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The Hawking Hubble Temperature as the Minimum Temperature, the Planck Temperature as the Maximum Temperature, and the CMB Temperature as Their Geometric Mean Temperature

Espen Gaarder Haug[1], **Eugene Terry Tatum**[2]

[1]Tempus Gravitational Laboratory, Ås, Norway
[2]Independent Researcher, Bowling Green, KY, USA
Email: espenhaug@mac.com, ett@twc.com

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Abstract

Using a rigorous mathematical approach, we demonstrate how the Cosmic Microwave Background (CMB) temperature could simply be a form of geometric mean temperature between the minimum time-dependent Hawking Hubble temperature and the maximum Planck temperature of the expanding universe over the course of cosmic time. This mathematical discovery suggests a re-consideration of $R_h = ct$ cosmological models, including black hole cosmological models, even if it possibly could also be consistent with the Λ-CDM model. Most importantly, this paper contributes to the growing literature in the past year asserting a tightly constrained mathematical relationship between the CMB temperature, the Hubble constant, and other global parameters of the Hubble sphere. Our approach suggests a solid theoretical framework for predicting and understanding the CMB temperature rather than solely observing it.[1]

Keywords

Hawking Temperature, Planck Temperature, CMB Temperature, Geometric Mean, Compton Wavelength, Hubble Sphere, Cosmological Models

1. All Electromagnetic Energy Can Be Expressed through the Compton Wavelength

Arthur Holly Compton [2], in 1923, gave us a formula for the Compton

[1]This paper is a strongly improved version of the preprint [1] put online on 2 Dec 2023.

wavelength of a particle

$$\lambda = \frac{h}{mc} \tag{1}$$

Furthermore, the reduced Compton wavelength is equal to $\bar{\lambda} = \frac{\hbar}{mc}$, where $\hbar$ is the reduced Planck constant $\hbar = \frac{h}{2\pi}$. Since pure energy can be expressed as equivalent rest-mass energy $m = \frac{E}{c^2}$, then we can also write

$$\bar{\lambda} = \frac{\hbar}{mc} = \frac{\hbar}{\frac{E}{c^2}c} = \frac{\hbar c}{E} \tag{2}$$

Haug [3] has recently at length discussed and demonstrated how the Compton wavelength likely is the true matter wavelength and how the de Broglie wavelength likely is only a mathematical derivative of this. We refer the reader to that paper for an in-depth discussion on this. Even if one does not agree with this, one simply needs to understand that the formulas just represented are valid.

2. The Maximum and Minimum Temperatures and Their Links to the Shortest and Longest Reduced Compton Wavelengths

The Planck [4] [5] temperature is given by

$$T_p = \frac{1}{k_b}\sqrt{\frac{\hbar c^5}{G}} = \frac{E_p}{k_b} = \hbar f_p \frac{1}{k_b} = \hbar \frac{c}{l_p}\frac{1}{k_b} \approx 1.42\times 10^{32}\ \text{K} \tag{3}$$

This means that the Planck temperature is the Planck frequency times the Planck constant, which is the Planck energy expressed as a temperature by dividing the Planck energy by the Boltzmann constant.

The Planck length is assumed by many physicists [6]-[9] to be the shortest meaningful length. This also means that the Planck frequency is the highest possible frequency, and this again indicates that the Planck energy is the highest possible energy for a photon: $E_p = m_p c^2 = \hbar \frac{c}{l_p} = \hbar f_p$, where f_p is the Planck frequency. Furthermore, since the Planck temperature simply is the Planck energy converted to the temperature scale of Kelvin by dividing it by the Boltzmann constant, this implies that the Planck temperature is likely the highest possible temperature, as also suggested by multiple researchers [10], even if there is still some question as to whether the maximum temperature could be somewhat lower [11] or somewhat higher [12] than the Planck temperature. It has recently been demonstrated that the Planck length is closely related to gravity, as the Planck length and other Planck units can be extracted directly from gravitational observations without knowledge of G or $\hbar$, see [13] [14].

The reduced Compton wavelength of the Planck energy is

$$\bar{\lambda} = \frac{\hbar}{m_p c} = \frac{\hbar}{E_p} = l_p \tag{4}$$

Alternatively, we could modify the Planck temperature slightly by assuming that the shortest possible wavelength is related to the radius of a Planck mass micro black hole, which has a Schwarzschild radius of $R_s = \frac{2Gm_p}{c^2} = 2l_p$ and a circumference of $2\pi R_s = 4\pi l_p$ with a corresponding (circumference) Planck temperature of:

$$T_{p,BH} = \hbar \frac{c}{4\pi l_p} \frac{1}{k_b} \approx 1.13 \times 10^{31} \text{ K} \tag{5}$$

This is exactly twice that of the Hawking temperature of a Planck mass micro black hole:

$$T_{Hw} = \frac{c^3 \hbar}{k_b 8\pi G m_p} = \frac{\hbar c}{k_b 4\pi \frac{2Gm_p}{c^2}} = \frac{\hbar c}{4\pi R_s} \frac{1}{kb} = \frac{1}{2} T_{p,BH} \tag{6}$$

If the maximum temperature is linked to the shortest possible reduced Compton wavelength, and if the Planck energy is the highest localized energy possible, then the lowest possible energy must be linked to the longest possible wavelength in the universe. We assert that the longest possible wavelength is the diameter of the Hubble sphere. This implies that the minimum temperature is related to what we can call the "Hubble frequency," and it must be given by:

$$T_{\min} = \hbar \frac{c}{2R_H} \frac{1}{k_b} \approx 8.3 \times 10^{-30} \text{ K} \tag{7}$$

The term $\frac{c}{2R_H}$ can be referred to as the Hubble frequency $f_H \approx 1.09 \times 10^{-18}$ per second. Consequently, the minimum temperature is simply given by $T_{\min} = \hbar f_H \frac{1}{k_b}$.

This represents the lowest frequency above zero that one can observe, as no wavelength can be longer than the diameter of the universe or certainly no longer than the circumference of the universe, as we will soon also look at.

As a function of time, the minimum temperature can be written as:

$$T_{\min}(t) = \hbar \frac{c}{2R_{H,t}} \frac{1}{k_b} = \hbar \frac{c}{2R_{s,t}} \frac{1}{k_b} = \frac{1}{2\pi} T_{Hw} \tag{8}$$

That is, the Hubble radius is time-dependent and equal to $R_{H,t} = \frac{Gm_p}{c^2} \frac{t}{t_p}$, or simply $R_H = ct$. This means that the minimum temperature at any time is equal to $\frac{1}{2\pi}$ times the Hawking temperature of the black hole universe at any given time.

In the Λ-CDM model, the diameter of the universe extends far beyond the Hubble radius R_H due to the assumption of the expansion of space, including accelerated expansion. However, there are various alternative cosmological models within the linear $R_H = ct$ category, as illustrated in, for example, [15]-[21]. Yet another concept to consider is the idea that the Hubble sphere is a black hole with an event horizon at the Hubble radius. In this scenario, no wavelength can be longer than the diameter of the Hubble sphere or, at the very most, the circumference of the Hubble sphere. This imposes a maximum limit on the wavelength of electromagnetic radiation and, consequently, a minimum energy above zero—a sort of "energy gap," and the lowest even theoretical measurable energy above zero. The temperature gap found here is in line with the new gravitational field theory of Haug [22] and also recent discoveries with respect to the Planck constant [23]. The notion that the observable universe could be inside a black hole is not a recent idea. It was proposed as early as 1972 by Pathria [24] and later in 1994 by Stuckey [25]. This idea, despite being in conflict with the Λ-CDM model, continues to be a topic of ongoing discussion, as evidenced by recent publications such as [26]-[28].

We observe that if we multiply this minimum temperature by $\frac{1}{2\pi}$, it is surprisingly identical to the Hawking temperature [29] when the mass in the Hawking temperature is the critical Friedmann mass $M_c = \frac{c^3}{2GH_0}$:

$$T_{Hw} = \frac{\hbar c^3}{k_b 8\pi G M_c} = \hbar \frac{c}{2R_H} \frac{1}{k_b} \frac{1}{2\pi} = \hbar f_H \frac{1}{k_b} \frac{1}{2\pi} \approx 1.32 \times 10^{-30} \text{ K} \tag{9}$$

The difference of $\frac{1}{2\pi}$ between the prediction from Equation (7) and the Hawking temperature formula could have multiple reasons. Even if not ideal, adjusting the end result with a factor like 2π is not abnormal. For example, Adler *et al.* [30] did so and simply called it a "calibration factor." The maximum reduced Compton wavelength could also be seen as the circumference of the Hubble sphere instead of the Hubble diameter, making the difference only a factor of 2 relative to Hawking temperature Equation (9) as we then get a minimum temperature of:

$$T_{\min} = \hbar \frac{c}{2\pi R_H} \frac{1}{k_b} \approx 2.64 \times 10^{-30} \text{ K} \tag{10}$$

So, we will assume, by Hawking temperature Equation (9), that the minimum temperature is the Hawking temperature.

The small difference mentioned above could be because the Hawking radiation is derived from the Schwarzschild metric. Other metrics, such as the Kerr [31] and Kerr-Newman [32] [33] metric, as well as the recent Haug and Spavieri metric [34], can likely also be used to derive similar temperatures; but, likely, there will only be small differences from the Hawking temperature. We do not have the final answer as to why there is a small difference of 2π or π in the formulas, so we will, like Adler *et al.*, for the moment, call whatever the small difference may be a calibration factor.

For now, let's assume that the Hawking temperature and the minimum temperature calculated from this alternative method, based on the Compton wavelength and the Compton frequency, indeed represents the minimum temperature inside a black hole. This would mean that the Hawking temperature possibly does not solely represent radiation emitted from a black hole, but could also be the minimum temperature (above zero) that could potentially be observed at any point inside the Hubble sphere. We can call this the Hawking Hubble temperature.

3. The CMB Temperature as a Geometric Mean of the Maximum and Minimum Temperature in the Hubble Sphere

Since the discoveries by Hubble [35] and Lemaitre [36], as well as the theoretical work of Friedmann, there has been great progress in cosmology, both experimental and theoretical. However, the standard Λ-CDM model, as well as most other cosmological models, has no way to predict the current CMB temperature.

Mean temperature plays an important role in various fields, including climate science, fluid dynamics, and biophysics [37]. To our knowledge, mean temperatures have not been linked to the CMB temperature, except that the 2.725K measurement can be called a type of mean (*i.e.*, average) temperature in all of the empty space of the Hubble sphere. To establish a more solid theoretical connection to the CMB as a mean temperature, a solid mathematical and physical foundation is necessary. It is not sufficient to simply take the mean of some temperatures and call it the mean temperature; as we will suggest, one must comprehend the Planck temperature, cosmic temperatures, and their inter-relationships in terms of the only variable that differentiates different energy levels—namely, the electromagnetic wavelength. Additionally, we know from mathematics and statistics that various types of means exist.

The geometric mean traces back to the Pythagoreans, who defined the three most commonly used means even of our time–namely, arithmetic mean, geometric mean, and harmonic mean (see [38]). The geometric mean, indicating a central tendency of a finite set of real numbers by using the product of their values, has wide applications across diverse fields, from economics, finance, engineering, nuclear medicine, informatics, ecology, surface and groundwater hydrology, geoscience, geomechanics, machine learning, and chemical engineering, see [39]. The geometric mean is utilized for various applications and challenges also in physics. For instance, Henderson [40] demonstrated the use of the geometric mean for problems in gas dynamics, Zhang *et al.* [41] employed the geometric mean density of states in one-dimensional nonuniform systems, and Yamagami [42] relied on the geometric mean of states and transition amplitudes, see also [43]. The potential role of geometric means in average temperatures should not be surprising. The reason that maximum and minimum temperatures in the Hubble sphere have not previously been linked to the CMB as a geometric mean temperature is likely to be that it has only recently been discovered that the reduced Compton wavelength plays a much more central role in energy, matter, and even gravity than previously thought.

Assume that the measured CMB temperature is somehow related to some type of mean value between the maximum and minimum allowed temperature in the Hubble sphere. The longest possible reduced Compton wavelength in the universe, or at least in a $R_H = ct$ universe as well as a black hole Hubble sphere, is the diameter of the Hubble sphere, so we have

$$\bar{\lambda}_{\text{Maximum}} = 2R_H. \tag{11}$$

Furthermore, the shortest possible reduced Compton wavelength is assumed to be the Planck length, so we have

$$\bar{\lambda}_{\text{Minimum}} = l_p. \tag{12}$$

The geometric mean of the shortest and longest reduced Compton wavelength is given by

$$\bar{\lambda}_{gm} = \sqrt{\bar{\lambda}_{\text{Maximum}}\bar{\lambda}_{\text{Minimum}}} = \sqrt{2R_H l_p} \tag{13}$$

we can call this wavelength: $\bar{\lambda}_{gm}$, the geometric mean reduced Compton wavelength of the observable universe. We find the temperature correlating with this wavelength by taking the energy of its frequency and simply dividing that by the Boltzmann constant. This gives

$$T_{gm} = \hbar \frac{c}{\bar{\lambda}_{gm}} \frac{1}{k_b} = \hbar f_m \frac{1}{k_b} \approx 34.27 \text{ K} \tag{14}$$

If we divide this by a calibration factor of 4π, and use the recent Hubble constant value given by Kelly *et al.* [44] of $66.6^{+4.1}_{-3.3}$ (km/s/Mpc), we obtain the CMB temperature of

$$T_{gm} = \hbar \frac{c}{\sqrt{\bar{\lambda}_{max}\bar{\lambda}_{min}}} \frac{1}{k_b 4\pi} = \hbar \frac{c}{\bar{\lambda}_{gm}} \frac{1}{k_b 4\pi} = \hbar f_m \frac{1}{k_b 4\pi} \approx 2.72^{+0.082}_{-0.069} \text{ K} \tag{15}$$

Or, if we use the circumference of the Hubble sphere instead of the diameter as the longest possible wavelength and the Planck circumference of a micro black hole instead of the Planck length as the minimum wavelength, then we get:

$$T_{gm} = \hbar \frac{c}{\sqrt{\bar{\lambda}_{max}\bar{\lambda}_{min}}} \frac{1}{k_b 2} = \hbar \frac{c}{\bar{\lambda}_{gm}} \frac{1}{k_b 2} \approx 2.72^{+0.082}_{-0.069} \text{ K} \tag{16}$$

So here, we only need to use a calibration factor of $\frac{1}{2}$ to get the Hubble temperature very close to measured CMB temperature values; see, for example, [45]-[48] (see also the **Appendix**). We are convinced that this is not simply a coincidence. The CMB temperature seems indeed to simply be related to the geometric mean of the shortest and longest wavelengths possible within the observable universe. Since the only thing that differentiates the different energy levels of electromagnetic radiation (a single beam of photons) is the wavelength, energy that corresponds to the geometric mean of the shortest and longest wavelengths, expressed as temperature (simply by dividing it by the Boltzmann constant), can be termed the geometric mean temperature. This suggests that the CMB temperature is simply the geometric mean temperature between the lowest and highest possible temperatures in the Hubble sphere, but through our reduced Compton wavelength approach. In our view, this is quite revolutionary, as it also points in the direction that $R_H = ct$ cosmological models, as well as black hole cosmological models, seem to be supported over the Λ-CDM model. However, as we will also show, the Λ-CDM model should not yet be excluded.

Our new discovery also seems to be related to recent breakthroughs in the theoretical foundation of the CMB temperature linked to the Planck scale. In their black hole cosmological model, Tatum *et al.* [49] proposed the following formula for the CMB temperature in 2015:

$$T_{CMB} = \frac{\hbar c^3}{k_b 8\pi G \sqrt{M_c m_p}} \tag{17}$$

In direct comparison to the Hawking temperature formula, the M in the denominator is simply changed to $\sqrt{M_c m_p}$, where M_c is the critical mass in the Friedmann [50] universe. Making use of the Schwarzschild formula, Tatum *et al.* also re-wrote their formula in the form of

$$T_{CMB} = \frac{\hbar c}{k_b 4\pi \sqrt{R_h R_{pl}}} \tag{18}$$

Their use of geometric means in these formulae is now obvious. Recently, it has been published that these Tatum *et al.* formulae are derivable also from the Stefan-Boltzmann law [51] [52]. Furthermore, it has been demonstrated that rewriting them to find the Hubble constant (by substituting H_0 for c/R_0) and then deriving the Hubble constant from published CMB temperature studies dramatically increases the accuracy in predicting the Hubble constant, see [53]. Tatum *et al.* define R_{pl} above as the Schwarzschild radius of the Planck mass, so this means $R_{pl} = R_s = \frac{2Gm_p}{c^2} = 2l_p$ which also means that their formula easily can be rewritten as

$$T_{CMB} = \frac{\hbar c}{k_b 4\pi \sqrt{2R_h l_p}} = \hbar f_m \frac{1}{k_b}\frac{1}{2} \approx 2.725\ \text{K} \tag{19}$$

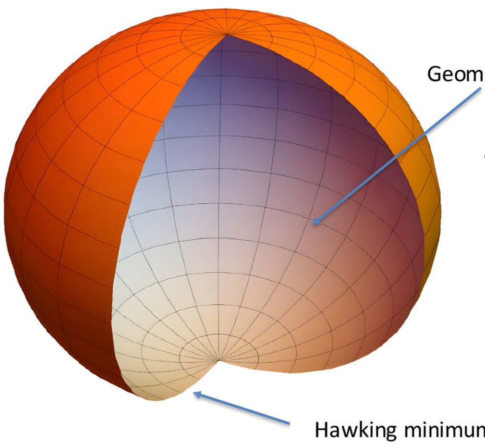
where f_m is the reduced Compton frequency of energy with a wavelength consisting of the geometric mean wavelength from the lowest and highest possible temperatures in the Hubble sphere. The Tatum *et al.* formula immediately above gives an identical result to that given in our last geometric mean temperature formula. One should remember that the use of the Schwarzschild formula for substitutions between mass and radius is allowable in a black hole cosmology model. One can even use such a substitution in the Hawking black hole temperature formula to arrive at the following Equation (by realizing that the mass in a Schwarzschild black hole must be $M = \frac{c^2 R}{2G}$):

$$T_{Hw} = \frac{\hbar c^3}{k_b 8\pi G M} = \frac{\hbar c^3 2G}{k_b 8\pi G c^2 R} = \hbar \frac{c}{2\pi R}\frac{1}{2k_b} \tag{20}$$

This result is identical to Equation (6), so no calibration factor is needed.

Our new geometric mean methodology in this paper arrives at the same formulae using a very different approach in comparison to Tatum *et al.*: by focusing on the Compton wavelength and Compton frequency in matter and energy, and by taking the geometric mean between the maximum and minimum reduced Compton wavelengths, we then examine the temperature to which this leads. This provides a new perspective on the Hawking temperature potentially being the minimum temperature in a black hole, the Planck temperature being the maximum temperature, and the average temperature inside the black hole being the geometric mean temperature in the way described in detail above.

Figure 1 demonstrates the Hubble sphere, where we show the minimum temperature that is likely in the Hubble sphere surface and the CMB temperature, which is the geometric mean temperature in the Hubble sphere. The CMB temperature varies according to which cosmic epoch we are considering. As with the Hubble sphere minimum temperature, the CMB temperature was a higher temperature in the past than it is now. Cosmic mass and radius values were presumably smaller the earlier the cosmic epoch.



$$T_{gm} = \hbar \frac{c}{\sqrt{\bar{\lambda}_{max}\bar{\lambda}_{min}}} \frac{1}{k_b 2} = \hbar \frac{c}{\bar{\lambda}_{gm}} \frac{1}{k_b 2} \approx 2.72^{+0.082}_{-0.069}\ K$$

$$T_{Hw} = \frac{\hbar c^3}{k_b 8\pi G M} = \hbar \frac{c}{2\pi R} \frac{1}{2k_b}$$

Figure 1. This figure illustrates one possible geometric interpretation of our approach. The Hawking temperature, besides representing Hawking radiation, also seems to be consistent with a minimum temperature in the Hubble sphere, which could be at the very surface of the Hubble sphere. The current CMB temperature is the geometric mean temperature related to the minimum possible wavelength (the Planck length) and the maximum possible wavelength, which is related to the Hubble diameter. This approach seems to be able to precisely predict the CMB temperature now and in the past. Such a geometric picture can likely be consistent with multiple cosmological models, in particular, black hole cosmological models, which will be discussed in the following section.

4. Hubble Sphere in Growing Black Hole Models

Two of the most interesting cosmological models likely to benefit from using a geometric mean CMB temperature approach are growing black hole models and $R_h = ct$ models. A growing black hole model is illustrated in **Figure 2**.

The Tatum *et al.* [49] model appears to have been the first cosmological model that accurately predicted the Hubble constant value using only the CMB temperature as an input value. This model basically assumes that the current Hubble sphere started out as a Schwarzschild black hole continually growing in mass and radius in a way consistent with a type of $R_h = ct$ model, where the Hubble radius is equal to the Schwarzschild radius ($R_h = R_s$) at any cosmic time *t*. On the other hand, the Lambda-CDM model does not have the mathematics or geometry that can predict today's CMB temperature from a known Hubble constant value or vice-versa. That said, we do not exclude yet that the geometric mean approach between minimum and maximum temperatures, which we introduce herein, might also be incorporated into the Lambda-CDM model. However, if so, we suspect that it might be an overly complicated process, something we leave to future research to find out.

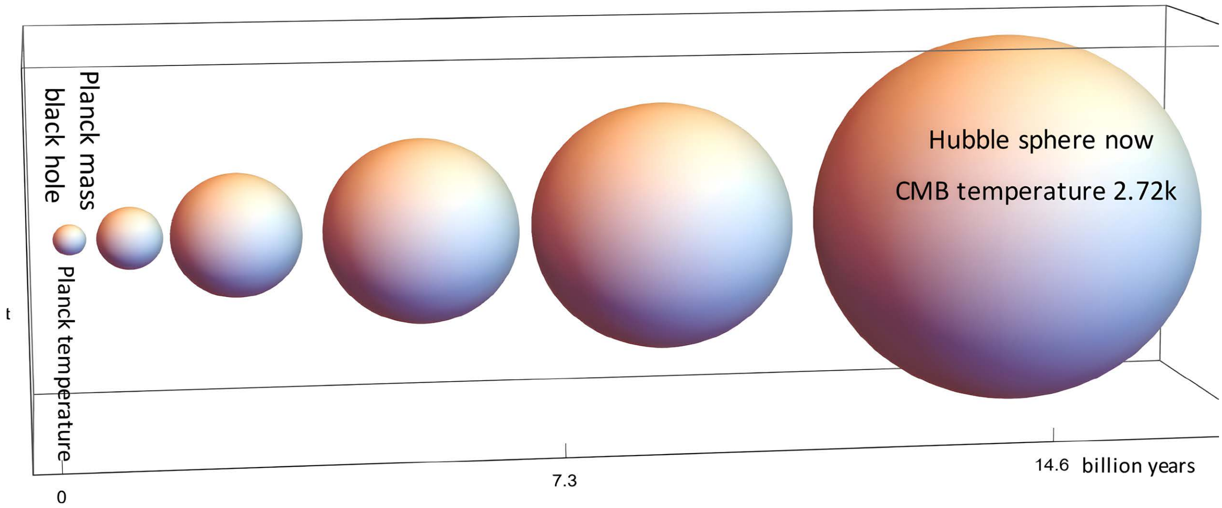


Figure 2. This figure illustrates a universe that started as a Planck mass black hole and then grew into today's Hubble sphere. The black hole radius, which is also the Hubble radius, grows at the speed of light *c*.

The growing black hole model of Tatum *et al.* is interesting in several other respects, not the least of which is the 4-axis log graph it implies with respect to the growth and expansion of the cosmological black hole. See **Figure 3**.

In such a model, expansion is commensurate with mass growth over cosmic time. This is very different from the Λ-CDM model, but it may be worth considering that there could be ongoing matter production coming from the cooling cosmic vacuum under continuing adiabatic expansion. If so, then this might suggest a new way to approach the cosmological constant problem. However, such is beyond the scope of the present paper.

Figure 4 illustrates such a growing black hole model. The universe in this model starts as a Planck mass black hole at the Planck temperature, and presumably arises from what we might call a "primordial Planck mass soup" at the Planck density. The radius of the black hole universe grows at the speed of light *c*, as one might expect for a $R_h = ct$ model. The CMB temperature, which is the temperature inside the black hole, is always simply the geometric mean between the

Planck temperature (maximum temperature) and the Hawking Hubble minimum temperature, which again is directly linked to the Hawking temperature at the continually expanding cosmic horizon surface of the Hubble sphere.

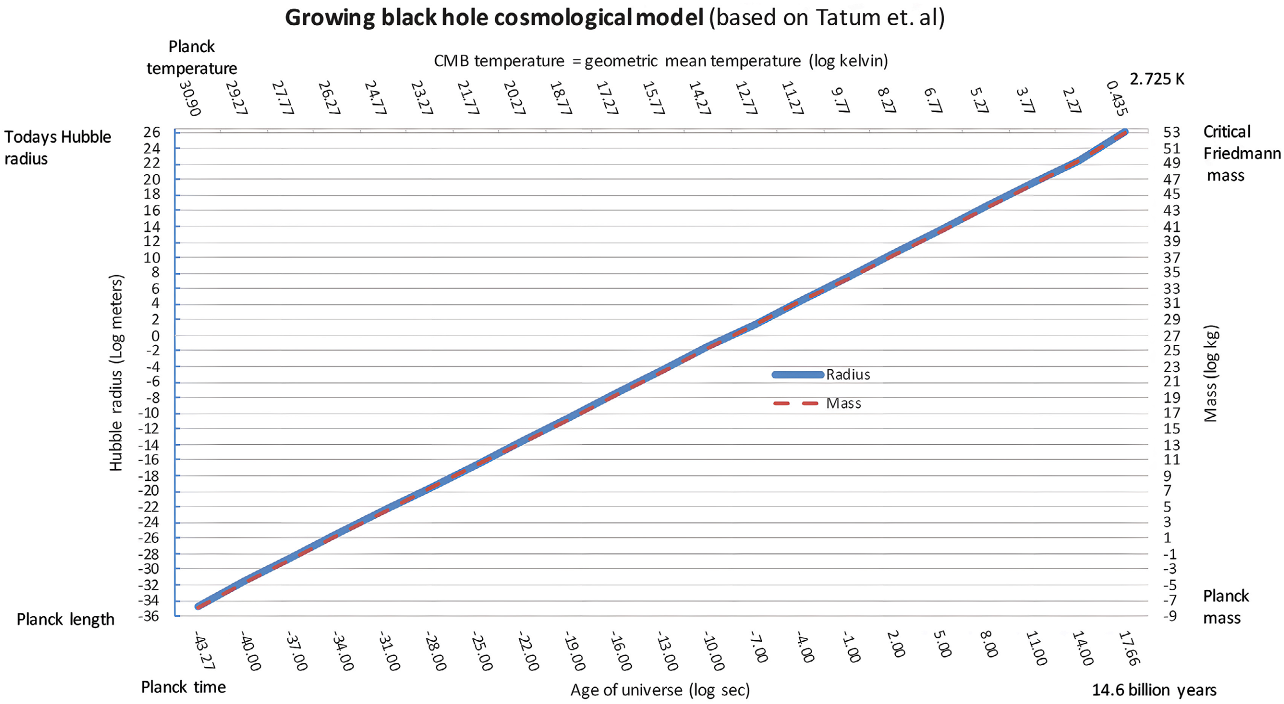


Figure 3. Growing black hole cosmology model.

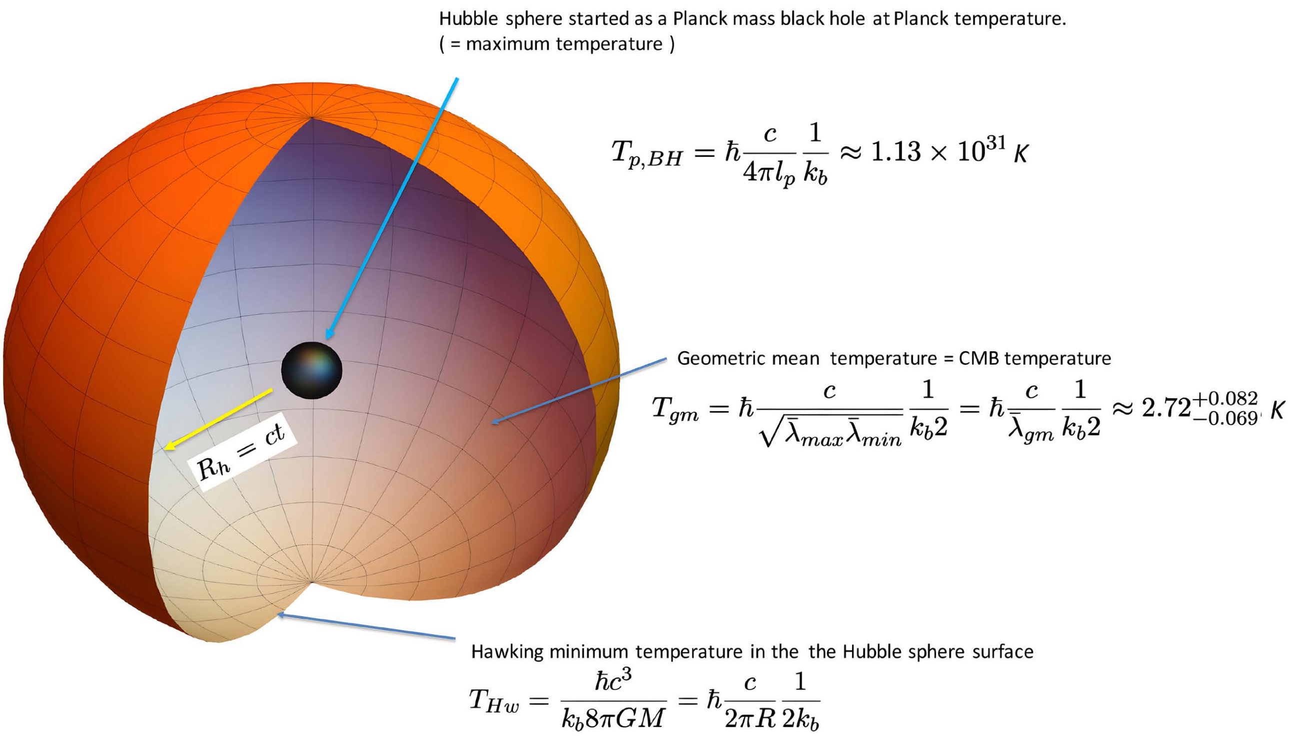


Figure 4. Hubble sphere that is a growing black hole.

Geometric methods are very general, as they rely on core geometric principles, specifically the geometric mean in this case. The same approach can, therefore, likely be applied to a series of different black hole models, such as Kerr [31] black holes, Kerr-Newman [32] black holes, and Haug-Spavieri metric [34] black holes. Most, if not all, of these could be extended to growing black hole models somewhat similar to that of Tatum *et al.* While this must be carefully investigated, the main framework for how to incorporate CMB temperature in them based on our geometric mean approach is already laid out in this paper. The general principles described herein should still hold, but the exact values and interpretations of different black hole universes will naturally vary to some extent. We leave that for further work and investigation.

5. Information Horizon Universes and Steady-State Black Hole Type Universes

As the geometric mean approach is mostly based on core principles in geometry and energy (such as simply $E = hf$), it is very likely that our approach can be applied to a series of different cosmological models. Each model must naturally be carefully investigated and checked against many observations, as well as other models such as Λ-CDM before one can come to any conclusions and a consensus view. However, we would like to mention a few more models that are likely consistent with the geometric mean temperature approach just to demonstrate its possible broad application for cosmological models.

Above, we briefly described growing black hole models, where one model, the Tatum *et al.* model, has already been linked to CMB and Hubble constant theoretical predictions that can also be explained by our geometric mean approach, both in the present and as a dynamic model wherein the Hubble sphere keeps expanding. Another perspective would be a steady-state black hole-type universe where every point in the universe is ultimately linked to Planck mass particles popping into and out of existence (see [22]), and where the Hubble radius is more of a type of information horizon. It is important to be aware that any universe with infinite spatial extension (extending infinitely in space), if it has even a minimum energy density, will, at least from a mathematical point of view, have an event horizon as seen from any point in space. Therefore, the observable universe will be different from the total universe.

The exact event horizon and its interpretations can naturally vary between different metric solutions to Einstein's [54] field equation. Such an event horizon could, under certain assumptions, simply also be interpreted as an information horizon, preventing any information from reaching us beyond such a radius.

Such a steady-state model of the universe, wherein the center is everywhere and there is an event horizon at the Hubble radius everywhere, simply due to the energy density of the universe, is illustrated in **Figure 5**. Such models would then need another form of dynamics to explain them, for example, as to why the predicted CMB temperature would vary as one looks farther out. One possibility could be that the energy density actually varies through the Hubble sphere, but other models of cosmological redshift might also suggest an alternative explanation. We just point this out to illustrate that our geometric mean approach could potentially be consistent with multiple cosmological models.

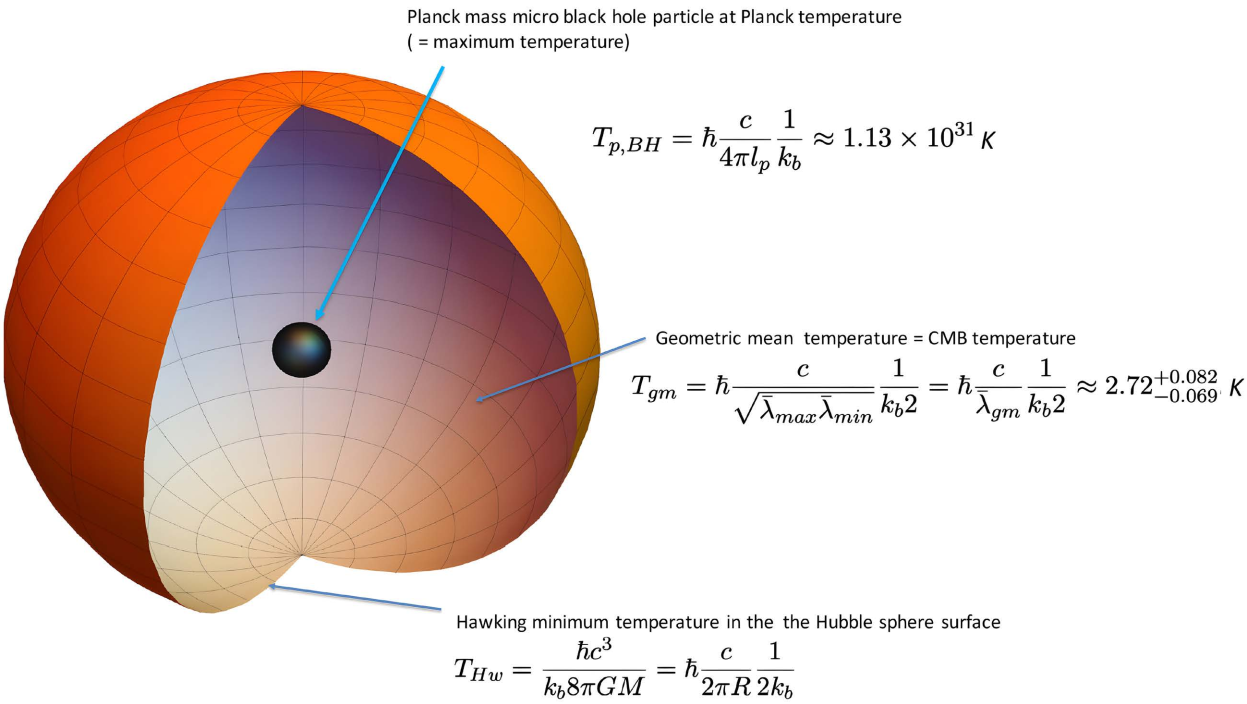


Figure 5. This figure illustrates a possible steady-state universe where the center of the universe is everywhere, and all matter is ultimately linked to Planck mass particles. In this model, the Hubble radius serves as an information horizon because the universe has a minimum energy density everywhere. We mention this to demonstrate that our geometric mean CMB temperature approach could potentially be applied to multiple types of cosmological models.

6. Multiple Possible Mechanisms by Which the CMB Temperature Could Be Linked to the Planck Temperature

Adler, Chen, and Santiago [30] claim that "*In the current standard viewpoint, small black holes are believed to emit black body radiation at the Hawking temperature.*" This is something we agree on, and Haug [22] [55] has recently indicated, based on quantum gravity theory, that all matter may ultimately be derived from Planck mass particles popping into and out of existence, lasting only the Planck time. These Planck mass particles should have the mathematical properties of micro black holes, and their energy could simply be the currency of such interchanges. Perhaps they are largely responsible for the proposed quantum fluctuations in vacuum energy. This speculation provides us with a possible underlying deeper theory of how the Cosmic Microwave Background (CMB) temperature could be related to the Planck temperature.

The Planck temperature, the maximum temperature that could exist in space, could be present only in Planck-sized areas of space, popping into and out of existence. Even an electron or proton is enormous in terms of spatial dimensions compared to the radius of a Planck mass particle micro black hole. Perhaps all of our measurements of the CMB temperature might be of photons radiating from Planck mass vacuum particles popping into and out of existence. Our approach appears to be consistent with a new way to quantize general relativity theory; Einstein's field equation, for example, can be rewritten as [52] [55]-[57]:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi l_p^2}{\hbar c} T_{\mu\nu}. \tag{21}$$

This rewritten form of the field equation gives all of the same results as before, but it leads, for example, to a rewritten Schwarzschild metric of the following form

$$\mathrm{d}s^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 \mathrm{d}t^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} \mathrm{d}r^2 + r^2 g\Omega^2$$

$$\mathrm{d}s^2 = -\left(1 - \frac{2l_p}{r}\frac{l_p}{\bar{\lambda}_M}\right) c^2 \mathrm{d}t^2 + \left(1 - \frac{2l_p}{r}\frac{l_p}{\bar{\lambda}_M}\right)^{-1} \mathrm{d}r^2 + r^2 g\Omega^2 \tag{22}$$

where $\bar{\lambda}_M$ is the reduced Compton wavelength of the mass *M*, and $g\Omega^2 = \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right)$. The term $\frac{l_p}{\bar{\lambda}_M}$ is the reduced Compton frequency per Planck time that represents the quantization of gravity. This new way to rewrite the Schwarzschild metric provides exactly the same predictions as the standard Schwarzschild solution, but it offers a deeper insight, in our view. It shows that gravity is ultimately linked to the Planck scale and establishes a connection between gravitational objects such as the Earth, the Sun, and even the mass or energy of the Hubble sphere and the Planck scale.

Again, it is important to be aware that the Planck length can be found from gravity observations without any knowledge of G or even $\hbar$; see [13]. Already in 1984, Cahill [58] [59] suggested that the Planck units could be more fundamental than the gravity constant; he simply rearranged the Planck mass formula $m_p = \sqrt{\frac{\hbar c}{G}}$ with respect to G and got $G = \frac{\hbar c}{m_p^2}$. However, Cohen [60] correctly pointed out that this just led to a circular problem, as no one at that time had demonstrated a way to find Planck units independent of *G*. This view has been held until recently; see, for example, the interesting paper by McCulloch [61]. Now, it is fully possible to find Planck units from gravity observations with no knowledge of G and $\hbar$. In our view, this means that the Planck scale has now been indirectly detected. This also indirectly may explain why we can predict the CMB temperature from theoretical considerations combined with practical knowledge of the shortest and longest possible wavelengths in the Hubble sphere.

Another view more consistent with the Λ-CDM model is that Planck mass particles (micro black holes) existed only just after the Big Bang, and that these particles decayed into today's known particles. Lloyd Motz [62]-[64] was likely the first to suggest the existence of a very fundamental particle with a mass equal to the Planck mass. However, he knew the Planck mass was way too massive in comparison to observed particles, such as protons and electrons. Motz tried to overcome this challenge by claiming that the Planck mass particles created just after the Big Bang had radiated most of their energy away and that this energy could have been the origin of the creation of particles such as protons and electrons. Others have suggested a similar idea that there were plenty of Planck mass-type particles just after the Big Bang, see De [65], but that such super-heavy particles have radiated away most of their energy. Therefore, one possibility is that the CMB is in some way a remnant of that epoch. Only further scrutiny of the CMB temperature can provide a final answer to these speculations. What is most important is that both the Stefan-Boltzmann law and this new geometric mean temperature approach essentially lead to the same conclusion and to the formulae suggested by Tatum *et al.* Now, the CMB temperature can clearly be predicted and described theoretically and not simply measured. This theoretical relationship, unknown to most astrophysicists at the time of this writing, seems to bind the different properties of the observable universe more closely together than before. It could be that our recent theoretical breakthroughs concerning the CMB temperature might lead to improved cosmological models.

Consistent with this view remains the possibility that the universe is a black hole-like object wherein the Hubble horizon has been, and continues to be, expanding at light speed. This would be a distinct class of $R_H = ct$ growing black hole cosmological models. Our new approach to an understanding of the CMB should naturally be carefully scrutinized by multiple researchers over time to check which cosmological models, as well as quantum gravity models, it best conforms with, or if our new theoretical CMB framework needs further modification. Surprisingly, there now appears to be a close theoretical link between CMB temperature, the Hubble constant, and other global parameters of the universe; see [49] [53].

7. Discussion Concerning How Well Our Model Stands up against Observations

The CMB formula presented herein was first introduced by Tatum *et al.* [49] in 2015. It has been recently proven to be consistent with the Stefan-Boltzmann law [51] [52]. We have now shown that the current CMB temperature can be derived as a geometric mean using the minimum and maximum temperatures. Such recent developments give the formula a solid theoretical foundation. However, a theoretical foundation alone is not sufficient. While the formula predicts the current CMB temperature very accurately, an important question is how well it performs for earlier cosmic epochs. The Hubble tension [66], which refers to the unexplained discrepancy in the Hubble constant inferred from local measurements, such as supernovae standard candles, and that inferred from direct measurement of the CMB, in our view, is related to this issue.

From observations, it is well established (see [67]-[69]) that the relation $T_t = T_0(1+z)$ is correct. Haug and Tatum [70] have recently shown that, in a black hole $R_h = ct$ cosmological model, the redshift must be $z = \sqrt{\frac{R_h}{R_t}} - 1$ for it to be consistent with their findings concerning the CMB temperature and its relationship to H_0, and not $z = \frac{R_h}{R_t} - 1$ as typically assumed. Haug and Tatum have demonstrated that, based on the CMB temperature and its relationships to H_0 and cosmological redshift, one can accurately match the full distance ladder of redshifts in the Union2 supernova database. From this, they derive a single H_0 value that corresponds to the H_0 value derived from CMB measurements, effectively solving the Hubble tension. See also [71].

While the Haug-Tatum model clearly appears to solve the Hubble tension inside $R_h = ct$ models, we naturally expect discussions to continue for some years, as supporters of the Λ-CDM model and other competing models are unlikely to quickly accept these claims. Naturally, extraordinary claims should be carefully scrutinized over time before being accepted. Nevertheless, one should be open-minded enough to actually investigate such claims when they are rooted in what we claim to be very solid mathematical derivations and tests against observational data.

An effective cosmological model must, of course, also be able to explain much more than the Hubble tension in order to challenge or surpass the Λ-CDM model. For example, how well the Haug-Tatum model fits observations from the early universe is an important question to address. While such is not the primary focus of this paper, it is a topic we have actively explored in other publications and preprints.

For example, the James Webb Space Telescope (JWST) has revealed that early galaxies observed at high z appear to be more developed than expected by the Λ-CDM model, which claims a cosmic age of about 13.8 billion years. The Haug-Tatum model offers a simple explanation for this "early galaxy maturation" phenomenon. Based upon the full distance ladder of the Union2 supernovae redshift database, Haug and Tatum [72] [73] have determined that the age of the universe is much closer to 14.6 billion years. This is approximately 800 million years older than the Λ-CDM model estimate. Additionally, the Haug-Tatum model, even when incorporating the full uncertainty in both observations and constants, produces a very narrow age range for the observable universe: $14622044825^{+415466}_{-415442}$ years. Such precision is significantly higher than that of any other model or observational study. The reason for this high precision is a new understanding of the relationship between the CMB temperature, the Hubble constant, cosmological redshifts, and the Planck scale. However, such is not the main topic of this paper, so we refer the reader to the above-mentioned works for more details on the Haug-Tatum model with respect to the early universe.

These Haug-Tatum findings strongly support a linear cosmic expansion based on $R_h = ct$. In contrast, the Λ-CDM model requires early inflation and a late accelerated expansion. Melia [74], in a recent 2024 paper, found that recent observations from the JWST disfavor the Λ-CDM model and strongly favor the $R_h = ct$ model. It is important to note that there are several cosmological models consistent with $R_h = ct$. Melia has also predicted an age of the universe of about 14.6 billion years, although his model does not seem to achieve the same level of precision as the Haug-Tatum model in this estimate. At present, the vigorous scientific debate concerning the Λ-CDM model versus the $R_h = ct$ model continues.

8. Conclusion

Using a rigorous mathematical approach, we have demonstrated how the CMB temperature could simply be a form of geometric mean temperature between the minimum time-dependent Hawking Hubble temperature and the maximum Planck temperature of the expanding universe over the course of cosmic time. This mathematical discovery suggests a re-consideration of $R_h = ct$ cosmological models, including black hole cosmological models, even if it possibly could also be consistent with the Λ-CDM model. Most importantly, this paper contributes to the growing literature in the past year asserting a tightly constrained mathematical relationship between the CMB temperature, the Hubble constant, and other global parameters of the Hubble sphere. Our approach suggests a solid theoretical framework for predicting and understanding the CMB temperature rather than solely observing it.

Conflicts of Interest

The authors declare no conflict of interest.

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Appendix: Wien's Law versus Planck's Law for Getting to the CMB Temperature

The CMB temperature has not been measured directly. Instead, it is the radiation frequency that is measured. There is a whole spectrum of radiation frequencies, but the peak frequency can be easily transformed into temperature by utilizing Wien's (approximation) law or, more exactly, Planck's law. The transformation from the measured peak wavelength to CMB temperature can be achieved by utilizing Wien's law. According to Wien's law, we can determine that

$$b = \frac{hc}{5k_b} \tag{23}$$

and the CMB temperature is then given as

$$T_{CMB} = \frac{b}{\lambda_{peak}} = \frac{hc}{5k_b}\frac{1}{\lambda_{peak}} \tag{24}$$

For a peak wavelength of 1.0634 mm, this gives a CMB temperature of $T_{CMB} = 2.706\text{ K}$.

From Planck's Law, we get

$$b = \frac{ch}{k_b\left(5 + W_0\left(-5/e^5\right)\right)} \approx 0.002897773 \tag{25}$$

where W_0 is the Lambert W function. If the peak wavelength is 1.0634 mm, this gives a CMB temperature of approximately

$$T_{CMB} = \frac{b}{\lambda_{peak}} \approx 2.725\text{ K} \tag{26}$$

This means that transforming measured peak wavelength from the CMB spectrum to CMB temperature using Wien's law will underestimate the CMB temperature by approximately 0.019 K compared to using the more exact Planck's law.