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The CMB Luminosity Distance and Cosmological Redshifts

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Abstract

We will outline the relationship between luminosity distance and cosmological redshifts, demonstrating that it is consistent with a new cosmological model recently proposed by Haug and Tatum [1], which appears to resolve the Hubble tension within the $R_h = ct$ cosmology.

Keywords

Luminosity Distance, Angular Distance, Co-Moving Distance

1. Luminosity Distance Consistent with the Haug and Tatum Cosmological Model

According to the Stefan-Boltzmann law [2] [3], the luminosity of a spherical black body is given by:

$$L = 4\pi R^2 \sigma T^4 \tag{1}$$

where T is the black body temperature and $\sigma = \frac{2\pi^5 k_b^4}{15c^2h^3}$ is the Stefan-Boltzmann constant (where k_b is the Boltzmann constant), R is the radius and L is the luminosity.

Let us assume this also holds true for a Schwarzschild [4] black hole (see [5]), where $R = R_s = \frac{2GM}{c^2}$, meaning the radius is equal to the Schwarzschild radius. Solving for the radius, one gets:

$$D_L = R = \sqrt{\frac{L}{4\pi\sigma T^4}} \tag{2}$$

Furthermore, the flux density is given by:

$$F = \frac{L}{4\pi R^2} \tag{3}$$

This means we also have:

$$D_L = R = \sqrt{\frac{L}{4\pi F}} \tag{4}$$

This is also called the luminosity distance. Moreover, we must have:

$$\frac{F_t}{F_H} = \frac{R_t^2}{R_H^2} \tag{5}$$

where F_t is the flux density in a growing black hole in the $R_h = ct$ cosmological model at time t and at radius R_t, and F_H is the flux density.

Haug and Tatum have demonstrated that if $T_t = T_0(1+z)$ (which seems to be supported by observations, see [6]-[8]), then in their type of $R_h = ct$ cosmology, one must have:

$$z = \left(\frac{R_h}{R_t}\right)^{\frac{1}{2}} - 1 \tag{6}$$

Solved for R_h, this gives:

$$R_h = R_t(1+z)^2 \tag{7}$$

where R_t is the co-moving transverse luminosity distance. This also means the luminosity distance must follow accordingly. We can rearrange this to:

$$R_t = \frac{R_h}{(1+z)^2}$$

$$R_h - R_t = R_h - \frac{R_h}{(1+z)^2}$$

$$R_h - R_t = \frac{c}{H_0}\left(1 - \frac{1}{(1+z)^2}\right)$$

$$R_h - R_t = \frac{2cz + cz^2}{H_0(1+z)^2} \tag{8}$$

We define $R_h - R_t = D$, which is the distance to the object emitting photons toward us, so we have:

$$D = \frac{2cz + cz^2}{H_0(1+z)^2} \tag{9}$$

This is the same formula that Haug and Tatum have presented, namely:

$$D = \frac{c}{H_0}\left(1 - \frac{1}{(1+z)^2}\right) = \frac{2cz + cz^2}{H_0(1+z)^2} \tag{10}$$

when $z \ll 1$ this can be approximated as:

$$D \approx \frac{2cz}{H_0} \tag{11}$$

The well-known Etherington [9] equation gives the relationship between the luminosity distance of standard candles and the angular diameter distance, which is given by:

$$D_L = (1+z)^2 D_A \tag{12}$$

Here, D_A represents the angular-diameter distance. We can see that this closely resembles our Equation (7). We have already proven that the luminosity distance in our $R_h = ct$ cosmology is identical to R_h (at present) and that before the present time, it corresponds to R_t. This implies that we have essentially demonstrated that the angular-diameter distance is identical to the luminosity distance in the Haug and Tatum cosmology model, which assumes $z = \left(\frac{R_h}{R_t}\right)^{\frac{1}{2}} - 1$. However, this should naturally be studied carefully over time; for example, one should carefully study whether the Etherington formula is also valid under $R_h = ct$ cosmology. We suspect it is, as our theory is consistent with the critical Friedmann equation [10]. It is also worth mentioning that there is a different variation of the Haug and Tatum cosmology, where they propose $z = \frac{R_h}{R_t} - 1$, which would yield different results that could also be studied further. However, as we will point out in the next section, the scaling $z = \left(\frac{R_h}{R_t}\right)^{\frac{1}{2}} - 1$ seems preferable.

2. $R_h = ct$ Model Distance and the Hubble Tension

The Hubble tension remains unresolved in standard cosmology, see Valentino *et al.* [11]. Krishnan *et al.* [12] have even suggested that the Hubble tension could even indicate a breakdown in standard cosmology, that is, in FLRW, which is the cornerstone of Λ-CDM cosmology.

Haug and Tatum [1] have shown, through a combination of trial-and-error methods and intelligent search algorithms, that the Hubble tension can be resolved within a specific class of $R_h = ct$ cosmology, where the Hubble sphere is essentially treated as a black hole. They achieve this by fitting their model to the complete distance ladder of observed Type Ia supernovae from the Union2 database, where they get a close to perfect fit without the need to rely on dark energy or any additional free parameters. This is made possible by a new exact mathematical relationship between the current (and past) CMB temperature and the Hubble constant, in conjunction with the $R_h = ct$ principle.

They achieved this for both a model with $z = \left(\frac{R_h}{R_t}\right)^{\frac{1}{2}} - 1$ and a model with $z = \frac{R_h}{R_t} - 1$. However, they have shown that only $z = \left(\frac{R_h}{R_t}\right)^{\frac{1}{2}} - 1$ predicts $T_t = T_0(1+z)$, which is consistent with observations. The more commonly assumed $z = \frac{R_h}{R_t} - 1$, is inside this framework instead consistent with $T_t = T_0(1+z)^{\frac{1}{2}}$ that do not seem supported by observational studies.

Haug [13] has recently extended on the Haug-Tatum model and provided a formal mathematical proof that the Hubble tension can be resolved by a closed-form solution for the more general redshift function $z=\left(\frac{R_h}{R_t}\right)^x-1$, where x can take any real value. The Haug-Tatum model is a special case of this general model, with $x=\frac{1}{2}$ and $x=1$. Resolving the Hubble tension alone is not sufficient for a model to be considered a good representation of the cosmos; it must also be consistent with various other cosmological aspects. Haug demonstrates that only when $x=\frac{1}{2}$ (the Haug-Tatum model) does it predict and remain consistent with the observed $T_t=T_0(1+z)$ relationship. Furthermore, only when $x=\frac{1}{2}$ do all three distances-luminosity distance, angular diameter distance, and comoving distance-become the same.

All of these $R_h=ct$ cosmological models based on the Haug-Tatum model will predict the same Hubble constant, but only one appears consistent with $T_t=T_0(1+z)$, which is also the model that offers maximum simplification, as the three different distances converge into one.

In conclusion, we can infer that the Λ-CDM model likely predicts incorrect distances for emitting photons. Additionally, its various types of distances for a given observed z seem overly complex, even if such complexity is required when assuming accelerated expansion of space. While this is internally consistent within their model, a much simpler model seems capable of explaining the cosmos. The Haug-Tatum model strongly suggests that no accelerated expansion is necessary and that a simpler model can address phenomena such as the Hubble tension.

3. The Mattig Formula

The Mattig [14] formula (see also [15]) is used to calculate the luminosity distance in terms of redshift and is inside FLRW cosmology given by:

$$R=\frac{c}{H_0}\frac{q_0z+(q_0-1)\left(\sqrt{1+2q_0z}-1\right)}{q_0^2(1+z)} \tag{13}$$

If we set $q_0=1$, we obtain:

$$d=R=\frac{cz}{H_0(1+z)} \tag{14}$$

This is the same distance as given by the Haug and Tatum model when they assume $z=\frac{R_h}{R_t}-1$. The first-term Taylor series expansion of this is the well-known distance formula often used in the Λ-CDM model:

$$d\approx\frac{cz}{H_0} \tag{15}$$

4. Speculative Equivalent Mattig Formula for $z = \left(R_h/R_t\right)^{\frac{1}{2}} - 1$ Scaling

We are suggesting a speculative modification to the Mattig [14] formula in an attempt to make it consistent with $z = \left(\frac{R_h}{R_t}\right)^{\frac{1}{2}} - 1$. Equation (16) is the only equation in this paper that is not strictly based on derivation, so it should simply be regarded as an initial guess (that could be wrong). The idea is that the Mattig formula may require modification to accommodate the redshift scaling $z = \left(\frac{R_h}{R_t}\right)^{\frac{1}{2}} - 1$ inside $R_h = ct$ cosmology. Based on a qualified assumption, we propose the following:

$$d = \frac{c}{H_0} \frac{2q_0 z + z^2 q_0^2 + \left(q_0 - 1\right)\left(\sqrt{1 + 2q_0 z} - 1\right)}{q_0^2 \left(1 + z\right)^2} \tag{16}$$

When $q_0 = 1$, we obtain:

$$d = \frac{2cz + cz^2}{H_0 \left(1 + z\right)^2} \tag{17}$$

This is equivalent to the distance Haug and Tatum [1] presented when they assume $z = \left(\frac{R_h}{R_t}\right)^{\frac{1}{2}} - 1$. Naturally, someone should derive an equivalent Mattig formula from scratch, as our speculative suggestion in this section could be wrong as it is not based on derivations but a speculative guess. It is simply meant to point to an additional idea that can be investigated.

5. Conclusion

The Haug-Tatum cosmological model, in which they propose the redshift scaling $z = \left(\frac{R_h}{R_t}\right)^{\frac{1}{2}} - 1$, leads to a co-moving luminosity distance relationship of $R_h = R_t \left(1 + z\right)^2$, as was already indirectly suggested in their paper. We have pointed out that the angular diameter distance in their model appears to be identical to this. In our $R_h = ct$ cosmological model, it seems that many types of cosmological distances converge to the same value. We suspect that some of the many different types of cosmological distances in the Λ-CDM model may result from an overly complex framework that does not represent the cosmos as accurately.

Conflicts of Interest

The author declares no conflict of interest.

References

[1] Haug, E.G. and Tatum, E.T. (2024) Solving the Hubble Tension Using the Union2

Supernova Database. https://doi.org/10.20944/preprints202404.0421.v1

[2] Boltzmann, L. (1884) Ableitung des Stefan'schen Gesetzes, betreffend die Abhängigkeit der Wärmestrahlung von der Temperatur aus der electromagnetischen Lichttheorie. *Annalen der Physik*, **258**, 291-294. https://doi.org/10.1002/andp.18842580616

[3] Stefan, J. (1879) Über die beziehung zwischen der Warmestrahlung und der temperatur. *Sitzungsberichte der Mathematisch-Naturwissenschaftlichen Classe der Kaiserlichen Akademie der Wissenschaften*, **79**, 391-428.

[4] Schwarzschild, K. (1916) Über das gravitationsfeld einer kugel aus inkompressibler flussigkeit nach der Einsteinschen theorie. Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse fur Mathematik, Physik, und Technik, 424.

[5] Haug, E.G. and Wojnow, S. (2024) The Blackbody CMB Temperature, Luminosity, and Their Relation to Black Hole Cosmology, Compared to Bekenstein-Hawking Luminosity. https://doi.org/10.20944/preprints202401.0173.v1

[6] de Martino, I., *et al.* (2012) Measuring the Redshift Dependence of the Cosmic Microwave Background Monopole Temperature with Planck Data. *The Astrophysical Journal*, **757**, Article ID: 144.

[7] Li, Y.Y., Hincks, A.D., Amodeo, S., Battistelli, E.S., Bond, J.R., Calabrese, E., *et al.* (2021) Constraining Cosmic Microwave Background Temperature Evolution with Sunyaev-Zel'dovich Galaxy Clusters from the Atacama Cosmology Telescope. *The Astrophysical Journal*, **922**, Article 136. https://doi.org/10.3847/1538-4357/ac26b6

[8] Riechers, D.A., Weiss, A., Walter, F., Carilli, C.L., Cox, P., Decarli, R., *et al.* (2022) Microwave Background Temperature at a Redshift of 6.34 from H_2O Absorption. *Nature*, **602**, 58-62. https://doi.org/10.1038/s41586-021-04294-5

[9] Etherington, I.M.H. (1933) Lx. on the Definition of Distance in General Relativity. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, **15**, 761-773. https://doi.org/10.1080/14786443309462220

[10] Haug, E.G. and Tatum, E.T. (2024) Friedmann Type Equations in Thermodynamic form Lead to Much Tighter Constraints on the Critical Density of the Universe. https://doi.org/10.20944/preprints202403.1241.v2

[11] Di Valentino, E., Mena, O., Pan, S., Visinelli, L., Yang, W., Melchiorri, A., *et al.* (2021) In the Realm of the Hubble Tension—A Review of Solutions. *Classical and Quantum Gravity*, **38**, Article ID: 153001. https://doi.org/10.1088/1361-6382/ac086d

[12] Krishnan, C., Mohayaee, R., Colgáin, E.Ó., Sheikh-Jabbari, M.M. and Yin, L. (2021) Does Hubble Tension Signal a Breakdown in FLRW Cosmology? *Classical and Quantum Gravity*, **38**, Article ID: 184001. https://doi.org/10.1088/1361-6382/ac1a81

[13] Haug, E.G. (2024) Closed form Solution to the Hubble Tension Based on $R_h = ct$ Cosmology for Generalized Cosmological Redshift Scaling of the Form: $z = (R_h/R_t)^x - 1$ Tested against the Full Distance Ladder of Observed SN Ia Redshift. https://doi.org/10.20944/preprints202409.1697.v2

[14] Mattig, W. (1957) Über den Zusammenhang zwischen Rotverschiebung und scheinbarer Helligkeit. *Astronomische Nachrichten*, **284**, 109-111. https://doi.org/10.1002/asna.19572840303

[15] Terrell, J. (1977) The Luminosity Distance Equation in Friedmann Cosmology. *American Journal of Physics*, **45**, 869-870. https://doi.org/10.1119/1.11065