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Relativistic Mechanics in Positive and Negative Subspace-Time according to the Inverse Relativity Model

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Abstract

In the second paper on the inverse relativity model, we explained in the first paper [1] that analyzing the four-dimensional displacement vector on space-time according to a certain approach leads to the splitting of space-time into positive and negative subspace-time. Here, in the second paper, we continue to analyze each of the four-dimensional vectors of velocity, acceleration, momentum, and forces on the total space-time fabric. According to the approach followed in the first paper. As a result, in the special case, we obtain new transformations for each of the velocity, acceleration, momentum, energy, and forces specific to each subspace-time, which are subject to the positive and negative modified Lorentz transformations described in the first paper. According to these transformations, momentum remains a conserved quantity in the positive subspace and increases in the negative subspace, while the relativistic total energy decreases in the positive subspace and increases in the negative subspace. In the general case, we also have new types of energy-momentum tensor, one for positive subspace-time and the other for negative subspace-time, where the energy density decreases in positive subspace-time and increases in negative subspace-time, and we also obtain new gravitational field equations for each subspace-time.

Keywords

4D Velocity Vector Analysis, Positive Subspace, Negative Subspace, Negative Relativistic Mechanics, Positive Tensor of Energy and Momentum, Inverse Theory of Relativity

1. Introduction

The idea of the inverse relativity model that was proposed in the first paper depends on dividing the total space-time in the special and general cases into a positive subspace-time and a negative subspace-time, Where the first paper included transformations of space and time coordinates for each subspace-time in the special case and the metric tensors for each subspace-time in the general case. Accordingly, we will try here in the second paper to describe each of the velocity, acceleration, momentum, energy, and relativistic forces in each of the positive and negative subspace times. In other words, we will try to formulate relativistic mechanics according to the new model (Inverse Relativity). As we know, the special theory of relativity introduced new types and concepts of energy and matter, such as the energy of a rest mass [2], which explained to us that matter is a form of energy and that a particle at rest contains energy. And there is also the relativistic total energy [3], which represents the sum of the relativistic kinetic energy of the particle and the energy of the rest mass. Not only that, but special relativity provided us with new mechanics, known as relativistic mechanics [4] page 81, which differs from classical mechanics in the number of dimensions of space. Where velocity, acceleration, momentum, energy, and forces are described in four-dimensional space [5] or in Minkowski space-time instead of the three-dimensional space followed in classical mechanics, this is because the previous quantities in Minkowski space-time are invariant under the Lorentz transformation, or in other words, they do not depend on the frame of reference. Special relativity also changed the mathematical formulas for the laws of mechanics in three-dimensional space. We now have new formulas for the law of relativistic kinetic energy, relativistic momentum, and relativistic forces [6] (pp. 43-44, 46-48). As a result of the change in mass at relativistic speed (speed close to the speed of light). Will the new model maintain the same previous concepts of energy and matter? Will we get the same mathematical formulas for the laws of relativistic mechanics? Or will we get new mathematical formulas as well as new relativistic mechanics? Specific to each subspace-time as a result of the geometric properties of that subspace-time, which were previously mentioned in the first paper. The second paper is also subject to the same analysis approach followed in the first paper, where each of the four-dimensional velocity, acceleration, momentum, and force vectors is analyzed on the total space-time fabric in the special and general cases, but according to the analysis conditions of the new model explained in the following points.

Analyzing the four-dimensional velocity vector, acceleration, momentum, or force on the space-time fabric is into two four-dimensional vectors by analyzing the components of this vector. For example, analyzing the four-dimensional velocity vector into two four-dimensional vectors is done by analyzing the component of this vector, which is the four-dimensional displacement vector, into two four-dimensional displacement vectors as in the first paper and representing the new four-dimensional velocity vectors in the four-dimensional subspaces shown in the first paper.

The analysis of the four-dimensional velocity, acceleration, momentum, or force vector is with respect to one of the observers or with respect to one of the reference frames and not in both together. Therefore, the transformation of each new four-dimensional vector from the reference frame (the frame containing the analysis) to another reference frame (a frame without analysis) is not subject to Lorentz transformations or the Lorentz transformation matrix, but will be subject to the positive and negative modified Lorentz transformations and their transformation matrices, which are also explained in the first paper.

After describing the velocity vectors, acceleration, momentum, and forces in each subspace-time, we can here perform further analysis of the four-dimensional vectors. But this time, with respect to both reference frames or to both observers in each subspace-time, to vectors of lower dimensions, three-dimensional or one-dimensional. It is the same approach used in special relativity [7], but with the positive and negative modified Lorentz transforms of the inverse relativity model. The purpose of this type of analysis is to study mechanical phenomena on the fabric of positive and negative subspace-time, such as the motion of the particle in the positive and negative 3D subspace, and also the motion of the particle in the time dimension of the positive and negative subspace-time. The approach of analysis and splitting space into sub-spaces is an approach followed in many theories, but with different analysis conditions specified by the author of the paper [8] [9]. As we explained, it is an approach followed in special relativity, where three-dimensional space is viewed as a sub-space of spacetime. But the problem is that the subspaces are of a mathematical nature, while at the physical level they are somewhat ambiguous [10]. Our model features a mathematical, geometric, and physical description of each subspace-time in the special and general case, as shown in the first paper, with drawings also for clarification. As we will explain here in the second paper, how the physical quantities are closely related to each subspace-time and the consistency of the model physically and geometrically.

2. Methods

2.1. Analysis of the Four-Dimensional Velocity Vector in Minkowski's Total Space-Time

If we have a particle of rest mass moving on the total space-time (Minkowski space-time), then if we want to describe the 4D velocity vector of this particle on the total space-time with respect to two observers O' and O belonging to the inertial reference frames S' and S, respectively [11], According to special relativity, it is equal to the 4D differential displacement vector with respect to each reference frame divided by the proper time [12] (which is the time with respect to the particle or with respect to a reference frame specific to the particle, an imaginary frame). Look at **Figure 1**.

$$S` \to x`\,y`\,z` \qquad S \to x\;y\;z \tag{1.1}$$

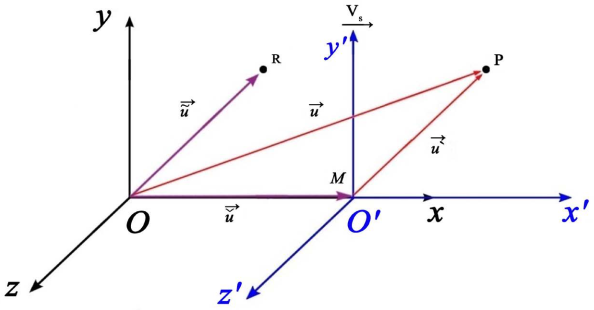

Figure 1. Shows the transformation of an instantaneous 3D velocity vector from one reference frame to another, with the analysis of the velocity vector in frame S into two 3D velocity vectors.

Therefore, the 4D velocity vector of the particle on the total space-time with respect to the observer O', is equal to the 4D differential displacement vector $\mathrm{d}A^{`\mu}$ with respect to the reference frame S' divided by the infinitesimal proper time $\mathrm{d}\tau$, and the 4D velocity vector is written in the following form [13] (p. 180).

$$u^{`\mu}=\frac{\mathrm{d}A^{`\mu}}{\mathrm{d}\tau}=\left(\frac{\mathrm{d}A^{`1}}{\mathrm{d}\tau},\frac{\mathrm{d}A^{`2}}{\mathrm{d}\tau},\frac{\mathrm{d}A^{`3}}{\mathrm{d}\tau},\frac{\mathrm{d}A^{`4}}{\mathrm{d}\tau}\right) \tag{1.2}$$

Because the proper time $\mathrm{d}\tau$ is the time relative to the particle, the transformation of time from the particle's reference frame to the reference frame S' is according to the following equation $\mathrm{d}t^{`}=\mathrm{d}\tau\gamma^{`}_{u}$, where $\gamma^{`}_{u}$ here depends on the speed of the particle $u^{`}$ for the reference frame S', that is, it is equal to $\gamma^{`}_{u}=1\Big/\sqrt{1-\frac{u^{`2}}{c^{2}}}$, By replacing the components of the 4D displacement vector in terms of the coordinates of the reference frame S' according to the notation shown. In the first paper, item 2.1, by also substituting the proper time in terms of time with respect to the reference frame S', we obtain the values of the components of the 4D velocity vector $u^{`\mu}$ ([14], pp. 116-118).

$$\begin{aligned}
u^{`1}&=\frac{\mathrm{d}A^{`1}}{\mathrm{d}\tau}=\gamma^{`}_{u}\frac{\mathrm{d}x^{`}}{\mathrm{d}t^{`}}=\gamma^{`}_{u}u^{`}_{x}\\
u^{`2}&=\frac{\mathrm{d}A^{`2}}{\mathrm{d}\tau}=\gamma^{`}_{u}\frac{\mathrm{d}y^{`}}{\mathrm{d}t^{`}}=\gamma^{`}_{u}u^{`}_{y}\\
u^{`3}&=\frac{\mathrm{d}A^{`3}}{\mathrm{d}\tau}=\gamma^{`}_{u}\frac{\mathrm{d}z^{`}}{\mathrm{d}t^{`}}=\gamma^{`}_{u}u^{`}_{z}\\
u^{`4}&=\frac{\mathrm{d}A^{`4}}{\mathrm{d}\tau}=\gamma^{`}_{u}\frac{c\mathrm{d}t^{`}}{\mathrm{d}t^{`}}=\gamma^{`}_{u}c
\end{aligned} \tag{2.2}$$

Therefore, we can write the 4D velocity vector $u^{`\mu}$ in the following form

$$u^{`\mu}=\gamma^{`}_{u}\left(u^{`}_{x},u^{`}_{y},u^{`}_{z},c\right)=\gamma^{`}_{u}\left(\vec{u}^{`},c\right) \tag{3.2}$$

As for the 4D velocity vector of the particle on the total space-time with respect to the observer O, it is equal to the 4D differential displacement vector $\mathrm{d}A^{\nu}$ with respect to the reference frame S divided by the infinitesimal proper time $\mathrm{d}\tau$, and it is written in the following formula

$$u^{\nu}=\frac{\mathrm{d}A^{\nu}}{\mathrm{d}\tau}=\left(\frac{\mathrm{d}A^{1}}{\mathrm{d}\tau},\frac{\mathrm{d}A^{2}}{\mathrm{d}\tau},\frac{\mathrm{d}A^{3}}{\mathrm{d}\tau},\frac{\mathrm{d}A^{4}}{\mathrm{d}\tau}\right) \tag{4.2}$$

The proper time transformation $\mathrm{d}\tau$ here is from the reference frame of the particle with respect to the reference frame S, and therefore it is equal to $\mathrm{d}t=\mathrm{d}\tau\gamma_{u}$ here γ_{u} depends on the speed of the particle u with respect to the reference frame S, and therefore, it is equal to $\gamma_{u}=1\Big/\sqrt{1-\frac{u^{2}}{c^{2}}}$, by substituting here also the components of the 4D displacement vector in terms of the coordinates of the reference frame S according to the notation described in the first paper, item 2.1, and by substituting the proper time in terms of the time of the reference frame S, we obtain the values of the components of the 4D velocity vector u^{ν}

$$\begin{aligned}
u^{1}&=\frac{\mathrm{d}A^{1}}{\mathrm{d}\tau}=\gamma_{u}\frac{\mathrm{d}x}{\mathrm{d}t}=\gamma_{u}u_{x}\\
u^{2}&=\frac{\mathrm{d}A^{2}}{\mathrm{d}\tau}=\gamma_{u}\frac{\mathrm{d}y}{\mathrm{d}t}=\gamma_{u}u_{y}\\
u^{3}&=\frac{\mathrm{d}A^{3}}{\mathrm{d}\tau}=\gamma_{u}\frac{\mathrm{d}z}{\mathrm{d}t}=\gamma_{u}u_{z}\\
u^{4}&=\frac{\mathrm{d}A^{4}}{\mathrm{d}\tau}=\gamma_{u}\frac{c\mathrm{d}t}{\mathrm{d}t}=\gamma_{u}c
\end{aligned} \tag{5.2}$$

Here, we can also write the 4D velocity vector u^{ν} in the following form

$$u^{\nu}=\gamma_{u}\left(u_{x},u_{y},u_{z},c\right)=\gamma_{u}\left(\vec{u},c\right) \tag{6.2}$$

Also, according to special relativity, the transformation from the vector u^{ν} to the vector u'^{μ} on Minkowski's total space-time is through the Lorentz matrix ([14], p. 113), described in the first paper, item 2.1, and as a result of the analysis of 4D displacement vectors on the total space-time fabric. And representing it in new four-dimensional spaces as well (positive subspace-time-negative subpace-time) described in the first paper, item 2.2, we also obtain an analysis of the 4D velocity vectors u^{ν} or u'^{μ} on the Minkowski space-time fabric, where analysis of the displacement vector $\mathrm{d}A^{\nu}$ into two vectors $\mathrm{d}A^{\nu}=\mathrm{d}\tilde{A}^{\epsilon}+\mathrm{d}\breve{A}^{\sigma}$, leads to analysis the velocity vector u^{ν} into two vectors according to the following equation.

$$u^{\nu}=\frac{\mathrm{d}A^{\nu}}{\mathrm{d}\tau}=\frac{\mathrm{d}\tilde{A}^{\epsilon}}{\mathrm{d}\tau}+\frac{\mathrm{d}\breve{A}^{\sigma}}{\mathrm{d}\tau} \tag{7.2}$$

2.2. The Four-Dimensional Velocity Vector in Positive Subspace-Time

The first expression from the right side of Equation 7.2 represents the 4D velocity vector of the particle $\tilde{u}^{\epsilon}$ in positive subspace-time, where $\mathrm{d}\tilde{A}^{\epsilon}/\mathrm{d}\tau=\tilde{u}^{\epsilon}$, and the 4D velocity vector $\tilde{u}^{\epsilon}$ is written in the following form

$$\tilde{u}^{\epsilon}=\frac{\mathrm{d}\tilde{A}^{\epsilon}}{\mathrm{d}\tau}=\left(\frac{\mathrm{d}\tilde{A}^{1}}{\mathrm{d}\tau},\frac{\mathrm{d}\tilde{A}^{2}}{\mathrm{d}\tau},\frac{\mathrm{d}\tilde{A}^{3}}{\mathrm{d}\tau},\frac{\mathrm{d}\tilde{A}^{4}}{\mathrm{d}\tau}\right) \tag{8.2}$$

By substituting the components of the 4D differential displacement vector $\mathrm{d}\tilde{A}^{\epsilon}$ in terms of the coordinates of the reference frame S according to the notation described in the first paper, item 2.3, and also by substituting the proper time in terms of the resultant vector time $\mathrm{d}t$ with respect to the reference frame S, and by replacing the time $\mathrm{d}t$ with the time $\mathrm{d}\tilde{t}$ because they are in the same reference frame and for the same event, we obtain here the values of the components of the 4D velocity vector $\tilde{u}^{\epsilon}$.

$$\begin{aligned}
\tilde{u}^1 &= \frac{\mathrm{d}\tilde{A}^1}{\mathrm{d}\tau} = \gamma_u \frac{\mathrm{d}\tilde{x}}{\mathrm{d}\tilde{t}} = \gamma_u \tilde{u}_x \\
\tilde{u}^2 &= \frac{\mathrm{d}\tilde{A}^2}{\mathrm{d}\tau} = \gamma_u \frac{\mathrm{d}\tilde{y}}{\mathrm{d}\tilde{t}} = \gamma_u \tilde{u}_y \\
\tilde{u}^3 &= \frac{\mathrm{d}\tilde{A}^3}{\mathrm{d}\tau} = \gamma_u \frac{\mathrm{d}\tilde{z}}{\mathrm{d}\tilde{t}} = \gamma_u \tilde{u}_z \\
\tilde{u}^4 &= \frac{\mathrm{d}\tilde{A}^4}{\mathrm{d}\tau} = \gamma_u \frac{\mathrm{d}\left(\tilde{V}\tilde{t}\right)}{\mathrm{d}\tilde{t}} = \gamma_u \tilde{V}
\end{aligned} \tag{9.2}$$

We can write the 4D velocity vector $\tilde{u}^{\epsilon}$ in the following form

$$\tilde{u}^{\epsilon} = \gamma_u \left(\tilde{u}_x, \tilde{u}_y, \tilde{u}_z, \tilde{V}\right) = \gamma_u \left(\vec{\tilde{u}}, \tilde{V}\right) \tag{10.2}$$

where $\tilde{u}$ represents the velocity of the particle in the positive subspace (3D subspace) with respect to the observer O, $\tilde{V}$ represents the speed of the particle in the time dimension of the positive subspace-time, by differentiating the 4D displacement vector transformation equation in positive space-time with respect to the proper time, Equation (17.1) described in the first paper, item 2.3, we obtain the transformation from the vector u'^{μ} to the vector $\tilde{u}^{\epsilon}$ in the positive subspace-time.

$$\frac{\mathrm{d}\tilde{A}^{\epsilon}}{\mathrm{d}\tau} = \tilde{\Lambda}^{\epsilon}_{\mu} \frac{\mathrm{d}A'^{\mu}}{\mathrm{d}\tau} \tag{11.2}$$

$$\tilde{u}^{\epsilon} = \tilde{\Lambda}^{\epsilon}_{\mu} u'^{\mu} \tag{12.2}$$

$$\begin{bmatrix} \tilde{u}^1 \\ \tilde{u}^2 \\ \tilde{u}^3 \\ \tilde{u}^4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u'^1 \\ u'^2 \\ u'^3 \\ u'^4 \end{bmatrix} \tag{13.2}$$

From the previous transformation equation, we obtain the transformations of the 4D velocity components in the positive subspace-time from the reference frame S' to the frame S, or from the observer O' to the observer O, and by substituting the value of each component shown in the set of Equations (2.2) and (9.2), we obtain the components transformations 4D velocity in reference coordinate form

$$\begin{aligned}
\tilde{u}^1 = u'^1 &\quad \Rightarrow \quad \tilde{u}_x \gamma_u = u'_x \gamma'_u \\
\tilde{u}^2 = u'^2 &\quad \Rightarrow \quad \tilde{u}_y \gamma_u = u'_y \gamma'_u \\
\tilde{u}^3 = u'^3 &\quad \Rightarrow \quad \tilde{u}_z \gamma_u = u'_z \gamma'_u \\
\tilde{u}^4 = u'^4 &\quad \Rightarrow \quad \tilde{V} \gamma_u = c \gamma'_u
\end{aligned} \tag{14.2}$$

By dividing the velocity transformation equation in the fourth dimension (the dimension of time) by the velocity transformation equations in the first, second, and third dimensions, then by substituting the value of $\tilde{V}$ from Equation (23.1) shown in the first paper, we obtain the transformations of the 3D velocity components in the positive subspace of the reference frame S' to S or from the observer O' to the observer O

$$\begin{aligned} &\frac{\tilde{u}_x\gamma_u}{\tilde{V}\gamma_u} = \frac{u`_x\gamma`_u}{c\gamma`_u} \Rightarrow \frac{\tilde{u}_x}{\tilde{V}} = \frac{u`_x}{c} \Rightarrow \tilde{u}_x = u`_x\sqrt{1-\frac{V_S^2}{c^2}} \\ &\frac{\tilde{u}_y\gamma_u}{\tilde{V}\gamma_u} = \frac{u`_y\gamma`_u}{c\gamma`_u} \Rightarrow \frac{\tilde{u}_y}{\tilde{V}} = \frac{u`_y}{c} \Rightarrow \tilde{u}_y = u`_y\sqrt{1-\frac{V_S^2}{c^2}} \\ &\frac{\tilde{u}_z\gamma_u}{\tilde{V}\gamma_u} = \frac{u`_z\gamma`_u}{c\gamma`_u} \Rightarrow \frac{\tilde{u}_z}{\tilde{V}} = \frac{u`_z}{c} \Rightarrow \tilde{u}_z = u`_z\sqrt{1-\frac{V_S^2}{c^2}} \end{aligned} \tag{15.2}$$

2.3. The Four-Dimensional Acceleration Vector in Positive Subspace-Time

By differentiating the 4D velocity vector $u^{`\mu}$ with respect to proper time again, we obtain the 4D acceleration vector $a^{`\mu}$ on the total space-time fabric with respect to the observer O' where $\mathrm{d}u^{`\mu}/\mathrm{d}\tau = a^{`\mu}$ [15]. By substituting for each acceleration component in terms of the component of the 4D velocity and time of the reference frame S', then by substituting the value of each 4D velocity component with the formula of the reference coordinates shown in the set of Equation (2.2), and by making the differentiation with respect to time $\mathrm{d}t`$, assuming that we are studying the acceleration of the particle at an instant when the speed of the particle is equal to the speed of the reference frame S', that is when $u`=0$ where $\mathrm{d}\gamma`_u = 1$ and also $\mathrm{d}\gamma`_u/\mathrm{d}t` = 0$, we finally obtain the values of the acceleration components of the vector $a^{`\mu}$ shown in the following set ([16], p. 141).

$$\begin{aligned} &a^{`1} = \gamma`_u\frac{\mathrm{d}}{\mathrm{d}t`}u^{`1} \Rightarrow a^{`1} = a`_x\gamma`^2_u + \frac{\mathrm{d}\gamma`_u}{\mathrm{d}t`}u`_x\gamma`_u \Rightarrow a^{`1} = a`_x\gamma`^2_u \\ &a^{`2} = \gamma`_u\frac{\mathrm{d}}{\mathrm{d}t`}u^{`2} \Rightarrow a^{`2} = a`_y\gamma`^2_u + \frac{\mathrm{d}\gamma`_u}{\mathrm{d}t`}u`_y\gamma`_u \Rightarrow a^{`2} = a`_y\gamma`^2_u \\ &a^{`3} = \gamma`_u\frac{\mathrm{d}}{\mathrm{d}t`}u^{`3} \Rightarrow a^{`3} = a`_z\gamma`^2_u + \frac{\mathrm{d}\gamma`_u}{\mathrm{d}t`}u`_z\gamma`_u \Rightarrow a^{`3} = a`_z\gamma`^2_u \\ &a^{`4} = \gamma`_u\frac{\mathrm{d}}{\mathrm{d}t`}u^{`4} \Rightarrow a^{`4} = \frac{\mathrm{d}\gamma`_u}{\mathrm{d}t`}c\gamma`_u \Rightarrow a^{`4} = 0 \end{aligned} \tag{16.2}$$

Therefore, write the 4D acceleration vector on the total space-time in the form of the following reference coordinates

$$a^{`\mu} = \gamma`^2_u\left(a`_x, a`_y, a`_z, 0\right) = \gamma`^2_u\left(\vec{a`}, 0\right) \tag{17.2}$$

By differentiating the 4D velocity vector $\tilde{u}^\epsilon$ also with respect to the proper time again, we obtain the 4D acceleration vector $\tilde{a}^\epsilon$ on the positive subspace-time with respect to the observer O where $\mathrm{d}\tilde{u}^\epsilon/\mathrm{d}\tau = \tilde{a}^\epsilon$, By following the same previous steps, replace each acceleration component in terms of the 4D velocity component, and replace the proper time in terms of the resultant time $\mathrm{d}t$ for the reference frame S, then the time $\mathrm{d}t$ is replaced by the time $\mathrm{d}\tilde{t}$ because they are for the same event and in the same frame of reference, then the value of each 4D velocity component in the form of reference coordinates shown in the set of Equation (9.2), and by making the differentiation with respect to time $\mathrm{d}\tilde{t}$ at an instant when the speed of the particle is equal to the speed of the reference frame S', that is, when $u = V_S$, where $\gamma_u = \gamma = cons$ because the reference frame in the special case does not accelerate. Therefore, $\mathrm{d}\gamma_u / \mathrm{d}\tilde{t} = 0$, we ultimately obtain the values of the acceleration components of the vector $\tilde{a}^{\epsilon}$ shown in the following set.

$$
\begin{aligned}
\tilde{a}^1 &= \gamma_u \frac{\mathrm{d}}{\mathrm{d}\tilde{t}}\tilde{u}^1 &&\Rightarrow\quad \tilde{a}^1 = \tilde{a}_x\gamma_u^2 + \frac{\mathrm{d}\gamma_u}{\mathrm{d}\tilde{t}}\tilde{u}_x\gamma_u &&\Rightarrow\quad \tilde{a}^1 = \tilde{a}_x\gamma_u^2 \\
\tilde{a}^2 &= \gamma_u \frac{\mathrm{d}}{\mathrm{d}\tilde{t}}\tilde{u}^2 &&\Rightarrow\quad \tilde{a}^2 = \tilde{a}_y\gamma_u^2 + \frac{\mathrm{d}\gamma_u}{\mathrm{d}\tilde{t}}\tilde{u}_y\gamma_u &&\Rightarrow\quad \tilde{a}^2 = \tilde{a}_y\gamma_u^2 \\
\tilde{a}^3 &= \gamma_u \frac{\mathrm{d}}{\mathrm{d}\tilde{t}}\tilde{u}^3 &&\Rightarrow\quad \tilde{a}^3 = \tilde{a}_z\gamma_u^2 + \frac{\mathrm{d}\gamma_u}{\mathrm{d}\tilde{t}}\tilde{u}_z\gamma_u &&\Rightarrow\quad \tilde{a}^3 = \tilde{a}_z\gamma_u^2 \\
\tilde{a}^4 &= \gamma_u \frac{\mathrm{d}}{\mathrm{d}\tilde{t}}\tilde{u}^4 &&\Rightarrow\quad \tilde{a}^4 = \frac{\mathrm{d}\gamma_u}{\mathrm{d}\tilde{t}}\tilde{V}\gamma_u &&\Rightarrow\quad \tilde{a}^4 = 0
\end{aligned}
\tag{18.2}
$$

We write the 4D acceleration vector in positive subspace-time in the form of the following reference coordinates

$$
\tilde{a}^{\epsilon} = \gamma_u^2\left(\tilde{a}_x, \tilde{a}_y, \tilde{a}_z, 0\right) = \gamma_u^2\left(\vec{\tilde{a}}, 0\right) \tag{19.2}
$$

As for the transformation from the acceleration vector $a^{`\mu}$ to the acceleration vector $\tilde{a}^{\epsilon}$ in the positive subspace-time, it is through differentiation of the 4D velocity vector transformation equation in the positive subspace-time, Equation (12.2) with respect to the proper time.

$$
\frac{\mathrm{d}\tilde{u}^{\epsilon}}{\mathrm{d}\tau} = \tilde{\Lambda}^{\epsilon}_{\mu}\frac{\mathrm{d}u^{`\mu}}{\mathrm{d}\tau} \tag{20.2}
$$

$$
\tilde{a}^{\epsilon} = \tilde{\Lambda}^{\epsilon}_{\mu}a^{`\mu} \tag{21.2}
$$

From the previous transformation equation, we obtain the transformation of the components of the 4D acceleration vector in the positive subspace-time from the reference frame S' to the frame S or from the observer O' to the observer O, where $\tilde{\Lambda}^{\epsilon}_{\mu}$ is a unit matrix, and by substituting the values of each component shown in the set of Equations (16.2) and (18.2), we obtain transformations of the values of the components of the 4D acceleration in the form of reference coordinates

$$
\begin{aligned}
\tilde{a}^1 &= a^{`1} &&\Rightarrow\quad \tilde{a}_x\gamma_u^2 = a`_x\gamma`^2_u \\
\tilde{a}^2 &= a^{`2} &&\Rightarrow\quad \tilde{a}_y\gamma_u^2 = a`_y\gamma`^2_u \\
\tilde{a}^3 &= a^{`3} &&\Rightarrow\quad \tilde{a}_z\gamma_u^2 = a`_z\gamma`^2_u \\
\tilde{a}^4 &= a^{`4} &&\Rightarrow\quad 0 = 0
\end{aligned}
\tag{22.2}
$$

By dividing the square of the velocity transformation equation in the time dimension (the fourth dimension equation in set (14.2)) by the 4D acceleration transformation equations in the first, second, and third dimensions. Then, by substituting the value of $\tilde{V}$ from Equation (23.1) shown in the first paper, we obtain the transformations of the 3D acceleration components in the positive subspace from the reference frame S’ to the frame S

$$\begin{aligned}
\frac{\tilde{a}_x\gamma_u^2}{\tilde{V}^2\gamma_u^2} &= \frac{a`_x\gamma`_u^2}{c^2\gamma`_u^2} \Rightarrow \frac{\tilde{a}_x}{\tilde{V}^2} = \frac{a`_x}{c^2} \Rightarrow \tilde{a}_x = a`_x\left(1-\frac{V_S^2}{c^2}\right)\\
\frac{\tilde{a}_y\gamma_u^2}{\tilde{V}^2\gamma_u^2} &= \frac{a`_y\gamma`_u^2}{c^2\gamma`_u^2} \Rightarrow \frac{\tilde{a}_y}{\tilde{V}^2} = \frac{a`_y}{c^2} \Rightarrow \tilde{a}_y = a`_y\left(1-\frac{V_S^2}{c^2}\right)\\
\frac{\tilde{a}_z\gamma_u^2}{\tilde{V}^2\gamma_u^2} &= \frac{a`_z\gamma`_u^2}{c^2\gamma`_u^2} \Rightarrow \frac{\tilde{a}_z}{\tilde{V}^2} = \frac{a`_z}{c^2} \Rightarrow \tilde{a}_z = a`_z\left(1-\frac{V_S^2}{c^2}\right)
\end{aligned} \tag{23.2}$$

2.4. The Four-Dimensional Momentum Vector in Positive Subspace-Time

By multiplying the 4D velocity vector $u`^{\mu}$ by the rest mass of the particle $m_0$, we obtain the 4D momentum vector $p`^{\mu}$ on the total space-time fabric with respect to the observer O’ where $m_0 u`^{\mu} = p`^{\mu}$ ([13], p. 182). By substituting each component of momentum in terms of the component of the 4D velocity and the rest mass, and then by replacing the value of each component of the 4D velocity with the formula of the reference coordinates shown in set of Equations (2.2), we obtain the values of the components of momentum for the vector $p`^{\mu}$ shown in the following set ([14], p. 118).

$$\begin{aligned}
p`^1 &= m_0 u`^1 = m`u`_x\\
p`^2 &= m_0 u`^2 = m`u`_y\\
p`^3 &= m_0 u`^3 = m`u`_z\\
p`^4 &= m_0 u`^4 = \frac{m`c^2}{c}
\end{aligned} \tag{24.2}$$

where the expression $m`c^2 = E`$ represents the relativistic total energy of the particle [17] with respect to the observer O’, so we write the vector $p`^{\mu}$ in the following formula

$$p`^{\mu} = \left(\vec{p}`, \frac{E`}{c}\right) \tag{25.2}$$

By also multiplying the 4D velocity vector $\tilde{u}^{\epsilon}$ by the rest mass, we obtain the 4D momentum vector $\tilde{p}^{\epsilon}$ on the positive subspace-time with respect to the observer O where $m_0\tilde{u}^{\epsilon} = \tilde{p}^{\epsilon}$. By following the same previous steps, we obtain the values of the momentum components $\tilde{p}^{\epsilon}$ shown in the following set

$$\begin{aligned}
\tilde{p}^1 &= m_0\tilde{u}^1 = m\tilde{u}_x\\
\tilde{p}^2 &= m_0\tilde{u}^2 = m\tilde{u}_y\\
\tilde{p}^3 &= m_0\tilde{u}^3 = m\tilde{u}_z\\
\tilde{p}^4 &= m_0\tilde{u}^4 = \frac{m\tilde{V}^2}{\tilde{V}}
\end{aligned} \tag{26.2}$$

where the expression $m\tilde{V}^2 = \tilde{E}$ also represents the relativistic energy of the particle with respect to the observer O. As we mentioned in the first paper item 2.3, the speed of light in positive subspace-time is not a universal constant, meaning that $\tilde{V}$ is the value of the speed of light in positive subspace-time, and therefore it is written the vector $\tilde{p}^{\epsilon}$ also has the following formula

$$\tilde{p}^{\epsilon} = \left(\vec{\tilde{p}}, \frac{\tilde{E}}{\tilde{V}}\right) \tag{27.2}$$

As for the transformation from the momentum vector $p^{`\mu}$ to the momentum vector $\tilde{p}^{\epsilon}$ in positive subspace-time, it is through multiplying both sides of the 4D velocity vector transformation equation in positive subspace-time, Equation (12.2) in the rest mass.

$$m_0\tilde{u}^{\epsilon} = \tilde{\Lambda}^{\epsilon}_{\mu} m_0 u^{`\mu} \tag{28.2}$$

$$\tilde{p}^{\epsilon} = \tilde{\Lambda}^{\epsilon}_{\mu} p^{`\mu} \tag{29.2}$$

From the previous transformation equation, we obtain the transformation of the 4D vector components of momentum in positive subspace-time from the reference frame S' to the frame S or from the observer O' to the observer O.

$$\begin{aligned} \tilde{p}^1 &= p^{`1} \\ \tilde{p}^2 &= p^{`2} \\ \tilde{p}^3 &= p^{`3} \\ \tilde{p}^4 &= p^{`4} \end{aligned} \tag{30.2}$$

By substituting the value of each component of the 4D momentum shown in the set of equations (24.2) and (26.2), we obtain the transformation of the 4D vector components of momentum and energy from the reference frame S' to the frame S or from the observer O' to the observer O, in terms of reference coordinates.

$$\begin{aligned} m\tilde{u}_x &= m`u`_x && \Rightarrow && \tilde{p}_x = p`_x \\ m\tilde{u}_y &= m`u`_y && \Rightarrow && \tilde{p}_y = p`_y \\ m\tilde{u}_z &= m`u`_z && \Rightarrow && \tilde{p}_z = p`_z \\ \frac{m\tilde{V}^2}{\tilde{V}} &= \frac{m`c^2}{c} && \Rightarrow && \frac{\tilde{E}}{\tilde{V}} = \frac{E`}{c} \end{aligned} \tag{31.2}$$

The first three equations represent the transformation of the components of positive relativistic momentum or the components of momentum in the positive subspace. This result is consistent with Noether's theorem [18], as we find that the amount of linear momentum in the positive subspace (3D subspace) is a conserved quantity as a result of the positive spatial structure symmetry between the reference frames for both observers. As for the momentum transformation equation in the fourth dimension, it represents the transformation of the positive relativistic energy of the particle. By substituting the value of $\tilde{V}$ from Equation (23.1) shown in the first paper, we obtain the transformation of the relativistic total energy from the reference frame S' to the frame S in the positive subspace.

$$\tilde{E} = E`\gamma^{-1} \quad \Rightarrow \quad \tilde{E} = E`\sqrt{1-\frac{V_S^2}{c^2}} \tag{32.2}$$

where we find that the positive relativistic energy with respect to the observer O decreases with the increase in the speed of the reference frame V_S. As we mentioned in the first paper item 2.3, time dilation reduces the speed of light from positive subspace-time, we find here that time dilation reduces the relativistic energy from positive subspace-time. This result is one of the opposite results of special relativity.

2.5. The Four-Dimensional Force Vector in Positive Subspace-Time

By differentiating the 4D momentum vector $p^{`\mu}$ with respect to proper time, we obtain the 4D force vector $F^{`\mu}$ on the total space-time fabric with respect to the observer O' where $\mathrm{d}m_0 u^{`\mu} / \mathrm{d}\tau = F^{`\mu}$ ([13], pp. 184-185). By substituting the value of each 4D velocity component into the form of the reference coordinates shown in the set of Equation (2.2), and also substituting the proper time in terms of time with respect to the reference frame S', and by performing the differentiation with respect to the time $\mathrm{d}t`$ with the same steps followed in item 2.3, we obtain the values of the components the forces of the vector $F^{`\mu}$ shown in the following set ([6], pp. 52-53).

$$
\begin{aligned}
F^{`1} &= \frac{\mathrm{d}m_0 u^{`1}}{\mathrm{d}\tau} = m`\gamma`_u a`_x \\
F^{`2} &= \frac{\mathrm{d}m_0 u^{`2}}{\mathrm{d}\tau} = m`\gamma`_u a`_y \\
F^{`3} &= \frac{\mathrm{d}m_0 u^{`3}}{\mathrm{d}\tau} = m`\gamma`_u a`_z \\
F^{`4} &= \frac{\mathrm{d}m_0 u^{`4}}{\mathrm{d}\tau} = \frac{\mathrm{d}m`c^2}{\mathrm{d}t`} \frac{\gamma`_u}{c}
\end{aligned}
\tag{33.2}
$$

where the expression $\mathrm{d}m`c^2 / \mathrm{d}t` = \dot{E}`$ represents the time rate of change in the relativistic total energy of the particle or the flow of relativistic total energy with respect to the observer O' on the total fabric of space-time, so the vector in $F^{`\mu}$ the following formula

$$
F^{`\mu} = \gamma`_u \left(\overrightarrow{F`}, \frac{\dot{E}`}{c} \right)
\tag{34.2}
$$

By also differentiating the 4D momentum vector $\tilde{p}^{\epsilon}$ with respect to the proper time, we obtain the 4D force vector $\tilde{F}^{\epsilon}$ on the positive subspace-time with respect to the observer O where $\mathrm{d}m_0 \tilde{u}^{\epsilon} / \mathrm{d}\tau = \tilde{F}^{\epsilon}$. By following the same steps in Set 33.2, replacing the time $\mathrm{d}t$ with the time $\mathrm{d}\tilde{t}$, we obtain the values of the components of the force vector $\tilde{F}^{\epsilon}$ in the following set.

$$
\begin{aligned}
\tilde{F}^{1} &= \frac{\mathrm{d}m_0 \tilde{u}^{1}}{\mathrm{d}\tau} = m\gamma_u \tilde{a}_x \\
\tilde{F}^{2} &= \frac{\mathrm{d}m_0 \tilde{u}^{2}}{\mathrm{d}\tau} = m\gamma_u \tilde{a}_y \\
\tilde{F}^{3} &= \frac{\mathrm{d}m_0 \tilde{u}^{3}}{\mathrm{d}\tau} = m\gamma_u \tilde{a}_z \\
\tilde{F}^{4} &= \frac{\mathrm{d}m_0 \tilde{u}^{4}}{\mathrm{d}\tau} = \frac{\mathrm{d}m\tilde{\mathrm{V}}^2}{\mathrm{d}\tilde{t}} \frac{\gamma_u}{\tilde{\mathrm{V}}}
\end{aligned}
\tag{35.2}
$$

where the expression $\mathrm{d}m\tilde{V}^2/\mathrm{d}\tilde{t} = \dot{E}$ represents the time rate of change in the positive relativistic energy of the particle with respect to the observer O in the positive subspace-time, and the vector $\tilde{F}^{\epsilon}$ is written in the following formula

$$\tilde{F}^{\epsilon} = \gamma_u \left(\vec{\tilde{F}}, \frac{\dot{\tilde{E}}}{\tilde{V}} \right) \tag{36.2}$$

As for the transformation from the force vector $F^{`\mu}$ to the force vector $\tilde{F}^{\epsilon}$ in positive subspace-time, it is through differentiation of the equation for transforming the 4D momentum vector in positive subspace-time, Equation (29.2) with respect to the proper time.

$$\frac{\mathrm{d}\left(m_0 \tilde{u}^{\epsilon}\right)}{\mathrm{d}\tau} = \tilde{\Lambda}^{\epsilon}_{\mu} \frac{\mathrm{d}\left(m_0 u^{`\mu}\right)}{\mathrm{d}\tau} \tag{37.2}$$

$$\tilde{F}^{\epsilon} = \tilde{\Lambda}^{\epsilon}_{\mu} F^{`\mu} \tag{38.2}$$

From the previous transformation equation, we obtain the transformation of the components of the 4D force vectors in positive subspace-time from the frame of reference S’ to the frame S. By substituting the value of each component of the 4D force vector shown in the set of Equations (33.2) and (35.2), we obtain the transformation of the components of the 4D forces in positive subspace-time from reference frame S’ to frame S or from observer O’ to observer O.

$$\begin{aligned}
\tilde{F}^1 &= F^{`1} & &\Rightarrow & m\gamma_u \tilde{a}_x &= m`\gamma`_u a`_x \\
\tilde{F}^2 &= F^{`2} & &\Rightarrow & m\gamma_u \tilde{a}_y &= m`\gamma`_u a`_y \\
\tilde{F}^3 &= F^{`3} & &\Rightarrow & m\gamma_u \tilde{a}_z &= m`\gamma`_u a`_z \\
\tilde{F}^4 &= F^{`4} & &\Rightarrow & \frac{\mathrm{d}m\tilde{V}^2}{\mathrm{d}\tilde{t}} \frac{\gamma_u}{\tilde{V}} &= \frac{\mathrm{d}m`c^2}{\mathrm{d}t`} \frac{\gamma`_u}{c}
\end{aligned} \tag{39.2}$$

By dividing the equation for the transformation of velocity in the time dimension (the fourth equation in set (14.2)) by the set of equations on the right side, and then also substituting the value of $\tilde{V}$ from Equation (23.1) shown in the first paper, we obtain the transformations of both the components of forces and the flow of positive relativistic energy.

$$\begin{aligned}
\frac{m\gamma_u \tilde{a}_x}{\tilde{V}\gamma_u} &= \frac{m`\gamma`_u a`_x}{c\gamma`_u} & &\Rightarrow & m\tilde{a}_x &= m`a`_x \sqrt{1 - \frac{V_S^2}{c^2}} & &\Rightarrow & \tilde{F}_x &= F`_x \sqrt{1 - \frac{V_S^2}{c^2}} \\
\frac{m\gamma_u \tilde{a}_y}{\tilde{V}\gamma_u} &= \frac{m`\gamma`_u a`_y}{c\gamma`_u} & &\Rightarrow & m\tilde{a}_y &= m`a`_y \sqrt{1 - \frac{V_S^2}{c^2}} & &\Rightarrow & \tilde{F}_y &= F`_y \sqrt{1 - \frac{V_S^2}{c^2}} \\
\frac{m\gamma_u \tilde{a}_z}{\tilde{V}\gamma_u} &= \frac{m`\gamma`_u a`_z}{c\gamma`_u} & &\Rightarrow & m\tilde{a}_z &= m`a`_z \sqrt{1 - \frac{V_S^2}{c^2}} & &\Rightarrow & \tilde{F}_z &= F`_z \sqrt{1 - \frac{V_S^2}{c^2}} \\
\frac{\mathrm{d}m\tilde{V}^2}{\mathrm{d}\tilde{t}} \frac{\gamma_u}{\tilde{V}^2\gamma_u} &= \frac{\mathrm{d}m`c^2}{\mathrm{d}t`} \frac{\gamma`_u}{c^2\gamma`_u} & &\Rightarrow & \dot{\tilde{E}} &= \dot{E}` \left(1 - \frac{V_S^2}{c^2}\right)
\end{aligned}$$

(40.2)

The first three equations represent the transformation of the components of positive relativistic forces, while the fourth equation represents the transformation of the flow of positive relativistic energy from the reference frame S’ to the frame S in the positive subspace-time, and we conclude from them that the positive relativistic forces with respect to the observer O decrease in the positive subspace. As the speed of the reference frame V_S increases, the same applies to the flow of positive relativistic energy. These are also results opposite to the results of special relativity.

2.6. The Four-Dimensional Velocity Vector in Negative Subspace-Time

The second expression from the right side of Equation (7.2) represents the 4D velocity vector $\breve{u}^{\sigma}$ of the particle in negative subspace-time, where $\mathrm{d}\breve{A}^{\sigma}/\mathrm{d}\tau=\breve{u}^{\sigma}$, and the 4D vector $\breve{u}^{\sigma}$ is written in the following form

$$\breve{u}^{\sigma}=\frac{\mathrm{d}\breve{A}^{\sigma}}{\mathrm{d}\tau}=\left(\frac{\mathrm{d}\breve{A}^{1}}{\mathrm{d}\tau},\frac{\mathrm{d}\breve{A}^{2}}{\mathrm{d}\tau},\frac{\mathrm{d}\breve{A}^{3}}{\mathrm{d}\tau},\frac{\mathrm{d}\breve{A}^{4}}{\mathrm{d}\tau}\right) \tag{41.2}$$

By substituting the value of the components of the 4D differential displacement vector $\mathrm{d}\breve{A}^{\sigma}$ in terms of the coordinates of the reference frame S according to the notation explained in the first paper item 2.4, and also by substituting the proper time in terms of the resultant vector time $\mathrm{d}t$ with respect to the reference frame S and by replacing the time $\mathrm{d}t$ with the time $\mathrm{d}\breve{t}$ because they are in the same frame and for the same event, we obtain the values of the components of the 4D velocity vector $\breve{u}^{\sigma}$.

$$\begin{aligned}
\breve{u}^{1}&=\frac{\mathrm{d}\breve{A}^{1}}{\mathrm{d}\tau}=\gamma_u\frac{\mathrm{d}\breve{x}}{\mathrm{d}\breve{t}}=\gamma_u\breve{u}_x\\
\breve{u}^{2}&=\frac{\mathrm{d}\breve{A}^{2}}{\mathrm{d}\tau}=\gamma_u\frac{\mathrm{d}\breve{y}}{\mathrm{d}\breve{t}}=\gamma_u\breve{u}_y\\
\breve{u}^{3}&=\frac{\mathrm{d}\breve{A}^{3}}{\mathrm{d}\tau}=\gamma_u\frac{\mathrm{d}\breve{z}}{\mathrm{d}\breve{t}}=\gamma_u\breve{u}_z\\
\breve{u}^{4}&=\frac{\mathrm{d}\breve{A}^{4}}{\mathrm{d}\tau}=\gamma_u\frac{\mathrm{d}\left(\breve{V}\breve{t}\right)}{\mathrm{d}\breve{t}}=\gamma_u\breve{V}
\end{aligned} \tag{42.2}$$

The 4D velocity vector $\breve{u}^{\sigma}$ is written in the following form

$$\breve{u}^{\sigma}=\gamma_u\left(\breve{u}_x,\breve{u}_y,\breve{u}_z,\breve{V}\right)=\gamma_u\left(\vec{\breve{u}},\breve{V}\right) \tag{43.2}$$

where $\breve{u}$ represents the velocity of the particle in the negative subspace (3D subspace), $\breve{V}$ represents the speed of the particle in the time dimension of negative subspace-time, or the value of the speed of light in negative subspace-time. By differentiating the equation for the transformation of the 4D displacement vector in negative subspace-time, Equation (29.1), which is explained in the first paper with respect to the proper time, we obtain the transformation from the vector $\grave{u}^{\mu}$ to the vector $\breve{u}^{\sigma}$ in negative subspace-time.

$$\frac{\mathrm{d}\breve{A}^{\sigma}}{\mathrm{d}\tau}=\breve{\Lambda}_{\mu}^{\sigma}\frac{\mathrm{d}\grave{A}^{\mu}}{\mathrm{d}\tau} \tag{44.2}$$

$$\breve{u}^{\sigma} = \breve{\Lambda}^{\sigma}_{\mu} u^{`\mu} \tag{45.2}$$

$$\begin{bmatrix} \breve{u}^1 \\ \breve{u}^2 \\ \breve{u}^3 \\ \breve{u}^4 \end{bmatrix} = \begin{bmatrix} \gamma-1 & 0 & 0 & \beta\gamma \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \beta\gamma & 0 & 0 & \gamma-1 \end{bmatrix} \cdot \begin{bmatrix} u^{`1} \\ u^{`2} \\ u^{`3} \\ u^{`4} \end{bmatrix} \qquad \beta = \frac{V_s}{c}, \quad \gamma = \frac{1}{\sqrt{1-\frac{V_S^2}{c^2}}} \tag{46.2}$$

From the previous transformation equation, we obtain the transformations of the 4D velocity components in negative subspace-time from the reference frame S’ to the frame S, or from the observer O’ to the observer O.

$$\begin{aligned} &\breve{u}^1 = u^{`1}(\gamma-1) + u^{`4}\beta\gamma \\ &\breve{u}^2 = 0 \\ &\breve{u}^3 = 0 \\ &\breve{u}^4 = u^{`1}\beta\gamma + u^{`4}(\gamma-1) \end{aligned} \tag{47.2}$$

By substituting the value of each component shown in set of Equations (2.2) and (42.2), and by taking each of $\gamma\gamma`_u$ as a common factor, we obtain transformations of the values of the 4D velocity components in the form of reference coordinates.

$$\begin{aligned} &\gamma_u \breve{u}_x = \gamma\gamma`_u\left(u`_x\left(1-\gamma^{-1}\right) + c\beta\right) \\ &\gamma_u \breve{u}_y = 0 \\ &\gamma_u \breve{u}_z = 0 \\ &\gamma_u \breve{V} = \gamma\gamma`_u\left(u`_x\beta + c\left(1-\gamma^{-1}\right)\right) \end{aligned} \tag{48.2}$$

By analyzing the velocity of the particle in the time dimension c into $\tilde{V}$ and $\breve{V}$ as followed in the first paper item 2.4 Equation (43.1), we obtain

$$\left(c-\tilde{V}\right)\gamma_u = \gamma\gamma`_u \frac{V_s u`_x}{c} + c\gamma\gamma`_u - c\gamma`_u \tag{49.2}$$

$$c\gamma_u = \gamma\gamma`_u \frac{V_s u`_x}{c} + c\gamma\gamma`_u - c\gamma`_u + \tilde{V}\gamma_u \tag{50.2}$$

From the fourth equation in the set of Equation (14.2), we find that $-c\gamma`_u + \tilde{V}\gamma_u = 0$

$$c\gamma_u = \gamma\gamma`_u\left(c + \frac{V_s u`_x}{c}\right) \tag{51.2}$$

$$\gamma_u = \gamma\gamma`_u\left(1 + \frac{V_s u`_x}{c^2}\right) \tag{52.2}$$

By dividing the last equation by the 4D velocity transformation equations in the first, second, and third dimensions in the set of Equation (48.2), we obtain the 3D velocity transformations in the negative subspace from the reference frame S’ to the frame S or from observer O’ to observer O.

$$\begin{aligned}
&\frac{\gamma_u \breve{u}_x}{\gamma_u}=\frac{\gamma\gamma`_u\left(u`_x\left(1-\gamma^{-1}\right)+c\beta\right)}{\gamma\gamma`_u\left(1+\dfrac{V_s u`_x}{c^2}\right)} &&\Rightarrow\quad \breve{u}_x=\frac{u`_x\left(1-\gamma^{-1}\right)+V_s}{1+\dfrac{V_s u`_x}{c^2}} \quad\Rightarrow\quad \breve{u}_x=V_S\\
&\frac{\gamma_u \breve{u}_y}{\gamma_u}=\frac{0}{\gamma\gamma`_u\left(1+\dfrac{V_s u`_x}{c^2}\right)} &&\Rightarrow\quad \breve{u}_y=0\\
&\frac{\gamma_u \breve{u}_z}{\gamma_u}=\frac{0}{\gamma\gamma`_u\left(1+\dfrac{V_s u`_x}{c^2}\right)} &&\Rightarrow\quad \breve{u}_z=0
\end{aligned} \tag{53.2}$$

As we previously made clear in the first paper, Equation (53.1), the right-hand expression in the first equation from the previous set of equations is equal to the speed of the reference frame V_S, regardless of whether $\breve{u}_x$ is the velocity of a particle or the velocity of light. Because both $\breve{u}_y=\breve{u}_z=0$, therefore $\breve{u}=\breve{u}_x=V_S$, and the same is true for light $\breve{V}=\breve{V}_x=V_S$, and this means that the speed of the particle in the negative subspace is equal to the speed of the particle in the time dimension, for negative subspace-time, equal to the speed of the reference frame that is, $\breve{u}=\breve{V}=V_S$. Therefore, Equation (43.2) represents the 4D velocity vector in negative subspace-time in a general form, that is, in the case of movement of the reference frames relative to each other along three axes. But when the relative motion between the reference frames is on only one axis, which is the $xx`$ axis, as shown above in **Figure 1**, in this case, we write the 4D velocity vector in negative subspace-time in the following formula:

$$\breve{u}^\sigma=\gamma_u\left(V_S,0,0,V_S\right) \tag{54.2}$$

2.7. The Four-Dimensional Acceleration Vector in Negative Subspace-Time

By differentiating the 4D velocity vector $\breve{u}^\sigma$ with respect to proper time again, we obtain the 4D acceleration vector $\breve{a}^\sigma$ on the negative subspace-time with respect to the observer O, where $\mathrm{d}\breve{u}^\sigma/\mathrm{d}\tau=\breve{a}^\sigma$. By following the same previous steps in set Equation (18.2) at the moment of acceleration mentioned above. We obtain the values of the components of the acceleration vector $\breve{a}^\sigma$

$$\begin{aligned}
&\breve{a}^1=\gamma_u\frac{\mathrm{d}}{\mathrm{d}\breve{t}}\breve{u}^1 &&\Rightarrow\quad \breve{a}^1=\breve{a}_x\gamma_u^2+\frac{\mathrm{d}\gamma_u}{\mathrm{d}\breve{t}}\breve{u}_x\gamma_u &&\Rightarrow\quad \breve{a}^1=\breve{a}_x\gamma_u^2\\
&\breve{a}^2=\gamma_u\frac{\mathrm{d}}{\mathrm{d}\breve{t}}\breve{u}^2 &&\Rightarrow\quad \breve{a}^2=\breve{a}_y\gamma_u^2+\frac{\mathrm{d}\gamma_u}{\mathrm{d}\breve{t}}\breve{u}_y\gamma_u &&\Rightarrow\quad \breve{a}^2=\breve{a}_y\gamma_u^2\\
&\breve{a}^3=\gamma_u\frac{\mathrm{d}}{\mathrm{d}\breve{t}}\breve{u}^3 &&\Rightarrow\quad \breve{a}^3=\breve{a}_z\gamma_u^2+\frac{\mathrm{d}\gamma_u}{\mathrm{d}\breve{t}}\breve{u}_z\gamma_u &&\Rightarrow\quad \breve{a}^3=\breve{a}_z\gamma_u^2\\
&\breve{a}^4=\gamma_u\frac{\mathrm{d}}{\mathrm{d}\breve{t}}\breve{u}^4 &&\Rightarrow\quad \breve{a}^4=\frac{\mathrm{d}\gamma_u}{\mathrm{d}\breve{t}}\breve{V}\gamma_u &&\Rightarrow\quad \breve{a}^4=0
\end{aligned} \tag{55.2}$$

We write the 4D acceleration vector in negative subspace-time in the form of the following reference coordinates

$$\breve{a}^\sigma=\gamma_u^2\left(\breve{a}_x,\breve{a}_y,\breve{a}_z,0\right)=\gamma_u^2\left(\vec{\breve{a}},0\right) \tag{56.2}$$

By differentiating the equation for transforming the 4D velocity vector in negative subspace-time, Equation (45.2), with respect to proper time, we obtain the transformation from the acceleration vector $\breve{a}^{\sigma}$ to the acceleration vector $a^{`\mu}$ in negative subspace-time.

$$\frac{\mathrm{d}\breve{u}^{\sigma}}{\mathrm{d}\tau}=\breve{\Lambda}_{\mu}^{\sigma}\frac{\mathrm{d}u^{`\mu}}{\mathrm{d}\tau} \tag{57.2}$$

$$\breve{a}^{\sigma}=\breve{\Lambda}_{\mu}^{\sigma}a^{`\mu} \tag{58.2}$$

From the previous transformation equation, we obtain the transformation of the components of the 4D acceleration vector in negative subspace-time from the reference frame S' to the frame S, or from the observer O' to the observer O. (In the same form as the set of Equations (47.2).

$$\begin{aligned}&\breve{a}^{1}=a^{`1}(\gamma-1)+a^{`4}\beta\gamma\\&\breve{a}^{2}=0\\&\breve{a}^{3}=0\\&\breve{a}^{4}=a^{`1}\beta\gamma+a^{`4}(\gamma-1)\end{aligned} \tag{59.2}$$

By substituting in the previous set the value of each acceleration component shown in set Equations (16.2) and (55.2), we obtain the transformations of the values of the 4D acceleration components in the form of reference coordinates.

$$\begin{aligned}&\breve{a}_{x}\gamma_{u}^{2}=a`_{x}\gamma`_{u}^{2}(\gamma-1)\\&\breve{a}_{y}\gamma_{u}^{2}=0\\&\breve{a}_{z}\gamma_{u}^{2}=0\\&0=0\end{aligned} \tag{60.2}$$

By dividing the square of Equation (52.2) by the first, second, and third equation in the previous set of equations, taking γ as a common factor in the first equation. We obtain the transformations of the 3D acceleration components in the negative subspace from the reference frame S' to the frame S, or from the observer O' to the observer O.

$$\begin{aligned}&\frac{\breve{a}_{x}\gamma_{u}^{2}}{\gamma_{u}^{2}}=\frac{a`_{x}\gamma`_{u}^{2}\gamma\left(1-\gamma^{-1}\right)}{\gamma^{2}\gamma`_{u}^{2}\left(1+\frac{V_{s}u`_{x}}{c^{2}}\right)^{2}}\quad\Rightarrow\quad\breve{a}_{x}=\frac{a`_{x}\left(1-\gamma^{-1}\right)}{\gamma\left(1+\frac{V_{s}u`_{x}}{c^{2}}\right)^{2}}\quad\Rightarrow\quad\breve{a}_{x}=0\\&\frac{\breve{a}_{y}\gamma_{u}^{2}}{\gamma_{u}^{2}}=\frac{0}{\gamma^{2}\gamma`_{u}^{2}\left(1+\frac{V_{s}u`_{x}}{c^{2}}\right)^{2}}\quad\Rightarrow\quad\breve{a}_{y}=0\\&\frac{\breve{a}_{z}\gamma_{u}^{2}}{\gamma_{u}^{2}}=\frac{0}{\gamma^{2}\gamma`_{u}^{2}\left(1+\frac{V_{s}u`_{x}}{c^{2}}\right)^{2}}\quad\Rightarrow\quad\breve{a}_{z}=0\end{aligned} \tag{61.2}$$

In the first equation in the previous set, we find that when the speed of the reference frame is much less than the speed of light $V_{s}\ll c$, the inverse of the Lorentz factor equals one, and therefore, the expression $\left(1-\gamma^{-1}\right)\approx 0$ is approximately equal to zero, which means that $\breve{a}_{x}=0$ in this case, but when $V_{s}\approx c$, we find in the denominator $\gamma \approx \infty$, and this also means that $\breve{a}_x = 0$. Therefore, here we can generalize this result for any values of the reference frame speed V_s, regardless of the value of $a`_x$, and we always find $\breve{a}_x = 0$. This result is consistent with the previous results, as we previously explained $\breve{u}_x = V_s$. Because the acceleration in the negative subspace $\breve{a}_x$ means the acceleration of the reference frame, and in the special case there is no acceleration of the frames. We can then reduce the 4D acceleration vector in negative subspace-time in the special case to the following reference coordinate formula.

$$\breve{a}^\sigma = \gamma_u^2 (0,0,0,0) \tag{62.2}$$

2.8. The Four-Dimensional Momentum Vector in Negative Subspace-Time

By multiplying the 4D velocity vector $\breve{u}^\sigma$ by the rest mass of the particle, we obtain the 4D momentum vector $\breve{p}^\sigma$ in negative subspace-time with respect to the observer O, where $m_0 \breve{u}^\sigma = \breve{p}^\sigma$. By substituting for each component of momentum in terms of the component of the 4D velocity and the rest mass, then by substituting the value of each component of the 4D velocity in the form of the reference coordinates shown in the set of Equation (42.2). We obtain the values of the momentum components of the vector $\breve{p}^\sigma$ shown in the following set

$$\begin{aligned} \breve{p}^1 &= m_0 \breve{u}^1 = m\breve{u}_x \\ \breve{p}^2 &= m_0 \breve{u}^2 = m\breve{u}_y \\ \breve{p}^3 &= m_0 \breve{u}^3 = m\breve{u}_z \\ \breve{p}^4 &= m_0 \breve{u}^4 = \frac{m\breve{V}^2}{\breve{V}} \end{aligned} \tag{63.2}$$

where the expression $m\breve{V}^2 = \breve{E}$ represents the negative relativistic energy of the particle with respect to the observer O. And $\breve{V}$ as we explained in item 2.6, is the value of the speed of light in negative subspace-time. Therefore, the vector $\breve{p}^\sigma$ is also written in the following form.

$$\breve{p}^\sigma = \left(\vec{\breve{p}}, \frac{\breve{E}}{\breve{V}} \right) \tag{64.2}$$

As for the transformation from the momentum vector $p`^\mu$ to the momentum vector $\breve{p}^\sigma$ in negative subspace-time, it is done by multiplying both sides of the 4D velocity vector transformation equation in negative subspace-time, Equation (45.2) in the rest mass.

$$m_0 \breve{u}^\sigma = \breve{\Lambda}^\sigma_\mu m_0 u`^\mu \tag{65.2}$$

$$\breve{p}^\sigma = \breve{\Lambda}^\sigma_\mu p`^\mu \tag{66.2}$$

From the previous transformation equation, we obtain the transformation of the 4D vector components of momentum and energy in negative subspace-time from the reference frame S' to the frame S or from the observer O' to the observer O (in the same form as the set of Equation (47.2)).

$$\begin{aligned}
\breve{p}^{1} &= p^{\backprime 1}(\gamma-1)+p^{\backprime 4}\beta\gamma \\
\breve{p}^{2} &= 0 \\
\breve{p}^{3} &= 0 \\
\breve{p}^{4} &= p^{\backprime 1}\beta\gamma+p^{\backprime 4}(\gamma-1)
\end{aligned} \tag{67.2}$$

By substituting the value of each 4D momentum component shown in the set of equations (24.2) and (63.2), while taking γ as a common factor in the first equation, we obtain the transformation of the 4D relativistic momentum components from the reference frame S' to the frame S or from the observer O' to the observer O.

$$\begin{aligned}
m\breve{u}_{x} &= m^{\backprime}u^{\backprime}_{x}(\gamma-1)+\frac{E^{\backprime}}{c}\beta\gamma && \Rightarrow && \breve{p}_{x}=\gamma\left(p^{\backprime}_{x}\left(1-\gamma^{-1}\right)+\frac{E^{\backprime}}{c}\beta\right) \\
m\breve{u}_{y} &= 0 && \Rightarrow && \breve{p}_{y}=0 \\
m\breve{u}_{z} &= 0 && \Rightarrow && \breve{p}_{z}=0 \\
\frac{m\breve{V}^{2}}{\breve{V}} &= m^{\backprime}u^{\backprime}_{x}\beta\gamma+\frac{m^{\backprime}c^{2}}{c}(\gamma-1) && \Rightarrow && \frac{\breve{E}}{\breve{V}}=p^{\backprime}_{x}\frac{V_{s}}{c}\gamma+\frac{E^{\backprime}}{c}(\gamma-1)
\end{aligned} \tag{68.2}$$

The first, second, and third equations represent the transformation of the components of negative relativistic momentum or relativistic momentum in the negative subspace. As for the transformation equation in the fourth dimension, it represents the transformation of the negative relativistic energy of the particle. By taking $\frac{1}{c}\gamma$ as a common factor, it can also be shortened to the following formula.

$$\frac{\breve{E}}{\breve{V}}=\frac{1}{c}\gamma\left(p^{\backprime}_{x}V_{s}+E^{\backprime}\left(1-\gamma^{-1}\right)\right) \tag{69.2}$$

By substituting for the value of $\breve{V}=V_{s}$, as we mentioned above in item 2.6.

$$\breve{E}=\frac{V_{s}}{c}\gamma\left(p^{\backprime}_{x}V_{s}+E^{\backprime}\left(1-\gamma^{-1}\right)\right) \tag{70.2}$$

In case the particle does not move along the $x^{\backprime}$-axis, that is, $u^{\backprime}_{x}=0$, the equation is also reduced to the following formula

$$\breve{E}=E^{\backprime}\frac{V_{s}}{c}(\gamma-1) \tag{71.2}$$

Equations (70.2) and (71.2) represent the transformation of the total relativistic energy from the reference frame S' to the frame S in negative subspace. They show that the negative relativistic energy with respect to the observer O increases with the increase in the speed of the reference frame V_{S}, until its value reaches infinity when the speed of the reference frame reaches the speed of light theoretically

2.9. The Four-Dimensional Force Vector in Negative Subspace-Time

By differentiating the 4D momentum vector in the negative subspace-time $\breve{p}^{\sigma}$ with respect to the proper time again as well. We obtain the 4D force vector $\breve{F}^{\sigma}$ for the particle in negative subspace-time with respect to the observer O, where

$\mathrm{d}\left(m_0\breve{u}^\sigma\right)/\mathrm{d}\tau = \breve{F}^\sigma$. By substituting the value of each 4D velocity component into the form of the reference coordinates shown in the set of Equation (42.2), by also substituting the proper time in terms of the resultant vector time $\mathrm{d}t$ with respect to the reference frame S, by replacing the time $\mathrm{d}t$ with the time $\mathrm{d}\breve{t}$ and performing the differentiation according to the conditions followed in item 2.5, we obtain here on the values of the components of the 4D force vector $\breve{F}^\sigma$

$$\begin{aligned}
\breve{F}^1 &= \frac{\mathrm{d}\left(m_0\breve{u}^1\right)}{\mathrm{d}\tau} = m\gamma_u\breve{a}_x \\
\breve{F}^2 &= \frac{\mathrm{d}\left(m_0\breve{u}^2\right)}{\mathrm{d}\tau} = m\gamma_u\breve{a}_y \\
\breve{F}^3 &= \frac{\mathrm{d}\left(m_0\breve{u}^3\right)}{\mathrm{d}\tau} = m\gamma_u\breve{a}_z \\
\breve{F}^4 &= \frac{\mathrm{d}\left(m_0\breve{u}^4\right)}{\mathrm{d}\tau} = \frac{\mathrm{d}m\breve{V}^2}{\mathrm{d}\breve{t}}\frac{\gamma_u}{\breve{V}}
\end{aligned} \tag{72.2}$$

where the expression $\mathrm{d}m\breve{V}^2/\mathrm{d}\breve{t} = \dot{\breve{E}}$ represents the time rate of change in the negative relativistic energy of the particle with respect to the observer O in the negative subspace-time, and the vector $\breve{F}^\sigma$ is written in the following formula.

$$\breve{F}^\sigma = \gamma_u\left(\vec{\breve{F}}, \frac{\dot{\breve{E}}}{\breve{V}}\right) \tag{73.2}$$

By differentiating the 4D momentum vector transformation equation in negative subspace-time with respect to proper time, Equation (66.2). We obtain an equation that transforms the force vector $F^{`\mu}$ into the force vector $\breve{F}^\sigma$ in the negative subspace-time from the reference frame S' to the frame S, or from observer O' to observer O.

$$\frac{\mathrm{d}\left(m_0\breve{u}^\sigma\right)}{\mathrm{d}\tau} = \breve{\Lambda}^\sigma_\mu\frac{\mathrm{d}\left(m_0u^{`\mu}\right)}{\mathrm{d}\tau} \tag{74.2}$$

$$\breve{F}^\sigma = \breve{\Lambda}^\sigma_\mu F^{`\mu} \tag{75.2}$$

From the previous transformation equation, we obtain the transformation of the components of the 4D vector forces in negative subspace-time from the reference frame S' to the frame S or from the observer O' to the observer O (in the same form as the set of Equation (47.2))

$$\begin{aligned}
\breve{F}^1 &= F^{`1}\left(\gamma-1\right)+F^{`4}\beta\gamma \\
\breve{F}^2 &= 0 \\
\breve{F}^3 &= 0 \\
\breve{F}^4 &= F^{`1}\beta\gamma+F^{`4}\left(\gamma-1\right)
\end{aligned} \tag{76.2}$$

We can obtain equations for the transformation of the components of 3D relativistic forces in the negative subspace-time, by substituting the values of each component of the 4D forces shown in set of Equations (33.2) and (72.2) in the previous set, taking $\gamma`_u\gamma\frac{\mathrm{d}}{\mathrm{d}t`}m`$ is a common factor in the first equation, by dividing the transformation equations in the first, second and third dimensions by Equation (52.2), then substituting the first equation in set Equation (53.2) into the first dimension equation. We obtain the transformations of the components of the forces. But we have another direct method where all components of the 3D acceleration shown in set Equation (61.2) are equal to zero, and thus, we can obtain the transformations of the components of the negative relativistic forces by directly substituting the value of the components of the acceleration in each dimension.

$$\begin{aligned}
&\frac{m\gamma_u\breve{a}_x}{\gamma_u}=\frac{\grave{\gamma}_u\gamma\frac{\mathrm{d}}{\mathrm{d}\grave{t}}\grave{m}\left(\grave{u}_x\left(1-\gamma^{-1}\right)+V_s\right)}{\grave{\gamma}_u\gamma\left(1+\frac{V_s\grave{u}_x}{c^2}\right)} &&\Rightarrow\quad m\breve{a}_x=\frac{\mathrm{d}}{\mathrm{d}\grave{t}}\grave{m}V_s &&\Rightarrow\quad \breve{F}_x=0\\
&\frac{m\gamma_u\breve{a}_y}{\gamma_u}=\frac{0}{\grave{\gamma}_u\gamma\left(1+\frac{V_s\grave{u}_x}{c^2}\right)} &&\Rightarrow\quad m\breve{a}_y=0 &&\Rightarrow\quad \breve{F}_y=0\\
&\frac{m\gamma_u\breve{a}_z}{\gamma_u}=\frac{0}{\grave{\gamma}_u\gamma\left(1+\frac{V_s\grave{u}_x}{c^2}\right)} &&\Rightarrow\quad m\breve{a}_z=0 &&\Rightarrow\quad \breve{F}_z=0
\end{aligned}\tag{77.2}$$

The previous three equations represent the transformation of components of negative relativistic forces in three dimensions from the reference frame S' to the frame S in the negative subspace. As for the relativistic forces in the fourth dimension, they represent the flow of negative relativistic energy.

$$\frac{\mathrm{d}m\breve{V}^2}{\mathrm{d}\breve{t}}\frac{\gamma_u}{\breve{V}}=\grave{m}\grave{\gamma}_u\grave{a}_x\beta\gamma+\frac{\mathrm{d}\grave{m}c^2}{\mathrm{d}\grave{t}}\frac{\grave{\gamma}_u}{c}(\gamma-1)\tag{78.2}$$

We can write the equation in the following form

$$\dot{\breve{E}}\frac{\gamma_u}{\breve{V}}=\grave{F}_x\grave{\gamma}_u\beta\gamma+\dot{\grave{E}}\frac{\grave{\gamma}_u}{c}(\gamma-1)\tag{79.2}$$

By taking $\grave{\gamma}_u\gamma\frac{1}{c}$ as a common factor

$$\dot{\breve{E}}\frac{\gamma_u}{\breve{V}}=\grave{\gamma}_u\gamma\frac{1}{c}\left(\grave{F}_xV_s+\dot{\grave{E}}\left(1-\gamma^{-1}\right)\right)\tag{80.2}$$

By dividing the previous equation by Equation (52.2)

$$\dot{\breve{E}}\frac{\gamma_u}{\breve{V}\gamma_u}=\frac{\grave{\gamma}_u\gamma}{\grave{\gamma}_u\gamma}\frac{1}{c}\frac{\left(\grave{F}_xV_s+\dot{\grave{E}}\left(1-\gamma^{-1}\right)\right)}{\left(1+\frac{V_s\grave{u}_x}{c^2}\right)}\tag{81.2}$$

Substituting the value of $\breve{V}=V_s$

$$\dot{\breve{E}}=\frac{V_s}{c}\frac{\left(\grave{F}_xV_s+\dot{\grave{E}}\left(1-\gamma^{-1}\right)\right)}{\left(1+\frac{V_s\grave{u}_x}{c^2}\right)}\tag{82.2}$$

In case the particle does not move along the $\grave{x}$-axis, meaning that, $\grave{u}_x=0$, $\grave{F}_x=0$, the previous equation is also reduced to the following formula.

$$\breve{\dot{E}} = \dot{E}`\frac{V_s}{c}\left(1-\gamma^{-1}\right) \tag{83.2}$$

Equations (82.2) and (83.2) represent the transformation of the total relativistic energy flow from the reference frame S' to the S frame in subspace-time. They show that the negative relativistic energy flow with respect to the observer O increases with the increase in the speed of the reference frame V_S, until it reaches the value of the flow to $\dot{E}`$ when the speed of the reference frame reaches the speed of light theoretically.

2.10. Analysis of Energy-Momentum Tensor on Total Space-Time

If we have an infinitesimal volume containing a number of non-interacting particles, identical in rest mass, at rest with respect to each other on the total fabric of space-time, which is commonly referred to as a "dust field", this field has a density $\rho_0 = n_0 m_0 / \left(V_{ol}\right)_0$, Where m_0 is the rest mass of the dust particle, n_0 is the number of particles, $\left(V_{ol}\right)_0$ is the dust proper volume, and ρ_0 represents the proper density of the mass on the total space-time fabric. If we wanted to describe both the density and flow of energy and the momentum of the dust on the total fabric of space-time for every observer, it would be through the energy-momentum tensor. For the observer, O' is according to the following equation ([14], pp. 176, 178).

$$T^{`kl} = \rho_0 u^{`k} u^{`l} \qquad k = l = 0,1,2,3 \tag{84.2}$$

where $T^{`kl}$ is the energy-momentum tensor, and $u^{`k}, u^{`l}$ are the contravariant components of the 4D velocity vector on the total fabric of space-time. With changing the index of the time dimension component followed in set of Equation (2.2) from $\left(u^{`4} = \gamma`_u c\right)$ to $\left(u^{`0} = \gamma`_u c\right)$, assuming here that the speed of the particles dust with respect to the reference frame S' is equal to zero $u` = 0$ and therefore $\gamma`_u = 1$. In this case, we write the 4D velocity vector $u^{`k}$ or $u^{`l}$ in the following formula.

$$u^{`k} = \gamma`_u\left(c, u^{`x}, u^{`y}, u^{`z}\right) = \left(c,0,0,0\right) \tag{85.2}$$

Because the tensor is of the second order, it consists of a 4 × 4 matrix, which is the product of multiplying the column matrix by the row matrix of the 4D velocity components $u^{`k} u^{`l}$, with ρ_0 as a common factor, and the result represents the tensor matrix of energy and momentum on the total space-time with respect to the observer O.

$$T^{`kl} = \rho_0 \begin{bmatrix} u^{`0} \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} u^{`0} & 0 & 0 & 0 \end{bmatrix} \tag{86.2}$$

$$T^{`kl} = \begin{bmatrix} T^{`00} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{87.2}$$

Because there is no movement of particles with respect to the observer O’, therefore, there is only one component in this tensor, which is $T^{`00} = \rho_0 c^2$, which represents the energy density of dust, where $\rho` = \rho_0$, because $n` = n_0$ and $V_{ol}^{`} = (V_{ol})_0$. We can also use this tensor in the general case as a local tensor where the total space-time remains flat under the influence of gravitational mass M with respect to the observer O’ and the speed of light is equal to c.

As for the observer O in the reference frame S, it will appear to him that the dust is flowing at the speed of the reference frame, that is, $u = V_s$ and therefore $\gamma_u = \gamma$. As for the energy-momentum tensor with respect to the frame of reference S, it will be according to the following equation.

$$T^{\mu\nu} = \rho_0 u^\mu u^\nu \qquad \mu = \nu = 0,1,2,3 \tag{88.2}$$

where $T^{\mu\nu}$ represents the energy-momentum tensor on the total space-time, and u^μ, u^ν are the contravariant components of the 4D velocity vector on the total space-time fabric with respect to the observer O, With changing the index of the time dimension component followed in set of Equation (5.2) from $\left(u^4 = \gamma_u c\right)$ to $\left(u^0 = \gamma_u c\right)$, in this case we write the 4D velocity vector u^μ or u^ν in the following formula.

$$u^\mu = \gamma_u\left(c\ , u^x, u^y, u^z\right) = \gamma\left(c, u^x, u^y, u^z\right) \tag{89.2}$$

As for the tensor matrix here, it also represents the product of multiplying a column matrix by a row matrix for the 4D velocity components $u^\mu u^\nu$, taking ρ_0 as a common factor. The result represents the tensor matrix of energy and momentum on the total space-time with respect to the observer O.

$$T^{\mu\nu} = \rho_0 \begin{bmatrix} u^0 \\ u^1 \\ u^2 \\ u^3 \end{bmatrix} \cdot \begin{bmatrix} u^0 & u^1 & u^2 & u^3 \end{bmatrix} \tag{90.2}$$

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{31} & T^{33} \end{bmatrix} \tag{91.2}$$

From the previous matrix, we obtain the value of the component $T^{00} = \rho_0 \gamma^2 c^2$, which here represents the total relativistic energy density of the dust to the observer O. By substituting the proper mass density in terms of the rest mass, the number of particles, and the proper volume, But from special relativity, we find that $m = m_0 \gamma$, and we also find $V_{ol} = V_{ol}^{`} \gamma^{-1}$ as a result of length contraction [19]. And to compensate for that as well

$$T^{00} = \rho_0 \gamma^2 c^2 = \frac{n m_0 \gamma}{V_{ol}^{`} \gamma^{-1}} c^2 = \frac{nm}{V_{ol}} c^2 = \rho c^2 \tag{92.2}$$

where ρ represents the relativistic mass density of dust with respect to the observer O

$$\rho c^2 = \rho_0 c^2 \gamma^2 \tag{93.2}$$

Equation (93.2) shows us that the relativistic energy density of dust to the observer O increases with increasing velocity of the reference frame. The transformation of the energy-momentum tensor from one frame of reference to another on the total space-time, whether in the special or general case, can be written through the general tensor transformation ([16] p. 79).

$$T^{\mu\nu} = T^{`kl}\left(\frac{\partial u^{\mu}}{\partial u^{`k}}\frac{\partial u^{\nu}}{\partial u^{`l}}\right) \tag{94.2}$$

From Equation (94.2), we find that the energy-momentum tensor, whether in the special or general case, depends on the 4D velocity. Therefore, the analysis of that velocity described in the first item 2-1 on flat, total space-time or even a curve will necessarily lead to an analysis of the energy-momentum tensor, and thus, we have energy-momentum tensor for positive subspace-time and another for negative subspace-time.

2.11. Energy-Momentum Tensors for Positive Subspace-Time

We can obtain the energy-momentum tensor for the positive subspace-time in the special case with respect to the observer O through the proper density of the dust and the 4D velocity vector $\tilde{u}^{\epsilon}$ of the dust particles on the positive subspace-time resulting from analyzing the original 4D velocity vector according to the following equation.

$$\tilde{T}^{\epsilon\tau} = \rho_0 \tilde{u}^{\epsilon}\tilde{u}^{\tau} \qquad \epsilon = \tau = 0,1,2,3 \tag{95.2}$$

where $\tilde{T}^{\epsilon\tau}$ is the energy -momentum tensors of the positive subspace-time, and $\tilde{u}^{\epsilon}, \tilde{u}^{\tau}$ are the contravariant components of the 4D velocity vector on the positive subspace-time. By substituting the values of the components of the 4D velocity from Equation (85.2) into the set of Equation (14.2), we obtain the values of the components of the velocity vector $\tilde{u}^{\epsilon}$ in the positive subspace-time, taking into account the change in the index of the time dimension component followed by $\left(\tilde{u}^4 = \gamma_u \tilde{V}\right)$ to $\left(\tilde{u}^0 = \gamma_u \tilde{V}\right)$, and also $\gamma_u = \gamma$, as our assumption is higher.

$$\tilde{u}^{\epsilon} = \gamma_u\left(\tilde{V}, \tilde{u}^x, \tilde{u}^y, \tilde{u}^z\right) = \gamma\left(\tilde{V}, 0, 0, 0\right) \tag{96.2}$$

Here, we can also represent the tensor matrix as the product of multiplying a column matrix by a row matrix of the 4D velocity vector components of the positive subspace-time $\tilde{u}^{\epsilon}, \tilde{u}^{\tau}$, taking ρ_0 as a common factor. The result represents the tensor matrix of energy and momentum on the positive subspace-time with respect to observer O.

$$\tilde{T}^{\epsilon\tau} = \rho_0 \begin{bmatrix} \tilde{u}^0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \tilde{u}^0 & 0 & 0 & 0 \end{bmatrix} \tag{97.2}$$

$$\tilde{T}^{\epsilon\tau} = \begin{bmatrix} \tilde{T}^{00} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{98.2}$$

We find here that the tensor matrix contains only one component, which is $\tilde{T}^{00}$, which represents the relativistic energy density of dust on the positive subspace-time fabric by substituting the proper density value. With the transformation of the rest mass into the relativistic mass

$$\tilde{T}^{00} = \rho_0 \tilde{V}^2 \gamma^2 = \frac{n m_0 \gamma}{\grave{V}_{ol}} \tilde{V}^2 \gamma = \frac{nm}{\grave{V}_{ol}} \tilde{V}^2 \gamma \tag{99.2}$$

As for the volume transformation in the positive subspace, it is constant as a result of the symmetry of the structure of the spatial space between observers, according to the modified Lorentz transformation for positive space-time, that is $\tilde{V}_{ol} = \grave{V}_{ol}$.

$$\tilde{T}^{00} = \frac{nm}{\tilde{V}_{ol}} \tilde{V}^2 \gamma = \tilde{\rho} \tilde{V}^2 \gamma \tag{100.2}$$

where $\tilde{\rho}\tilde{V}^2$ represents the positive relativistic energy density, and $\tilde{\rho}$ is the positive relativistic mass density, by substituting the values of the previous tensor components in Equation (98.2), we get the following formula for the tensor matrix:

$$\tilde{T}^{\epsilon\tau} = \begin{bmatrix} \tilde{\rho}\tilde{V}^2\gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{101.2}$$

We can also use the tensor shown in the previous equation in the general case, but as a local tensor, *i.e.*, at a point on the positive subspace-time, under the influence of gravitational mass M, where $\gamma = 1\big/\sqrt{2MG/rc^2}$, according to the proper time transformation equation in positive subspace-time in the general case Equation (71.1) shown in the first paper.

The transformation of the energy-momentum tensor from one frame of reference to another on the positive subspace-time, whether in the special or general case, is also done through the general tensor transform.

$$\tilde{T}^{\epsilon\tau} = T^{`kl} \left( \frac{\partial \tilde{u}^{\epsilon}}{\partial u^{`k}} \frac{\partial \tilde{u}^{\tau}}{\partial u^{`l}} \right) \tag{102.2}$$

Because the structure of the positive subspace-time is symmetrical (that is, all components of the displacement vector are invariant under the modified Lorentz transformation) with respect to both frames of reference in the special and general cases as well, according to the first paper item 2.6. The proper time of the particles is also uniform, and therefore, each of the 4D velocity components is also equal. This means that the energy-momentum tensor on positive subspace-time is invariant under transformation.

$$\tilde{T}^{\epsilon\tau} = T^{`kl} \qquad \left(\frac{\partial\tilde{u}^{\epsilon}}{\partial u^{`k}}\frac{\partial\tilde{u}^{\tau}}{\partial u^{`l}}\right) = 1 \qquad \frac{\mathrm{d}\tau}{\mathrm{d}\tau}\left(\frac{\partial\tilde{A}^{\epsilon}}{\partial A^{`k}}\frac{\partial\tilde{A}^{\tau}}{\partial A^{`l}}\right) = 1 \tag{103.2}$$

From the previous equation, we obtain the transformation of the relativistic energy density with respect to the observer O in the positive subspace.

$$\tilde{T}^{00} = T^{`00} \quad \Rightarrow \quad \tilde{\rho}\tilde{V}^{2}\gamma = \rho_0 c^2 \quad \Rightarrow \quad \tilde{\rho}\tilde{V}^{2} = \rho_0 c^2 \gamma^{-1} \tag{104.2}$$

Equations (104.2) show us that energy density on the positive subspace-time fabric decreases with respect to the observer O with the increase in the speed of the reference frame, or in other words, time dilation reduces energy from the positive subspace. This is an opposite result to the result of special relativity shown in Equation (93.2).

2.12. Energy-Momentum Tensors for Negative Subspace-Time

To obtain the energy-momentum tensor on negative subspace-time in the special or general case, we analyze all the contravariant components of 4D velocity vectors on the total space-time fabric, according to the following equations $\partial u^{\mu} = \partial\tilde{u}^{\epsilon} + \partial\breve{u}^{\sigma}$ and $\partial u^{\nu} = \partial\tilde{u}^{\tau} + \partial\breve{u}^{\rho}$. Substituting this in Equation (94.2):

$$T^{\mu\nu} = T^{`kl}\left[\frac{\partial\tilde{u}^{\epsilon} + \partial\breve{u}^{\sigma}}{\partial u^{`k}} \cdot \frac{\partial\tilde{u}^{\tau} + \partial\breve{u}^{\rho}}{\partial u^{`l}}\right] \tag{105.2}$$

$$T^{\mu\nu} = T^{`kl}\left[\left(\frac{\partial\tilde{u}^{\epsilon}}{\partial u^{`k}} + \frac{\partial\breve{u}^{\sigma}}{\partial u^{`k}}\right) \cdot \left(\frac{\partial\tilde{u}^{\tau}}{\partial u^{`l}} + \frac{\partial\breve{u}^{\rho}}{\partial u^{`l}}\right)\right] \tag{106.2}$$

$$T^{\mu\nu} = T^{`kl}\left[\frac{\partial\tilde{u}^{\epsilon}}{\partial u^{`k}}\frac{\partial\tilde{u}^{\tau}}{\partial u^{`l}} + \frac{\partial\tilde{u}^{\epsilon}}{\partial u^{`k}}\frac{\partial\breve{u}^{\rho}}{\partial u^{`l}} + \frac{\partial\breve{u}^{\sigma}}{\partial u^{`k}}\frac{\partial\tilde{u}^{\tau}}{\partial u^{`l}} + \frac{\partial\breve{u}^{\sigma}}{\partial u^{`k}}\frac{\partial\breve{u}^{\rho}}{\partial u^{`l}}\right] \tag{107.2}$$

From the previous equation, we find that the following quantities are identical and represent the dot product of the tangent vectors $\mathrm{d}\tilde{u}^{\epsilon}, \mathrm{d}\breve{u}^{\sigma}$, so they can be abbreviated to the following formula:

$$\frac{\partial\tilde{u}^{\epsilon}}{\partial u^{`k}}\frac{\partial\breve{u}^{\rho}}{\partial u^{`l}} = \frac{\partial\breve{u}^{\sigma}}{\partial u^{`k}}\frac{\partial\tilde{u}^{\tau}}{\partial u^{`l}} = 2\left(\frac{\partial\tilde{u}^{\epsilon}}{\partial u^{`k}}\frac{\partial\breve{u}^{\rho}}{\partial u^{`l}}\right)\cos\theta \tag{108.2}$$

where θ represents the angle between the tangent vectors $\mathrm{d}\tilde{u}^{\epsilon}, \mathrm{d}\breve{u}^{\sigma}$ by substituting from 108.2 into 107.2

$$T^{\mu\nu} = T^{`kl}\left(\frac{\partial\tilde{u}^{\epsilon}}{\partial u^{`k}}\frac{\partial\tilde{u}^{\tau}}{\partial u^{`l}}\right) + T^{`kl}\left(\frac{\partial\breve{u}^{\sigma}}{\partial u^{`k}}\frac{\partial\breve{u}^{\rho}}{\partial u^{`l}}\right) + 2T^{`kl}\left(\frac{\partial\tilde{u}^{\epsilon}}{\partial u^{`k}}\frac{\partial\breve{u}^{\rho}}{\partial u^{`l}}\right)\cos\theta \tag{109.2}$$

The first expression on the right side of the equation represents the energy-momentum tensor for the positive subspace-time in terms of the contravariant components of velocity, as we explained above, while the second expression represents the energy-momentum tensor for the negative subspace-time in terms of the contravariant components of the velocity, the third expression represents the energy and momentum tensors composed of both vectors $\mathrm{d}\tilde{u}^{\epsilon}, \mathrm{d}\breve{u}^{\sigma}$, because the angle can take any value from zero to 180, where there is no distinct angle, so here we take the average value of the angle, which is angle 90, by substituting all of that in the previous equation, taking into account that $\cos 90 = 0$, we get

$$T^{\mu\nu} = \tilde{T}^{\epsilon\tau} + \breve{T}^{\sigma\rho} \tag{110.2}$$

$$\breve{T}^{\sigma\rho} = T^{\mu\nu} - \tilde{T}^{\epsilon\tau} \tag{111.2}$$

Equation (111.2) represents the energy-momentum tensor of negative subspace-time in the special or general case in terms of the total and positive energy-momentum tensor. By substituting the matrix of each tensor from Equations (91.2) and (98.2), we obtain the energy-momentum tensor matrix for the negative subspace-time in the special case.

$$\breve{T}^{\sigma\rho} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{31} & T^{33} \end{bmatrix} - \begin{bmatrix} \tilde{T}^{00} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{112.2}$$

$$\breve{T}^{\sigma\rho} = \begin{bmatrix} \breve{T}^{00} & \breve{T}^{01} & \breve{T}^{02} & \breve{T}^{03} \\ \breve{T}^{10} & \breve{T}^{11} & \breve{T}^{12} & \breve{T}^{13} \\ \breve{T}^{20} & \breve{T}^{21} & \breve{T}^{22} & \breve{T}^{23} \\ \breve{T}^{30} & \breve{T}^{31} & \breve{T}^{31} & \breve{T}^{33} \end{bmatrix} \tag{113.2}$$

From the previous two equations, we can obtain the negative energy density component $\breve{T}^{00}$ in terms of the total energy density and the positive energy density according to the following equation:

$$\breve{T}^{00} = T^{00} - \tilde{T}^{00} \tag{114.2}$$

By substituting the values of each of $T^{00}, \tilde{T}^{00}$ in the previous equation

$$\breve{T}^{11} = \rho_0 \gamma^2 c^2 - \rho_0 \gamma^2 \tilde{V}^2 = \rho_0 \gamma^2 \breve{V}^2 \tag{115.2}$$

By substituting the proper density value in terms of the rest mass, the proper volume, and the number of particles, where the mass transformation $m = m_0 \gamma$,

$$\breve{T}^{00} = \frac{n m_0 \gamma}{\grave{V}_{ol}} \breve{V}^2 \gamma = \frac{nm}{\grave{V}_{ol}} \breve{V}^2 \gamma \tag{116.2}$$

As for the volume transformation in the negative subspace is according to the negative modified Lorentz transformations, so we write the volume in terms of both the total and positive relativistic volume according to the following equation $\breve{V}_{ol} = V_{ol} - \tilde{V}_{ol}$, but $V_{ol} = \grave{V}_{ol} \gamma^{-1}$ and $\tilde{V}_{ol} = \grave{V}_{ol}$, by substituting this, we also get the volume transformation in the negative subspace $V_{ol} = \grave{V}_{ol} \left(\gamma^{-1} - 1 \right)$, by adding the amount $\left(\gamma^{-1} - 1 \right)$ in the numerator and denominator in Equation (116.2):

$$\breve{T}^{00} = \frac{nm}{\grave{V}_{ol} \left(\gamma^{-1} - 1 \right)} \breve{V}^2 \gamma \left(\gamma^{-1} - 1 \right) = \breve{\rho} \breve{V}^2 (1 - \gamma) \tag{117.2}$$

where $\breve{\rho} \breve{V}^2$ represents the negative relativistic energy density, $\breve{\rho}$ the negative relativistic mass density. In the special case, as we explained above in Section 9.2, there are no forces or changes in momentum in the negative subspace-time. So, all other components of the tensor are zero, by substituting this in Equation (113.2), we obtain the following formula for the negative subspace-time tensor matrix:

$$\breve{T}^{\sigma\rho} = \begin{bmatrix} \breve{\rho}\breve{V}^2(1-\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{118.2}$$

In the general case, we use the tensor shown in the Equation (113.2), where the reference frame is accelerating. We can also use the tensor shown in Equation (118.2) in the general case, but as a local tensor, *i.e.*, at a point on the negative Subspace-time curved under the influence of gravitational mass M, where $\gamma = 1\big/\sqrt{2MG/rc^2}$.

By substituting back into Equation (114.2), the value of $\breve{T}^{00}$ from Equation (117.2) and the values of each of $T^{00}, \tilde{T}^{00}$ shown above in terms of ρ_0 in Equations (93.2) and (104.2).

$$\breve{\rho}\breve{V}^2(1-\gamma) = \rho_0 c^2 \gamma^2 - \rho_0 c^2 \tag{119.2}$$

$$\breve{\rho}\breve{V}^2 = \rho_0 c^2 \frac{\gamma^2 - 1}{1-\gamma} \qquad \breve{\rho}\breve{V}^2 \approx -\rho_0 c^2 \gamma \qquad V_S < c \tag{120.2}$$

The previous equation shows us that the relativistic energy density of the dust on the negative subspace-time fabric with respect to the observer O in the special case increases with the increase in the velocity of the reference frame but with a negative value.

2.13. Gravitational Field Equations for Positive and Negative Subspace-Time

The general theory of relativity describes gravity as a curvature in the total fabric of space-time through the Einstein field equations [20]. But as a result of the splitting of the total curved fabric of space-time in the general case into positive and negative subspace-times, which is explained in the first paper, item 2.5, we, therefore, need to describe gravity in each subspace-time using new field equations specific to each subspace-time.

$$R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu} \tag{121.2}$$

where $R^{\mu\nu}$ is the Ricci tensor for the total curvature of space-time, R is scalar curvature (or the Ricci scalar) for the total of space-time ([14], pp. 161-162), $g^{\mu\nu}$ is the metric tensor for the total space-time, G is the universal gravitational constant, $T^{\mu\nu}$ is the energy-momentum tensor on the total space-time as we previously explained. The result of the analysis of the metric tensor in the general case shown in the first paper, Equation (81.1), as well as the analysis of the energy-momentum tensor in the general case shown here in Equation (110.2), by substituting for this in the Einstein gravitational field equations.

$$R^{\mu\nu} - \frac{1}{2} R\left(\tilde{g}^{\epsilon\tau} + \breve{g}^{\sigma\rho}\right) = \frac{8\pi G}{c^4}\left(\tilde{T}^{\epsilon\tau} + \breve{T}^{\sigma\rho}\right) \tag{122.2}$$

Here, we also impose an analysis of the Ricci tensor into two tensors, where each new tensor is proportional to the properties of each subspace-time.

$$\tilde{R}^{\epsilon\tau}+\breve{R}^{\sigma\rho}-\frac{1}{2}R\tilde{g}^{\epsilon\tau}+\frac{1}{2}R\breve{g}^{\sigma\rho}=\frac{8\pi G}{c^4}\tilde{T}^{\epsilon\tau}+\frac{8\pi G}{c^4}\breve{T}^{\sigma\rho} \tag{123.2}$$

We can now look at the last equation as representing the sum of two equations, where each similar tensor set in the upper index represents an independent equation.

$$\tilde{R}^{\epsilon\tau}-\frac{1}{2}R\tilde{g}^{\epsilon\tau}=\frac{8\pi G}{c^4}\tilde{T}^{\epsilon\tau} \tag{124.2}$$

The first equation represents the gravitational field equation in positive subspace-time, and it is called the positive field equation. Because the positive subspace-time in the general case is flat space-time as explained in the first paper item 2.6, therefore $\tilde{R}^{\epsilon\tau}=0$ and the previous equation is reduced to the following formula.

$$-\frac{1}{2}R\tilde{g}^{\epsilon\tau}=\frac{8\pi G}{c^4}\tilde{T}^{\epsilon\tau} \tag{125.2}$$

As for the second equation, it represents the gravitational field equation in negative subspace-time, and it is called the negative field equation and is written in the following formula

$$\breve{R}^{\sigma\rho}-\frac{1}{2}R\breve{g}^{\sigma\rho}=\frac{8\pi G}{c^4}\breve{T}^{\sigma\rho} \tag{126.2}$$

The new field equations for the gravitational field in the inverse relativity model do not represent a substitute for the field equations in general relativity, but they are a logical result after analyzing the metric tensor, energy-momentum tensor, and Ricci tensor. It is an attempt to study the gravitational field in each subspace-time instead of the total space-time. For example, when looking at the first equation, the positive field equation, which describes gravity in the positive subspace-time, as we explained in the first paper, the positive subspace-time is a flat space-time despite the presence of masses that have attraction between them, and it also represents the space-time of causality. Therefore, gravity does not appear in this subspace-time as a curvature, but rather appears as a force that causes attraction. It represents the most appropriate space-time to describe the quantum field of gravity and the behavior of the quantum particles that create the effect of gravity. Thus, the solution of the equation represents a description of a quantum field. While the second equation is the negative field equation, we find that it describes gravity as a curvature only without the presence of a force because the negative subspace-time is a curved spacetime and without causality, as we explained in the first paper. Therefore, the solution to this equation is the same steps as the solution to the field rates in general relativity, *i.e.*, through the scalar curvature, Christopher symbols, etc. We can say that the positive subspace-time is the background that describes the behavior of the quantum field of gravity, while the negative subspace-time describes the curvature arising from the behavior of the quantum field.

3. Results

The result of analyzing the 4D displacement vector on the total space-time fabric in the special and general cases and the splitting of the total space-time fabric into a positive subspace-time and a negative subspace-time is described in the first paper. Here, we also get an analysis of each of the 4D vectors (velocity, acceleration, momentum, and energy-force) on the total space-time fabric into two vectors, one on the positive subspace-time and the other on the negative subspace-time. Through the positive and negative modified Lorentz matrices also shown in the first paper, we obtain the following transformations. Velocity transformations where the speed of the particle decreases in the positive subspace and increases in the negative subspace with respect to a fixed observer (which is a result that contradicts special relativity). See comparison **Table 1** and also the set of Equations (15.2) and (53.2). Relativistic momentum transformations, where the momentum remains constant in the positive subspace (which is a result that contradicts special relativity) and increases in the negative subspace with the increase in the speed of the reference frame with respect to a fixed observer. See set of Equations (31.2) and (68.2). Relativistic total energy transformations, where energy decreases in the positive subspace (this is the opposite result of special relativity) and increases in the negative subspace with respect to a fixed observer, see comparison **Table 2** and also Equations (32.2) and (71.2). Force transformations, where the forces decrease in the positive subspace (this is also a result opposite to special relativity), and are non-existent in the negative subspace with respect to a fixed observer. See the set of Equations (40.2) and (77.2). The previous transformations in each subspace represent new relativistic mechanics that differ from relativistic mechanics in the total space of special relativity in terms of the formulation of laws and results as well. In the general case, we also have new types of energy-momentum tensors, one for positive subspace-time and the other for negative subspace-time. See equations (101.2) and (118.2), where we find that the energy density decreases in positive subspace-time (this is an opposite result of general relativity) and increases in negative subspace-time., and we also get new gravitational field equations in each subspace-time. See comparison **Table 3** and look at Equations (125.2) and (126.2), which differ from Einstein's gravitational field equations in general relativity in terms of mathematical formulation and solution methods as well.

4. Discussions

In the positive subspace, we find that physical quantities such as velocity, acceleration, momentum, and forces with respect to the observer O, shown in the following set of Equation (127.2), change with their counterparts changing with respect to the observer O' as a result of any event or causality that occurs to the particle. For example, when a collision between two particles occurs, the velocity of each particle changes, it has an instantaneous acceleration, the momentum of the particle also changes, and it has a force with respect to the observer O’. We also find a similar change in the velocity, momentum, and forces of each particle with respect to the observer O in the positive subspace. This means that the relativistic mechanics of the particles here are linked to the causality that occurs in this subspace (collision), that is, it possesses the geometric properties of the positive subspace described in the first paper, item 2.3. Thus, we have obtained a new relativistic mechanics that we call positive relativistic mechanics.

$$\begin{array}{llll} \tilde{u}_x = u`_x \gamma^{-1} & \tilde{a}_x = a`_x \gamma^{-2} & \tilde{p}_x = p`_x & \tilde{F}_x = F`_x \gamma^{-1} \\ \tilde{u}_y = u`_y \gamma^{-1} & \tilde{a}_y = a`_y \gamma^{-2} & \tilde{p}_y = p`_y & \tilde{F}_y = F`_y \gamma^{-1} \\ \tilde{u}_z = u`_z \gamma^{-1} & \tilde{a}_z = a`_z \gamma^{-2} & \tilde{p}_z = p`_z & \tilde{F}_z = F`_z \gamma^{-1} \end{array} \tag{127.2}$$

As for the negative subspace, we find that the physical quantities such as velocity, acceleration, momentum, and forces with respect to the observer O, shown in the following Equation (128.2), are constant while the velocity of the reference frame remains constant, that is, they do not change with the change of their counterparts with respect to the observer O’. This means that these physical quantities of the particle here are not linked to any causality that happens to this particle. In other words, the relativistic mechanics of the particles here have the geometric properties of the negative subspace described in the first paper, item 2.4. Thus, we have also obtained a new relativistic mechanics that we call negative relativistic mechanics.

$$\begin{array}{llll} \breve{u}_x = V_s & \breve{a}_x = 0 & \breve{p}_x = mV_s & \breve{F}_x = 0 \\ \breve{u}_y = 0 & \breve{a}_y = 0 & \breve{p}_y = 0 & \breve{F}_y = 0 \\ \breve{u}_z = 0 & \breve{a}_z = 0 & \breve{p}_z = 0 & \breve{F}_z = 0 \end{array} \tag{128.2}$$

Table 1. Comparison between velocity transformations in special relativity and inverse relativity.

Transformations of Velocity	Special Relativity	Inverse Relativity
Equations	Velocity Transformation in Total space $u_x = \frac{V_s + u`_x}{1 + \frac{V_s u`_x}{c^2}}$ $u_y = \frac{u`_y}{\gamma\left(1 + \frac{V_s u`_x}{c^2}\right)}$ $u_z = \frac{u`_z}{\gamma\left(1 + \frac{V_s u`_x}{c^2}\right)}$	Velocity Transformation in Positive subspace $\tilde{u}_x = u`_x \gamma^{-1}$ $\tilde{u}_y = u`_y \gamma^{-1}$ $\tilde{u}_z = u`_z \gamma^{-1}$ Velocity Transformation in Negative subspace $\breve{u}_x = V_s$ $\breve{u}_y = 0$ $\breve{u}_z = 0$

Table 2. Comparison between the total relativistic energy in special relativity and inverse relativity.

Transformations of Total Energy	Special Relativity	Inverse Relativity
Equations	Total Relativistic Energ $u\grave{}_x = 0$ $E = E\grave{}\gamma$ $\gamma = \frac{1}{\sqrt{1 - \frac{V_S^2}{c^2}}}$	Positive Relativistic Energy $\tilde{E} = E\grave{}\gamma^{-1}$ Negative Relativistic Energy $u\grave{}_x = 0$ $\breve{E} = E\grave{}\frac{V_s}{c}(\gamma - 1)$

Table 3. Comparison between the gravitational field equations in general relativity and inverse relativity.

Gravitational Field	General Relativity	Inverse Relativity
Equations	Total space-time $R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = \frac{8\pi G}{c^4}T^{\mu\nu}$	Positive subspace-time $-\frac{1}{2}R\tilde{g}^{\epsilon\tau} = \frac{8\pi G}{c^4}\tilde{T}^{\epsilon\tau}$ Negative subspace-time $\breve{R}^{\sigma\rho} - \frac{1}{2}R\breve{g}^{\sigma\rho} = \frac{8\pi G}{c^4}\breve{T}^{\sigma\rho}$

In the first paper, we explained that the inverse relativity model paves the way for solving problems in which both special and general relativity failed through the geometric properties of each subspace-time. However, the first paper included transformation coordinates of the space and time for each subspace-time in the special case, and the metric tensor for each subspace-time in the general case, and this is not sufficient to solve problems such as relativistic thermodynamics [21] or quantum gravity, Whereas, treating thermodynamics according to the new model requires relativistic mechanics specific to each subspace-time, and treating quantum gravity according to the new model also requires describing the energy-momentum tensor in each subspace-time, as well as the gravitational field equations specific to each subspace-time. Therefore, the second paper represents a completion of the inverse relativity model.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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