to a particle impulse k and energy E

$$\psi\left(x\right) = \mathrm{e}^{-i(kx-\omega t)},\ \ k = p/\hbar,\ \ \omega = E/\hbar;\ \ E = p^2/2m. \quad (6.5)$$

Some controversies of this harmonic representation will be discussed later.

Quantum Mechanical Atomization Theorem. Schrödinger operator L_S preserves polynomiality and analyticity, and being applied to an analytic function yields another analytic function. Polynomial solutions of Schrodinger equations with polynomial potential $V\left(x_i,t\right)$ can be exactly represented/atomized via Atomic Series with a limited number of multidimensional Atomic and AString Functions, while analytic solutions for analytic potential $V\left(x_i,t\right)$ can be represented via converging Atomic Series with any predefined degree of precision.

Proof. For polynomial potential $V\left(x_i,t\right) = V_{kl}x_i^k t^l$, the linear Schrödinger equation can be solved via polynomials. Really, the linear Schrödinger operator L_S contains first, and second derivatives and multiplications which preserve polynomiality and, applied to a polynomial wave function $\psi\left(x_i,t\right) = \psi_{kl}x_i^k t^l$ yields another polynomial $L_S\left(\psi\left(x_i,t\right)\right) = L_{pq}x_i^p t^q$ the coefficients L_{kl} of which can be recurrently chosen to match the right part of the Schrödinger equation having

polynomials on both sides:

$$L_S(\psi) = V\psi;\ L_{pq}x_i^p t^q = V_{kl}x_i^k t^l \psi_{mn}x_i^m t^n. \tag{6.6}$$

So, the wave function is resolved via polynomials $\psi(x_i,t) = \psi_{kl}x_i^k t^l$, which can be exactly represented by Atomic Series via Atomic Functions due to (4.12):

$$x_i^n \equiv C_k up\left(x_i - k2^{-n}\right);\ t^l \equiv D_k up\left(t - k2^{-l}\right). \tag{6.7}$$

Therefore, the polynomial wave function $\psi(x_i,t)$, which can be split into spatial and temporal parts

$$\psi(x_i,t) = \psi(x_i)\psi_t(t) \tag{6.8}$$

can be exactly representable via the superposition of localized finite atomic functions, which are the combination of AStrings:

$$\begin{aligned}\psi(x_i) &\equiv \sum\nolimits_k up(x_i,a_k,b_k,c_k) = \sum\nolimits_n AString(x_i,a_n,b_n,c_n) = A_k(x_i)\psi_i^{(k)}.\\ \psi_t(t) &\equiv \sum\nolimits_k up(t,a_k,b_k,c_k) = \sum\nolimits_n AString(t,a_n,b_n,c_n) = A_k(t)\psi_t^{(k)}.\end{aligned} \tag{6.9}$$

Due to (6.7) the polynomial wave function $\psi(x_1,x_2,x_3,t)$, which can be split into a multiplication of 1D polynomials, would be the multiplication of Atomic Series over the atomic functions exactly representing those polynomials or using multidimensional atomic functions (4.6)

$$\begin{aligned}\psi(x_1,x_2,x_3,x_4=t) &\equiv \prod\nolimits_{i=1}^4 P_n(x_i) = \sum\nolimits_k \prod\nolimits_{i=1}^4 up_i(x_i,a_{ik},b_{ik},c_{ik})\\ &= \sum\nolimits_k \boldsymbol{UP}(x_1,x_2,x_3,t,\boldsymbol{a}_k,\boldsymbol{b}_k,\boldsymbol{c}_k)\\ &= \sum\nolimits_l \boldsymbol{AString}(x_1,x_2,x_3,t,\boldsymbol{a}_l,\boldsymbol{b}_l,\boldsymbol{c}_l)\\ &= A_k(x_1,x_2,x_3,t)\psi^{(k)}.\end{aligned} \tag{6.10}$$

Next, the QM Atomization theorem can be generalized to a wider class of analytic functions representable by definition [33] by converging Taylor series via polynomials exactly atomizable via finite Atomic Functions. Schrödinger operator containing derivatives and multiplications preserves analyticity and, being applied to an analytic function, yields another analytic function representable by converging Taylor series via polynomials, which in turn are representable by Atomics:

$$\begin{aligned}\psi(x_1,x_2,x_3,t) &= \sum\nolimits_k \boldsymbol{UP}(x_1,x_2,x_3,t,\boldsymbol{a}_k,\boldsymbol{b}_k,\boldsymbol{c}_k)\\ &= \sum\nolimits_l \boldsymbol{AString}(x_1,x_2,x_3,t,\boldsymbol{a}_l,\boldsymbol{b}_l,\boldsymbol{c}_l)\\ &= A_k(x_1,x_2,x_3,t)\psi^{(k)}.\end{aligned} \tag{6.11}$$

While polynomial wave functions can be represented exactly, analytic wave functions can be represented by converging atomic series with any predefined degree of precision by increasing the number of atomic pulses on a lattice.

Proof obtained. This theorem can be understood intuitively, noting that for analytic potentials, the linear Schrodinger Equation (6.2) can be resolved via analytic functions representable by polynomials exactly composable by Atomic finite splines. Let's note that this cannot be achieved with widely-used polynomial splines, which, according to Streng-Fix theorems, can exactly reproduce only polynomials of a spline's order, but not higher [1]-[9]. Atomic splines are more universal and can

exactly compose polynomials of any order, so better suitable as 'building blocks' of quantum fields rather than approximating those fields. Let's note that similar atomization theorems are proven in [1] [2] for the spacetime field from General Relativity, opening the prospects of combining QM and GR.

Let's also note that Atomic Series is different from the widely-known Taylor's or Fourier Series operating with continuous functions because they are based on finite atomic functions resembling small 'quanta' or 'atoms' as 'building blocks' of fields, including quantum mechanical fields.

7. Atomic Interpretations of Quantum Mechanics

Formulated Atomization Theorems provide the theoretical foundation for introducing finite Atomic Functions into QM and formulating some interpretations below. They are especially simple yet descriptive in 1D cases where the spatial part $\psi(x)$ of the wave function $\psi(x,t)=\psi(x)\psi_t(x)$ is represented via 1D Atomic Series (6.9), (5.3).

$$\psi(x)=\sum\nolimits_k up(x,a_k,b_k,c_k)=\sum\nolimits_k \psi_k up\left(\frac{x-b_k}{a_k}\right);\ c_k=\psi_k. \tag{7.1}$$

7.1. Quantum Wave Function as a Superposition of Atomic Wave Functions

Let us analyze in more detail the atomic representation

$$\psi(x)=\sum\nolimits_k \psi_k up\left(\frac{x-b_k}{a_k}\right). \tag{7.2}$$

First, the Quantum Wave Function is representable/atomizable via superposition/sum of solitary Atomic Function pulses $\psi up\left((x-b)/a\right)$ with intensity/height ψ, width a shifted to location $x=b$. Having the same unit of measurement as the wave function and the shape of the Atomic Function $up(x)$ (**Figure 1**), we can introduce an *Atomic Wave Function* $\psi up\left((x-b)/a\right)$ stating that the Quantum Wave Function $\psi(x)$ is a superposition (7.2) of Atomic Wave Functions (AWF). AWFs are infinitely smooth, non-analytical, finite/localized, and capable of overlapping with a few neighbors to compose polynomials and analytical wave functions as the solutions of Schrödinger Equations (6.2).

Unlike continuous polynomial and trigonometric functions, which are the base of the Taylor and Fourier series capable of composing wave functions from waves and swings, Atomic Wave Functions are localized/finite and can, like in the Lego game, compose wave functions (**Figure 5**, **Figure 6**) from "building blocks", or "mathematical atoms", as atomic functions were sometimes called in the 1970s. Let's note it is impossible to derive similar finite infinitely smooth objects from other pulse-like functions like polynomial splines, Gaussian functions or the Fourier series, which are widely used in QM. Polynomial splines of n-order cannot exactly represent polynomials of order higher than n. Fourier series over Gaussian functions are used in QM to represent wave packets but, with a limited number of harmonics, cannot represent even a simple polynomial (a constant or a line). So,

atomic series over finite functions $\psi up\left(\left(x-b\right)/a\right)$ are unique and have certain theoretical significance in interpretations discussed hereafter.

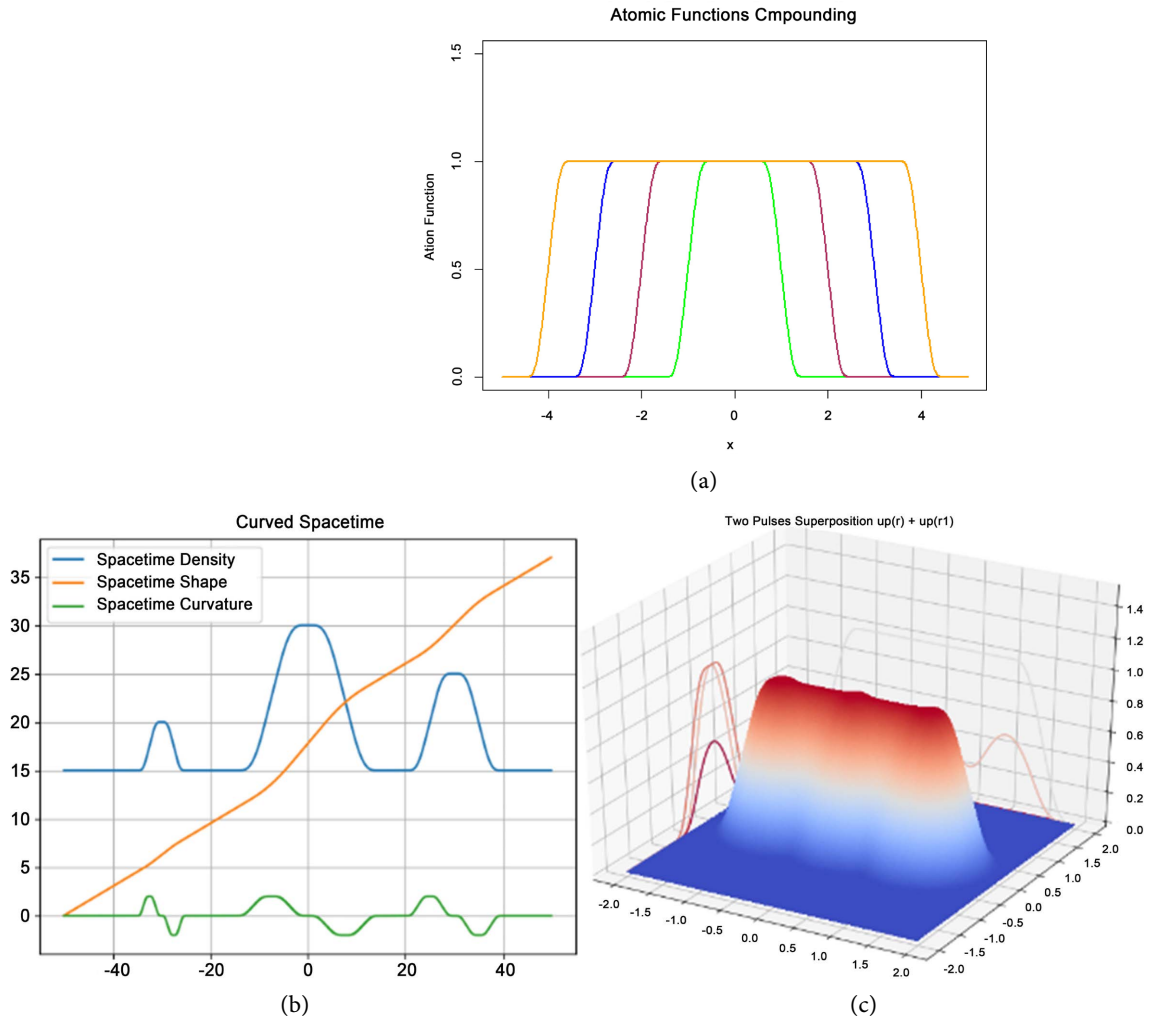
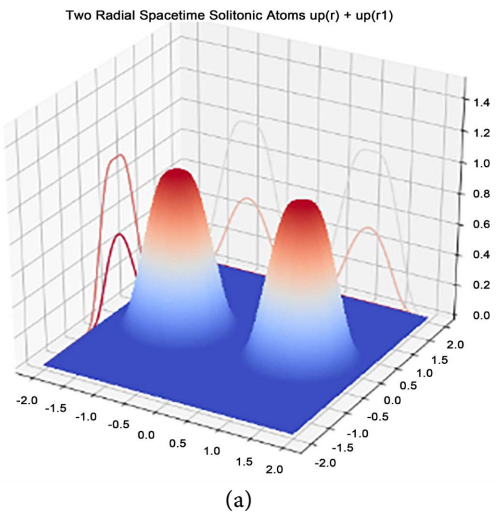


Figure 5. Representation of uniform and variable wave functions via atomic functions.



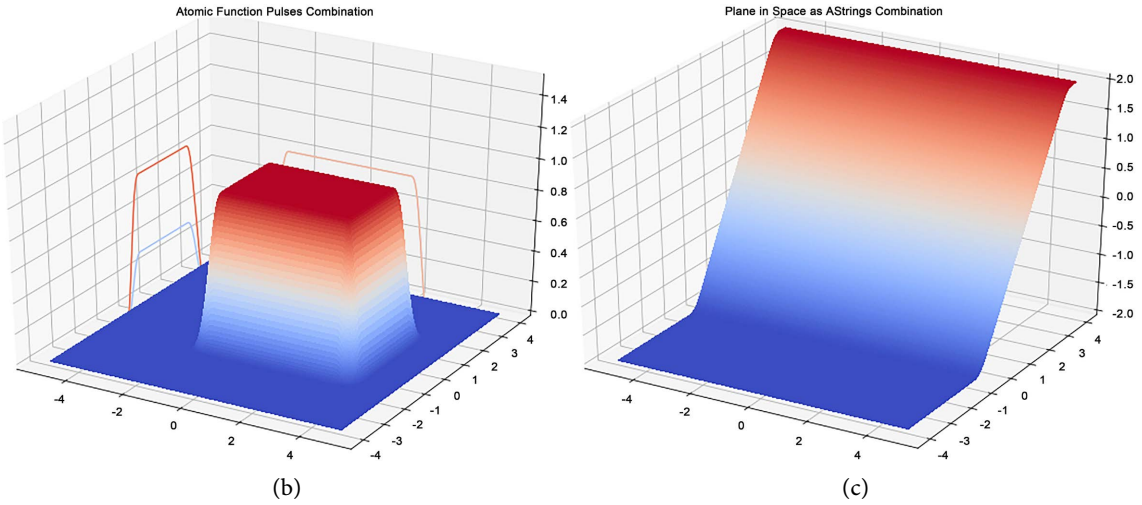


Figure 6. (a) Two localized wave function pulses; (b) Constant function as a superposition of a few atomic pulses; (c) Composing a growing field from atomic wave functions.

Let's note that it is possible to represent quantum wave functions via more complex atomic functions [10]-[19] discussed in [1] [2] [33] as was done in [33]. Basically, for every QM state function and the type of Schrodinger Equation (6.2), it is possible to invent its own complex atomic function. However, this method hides the universality of the most important and simplest atomic function $up(x)$, also noting that more complex atomic functions can be ultimately expressed via the 'main' function $up(x)$ using convolutions [10]-[15], making them somewhat secondary.

7.2. Atomization is not a Quantum Mechanical Quantization

Let's note that atomization (7.2) has a different meaning from 'standard' QM quantization of representing a wave function via superposition of QM states (eigenvalue modes).

$$\psi_x(x) \equiv \sum_k d_k \psi_{xk}(x). \tag{7.3}$$

Here, $\psi_{xk}(x)$ are the continuous functions related to some discrete energy levels while $up(x)$ are localized finite functions related to the special smooth discretization of space where the wave functions are defined upon. Moreover, analytic QM state functions $\psi_{xk}(x)$ themselves can be represented via Atomic Wave Functions $\psi up\left((x-b)/a\right)$.

7.3. Wave Function Collapse and Localized Atomic Wave Functions

Wave function collapse [20] is a typical QM concept when the QM Wave Function 'collapses' into a point during the observation or interaction with the matter, like an electron hitting the screen in the double-slit experiment [20]. Typically, it is described by using delta-functions (called Dirac Delta Functions) [20]

$$\delta(x-x') = \infty,\ x = x',\ \ \delta(x-x') = 0,\ x = x';$$

$$\int_{-\infty}^{\infty} \delta(x-x')\psi(x') = \psi(x);\ \int_{-\infty}^{\infty} \delta(x) = 1. \tag{7.4}$$

which are often [20] approximated by a Gaussian

$$\delta(x,a)\approx\frac{1}{a\sqrt{\pi}}\exp\left(-\left(\frac{x}{a}\right)^2\right),\ \int_{-\infty}^{\infty}\delta(x,a)=1. \tag{7.5}$$

Interestingly, instead of a continuous Gaussian function, it is possible to offer a Localized Atomic Wave Function $cup(x/a)$ on the small interval $[-a,a]$ with typical normalization (4.2), (4.6):

$$\delta(x,a)=\frac{1}{a}up\left(\frac{x}{a}\right);\ \int_{-\infty}^{\infty}up(x,a)=1. \tag{7.6}$$

This way not only offers a consistent approach to composing QM functions with one set of Atomic Functions but also avoids using a concept of a collapsing to a 'point' in space in favor of a more generic concept of a finite region of space, like in string theory.

Similarly, the concept of Localized Atomic Wave Functions can be generalized towards well-known QM models of many particles [20] when the combined wave function of a system $\psi_x(x)$ splits into many functions ψ_{xl} associated with individual particles, as shown in (**Figure 5**, **Figure 6**).

$$\psi_x(x)\equiv\sum_{kl}\psi_{xl}(x,a_k,b_k,c_k); \tag{7.7}$$

$$\psi_{xl}(x,a_k,b_k,c_k)=up(x,a_k,b_k,c_k)=c_k up\left(\frac{x-b_k}{a_k}\right). \tag{7.8}$$

7.4. Gaussian vs Atomic Function Distributions

Gaussian function $e^{-x^2/2}$ is often appears in QM to describe the normal probability distribution of finding a particle at a given location, approximations of Dirac delta functions (7.5), and 'ground states' for QM Harmonic Oscillator [20]. Interestingly, Gaussian, being an analytical function, can not only be represented via Atomic Functions by Atomization Theorems (5.2)

$$g(x)=e^{-x^2/2}=\sum_{k=0}^{\infty}\frac{2^{-k}\left(-x^2\right)^k}{k!}=\sum_n up(x,a_n,b_n,c_n); \tag{7.9}$$

but be replaced by an Atomic Function, which also has a probabilistic interpretation (4.7) and is capable of describing the localization of wave functions (7.2)

$$\psi(x)=\sum_k\psi_k up\left(\frac{x-b_k}{a_k}\right). \tag{7.10}$$

It seems more natural to describe localized objects with localized finite atomic functions as opposed to continuous Gaussian functions spreading to infinity. So, normal Gaussian distributions, known as density functions of random variables, $\xi=\sum_{k=1}^{n}\xi_k$ can be interpreted as a superposition of more granular weighted atomic statistical distributions $\xi=\sum_{k=1}^{n}\xi_k 2^{-k}$ (4.7).

7.5. Heisenberg Uncertainly Principle and Atomic Functions

Introducing Atomic Functions in QM also allows incorporation of the core of

Heisenberg's Uncertainly Principle [20], which states that the uncertainty Δx (measured as the width of a probability pulse) in the position of a particle multiplied by the uncertainty Δp in its momentum is higher than half of a reduced Planck's constant:

$$\Delta x \Delta p \geq \frac{\hbar}{2}. \tag{7.11}$$

Analyzing the origins of this relation [20] allows us to conclude that it is related to the Fourier transformation for localized quantum wave packets where the same Gaussian-based function $\psi(p)$ is used to represent a wave packet in both position and momentum space [20]

$$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(p) \mathrm{e}^{-ipx/\hbar} \mathrm{d}p. \tag{7.12}$$

Interestingly, replacing the Gaussian wave packet with the Atomic Function packet described later does not violate the procedure, especially taking into consideration that uncertainties Δx and Δp can be easily associated with the widths of atomic pulses $\psi up\left((x-b)/a\right)$ upholding Heisenberg's principle.

7.6. Quantum Particle in a Box and Wave Function Boundary Problem

Particle-in-a-box is another well-known QM model [20] describing the probabilistic nature of a particle, a free particle moving between impenetrable walls $x \in [-l, l]$ which can be modeled by infinite potential $V(x) = \infty$, $|x| \geq l$; $V(x) = 0$, $|x| < l$ in Schrödinger Equations (7.1). Solutions shown in **Figure 7** have the simple trigonometric form:

$$\psi_n(x,t) = A\sin\left(k_n(x-l)\right)\mathrm{e}^{-i\omega_n t}, |x| < l;\ \psi_n(x,t) = 0, |x| \geq l \tag{7.13}$$

$$k_n = 2n\pi/l;\ \ E_n = \hbar\omega_n = \frac{n^2\pi^2\hbar^2}{2m(2l)^2}. \tag{7.14}$$

where k_n is the quantized discrete wavenumber, E_n is quantized energy levels, and n is a positive integer $(1, 2, 3, \cdots)$.

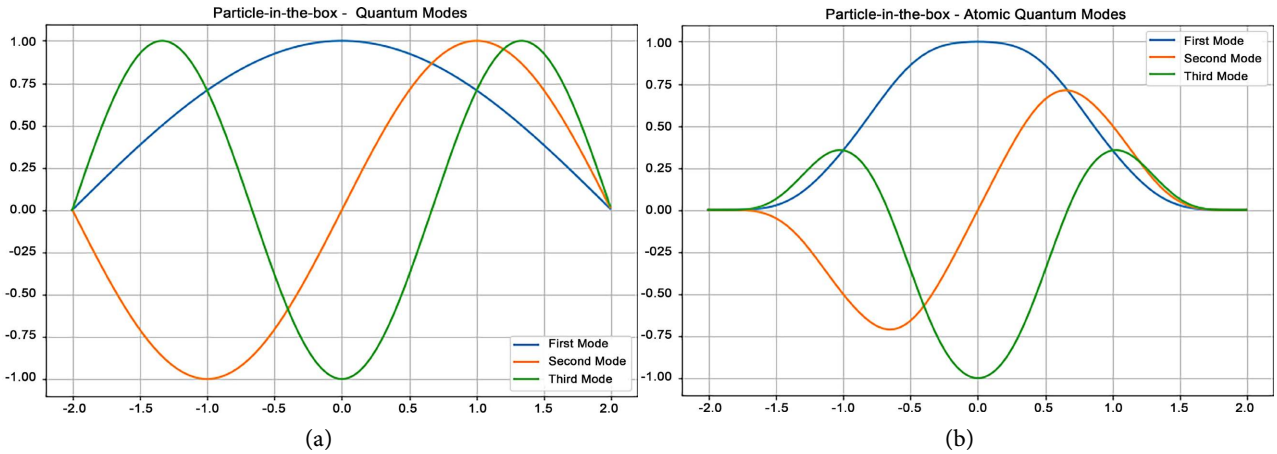


Figure 7. Wave function modes for a particle in a box: (a) Harmonic modes; (b) Atomic function modes.

Despite upholding the quantization idea, this simple model creates a well-known boundary problem related to the fact that derivatives of waveforms (7.13) $\psi_n'(x,t)$ would not be equal to 0 at $x=\pm l$, hence creating a discontinuity problem described in [35] as follows. "*Usually in quantum mechanics, it is also demanded that the derivative of the wavefunction in addition to the wavefunction itself be continuous… The wavefunction is not a differentiable function at the boundary of the box, and thus it can be said that the wavefunction does not solve the Schrödinger equation at the boundary points* (*but does solve everywhere else*)." The root cause of this problem is related to the very structure and limitations of the linear Schrödinger Equation (6.1), which from the very beginning enforces the hypothesis that a wave function has a trigonometric form [20]

$$\psi(x)=\mathrm{e}^{-i(kx-\omega t)}. \tag{7.15}$$

for which it is impossible to demand that both $A\sin\left(k_n(x-l)\right)$ from (7.13) and its cos derivative be zero on the boundary.

This boundary QM problem can be resolved by seeking a wave function solution with the factor as an atomic function (or generally another function), which vanishes at the box boundary along with its derivatives:

$$\psi(x)=up(x/l)\psi_1(x);\ up(x/l)=0,\ up'(x/l)=0,\ x=\pm l. \tag{7.16}$$

Applying this modulation over 'standard' sinusoidal modes (7.13) would produce a composite solution similar to a wave packet shown in **Figure 7(b)**, which resolves the abovementioned boundary problem [35].

7.7. Atomization of Quantum Harmonic Oscillator

One of the most important concepts of Quantum Mechanics is the Quantum Harmonic Oscillator (HO), described by L. Susskind [20] as a "…second basic ingredient of Quantum Mechanics" (after qubit). The classical HO model [20] describes an oscillation $y(t)$ of a mass particle on a spring with linear restoring force yielding a simple harmonic solution

$$y''-k^2y=0;\ y=A\sin(kt+c). \tag{7.17}$$

Quantum HO described by Schrödinger Equation (6.4) with quadratic potential [20] [36]

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+V(x)\psi(x)=E\psi(x),\ V(x)=1/2\,\omega^2x^2 \tag{7.18}$$

leads to quantized eigenvalue solutions for energy and eigenmodes expressible via Gaussians and Hermite polynomials $H_n(x)$ [20] [36]:

$$E_n=\hbar\omega\left(n+\frac{1}{2}\right);\ \psi_n(x)=A_nH_n(x)\mathrm{e}^{-B_nx^2}. \tag{7.19}$$

Both classical and quantum HO models yield analytic functions which, according to the QM atomization theorem (§6), can be represented/atomized via the superposition of finite atomic functions (6.9), (7.1):

$$\psi_n(x) \equiv \sum_k up(x, a_k, b_k, c_k) = A_{nk}(x)\psi_n^{(k)}. \quad (7.20)$$

It leads to the atomized interpretation of harmonic oscillators distributed in spacetime as an overlapping superposition of solitary atomic wave functions in line with generic atomic QM interpretation (§7.2). Due to the statistical properties of atomic functions, quantum probabilistic distributions can be represented as a superposition of atomic probabilistic distributions (4.7).

Generalization of quantum harmonic oscillator toward the chain of many rigid balls connected by elastic springs can be used according to [20] as a foundation model for Quantum Fields Theories [20]-[30] and other fields. While being universal, this model has some pitfalls. Rigid balls/nodes imply linear displacements between balls, which is mathematically equivalent to approximating a continuous function with linear splines:

$$\psi(x) = N_1(x)\psi(x_1) + N_2(x)\psi(x_2) + N_3(x)\psi(x_3);\ N_i(x) = a_i x + b_i. \quad (7.21)$$

For smooth fields, this creates an unphysical discontinuity in deformations of springs between nodes $\varepsilon_{12} = (u_2 - u_1)/a \neq \varepsilon_{23} = (u_3 - u_2)/a$. The problem can be resolved by using Atomic AString Functions splines $AN_i(x)$, which not only provide smooth connections between HO nodes but also represent via Atomization Theorems (§5) a polynomial of any order and solutions of differential equations describing quantum fields:

$$\begin{aligned}\psi(x) &\equiv \sum_k up(x, a_k, b_k, c_k) = \sum_n AString(x, a_n, b_n, c_n)\\ &= AN_1(x)\psi(x_1) + AN_2(x)\psi(x_2) + AN_3(x)\psi(x_3) + \cdots;\end{aligned} \quad (7.22)$$

This Atomic Oscillator model suggests the improvement of fundamental HO models by replacing 'rigid balls' with interacting quanta/atoms, with more physical preservation of smoothness with Atomic Wave Functions, as shown in **Figures 1-7**.

7.8. Limitations of Schrödinger Equations and Use of Atomic Functions

The abovementioned particle-in-a-box model and harmonic oscillator discussion highlight the following limitations of the 'standard' linear Schrödinger Equation (6.2) and Quantum Mechanical models based on it.

1) The Schrödinger equation was derived with the assumption (6.5) (hypothesis originally introduced by L. de Broglie in the 1920s [20] with his wave-particle duality), that wave function is a continuous infinite harmonic wave function (6.5),(7.15) $\psi(x) = e^{-i(kx-\omega t)}$ which locks the mathematical model in the domain of trigonometric functions which as demonstrated (§7.6) was quite limited to satisfy some complex boundary conditions involving both the function and its derivatives on a boundary. Meanwhile, using representation $\psi(x) = f(kx - \omega t)$ for a more generic periodic function would yield another Schrödinger-like equation that may be more suitable for resolving boundary problems.

2) Wave function in the original form $\psi(x) = e^{-i(kx-\omega t)}$ are infinitely-defined

($-\infty < x < +\infty$), while quantum systems with discrete eigenvalues are typically confined. It sometimes creates hidden contradictions (not only related to QM) in describing finite physical objects with infinite functions—the problem also noted by A. Einstein [1] [4] [21]. Finite Atomic Functions are capable of both composing any analytic function and satisfying complex boundary conditions, so they can be useful in many physical theories dealing with finite systems, including Quantum Mechanics.

3) There is a linearity problem—with the originating function $\psi(x) = e^{-i(kx-\omega t)}$ satisfying linear differential equation for trigonometric functions, the Schrödinger Equation (6.1), (6.2) becomes linear, with problems to describe typically nonlinear wave packets like Gaussian (7.12) and atomic packets (7.16) shown in **Figure 7**. To resolve this, Schrödinger himself introduced the nonlinear Schrödinger equations yielding wave packet solutions with secant envelope functions [36], which seems more adequate description of nonlinear particle localization than standard linear waves suitable mostly for infinite continua. Atomic packets (7.16) are also nonlinear, with the benefits of consistent usage of the same atomic function $up(x)$ for linear QM wave functions atomization (7.2) as well as for nonlinear envelope packets (7.16).

4) In QM, wave functions as the solution of the linear eigenvalue problem for quantized energy levels (6.4) are represented by the superstition of linear vibrating modes like in Fourier Series (7.3), (7.12), which has some well-known problems to represent polynomials and even a simple constant with a limited number of harmonics. The problem can be resolved by using Atomic Functions, which, with Atomisation Theorems (§5), can compose constants, polynomials, and harmonic functions too.

5) The linearity of the Schrödinger Equation (6.1) with originating function (6.5) $\psi(x) = e^{-i(kx-\omega t)}$ becomes the well-known obstacle to resolving the fundamental problem of quantum gravity [22]-[28]—to combine nonlinear Einstein's General Relativity where spacetime is curved with linear Schrödinger equations where spacetime is flat [22]. This mutual incompatibility of equations can be resolved by reformulating QM equations in curved space like it is done in [22] and other papers [23]-[28] and finding common mathematical terms linking two theories as it is discussed in [1] [2] [22].

Let's note that the above-listed problems do not disregard the merits of Quantum Mechanics as one of the most successful and testable physical theories. It rather highlights the possibility of the theory enhancements based on field finite atomization and novel interpretations based on finite Atomic Functions, which can be used for both Quantum Mechanics and General Relativity [1]-[9].

8. Summary and Future Research Directions

This paper describes how the Atomic Functions theory [1]-[19] started after pioneering works in the 1970s [10]-[18] and evolved since 2017 towards major physical theories [1]-[9] can be introduced into Quantum Mechanics to represent/atomize

a wave function via the superposition of localized finite smooth spline-like atomic pulses often called 'mathematical atoms'. It offers the novel 'atomic interpretation' of QM where wave functions are the superpositions of localized Atomic Wave Functions, which can also describe collapsed wave functions, represent widely-used Gaussians, uphold Heisenberg's uncertainly principle and quantum harmonic oscillator concepts. Also, Atomic Functions can help solve the wave function discontinuity boundary problem for a typical particle-in-a-box and other QM models by seeking a solution as an atomic wave packet vanishing at the boundary along with its derivatives. These kinds of boundary problems highlight some limitations of the QM Schrödinger equation originating from infinite harmonic trigonometric representation of a wave function, which may be not flexible enough to satisfy complex boundary conditions for typically confined QM systems. Finding a more generic nonlinear Schrödinger equation yielding atomic wave packets is an interesting future research direction.

Introducing Atomic Functions in QM has another theoretical significance as it follows more generic research started in 2017 on Atomic Spacetime and fields theories [1]-[9] where Atomic AString Functions capable of composing smooth fields from 'finite objects' resembling quanta/metriants/solitonic atoms/elementary statistical distributions are suggested to describe some fundamental 'building blocks' of fields similar to loops and strings from string and loop quantum gravity theories [23]-[29]. It offers a unified mathematical framework for the 'theories of everything', including Quantum Gravity [22]-[28], with ongoing research in this important direction.

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The theory described here expands the 50 years of history of Atomic Functions pioneered in Ukraine by the author's teacher, Academician NAS of Ukraine V.L. Rvachev (Ukrainian V. L. Rvachov) (1926-2005) and Professor V. A. Rvachov from the National Aerospace University 'Kharkiv Aviation Institute', and this paper is dedicated to those remarkable scientists of Ukraine.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Eremenko, S.Y. (2024) Theory and Properties of Atomic Spacetime. *Journal of Applied Mathematics and Physics*, **12**, 1967-2006. https://doi.org/10.4236/jamp.2024.126121

[2] Eremenko, S.Y. (2023) Atomization Theorems in Mathematical Physics and General Relativity. *Journal of Applied Mathematics and Physics*, **11**, 158-191. https://doi.org/10.4236/jamp.2023.111012

[3] Eremenko, S.Y. (2022) Atomic Spacetime Model Based on Atomic AString Functions. *Journal of Applied Mathematics and Physics*, **10**, 2604-2631. https://doi.org/10.4236/jamp.2022.109176

[4] Eremenko, S.Yu. (2018) Atomic Solitons as a New Class of Solitons. *Journal Nonlinear World*, **16**, 39-63. https://www.researchgate.net/publication/329455498.

[5] Eremenko, S.Yu. (2018) Atomic Strings and Fabric of Spacetime. *Achievements of Modern Radioelectronics*, No. 6, 45-61. https://www.researchgate.net/publication/320457739

[6] Eremenko, S.Yu. (2020) Soliton Nature: Discover Beautiful Nature with 200 Images and Video Channel. Book Writing Inc. https://www.amazon.com/dp/1951630777

[7] Eremenko, S.Yu. (2022) AString Functions in Theoretical Physics. *International Conference on Atomic and R-Functions* (*ICAR*), *Virtual Workshop*, Split, 1-14. https://www.researchgate.net/publication/358264308

[8] Eremenko, S.Yu. (2021) Atomic String Functions and Spacetime Quantization. *Proceedings of* 5*th International Conference on Multi-Scale Computational Methods for Solids and Fluids*, Split, 30 June-2 July 2021, 112-121. https://www.researchgate.net/publication/352878876

[9] Eremenko, S.Yu. (1992) Natural Vibrations and Dynamics of Composite Materials and Constructions. Naukova Dumka.

[10] Rvachev, V.L. and Rvachev, V.A. (1971) About one Finite Function. *DAN URSR A*, No. 6, 705-707.

[11] Rvachev, V.L. (1982) Theory of R-Functions and Their Applications. Naukova Dumka.

[12] Rvachev, V.L. and Rvachev, V.A. (1982) Non-Classical Methods in the Theory of Approximations in Boundary Value Problems. Naukova Dumka.

[13] Kravchenko, V.F. and Rvachev, V.A. (1996) Application of Atomic Functions for Solutions of Boundary Value Problems in Mathematical Physics. *Foreign Radioelectronics*: *Achievements of Modern Radioelectronics*, **8**, 6-22.

[14] Kravchenko, V.F., Kravchenko, O.V., Pustovoit, V.I., Churikov, D.V., Volosyuk, V.K. and Pavlikov, V.V. (2016) Atomic Functions Theory: 45 Years behind. 2016 9*th International Kharkiv Symposium on Physics and Engineering of Microwaves*, *Millimeter and Submillimeter Waves* (*MSMW*), Kharkiv, 20-24 June 2016, 1-4. https://doi.org/10.1109/msmw.2016.7538216

[15] Gotovac, B. and Kozulic, V. (1999) On a Selection of Basis Functions in Numerical Analyses of Engineering Problems. *International Journal for Engineering Modelling*, **12**, 25-41.

[16] Kamber, G., Gotovac, H., Kozulić, V., Malenica, L. and Gotovac, B. (2020) Adaptive Numerical Modeling Using the Hierarchical FUP Basis Functions and Control Volume Isogeometric Analysis. *International Journal for Numerical Methods in Fluids*, **92**, 1437-1461. https://doi.org/10.1002/fld.4830

[17] Eremenko, S.Yu. (1991) Finite Element Methods in Solid Mechanics. Osnova. https://www.researchgate.net/publication/321171685 https://books.google.com.au/books?id=dyTLDwAAQBAJ

[18] Eremenko, S.Yu. (2022) Novel Model of Atomic Spacetime Based on Atomic AString Metriant Functions. *International Journal of Innovative Research in Sciences and Engineering Studies*, **2**, 1-15.

[19] Eremenko, S.Yu. (2022) Atomic Function and AString Function (Python Code).

[20] Susskind, L. and Friedman, A. (2014) Quantum Mechanics: The Theoretical Minimum. Basic Books.

[21] Einstein, A. (1933) On the Method of Theoretical Physics. Oxford University Press. https://openlibrary.org/books/OL6292654M

[22] Eremenko, S.Y. (2023) Quantum Gravity Based on Generalized Thermodynamics. *Journal of Applied Mathematics and Physics*, **11**, 988-1028. https://doi.org/10.4236/jamp.2023.114066

[23] Rovelli, C. and Smolin, L. (1988) Knot Theory and Quantum Gravity. *Physical Review Letters*, **61**, 1155-1158. https://doi.org/10.1103/physrevlett.61.1155

[24] Rovelli, C. and Vidotto, F. (2014). Covariant Loop Quantum Gravity. Cambridge University Press. https://doi.org/10.1017/cbo9781107706910

[25] Rovelli, C. (2007) Quantum Gravity. In: Butterfield, J. and Earman, J., Eds., *Philosophy of Physics*, Elsevier, 1287-1329. https://doi.org/10.1016/b978-044451560-5/50015-4

[26] Smolin, L. (2000) The Cubic Matrix Model and a Duality between Strings and Loops.

[27] Penrose, R. (1967) Twistor Algebra. *Journal of Mathematical Physics*, **8**, 345-366. https://doi.org/10.1063/1.1705200

[28] Wald, R.M. (1994) Quantum Field Theory on Curved Spacetime and Black Hole Thermodynamics. University of Chicago Press.

[29] Hartle, J.B. and Hawking, S.W. (1983) Wave Function of the Universe. *Physical Review D*, **28**, 2960-2975. https://doi.org/10.1103/physrevd.28.2960

[30] Hartle, J. (1995) Spacetime Quantum Mechanics and the Quantum Mechanics of Spacetime. In: Julia, B. and Zinn-Justin, J., Eds., *Proceedings on the* 1992 *Les Houches School*, *Gravitation and Quantisation*, Elsevier Science, 285.

[31] Fabius Function. https://en.wikipedia.org/wiki/Fabius_function

[32] Analytic Function. https://en.wikipedia.org/wiki/Analytic_function

[33] Kravchenko, V.F., Kravchenko, O.V. and Safin, A.R. (2010) A New Construction of Solution of Schrodinger Equation on the Basis of Atomic Functions. 2010 *International Kharkov Symposium on Physics and Engineering of Microwaves*, *Millimeter and Submillimeter Waves*, Kharkiv, 21-26 June 2010, 1-3. https://doi.org/10.1109/msmw.2010.5546001

[34] Particle in a Box. https://en.wikipedia.org/wiki/Particle_in_a_box

[35] Nonlinear Schrödinger Equation. https://en.wikipedia.org/wiki/Nonlinear_Schr%C3%B6dinger_equation

[36] Quantum Harmonic Oscillator. https://en.wikipedia.org/wiki/Quantum_harmonic_oscillator