International Journal of Modern Nonlinear Theory and Application, 2025, 14(3), 43-58
https://www.scirp.org/journal/ijmnta
ISSN Online: 2167-9487
ISSN Print: 2167-9479

Gyroscope Chaos Control Using Backstepping Control Design

Rostand Martial Davy Loembe Souamy[1,2,3,4,5,6*], Pingjiang Guo[2], Xianfan Chun[2], Baowen Xu[3], Honghua Wang[4], Christian Tathy[6]

[1]Laboratory of Electrical and Electronic Engineering (LGEE), National Higher Polytechnic School Marien Ngouabi University, Brazzaville, Republic of the Congo
[2]School of Automation and Artificial Intelligence, Nanjing University of Posts and Telecommunications, Nanjing, China
[3]Jiangsu Province Key Laboratory for Novel Technology, Department of Computers and Technology, Nanjing University, Nanjing, China
[4]Laboratory of Control Theory and Control Engineering, Hohai University, College of Energy and Electrical Engineering, Nanjing, China
[5]Laboratory of Nanomaterials and Nanotechnologies, National Institute for Research in Exact and Naturel Sciences (IRSEN) P.O. Brazzaville, Republic of the Congo
[6]Laboratory of Mechanical, Energy and Engineering, National Higher Polytechnic School, Marien Ngouabi, University, Brazzaville, Republic of the Congo
Email: *loembesouamy@gmail.com, jianggp@nupt.edu.cn, fancx@njupt.edu.cn

How to cite this paper: Souamy, R.M.D.L., Guo, P.J., Chun, X.F., Xu, B.W., Wang, H.H. and Tathy, C. (2025) Gyroscope Chaos Control Using Backstepping Control Design. *International Journal of Modern Nonlinear Theory and Application*, **14**, 43-58.
https://doi.org/10.4236/ijmnta.2025.143003

Received: July 23, 2025
Accepted: September 26, 2025
Published: September 29, 2025

Abstract

The study proposes a backstepping controller that omits cubic (third-order) terms to suppress chaotic motion in a two-state symmetric gyroscope. A Lyapunov-based proof claims global asymptotic stability, and numerical simulations illustrate state convergence under the new law of control and Lyapunov stability of theory respectively. The work aims to reduce controller complexity compared with earlier designs that retained third-order terms. A rigorous analysis shows that the controller will converge asymptotically. Numerical simulations are given to verify the effectiveness of the proposed backstepping controller design.

Keywords

Chaotic-Gyroscope, Lyapunov Method, Backstepping Method, Control Design

1. Introduction

The first classical chaotic system is found by Edward Lorenz system when he studied the atmospheric convection in 1963, in [1]. It is a nonlinear system of three differential equations. With the most commonly used values of three parameters, there are two unstable critical points. The solutions remain bounded, but orbit chaotically around these two points.

Rossler carried out a most important work which brought the interest inaccurate nonlinear of dynamic system in 1976, in [2]. Rossler himself proposed an advanced system in 1979, in [3]. Otto Grebogi *et al.*, controlling chaos in [3] [4]. Sprott embarked upon an extensive search in [5] for autonomous three states chaotic systems. Chen made another chaotic system in [5], which nevertheless is not structurally equivalent to the Lorenz' system in [1]-[8].

A chaotic-gyroscope system can be chaotic dynamic whenever its evolution sensitively depends on the initial conditions in [9]. Chaos control refers to manipulating the dynamical behavior of chaotic system, in which the goal is to suppress chaos when it is harmful or create chaos when it is beneficial in [1]-[9]. It typically consists of a spinning wheel or a disc mounted on a base in such a way that its axis can freely rotate.

When the wheel or disc spins, it exhibits properties of angular momentum, which helps it resist changes in orientation. Gyroscopes are used in various applications as follows as a particularly form of nonlinear system, including navigation systems, aircraft and spacecraft control, stabilization systems for cameras and sensors, and even in some consumer devices like smartphones for motion sensing. They play a crucial role in maintaining stability and accuracy in these systems by providing a reference for orientation and angular velocity, which have been widely to evaluate control schemes of chaotic system in [8]-[10].

A variety of approaches have been proposed for solving the gyros chaos control problem. These methods include active control in [11], based on dynamical behaviors and chaos control in [12], based on variable structure control in [13], based on fuzzy sliding mode control in [14], based on via backstepping control in [15]. Based on an improved backstepping method in [16]. Designing to stabilize gyro chaotic system . Chaos control and modified projective synchronization of unknown heavy symmetric chaotic system in [17]. Based on adaptive control for the stabilization and synchronization of nonlinear gyroscopes in [18]. Based on robust nonlinear dynamic inversion in [19]. Based on adaptive robust finite-time in [20]. LOEMBE SOUAMY *et al.* designed on backstepping control design in [21], LOEMBE SOUAMY *et al.* based on adaptive backstepping control design in [22], Based on passivity- based synchronization in [23], based in secure communication with a chaotic system in [24], and others in [25] [26], etc.

In this paper, we adopt the backstepping controller technique to realize the control of chaos in nonlinear gyros with two states, and propose a novel control method based on backstepping control design without three order terms, which is different from the existing methods in [9]-[26]. The proposed method shows a novel controller without cubic (third order terms) can reduce the complexity of gyros chaos control and increase the effectiveness and feasibility of a backstepping controller design technique, which will be supported by theoretical analysis and simulations results.

The rest parts of this paper are organized as follows. In Section 2, a brief description of the gyro system with some uncertainties are introduced. In Section 3, we discuss the design of the backstepping controller and verify the stability of the error system by using the Lyapunov stability theory. In Section4, numerical simulations are given for illustration of the effectiveness of the backstepping control technique. Some conclusions are presented in Section 5.

2. Mathematical of Modeling of Gyroscope Dynamics

2.1. System Description

The geometry of the problem under consideration is depicted in **Figure 1**. Chaotic motion of a symmetric gyroscope subject to a harmonic dynamical system and nonlinear system analysis. Introduction to nonlinear systems, is distinctive or distinguishes between linear system and nonlinear system. Motivation in nonlinear systems as an example of Chaos and Bifurcation. Many times, we see that a dynamic system behaves nicely at some operating point. Suddenly becomes haywire without any apparent warning. The forthcoming slides give many examples of such phenomena.

How can a differential equation with a continuous dynamical system cause such abrupt change in behavior of gyroscope?

The symmetric gyroscope mounted on a vibrating base is shown in **Figure 1**. The dynamics of a symmetrical gyroscope with linear-plus-cubic damping of the angle can be expressed as it follows that a significantly of system. The motion of a symmetrical gyroscope mounted on a vibrating base can be described by Euler angles θ, ϕ, and ψ.

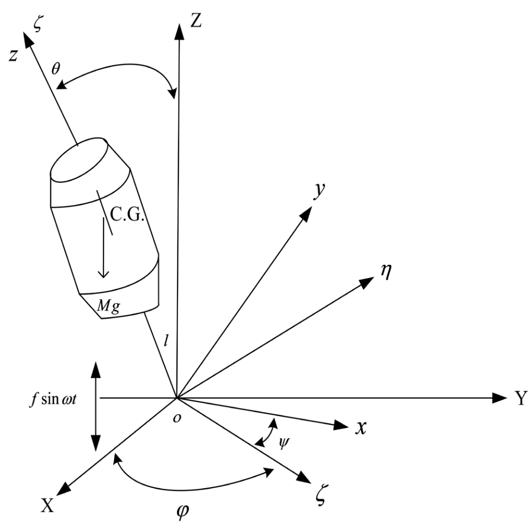


Figure 1. A schematic diagram of a symmetric gyroscope in [9].

l is the distance between the center of gravity and O; $\bar{l}$ is the magnitude of ex ternal excitation disturbance; M_g is the force of gravity; ω is the frequency of the external excitation disturbance; θ is the nutation angle; ϕ is the precession angle; ψ is a rotation angle around the gyroscope's symmetry axis. By using

Lag grangian approach, the Lagrangian has the expression as followed as:

$$L=\frac{1}{2}I_1\left(\theta^2+\dot{\phi}^2+\sin^2\theta\right)+\frac{1}{2}I_3\left(+\dot{\phi}\cos\theta+\dot{\psi}\right)^2-M_g\left(l-\overline{l}\sin\omega t\right)\cos\theta \quad (1)$$

where I_1 and I_3 are the polar and equatorial moments of inertial of the symmetric gyroscope respectively. From the analysis of Equation (1), it is observed that ϕ and ψ represent cyclic coordinates. Thus, the first integrals are:

$$P_\phi=\frac{\partial L}{\partial\psi}=I_1\sin^2\theta+I_3\left(\dot{\phi}\cos\theta+\dot{\psi}\right)\cos\theta=\beta_\psi, \quad (2)$$

$$P_\psi=\frac{\partial L}{\partial\psi}=I_3\left(\dot{\phi}\cos\theta+\dot{\psi}\right)\cos\theta=\beta_\phi=I_3\omega_Z, \quad (3)$$

The development of coherent theory and dynamical system without three order terms denotes a many practical parameter excitation, is the frequency of the external excitation disturbance an application of gyroscope and we introduce the coordinate system such that.

$$\begin{aligned} x' &= \cos\psi x''-\sin\psi y'' \\ y' &= \sin\psi x''+\cos\psi y'' \\ z' &= \cos\theta z'+\sin\theta\hat{y}' \end{aligned} \quad (4)$$

$\vec{\omega}=\dot{\theta}\hat{x}'+\dot{\phi}z'+\psi z''\Rightarrow$ then

$z'=\cos\theta z'+\sin\theta\hat{y}'=\sin\theta\sin\psi x''+\sin\theta\cos\psi y''+\cos\theta z''$ The orientation as follows as:

$$\begin{aligned} \omega_{x'} &= \dot{\theta}\cos\psi-\dot{\phi}\sin\theta\sin\psi \\ \omega_{y'} &= -\dot{\theta}\sin\psi+\dot{\phi}\sin\theta\cos\psi \\ \omega_{z'} &= \dot{\psi}+\dot{\phi}\cos\theta \end{aligned} \quad (5)$$

We used the kinetic energy and obtaining $T=\frac{1}{2}I_{xx}\omega_x^2+\frac{1}{2}I_{yy}\omega_y^2+\frac{1}{2}I_{zz}\omega_z^2\Rightarrow$

Note that this is an orthogonal coordinate system, we assume that $I_{x''x''}=I_{y''y''}$ then P_ψ,P_ϕ are constant.

$$T=\frac{1}{2}I_{xx}\left(\dot{\theta}^2+\dot{\phi}^2\sin^2\theta\right)+\frac{1}{2}I_{zz}\left(\dot{\psi}+\dot{\phi}\cos\theta\right)^2 \quad (6)$$

That using Larangian approach, to obtain this system Equation (6), where the velocity describes $V=mgl\cos\theta$,the rotation changes and angular momentum as follows as:

$$L=\frac{1}{2}I_{xx}\left(\dot{\theta}^2+\dot{\phi}^2\sin^2\theta\right)+\frac{1}{2}I_{zz}\left(\dot{\psi}+\dot{\phi}\cos\theta\right)^2-mgl\cos\theta \quad (7)$$

Angular momentum is the same at every point on an orbit, it is closer, it increases speed therefore the derivatives of system (7) as shown in system Equation (8) then we obtain as follows as:

$$\frac{\partial L}{\partial\psi}=0\Rightarrow P_\psi=I_{zz}\left(\dot{\psi}+\dot{\phi}\cos\theta\right)=I_{zz}\omega_z$$

$$\frac{\partial L}{\partial\phi}=0\Rightarrow P_\phi=I_{xx}\dot{\phi}\sin^2\theta+I_{zz}\cos\theta\left(\dot{\psi}+\dot{\phi}\cos\theta\right)=I_{xx}\omega_x$$

$$\frac{\partial L}{\partial t}=0\Rightarrow E=\frac{1}{2}I_{xx}\left(\dot{\theta}^2+\dot{\phi}^2\sin\theta\right)+\frac{1}{2}I_{zz}\left(\dot{\psi}+\dot{\phi}\cos\theta\right)^2+mgl\cos\theta \quad (8)$$

According to the conservation of angular momentum because of this gyroscope are useful for measuring or maintaining L_{spin} when rotating a gyroscope system. The direction is given by the right hand rule which would give L_{spin} the direction out of the diagram. For an orbit, angular momentum is conserved and this leads to one of Kepler's law. The Routh's procedure is adopted along with the above mentioned relation, the Routhian of the system becomes.

$$\begin{aligned}R&=L-\beta_\phi\phi-\beta_\psi\psi\\&=\frac{1}{2}I_2\theta^2-\left[\frac{\left(\beta_\phi-\beta_\psi\cos\theta\right)^2}{2I_1\sin^2\theta}+\frac{\beta_\phi^2}{2I_3}+M_g\left(l+\overline{l}\sin\omega t\right)\cos\theta\right]\end{aligned} \quad (9)$$

The equation above depends on the angle θ alone. According to Grantmacher in [12], $\beta_\phi=\beta_\psi$, when $\theta=0$. The dissipative force is also assumed to be in linear-plus-cubic form which is

$$F=D_1\dot{\theta}-D_2\dot{\theta}^3 \quad (1.10)$$

where D_1 and D_2 are positive constants. The equation above allows the system to be viewed as a single-degree-of-freedom system. The equation governing the gyroscope is given by Equation (1.11) as follows as:

$$\ddot{\theta}+\frac{\beta_\phi^2}{I_1^2}\frac{\left(1-\cos\theta\right)^2}{\sin^3\theta}+\frac{D_1}{I_1}\dot{\theta}+\frac{D_2}{I_1}\dot{\theta}^3-\frac{M_g l}{I_1}\sin\theta=\frac{M_g\overline{l}}{I_1}\sin\omega t\sin\theta \quad (1.11)$$

The normalized equation in convenient-first order form are under bellowed.

2.2. Gyroscope and Its Chaos Control in the Integrity of the Specifications

This section presents chaos control of chaos gyroscope, we generally consider two-state systems that are coupled. The evolution is chaotic, and we are interested in the conditions such that the component system execute the same motion, the background and the motivation of the proposed of method continuing with a short overview of chaotic dynamic and a list of the main contributions, and finish with the system of the proposed of method. Chaos Control problem has been concerned since early 1990's in [4].

Many control methods have been applied to control chaos, such as chaos and synchronizations. These methods include active control in [11], based on dynamical behaviors and chaos control in [12], based on variable structure control in [13], based on fuzzy sliding mode control in [14], based on via backstepping control in [15]. Based on an improved backstepping method in [16]. Designing to stabilize gyro chaotic system. Chaos control and modified projective synchronization of unknown heavy symmetric chaotic system in [17]. Based on adaptive control for the stabilization and synchronization of nonlinear gyroscopes in [18]. Based on robust nonlinear dynamic inversion in [19]. Based on adaptive robust finite-time in [20]. LOEMBE SOUAMY *et al.* designed on backstepping control design in [21], LOEMBE SOUAMY *et al.* based on adaptive backstepping control design in [22], Based on passivity-based synchronization in [23], based in secure communication with a chaotic system in [24], and others in [25] [26] etc.

Backstepping method has been successfully used to control chaos in [15]-[26], where the controller contains a cubic (third-order terms) which is complicated. Why are we explaining and eliminating cubic terms in our controller simplifies implementation?

Eliminating cubic terms in controller design significantly simplifies implementations (e.g. reduced computations or actuator demand) and quantify any performances trade-offs versus earlier controllers by reducing computational complexity and easing the real-time processing burden, which is particularly beneficial for system with limited hardware resources such as embedded processors or microcontrollers. Without cubic nonlinearities, the control law involves fewer multiplications and power operations, leading to faster computations and lower power consumptions.

Additionally, linear or lower-order controllers typically generate smoother control signals, which reduces actuator wear and improves long-term system reliability. However, this simplification may come at the cost of reduced accuracy in tracking or disturbance rejection for highly nonlinear system, as the controller may no longer fully capture or compensate for the system's inherent nonlinear dynamics. Quantitatively, this can result in performance trade-offs such as a 5 - 10% increase in steady-state error or slower transient response when compared to earlier, more complex controller that retained the full nonlinear terms. The symmetric gyroscope mounted on a vibrating base is shown in **Figure 1**. The dynamics of a symmetrical gyroscope with linear-plus-cubic damping of the angle θ can be expressed as it follows that a significantly of system Equation as follows as:

$$\ddot{\theta}+\alpha^2\frac{(1-\cos\theta)^2}{\sin^3\theta}-\beta\sin\theta+c_1\dot{\theta}+c_2\dot{\theta}^3=f\sin\omega t\sin\theta \tag{1.12}$$

Let $x_1=\theta$, $x_2=\dot{\theta}$, $g(\theta)=-\alpha^2\left((1-\cos\theta)^2/\sin^3\theta\right)$. How does a gyroscope work in the integrity of the specifications? And consider the uncertainty, this system Equation (1.12) can be transformed into the following form Equation (1.13) as follows as:

$$\begin{cases}\dot{x}_1=x_2,\\ \dot{x}_2=g(x_1)-c_1x_2-c_2x_2^3+(\beta+f\sin\omega t)\sin(x_1)\end{cases} \tag{1.13}$$

The complex system (1.13) has been studied by Chen in [9], For the value of f in the range of $32<f<36$ system f and constant values of system as follows as; $\alpha^2=100$, $\beta=1$, $c_1=0.5$, $c_2=0.05$, $\omega=2$. Where α (rad/s²): Damping coefficient representing the rate of energy dissipation in the system due to internal friction or resistance. Where β (rad/s²): Nonlinear stiffness coefficient accounting for the restoring torque proportional to the cube of the angular displacement. Where c_1, c_2 (N.m.s/rad): Coupling coefficients reflecting the interaction strength between multiple gyroscope axes or between coupled gyroscopes ping coefficient representing. Where f (N.m): External excitation torque amplitude to the gyroscope. Where ω (rad/s): Excitation frequency representing the angular frequency of the applied torque. These parameters directly correspond to the mechanical and dynamic characteristics of actual gyroscopes, influencing their oscillatory behavior and response to external forces. In **Figure 2** illustrates the irregular motion exhibited by system (1.13) for $f = 35.5$ and initial conditions of systems $x_1(0) = -1$ and $x_2(0) = 2$. In **Figure 3** illustrates the time history, and we see that a dynamical system is behaving nicely in some operating point.

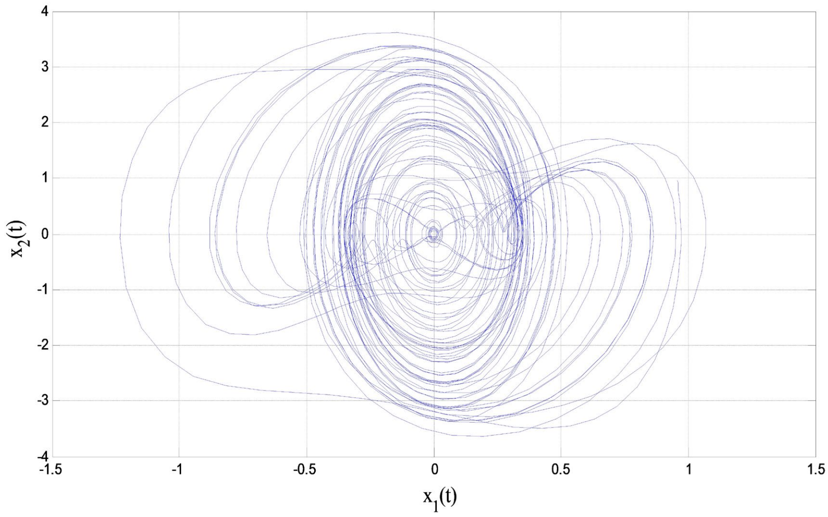


Figure 2. Phase plane trajectory of chaotic gyroscope system x_1 and x_2.

Bifurcation analysis, is the study of qualitative change in behavior of the system trajectories with changes in *r*, consider a dynamical system $\dot{x} = f(x, r)$ where $x \in \mathbb{R}^n$ is the state and $r \in \mathbb{R}^m$ is a parameter of the system, as shown in **Figure 3**, it follows as:

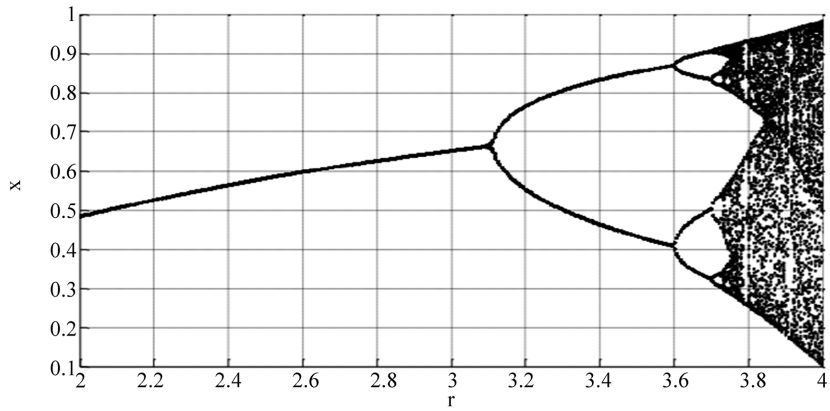


Figure 3. Illustrates the bifurcation diagram for specific value *x* steady state angular position.

Lyapunov exponent shows a dynamical system, is quantity that characterize the rate of separation of infinitesimally close trajectories, as shown in **Figure 4**, it follows as:

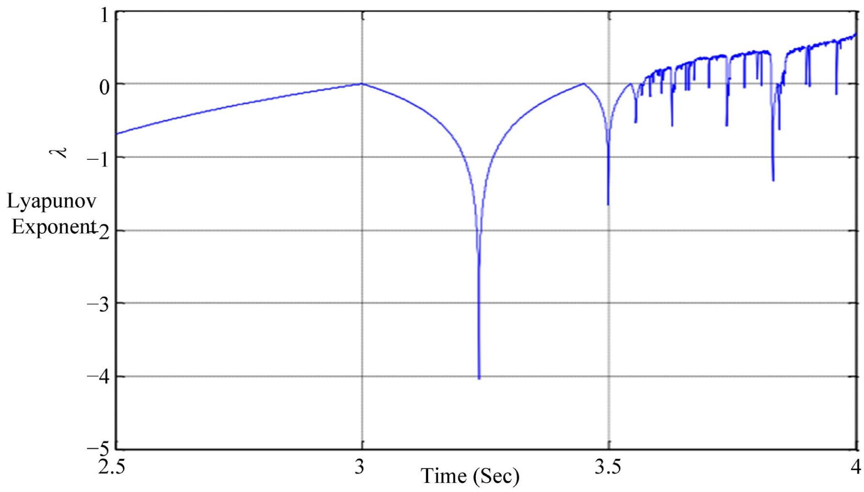


Figure 4. Illustrates Lyapunov exponent shows a dynamical system, is quantity that characterize the rate of separation of infinitesimally close trajectories.

Two useful assumptions are introduced as follows as:

Assumptions 2.3: Suppose that there exists a constant $l > 0$, $i = 1,2$, the such that following holds, $|g(x_1)| \le l_1 |x_1|$, where $g(x_1) = -\alpha^2 \left((1-\cos x_1)^2 / \sin^3 x_1 \right)$.

To achieve is now, we introduce a control signal to system (1.10) and rewrite the system in strict-feedback control form as follows as:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = g(x_1) - c_1 x_2 - c_2 x_2^3 + (\beta + f \sin \omega t)\sin(x_1) + u(t). \end{cases} \tag{1.14}$$

where our objective denotes the designer of controller based on backstepping control design $u(t)$ to stabilize chaotic gyroscope in presence of disturbance system (1.10) and it follows as the control signal, which is needed to be determined later. However the control $u(t)$ is to force the chaotic system (1.13) to follow for any initial conditions, *i.e.* $\lim_{t\to\infty} \|x(t)\| = 0$ where $x(t)$ the state is vector as $x(t) = [x_1(t), x_2(t)]^{\mathrm{T}}$ and $\|\cdot\|$ is the Euclidean norm of a vector.

3. Backstepping Control Design

This section presents gyroscope chaos control; we generally shall be considering two systems that are coupled. The evolution is chaotic, and we are interested in the conditions such that the component system execute the same motion, the background and the motivation of this research continuing with a short overview of a chaotic dynamic and the proposed of method which are main contributions, and overcoming their backstepping of control design.

3.1. Stability Analysis on Backstepping Control Design by Lyapunov Method Theory

The inversion design method, also called backstepping control design, is usually used in conjunction with the Lyapunov function, considering the control law so that the whole closed-loop system satisfies the expected dynamic and static performance. Consider the following two-dimensional nonlinear systems;

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x_1) + u(t) \end{cases}, \tag{1.15}$$

where $x_i \quad i = 1,2$ are system states, $f(x_1)$ is the unknown nonlinear function and $u(t)$ is input of the system. Backstepping design introduces a virtual control in every step and finally gets the real controller through reverse recursion. Firstly, a virtual variable is defined as follows as:

$$z_1 = x_1, \tag{1.16}$$

Then, we choose a scalar of Lyapunov function

$$V_1 = \frac{1}{2} z_1^2. \tag{1.17}$$

Define the second virtual variable as follows as:

$$z_2 = x_2 + \varphi_1, \tag{1.18}$$

where $\varphi_1 = c_1 z_1$ is a smooth function, $c_1 \in (0, +\infty)$.The parameter of system is given by $\dot{z}_1 = x_2$. We rewrite Equation (1.17) as the derivative of Lyapunov function and obtain as follows as:

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1 (z_2 - \varphi_1) = -c_1 z_1^2 + z_1 z_2. \tag{1.19}$$

We choose the second scalar of Lyapunov function:

$$V_2 = V_1 + \frac{1}{2} z_2^2. \tag{2.20}$$

We rewrite Equation (2.20) as a derivative of Lyapunov function then we obtain as follows as:

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + z_2 \dot{z}_2 = -c_1 z_1^2 + z_2 (z_1 + \dot{z}_2) \\ &= -c_1 z_1^2 - c_1 z_2^2 + z_2 (z_1 + c_2 z_2 + f(x) + u + \dot{\varphi}_1), \end{aligned} \tag{2.21}$$

where $c_2 \in (0, +\infty)$. According to the above principle, our controller designs as follows as:

$$u(t) = -(z_1 + c_2 z_2 + f(x) + \dot{\varphi}_1). \tag{2.23}$$

Adding the controller $u(t)$, from Equation (2.22) to system Equation (2.21) then we obtain the inequality Equation (2.23), it follows as:

$$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2 \le 0. \tag{2.23}$$

Then $\lim_{t \to \infty} x_1(t) = \lim_{t \to \infty} x_2(t) = 0$, the system will be asymptotically stable at the equilibrium point $(0,0)$.

3.2. Controlling of Two Chaotic Gyroscopes Using Backstepping Control Design

The inversion design method, also called backstepping control design, usually used in conjunction with the Lyapunov function, the symmetric gyroscope mounted on a vibrating base is shown in Figure 1. According to the study by Chen, the dynamics of symmetrical gyroscope with linear-plus-cubic damping of the angle θ can be expressed as follows as in **Figure 1**. The gyroscope system Equation (1.13) considering the two chaotic nonlinear gyroscope with controller without third-orders terms based on backstepping design. Consider the gyroscope chaotic system which is described as following as:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = g(x_1) - c_1 x_2 - c_2 x_2^3 + (\beta + f \sin \omega t) \sin x_1 + u(t) \end{cases} \tag{2.24}$$

where $c_2 = 0.05$, $x = [x_1 \quad x_2]^{\mathrm{T}} \in R^2$ is state of gyroscope systems, nonlinear systems $g(x_1) = \frac{-\alpha^2 (1 - \cos x_1)^2}{\sin^3 x_1}$, $f \sin \omega t$ denotes a parameter excitation, ω is the frequency of external disturbance, $c_1 x_2$ and $c_2 x_2^3$ are linear and nonlinear damping terms, and $u(t) \in R$ is the controller to be designed later. When for the value of f in the range of $32 < f < 36$, and constant values of $\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, and $\omega = 2$, systems Equation (1.13) is chaotic system .

Assumptions 2.4. We suppose that there exists a constant $l_1 > 0$, such that following inequality holds, $|g(x_1)| \le l_1 |x_1|$.

There is positive constant l_2 such that following inequality holds

$$|x_2^3| \le l_2 |x_2| \tag{2.25}$$

Because the states of chaotic system are bounded. The objective of this section is to design a controller $u(t)$ by using of backstepping method such that system (2.24) is asymptotically stable at the equilibrium point $(0,0)^{\mathrm{T}}$. Based on the backstepping control designs, we design a virtual controller $\alpha(x_1)$ and an error variable parameter ω_2 as follows as $\omega_2 = x_2 - \alpha(x_1)$, where $\alpha(x_1) = -x_1$.

Then, we can get the following system:

$$\begin{cases} \dot{x}_1 = \omega_2 - x_1, \\ \dot{\omega}_2 = g(x_1) - c_1 x_2 - c_2 x_2^3 + \omega_2 - x_1 + (\beta + f \sin \omega t) \sin x_1 + u(t) \end{cases} \tag{2.26}$$

The stability of system Equations (2.26) is equivalent to that of the state system Equation (2.24).

In the following, we design an advanced controller on backstepping design $u(t)$ without cubic terms (third-order-terms), which is different from the others controllers as in [7]-[11] [14]-[20] [26], and so on.

Theorem 2.5. We suppose that Assumption 2.4 holds. Use the following control laws as follows as:

$$\begin{aligned} u(t) = & -\left(\frac{1}{2}\varepsilon_1 l_1^2 + k\right)(x_1 + x_2) - c_2 l_2 |x_2| \operatorname{sign}(x_1 + x_2) \\ & + c_1 x_2 - (\beta + f \sin \omega t) \sin(x_1) \end{aligned} \tag{2.27}$$

where $k > 0$ is the feedback gain, $\varepsilon_1 > \frac{1}{2}$, $l_1 > 0$ and $l_2 > 0$ are constants parameters. Then, the chaotic gyroscope system Equation (2.24) can be asymptotically stabilized, which means Equation (2.24) is asymptotically stable to the origin.

Proof: We select the Lyapunov functions as follows as;

$$V_2(x_1,\omega_2)=V_1+\frac{1}{2}\omega_2^2, \tag{2.28}$$

where $V_1=\frac{1}{2}x_1^2$. Then we complete the proof of Theorem 2.5. From the following two steps.

Step 1. Get the derivative of V_1 along system (2.4) as the following:

$$\dot{V}_1(t)=x_1\dot{x}_1=x_1\alpha(x_1)+x_1\omega_2 \tag{2.29}$$

Since $\alpha(x_1)=-x_1$, then we can obtain $\dot{V}_1(t)<-x_1^2<0$ when $\omega_2=0$, that is, $x_1(t)$ is asymptotically stable. Therefore, we need to prove $\lim_{t\to+\infty}\omega_2(t)=0$.

Step 1. Get the derivative of V_2 along system (2.24) as the following:

$$\begin{aligned}\dot{V}_2(t)&=x_1\dot{x}_1+\omega_2\dot{\omega}_2\\&=x_1(\omega_2-x_1)+\omega_2\big(g(x_1)-c_1x_2-c_2x_2^3\\&\quad+(\beta+f\sin\omega t)\sin(x_1)-x_1+\omega_2+u(t)\big)\\&=-x_1^2+x_1\omega_2+\omega_2g(x_1)-c_1\omega_2x_2-c_2x_2^3\omega_2\\&\quad+\omega_2(\beta+f\sin\omega t)\sin(x_1)-x_1\omega_2+\omega_2^2+u(t)\omega_2\\&=-x_1^2+\omega_2g(x_1)-c_1\omega_2x_2-c_2x_2^3\omega_2\\&\quad+\omega_2(\beta+f\sin\omega t)\sin(x_1)+\omega_2^2+u(t)\omega_2\end{aligned} \tag{2.30}$$

From Assumption 2.4. we can get the following inequality:

$$\begin{aligned}\omega_2g(x_1)&\le l_1|\omega_2|\cdot|x_1|\\&\le\frac{1}{2}\varepsilon_1l_1^2|\omega_2|^2+\frac{1}{2\varepsilon_1}|x_1|^2\\&=\frac{1}{2}\varepsilon_1l_1^2\omega_2^2+\frac{1}{2\varepsilon_1}x_1^2\end{aligned} \tag{2.31}$$

Hence, from (2.31) we have the following inequalities:

$$\begin{aligned}\dot{V}_2(t)&\le-x_1^2+|\omega_2|\cdot|g(x_1)|-c_1\omega_2x_2+c_2\left|x_2^3\right|\cdot|\omega_2|\\&\quad+\omega_2(\beta+f\sin\omega t)\sin(x_1)+\omega_2^2+\omega_2u(t)\\&\le-x_1^2+l_1|\omega_2|\cdot|x_1|-c_1\omega_2x_2+c_2l_2|x_2|\cdot|\omega_2|\\&\quad+\omega_2(\beta+f\sin\omega t)\sin(x_1)+\omega_2^2+\omega_2u(t)\\&\le-x_1^2+\frac{1}{2\varepsilon_1}x_1^2+\frac{1}{2}\varepsilon_1l_1^2\omega_2^2-c_1\omega_2x_2+c_2l_2\omega_2|x_2|\operatorname{sign}(\omega_2)\\&\quad+\omega_2(\beta+f\sin\omega t)\sin(x_1)+\omega_2^2+\omega_2u(t)\end{aligned} \tag{2.32}$$

Substitute the controller $u(t)$ (see Equation (2.27)), inequality (2.32) can be rewritten as the following:

$$\dot{V}_2(t)\le-x_1+\frac{1}{2\varepsilon_1}x_1^2-k\omega_2^2 \tag{2.33}$$

If we choose an appropriate constant ε_1 such that $\frac{1}{2\varepsilon_1}<1$, we can obtain that $\dot{V}_2(t)<0$.

Then the gyroscope chaos in Equation (2.24) is asymptotically stable at origin $(0,0)$.

The proof is completed.

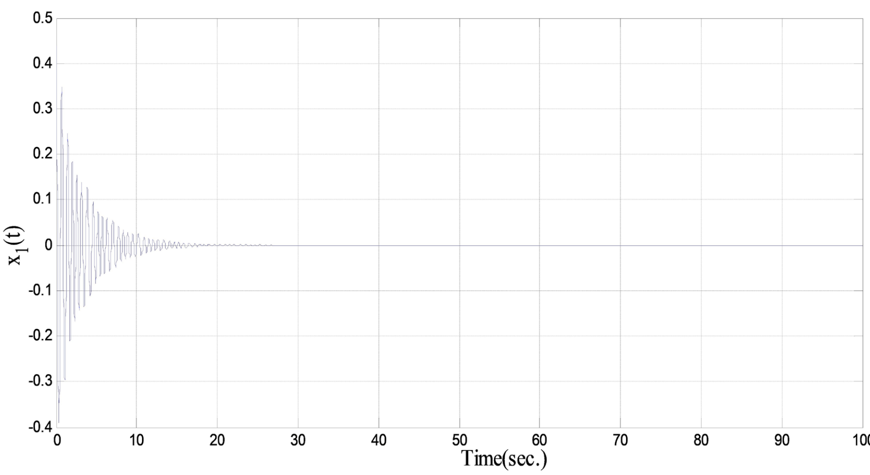
Remark 2.6. As in [7]-[11] [14]-[20] [26]. The authors proposed a controller with third-order terms. For chaos synchronization in gyroscope system while we already have removed these terms in the proposed method a novel controller without third-order terms can because of reducing the complexity of behavior of gyroscope on chaos control into two states and increase the feasibility of backstepping controller design approach.

4. Simulations and Analysis Results

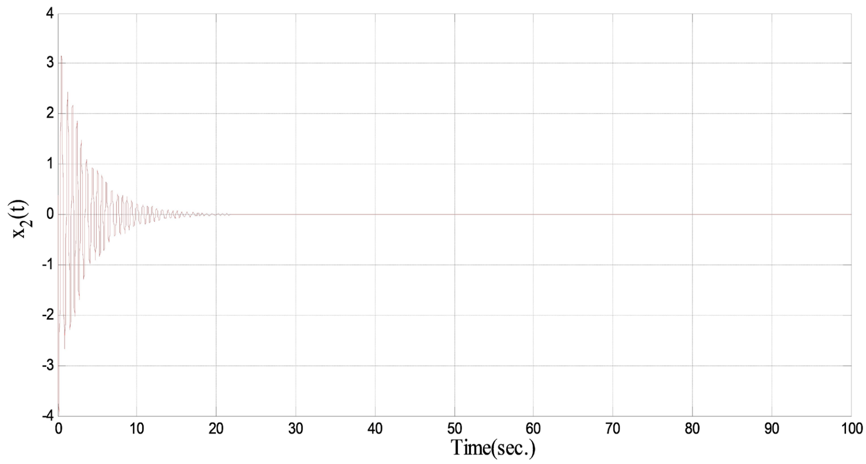
In this section, some numerical simulations are provided to verify the effectiveness of the proposed of gyroscope chaos control using backstepping control design method. Parameters of the nonlinear gyroscope are given as follows as:

$\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, $\omega = 2.5$, $f = 36$ which are shown in Section. In this way, the gyroscope system will show its chaotic characteristics and assuming the characteristics of dynamical system and its solution, one can determine the behavior of the two states. The initial conditions are defined in the following: $x_1(0) = 1$ and $x_2(0) = -1$, some control parameters are shown: As in $\varepsilon_1 = 2$, $l_1 = 4$, $l_2 = 10$, $k = 10$. According to Theorem 2.5, the gyroscope chaos can be stabilized as in **Figure 5** shows that the time responses of states of controlled chaotic gyroscope systems. The controller $u(t)$ under the new law of control and Lyapunov stability theory respectively. The work aims to reduce controller complexity compared with earlier designs that retained third-order terms, is shown in **Figure 6**. This result demonstrates that states of the chaotic gyroscope system can be stabilized asymptotically at the equilibrium point $(0,0)$.

This result demonstrates that states of the chaotic gyroscope system can be stabilized asymptotically at the equilibrium point $(0,0)$. Based on the backstepping control method, this paper has developed an effective controller without a cubic term (third-order terms) to control the chaotic gyroscope system in **Figure 6**. Compared with the existing control schemes, our design can reduce cause the complexity of the controller behavior on gyroscope chaos control.



(a)



(b)

Figure 5. Time responses of states of the controlled chaotic gyroscope system.

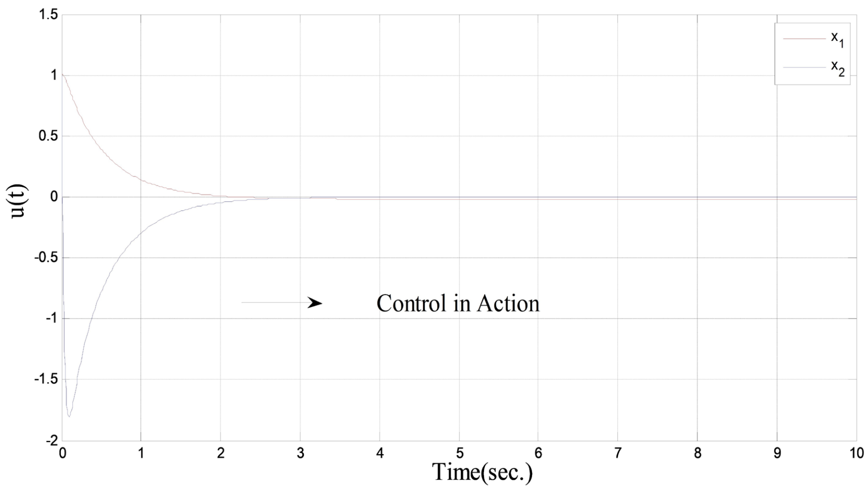


Figure 6. Variation of the control action $u(t)$ over time

A rigorous analysis has shown that by introducing control design techniques. Using backstepping control law, the control of the chaotic system can be achieved. A numerical simulation shows the effectiveness and feasibility of the proposed controller based on backstepping control design.

5. Conclusions

Based on Lyapunov stability theory a controller which employed the backstepping approach has been designed for gyroscope chaos control using backstepping control design. The backstepping technique, we have applied, allows for flexibility in the controller without cubic terms (third-order terms) design and global stability based on the appropriate choice of Lyapunov functions. Some useful results are achieved on the gyroscope chaos including control in this paper. However, gyroscope system is important in some engineering field such as navigation and aeronautics and some behaviors of gyroscope system have not research clearly when this system has been increasingly focused on. We will contribute continuously to the gyroscope system in the future and develop our research work from the following. Firstly; the more unmodelling dynamics will be concerned for gyroscope system, such as stochastic disturbance in electronic device. These unmodeling dynamics can decrease even destroy the stability of gyroscope system. In order to eliminate the negative effect, a novel control scheme is needed.

The simulation results show that the gyroscope chaos control system schemes of the backstepping approach is effective and have low complexity. Compared with the existing gyroscope chaos control scheme, our design avoids the complexity of behavior on the chaos controller and therefore has a lower implementation cost.

Available of Data and Materials

All data generated or analyzed during this study are included in this published article.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Lorenz, E.N. (1963) Deterministic Nonperiodic Flow. *Journal of the Atmospheric Sciences*, **20**, 130-141. https://doi.org/10.1175/1520-0469(1963)020<0130:dnf>2.0.co;2

[2] Rössler, O.E. (1976) An Equation for Continuous Chaos. *Physics Letters A*, **57**, 397-398. https://doi.org/10.1016/0375-9601(76)90101-8

[3] Rössler, O.E. (1979) Continuous Chaos—Four Prototype Equations. *Annals of the New York Academy of Sciences*, **316**, 376-392. https://doi.org/10.1111/j.1749-6632.1979.tb29482.x

[4] Ott, E., Grebogi, C. and Yorke, J.A. (1990) Controlling Chaos. *Physical Review Letters*, **64**, 1196-1199. https://doi.org/10.1103/physrevlett.64.1196

[5] Sprott, J.C. (1994) Some Simple Chaotic Flows. *Physical Review E*, **50**, R647-R650. https://doi.org/10.1103/physreve.50.r647

[6] Chen, G. and Dong, X. (1998) From Chaos to Order-Methodologies, Perspectives and Applications. World Scientific Publishing Co. Pte. Ltd. https://doi.org/10.1142/9789812798640

[7] Chen, G. and Ueta, T. (1999) Yet Another Chaotic Attractor. *International Journal of Bifurcation and Chaos*, **9**, 1465-1466. https://doi.org/10.1142/s0218127499001024

[8] Ge, S.S., Wang, C. and Lee, T.H. (2000) Adaptive Backstepping Control of a Class of Chaotic Systems. *International Journal of Bifurcation and Chaos*, **10**, 1149-1156. https://doi.org/10.1142/s0218127400000815

[9] Chen, H.-K. (2002) Chaos and Chaos Synchronization of a Symmetric Gyro with Linear-Plus-Cubic Damping. *Journal of Sound and Vibration*, **255**, 719-740. https://doi.org/10.1006/jsvi.2001.4186

[10] Van Dooren, R. (2003) Comments on "Chaos and Chaos Synchronization of a Symmetric Gyro with Linear-Plus-Cubic Damping". *Journal of Sound and Vibration*, **268**, 632-634. https://doi.org/10.1016/s0022-460x(03)00343-2

[11] Lei, Y., Xu, W. and Zheng, H. (2005) Synchronization of Two Chaotic Nonlinear Gyros Using Active Control. *Physics Letters A*, **343**, 153-158. https://doi.org/10.1016/j.physleta.2005.06.020

[12] Zhang, Y., Zhang, Q., Zhao, L. and Yang, C. (2007) Dynamical Behaviors and Chaos Control in a Discrete Functional Response Model. *Chaos, Solitons & Fractals*, **34**, 1318-1327. https://doi.org/10.1016/j.chaos.2006.04.032

[13] Yan, J., Hung, M., Lin, J. and Liao, T. (2007) Controlling Chaos of a Chaotic Nonlinear Gyro Using Variable Structure Control. *Mechanical Systems and Signal Processing*, **21**, 2515-2522. https://doi.org/10.1016/j.ymssp.2006.07.002

[14] Yau, H. (2008) Chaos Synchronization of Two Uncertain Chaotic Nonlinear Gyros Using Fuzzy Sliding Mode Control. *Mechanical Systems and Signal Processing*, **22**, 408-418. https://doi.org/10.1016/j.ymssp.2007.08.007

[15] Idowu, B.A., Vincent, U.E. and Njah, A.N. (2008) Control and Synchronization of Chaos in Nonlinear Gyros via Backstepping. *International Journal of Nonlinear Science*, **5**, 1-19.

[16] Alireza Sahab, M.H.Z. (2009) Improve Backstepping Method to GBM. *World Applied Science Journal*, **6**, 1399-1403.

[17] Farivar, F., Aliyari Shoorehdeli, M., Nekoui, M.A. and Teshnehlab, M. (2012) Chaos Control and Modified Projective Synchronization of Unknown Heavy Symmetric Chaotic Gyroscope Systems via Gaussian Radial Basis Adaptive Backstepping Control. *Nonlinear Dynamics*, **67**, 1913-1941. https://doi.org/10.1007/s11071-011-0118-z

[18] Idowu, B.A., Guo, R. and Vincent, U.E. (2013) Adaptive Control for the Stabilization and Synchronization of Nonlinear Gyroscopes. *International Journal of Chaos, Control, Modelling and Simulation*, **2**, 27-43. https://doi.org/10.5121/ijccms.2013.2204

[19] Yang, I. and Lee, D. (2013) Synchronization of Chaos Gyros Based on Robust Nonlinear Dynamic Inversion. *Mathematical Problems Engineering*, **2013**, Article ID: 519796.

[20] Aghababa, M.P. and Aghababa, H.P. (2013) Chaos Synchronization of Gyroscopes Using an Adaptive Robust Finite-Time Controller. *Journal of Mechanical Science and Technology*, **27**, 909-916. https://doi.org/10.1007/s12206-013-0106-y

[21] Loembe-Souamy, R.M.D., Jiang, G., Fan, C. and Wang, X. (2015) Chaos Synchronization of Two Chaotic Nonlinear Gyros Using Backstepping Design. *Mathematical Problems in Engineering*, **2015**, Article ID: 850612. https://doi.org/10.1155/2015/850612

[22] Davy, L.R.M., Jiang, G., Fan, C., Wang, X. and Wu, X. (2016) Chaos Synchronization of Two Uncertain Chaotic Nonlinear Gyros Using Adaptive Backstepping Design. 2016 *Chinese Control and Decision Conference* (*CCDC*), Yinchuan, 28-30 May 2016, 928-931. https://doi.org/10.1109/ccdc.2016.7531116

[23] Kocamaz, U.E., Çiçek, S. and Uyaroğlu, Y. (2017) Secure Communication with Chaos and Electronic Circuit Design Using Passivity-Based Synchronization. *Journal of Circuits, Systems and Computers*, **27**, Article 1850057. https://doi.org/10.1142/s0218126618500573

[24] Çiçek, S., Kocamaz, U.E. and Uyaroğlu, Y. (2018) Secure Communication with a Chaotic System Owning Logic Element. *AEU-International Journal of Electronics and Communications*, **88**, 52-62. https://doi.org/10.1016/j.aeue.2018.03.008

[25] Gokyildirim, A., Kocamaz, U.E., Uyaroglu, Y. and Calgan, H. (2023) A Novel Five-Term 3D Chaotic System with Cubic Nonlinearity and Its Microcontroller-Based Secure Communication Implementation. *AEU-International Journal of Electronics and*

Communications, **160**, Article 154497. https://doi.org/10.1016/j.aeue.2022.154497

[26] Aguessivognon, J.M., Miwadinou, C.H. and Monwanou, A.V. (2023) Effect of Bi-harmonic Excitation on Complex Dynamics of a Two-Degree-of-Freedom Heavy Symmetric Gyroscope. *Physica Scripta*, **98**, Article 095230. https://doi.org/10.1088/1402-4896/aceb3d