

The Implementation of Desirable Transformations of the Registered Seismic Waves

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Abstract

A critical phase in seismic research is the transformation of the recorded seismic waves into new ones, more suitable for further interpretation or processing. When the propagation of the recorded and desired after the transformation waves can be described by analytical functions, the use of the method of stationary-phase transformations of seismic data proposed by the authors earlier seems justified and appropriate. This method relies on a unified mathematical approach (finding the envelope of a family of functions), is fairly versatile, and is relatively simple to apply. Nearly 30 years of experience in its application, the creation, and the use of 16 procedures aimed at implementing the desired transformations of waves recorded as a result of seismic observations. These procedures include: poststack time and depth migrations using average velocities; transformation waves with hyperbolic traveltimes curves into waves with linear curves without information about velocities; prestack and poststack time migrations using stacking velocities; prestack time and depth migrations utilizing average velocities; transformation of input data into time sections employing average velocities; poststack depth migration using a depth-linear velocity function; transformation of VSP data into time sections, time migrated sections, and depth migrated sections with using average velocities; transformation refracted waves (headwaves) into time sections, time migrated sections, and depth migrated sections applying average velocities; transformation of input data into “floating” time sections simultaneously with obtaining information on the average velocities of waves propagation. Many of these procedures have no analogues, and those that do differ from them significantly in dynamic features. The procedures mentioned above were developed using the stationary-phase transformations method and are undoubtedly beneficial for broadening the potential of seismic research. Moreover, this

method itself serves as an impactful and promising tool for addressing numerous ongoing and future challenges in seismic exploration.

Keywords

Transformation, Stationary-Phase, Data, Migration, Traveltime, Section, Family of Functions, Envelope

1. Introduction

This work is, mostly, devoted to obtaining with help of the method of stationary-phase transformations of seismic data desirable and helpful procedures for transformations of the registered seismic waves.

Many researchers [1]-[22] have worked and continue to work to implement such transformations.

Nevertheless, there is still a great demand for them, both due to emergence of new tasks in the practice of seismic research (for example, 3-D seismic exploration) and to detail previously performed seismogeological constructions.

In this context, the authors advocate using the stationary-phase transformations method, which is a practical and enough versatile approach.

This method was proposed and developed [23]-[27]. In all the years following these publications, the method of stationary-phase transformations was actively developed and used.

Most of the procedures that were implemented by the method of stationary-phase transformations have no analogues, and those that have (for example, Kirchhoff's poststack time migrations) differ from them in some dynamic features.

As the name implies, the stationary-phase transformations method is based on the stationary phase method well-known in physics and mathematics. Thomson (1887) [28] was the first to justify it for high-frequency waves, but its applicability for sound and seismic waves has been demonstrated and successfully used by many researchers [3] [4] [17] [19] [29]-[32].

The method rests on two fundamental principles [5] [14] [19]:

- The point of tangency between the traveltime curve for the seismic wave and the integration line, as well as its surroundings, significantly contributes to the integration result.
- The difference between the second derivatives of the integration line and traveltime curve at the point of tangency has a significant impact on the amplitude of the wave after integration.

The authors of the method of stationary-phase transformations of the seismic data proposed to solve the kinematic part of the problem using the apparatus of finding the envelope of a family of functions depending on parameters [3] [33]-[36]. As will be shown below, when such families describe the traveltime curves

for the waves recorded and desired after the transformation, there is a third family of functions, by summing up along the envelope of which the desired transformations can be achieved.

They also suggested keeping the difference between the second derivatives of the wave traveltime curve and the integration line at the tangency point the same during the integration process. In this case, the wave distortions that inevitably arise as a result of the transformations will be almost identical and will not significantly affect the wave correlation. In addition, by trying out possible values of this difference, can be found such one that provides the desired compromise between noise immunity and transformation resolution.

All seismic data transformation procedures described in this study have been rigorously tested on synthetic materials, confirming their functionality and compliance with theoretical assumptions. Some of these transformations have also been tested on real data.

It is noteworthy that the envelopes used in the procedures for obtaining time migrated sections correspond to the propagation time curves of the diffracted waves, which is consistent with the Kirchhoff method. This circumstance confirms the theoretical correctness of the proposed approach.

In this work the same notation was used for the construction of various transformations. So, the equation of the family of the traveltime curves for the recorded waves is denoted as T ; the equation of the family of the traveltime curves for the waves after the transformation is \tilde{T}_0 ; the equation of the family of functions that connects both previous families is \tilde{T} , and the envelope of this family is $\tilde{\tilde{T}}$; T_0 is the two-way normal traveltime for the reflected wave at the beginning of the coordinate system; V is the average wave propagation velocity; η is the location of the recorded trace and ξ is the location of the result trace; t is the angular coefficient of the T_0 line on time sections, and φ is dip angle of the boundary. The distance between the traces in all the figures presented in the work is 15 m. If the listed designations are given a different meaning or new designations are used, this will be reported in the text.

2. Method

2.1. Method of the Stationary-Phase Transformations of Seismic Data

In most practical situations, the traveltime curves for recorded waves can be described analytically as a family of functions that rely on several variables and parameters:

$$T = F(x_1, x_2, \dots, c_1, c_2, \dots, c_i, \dots), \quad (1)$$

where x_1, x_2, \dots are variables and $c_1, c_2, \dots, c_i, \dots$ are independent parameters.

The desired traveltime curves for transformed waves can also be described as a family of functions that are dependent on a several of variables and parameters:

$$\tilde{T}_0 = P(y_1, y_2, \dots, d_1, d_2, \dots, d_j, \dots), \quad (2)$$

where y_1, y_2, \dots are variables and $d_1, d_2, \dots, d_j, \dots$ are independent parameters.

If at least one of these parameters is the same for both families $d_j = c_i$, we can express d_j using the remaining terms of the Equation (2)

$$d_j = R(\tilde{T}_0, y_1, y_2, \dots, d_1, d_2, \dots, d_{j-1}, d_{j+1}, \dots) \quad (3)$$

and substitute it into Equation (1) to obtain the new family of functions

$$\tilde{T} = F\left(R(\tilde{T}_0, y_1, y_2, \dots, d_1, d_2, \dots, d_{j-1}, d_{j+1}, \dots), x_1, x_2, \dots, c_1, c_2, \dots, c_{i-1}, c_{i+1}, \dots\right), \quad (4)$$

that connects families (1) and (2).

The envelope of the family (4) \tilde{T} can be obtained by excluding any parameters of this family using the necessary condition for the existence of the envelope [32] [36] and so on:

$$\tilde{T}'_{r_1} = 0, \dots, \tilde{T}'_{r_k} = 0, \quad (5)$$

where r_k is an independent parameter of the family (4).

To perform stationary-phase transformations, seismic data must be integrated along the envelope of the family (4) and the outcome assigned to point \tilde{T}_0 of the some resultant trace located at point ξ . In this scenario, there exists a certain value of \tilde{T}_0 at which the condition $d_j = c_i$ is met. Consequently, one of the functions from family (1) will be coincide to one of the functions from family (4), and the envelope of the family (4) will be in touching with this function. During integration seismic data along \tilde{T} , the vicinity of the point of tangency is shifted to the desired location.

It is proposed that the difference, which is named by us “touch character”, between the second derivatives at the point of contact be kept constant during the integration. In this case, the wave distortions that unavoidably occur as a consequence of the transformations will be almost identical and will have little influence on the outcome of future correlation of the transformed waves. The concrete value of “touch character” must be determined experimentally.

If we denote “touch character” as α then:

$$\alpha = \tilde{T}'' - T'' = const. \quad (6)$$

If seismic conditions are almost similar, α is a very stable parameter that, once established, can be used for various transformations.

The “touch character” constancy can be established during the transformation process by uneven summing of seismic records. For certain simple stationary-phase transformations, the necessary summation order can be determined analytically using the difference of total differentials or a second-order Taylor series expansion around the point of tangency.

In most circumstances, however, an approximate technique is utilized, which consists of the stages listed below. Assuming that some point on the summation line (the envelope of the family of functions (4)) represents the point of tangency, will choose the next point so that the time on the summation (integration) curve varies from the predicted time on the travelttime curve by the value of “touch char-

acter”. In this case, the summation must be performed in sufficiently small steps to prevent missing the desired point.

The impact of the value of the “touch character” on the quality of the transformation is shown in **Figures 1-4**.

Figure 1 depicts a fragment of the CMP timesection. **Figures 2-4** depict the same fragment after prestack time migrations with “touch character” values of 5000, 500, and 10, respectively. According to the authors, the transformation resolution in **Figure 2** is high, but noise immunity is low. **Figure 4** depicts the opposite situation-low resolution and high noise immunity. Only in **Figure 3** do we see an acceptable compromise between them.

The stationary-phase transformations are implemented using the same scheme for any transformation. First, equations for families of functions that describes the traveltimes curves for waves recorded and sought after the transformation should be found. At least one parameter from each family should be shared. We express

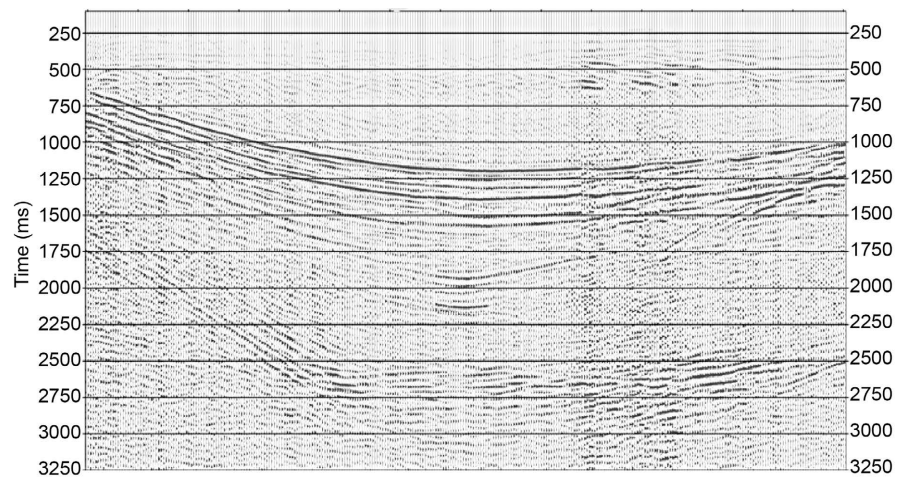


Figure 1. Fragment of the CMP time section.

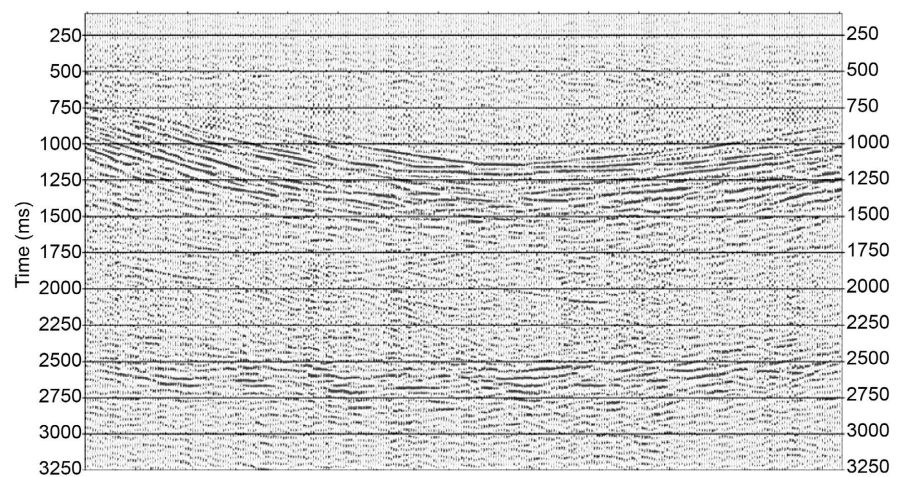


Figure 2. Fragment of the prestack time migrated section. Value of the “touch character” is 5000.

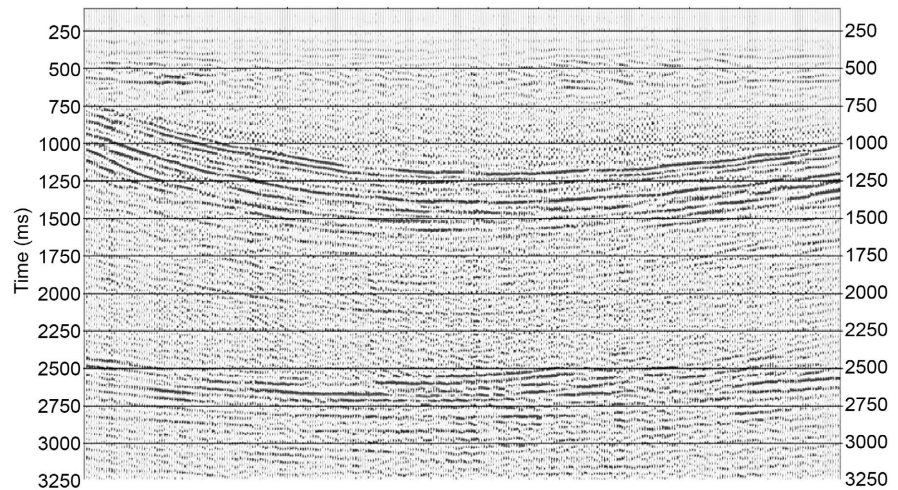


Figure 3. Fragment of the prestack time migrated section. Value of the “touch character” is 500.

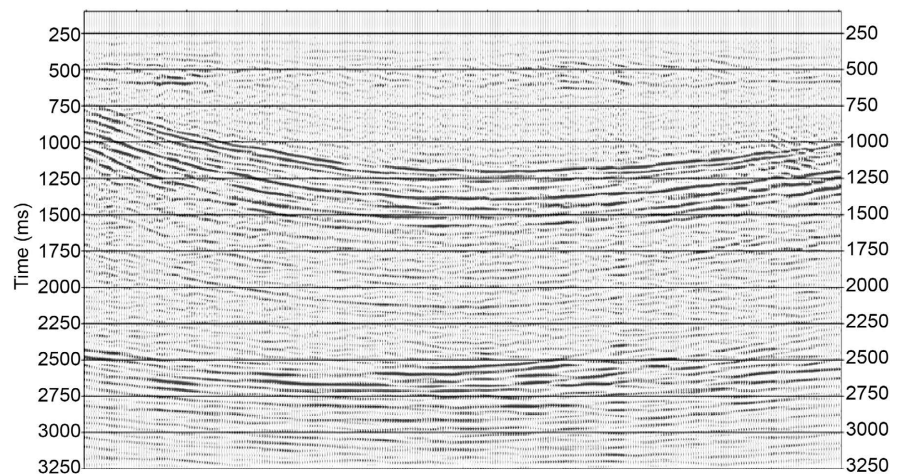


Figure 4. Fragment of the prestack time migrated section. Value of the “touch character” is 10.

this parameter using the parameters and variables from the second equation and then plug the resulting expression in the first equation.

The new family of functions connects both preceding families. Then should be find the envelope of this family and summarize the recorded seismic data along it unevenly. The order of summation is determined by the difference of total differentials or the decomposition of functions according to the Taylor formula, or approximately as shown above.

So, the sum of irregularly located seismic data is used to perform stationary-phase transformations. Because of this unevenness, the difference in the second derivatives at the point of tangency between the wave’s traveltime curve and the summation line may be preserved as a constant, allowing for a compromise between wave transformation resolution and noise immunity.

The technique of stationary-phase transformations will be illustrated in more

detail below, using the example of an enough simple and extensively used procedure for transforming waves reflected from a flat boundary and recorded on time sections into time migrated sections.

The “touch character” value is determined experimentally and, to some extent, subjectively.

2.2. Illustration of the Method: Poststack Time and Depth Migrations Using Average Velocity

Time and depth migrations techniques are widely accepted. The works of [2] [3] [5] [15] [17] [18] and others played and are playing a significant role in the formation and promotion of this approach.

The corresponding procedure created by the stationary-phase transformation method is quite simple and serves as a good illustration of the implementation of this method.

The equation of the family of functions describing the reflection traveltime curves for the average velocity model with plane reflectors recorded on time sections has the form

$$T = T_0 + t * \eta, \tag{7}$$

where η is the location of the time section trace (Figure 5).

Following the formulation of the problem, the transformation’s result must be located at the point (\tilde{T}_0, ξ) , where \tilde{T}_0 is the double vertical depth in the time scale and ξ is the location of the migrated section trace.

Because the cosine of the reflector dip angle is equal to $\sqrt{4 - t^2 * V^2} / 2$ ([32] [37] and so on) the equation of the family of traveltime curves for the reflected waves on migrated time sections is:

$$\tilde{T}_0 = 2 * (T_0 + t\xi) / \sqrt{4 - t^2 * V^2}. \tag{8}$$

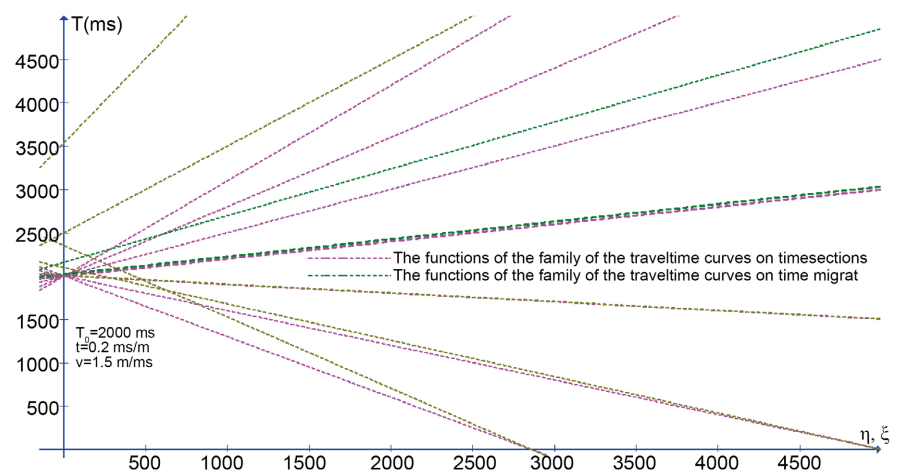


Figure 5. Traveltime curves for the reflected waves recorded on time and migrated sections.

On Figure 5 for $T_0 = 2000$ ms, $t = 0.2$ ms/m and $V = 1.5$ m/ms we can see

some functions of the family of the reflection traveltimes on time section and them also on time migrated section.

Here, we express the parameter T_0 in terms of the rest of the family's parameters and variables of the family (8):

$$T_0 = \tilde{T}_0 * \sqrt{4 - t^2 * V^2} / 2 - t\xi \tag{9}$$

and substitute it into (7):

$$\tilde{T} = \tilde{T}_0 * \sqrt{4 - t^2 * V^2} / 2 + t(\eta - \xi). \tag{10}$$

The acquired family connects the family of traveltimes curves for reflected waves (7) with the desired family of traveltimes curves for transformed waves (8) on time migrated sections.

It is easy to make sure that, if the value of \tilde{T}_0 is (8), then the equation of the function of the family \tilde{T} equal $\tilde{T} = T_0 + t * \eta$ regardless of the concrete value of ξ and the envelope of the family (10) will be in touching with this function.

To find the envelope of the family (10) let's use the condition $T'_t = 0$ for excluding the parameter t :

$$t = 4 * (\eta - \xi) / \left(V * \sqrt{\tilde{T}_0^2 * V^2 + 4 * (\eta - \xi)^2} \right). \tag{11}$$

If t is substituted into (10), the envelope of the family (10) will be obtained

$$\tilde{T} = \sqrt{\tilde{T}_0^2 + 4 * (\eta - \xi)^2} / V^2. \tag{12}$$

For $t = 0.2$ ms/m and $\xi = 2500$ m on **Figure 6** you can see some functions of the family (10) and coincidence of the function $T = 2000 + 0.2 * \eta$ from the family (7) with a function from the family (10) and touching this function with the envelope (12).

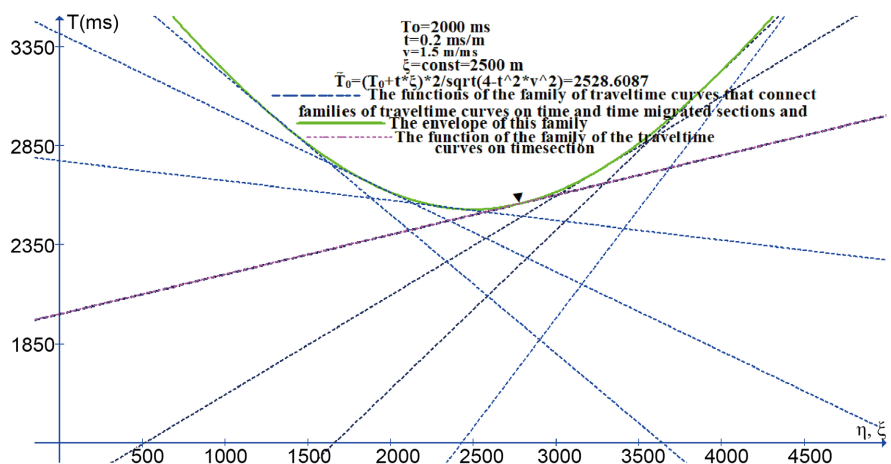


Figure 6. Touching the summation line with the traveltime curve for the reflected wave recorded on the timesection.

To satisfy condition (6) let's expand (7) and (12) in a Taylor series at the point

η and yields:

$$\begin{aligned} T'_\eta &= t, \\ T''_{\eta^2} &= 0, \end{aligned} \tag{13}$$

$$\tilde{T}'_\eta = \frac{4 * (\eta - \xi)}{V^2 * \sqrt{\tilde{T}_0^2 + \frac{4 * (\eta - \xi)^2}{V^2}}}, \tag{14}$$

$$\tilde{T}''_{\eta^2} = \frac{4 * \tilde{T}_0^2}{V^2 * \left(\sqrt{\tilde{T}_0^2 + \frac{4 * (\eta - \xi)^2}{V^2}} \right)^3}, \tag{15}$$

$$T(\eta_i + \Delta\eta_i) = \tilde{T}_0 * \sqrt{4 - t^2 * V^2} / 2 + t(\eta_i - \xi) + \Delta\eta_i * t, \tag{16}$$

$$\begin{aligned} \tilde{T}(\eta_i + \Delta\eta_i) &= \sqrt{\tilde{T}_0^2 + 4 * (\eta_i - \xi)^2 / V^2} + \frac{4 * \Delta\eta_i * (\eta_i - \xi)}{V^2 * \sqrt{\tilde{T}_0^2 + 4 * (\eta_i - \xi)^2 / V^2}} \\ &+ 2 * \frac{(\Delta\eta_i)^2 * (\tilde{T}_0)^2}{V^2 * \sqrt{\tilde{T}_0^2 + 4 * (\eta_i - \xi)^2 / V^2}^3}. \end{aligned} \tag{17}$$

If subtract (16) from (17), given that η_i is the point of tangency, and equates this difference to $\alpha = const$, we will get:

$$\Delta\eta_i = \sqrt{\alpha * V^2 * \sqrt{\tilde{T}_0^2 + 4 * (\eta_i - \xi)^2 / V^2}^3 / (2 * \tilde{T}_0)}. \tag{18}$$

Therefore, to transform time section into time migrated section, we should sum the origin's traces along (12) and put the results into the point (\tilde{T}_0, ξ) for all values, \tilde{T}_0, η_i and ξ (Figure 7).

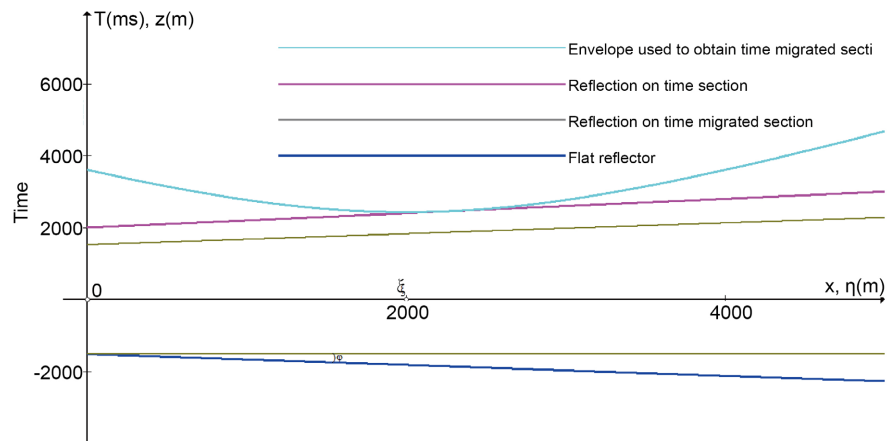


Figure 7. Poststack time and depth migrations using average velocities.

The Equation (12) is the equation of the diffracted wave with the diffraction center at point (\tilde{T}_0, ξ) . Thus, the stationary-phase method for the average-velocity model of the medium in kinematic terms agrees with the Kirchhoff method

and differs from it in dynamic features.

From **Figure 7** we can see that the equation of the reflecting boundary is described as

$$z = h_0 + k * \xi, \quad (19)$$

where h_0 is vertical distance from the source to the reflector and $k = tg\varphi = t * V / \sqrt{4 - t^2 * V^2}$.

Let's use (19) to build the depth migration procedure.

If we take into account that

$$h_0 = T_0 * V / \sqrt{4 - t^2 * V^2}, \quad (20)$$

then the equation of the flat reflector is:

$$z = (T_0 + t * \xi) * V / \sqrt{4 - t^2 * V^2}. \quad (21)$$

Let's represent parameter T_0 by utilizing other parameters and variables from the family (21):

$$T_0 = z * \sqrt{4 - t^2 * V^2} / V - t * \xi \quad (22)$$

and substitute it into (7):

$$\tilde{T} = \left[z * \sqrt{4 - t^2 * V^2} + t * V * (\eta - \xi) \right] / V. \quad (23)$$

For finding the envelope of this family excluding parameter t , using the next condition:

$$t = 2 * (\eta - \xi) / \left[V * \sqrt{z^2 + (\eta - \xi)^2} \right] \quad (24)$$

$$\tilde{T}'_t = -z * t * V / \sqrt{4 - t^2 * V^2} + (\eta - \xi) = 0, \quad (25)$$

Substituting (25) into (23) to obtain the equation of the envelope of the family (23):

$$\tilde{T} = 2 * \sqrt{z^2 + (\eta - \xi)^2} / V, \quad (26)$$

which will be used for the receiving depth migration section.

As can be seen from **Figure 7**, the envelopes used to realize time and depth migration coincide, but the results of summation along these envelopes refer to different points \tilde{T}_0 and z .

To keep the "touch character" during summation approximately constant, let us expand functions (7) and (26) by Taylor series at the point of tangency by analogy with (13)-(17).

In this case we will obtain

$$\Delta \eta_i = \sqrt{\alpha * V * \sqrt{z^2 + (\eta_i - \xi)^2}^3} / 2. \quad (27)$$

The result of testing this procedure is shown in **Figure 8**.



Figure 8. Synthetic time section ($T_0 = 2000$ ms, $t = 0.2$ ms/m) and poststack time, and depth migrations using average velocity $V = 1.5$ m/ms.

3. Implementation of the Method

3.1. Transformation Reflected Waves Recorded on CMP Seismograms with Hyperbolic Traveltime Curves into Waves with Linear Traveltime Curves

The traveltime curve for the wave reflected from a flat reflector in the CMP system is described by a hyperbola of the following shape [32] [37]:

$$T = \sqrt{T_0^2 + 4 * \eta^2 / V_{stk}^2} , \tag{28}$$

where η is half of the source-receiver offset (Figure 9).

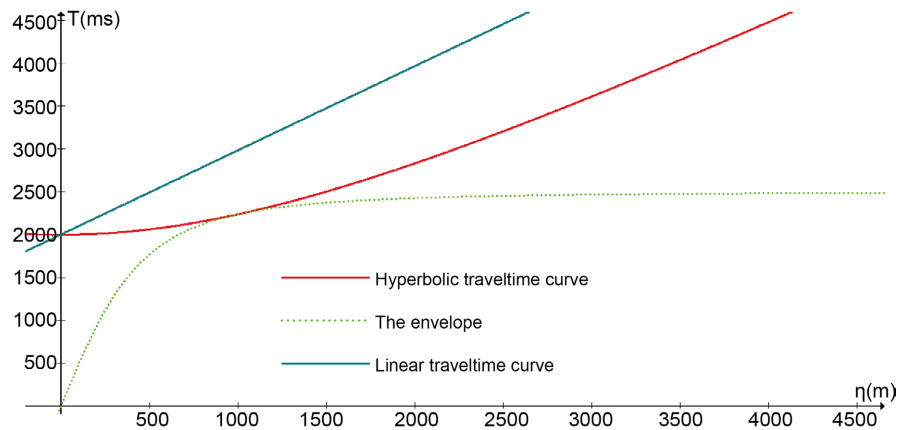


Figure 9. The linearization of hyperbolic traveltimes curves for reflected waves on CMP seismograms.

Sometimes it is preferable to transform waves with hyperbolic traveltime curves into a wave with linear traveltime curves (for example, to increase the resolution of the CMP summation) of the following shape:

$$\tilde{T}_0 = T_0 + 2 * \xi / V_{stk} , \tag{29}$$

where ξ is the location of the result trace.

T is the family of functions that depends on the parameters (T_0, V_{stk}) , and \tilde{T}_0 is the family of functions that also depends on parameters (T_0, V_{stk}) .

Let us express T_0 through other parameters and variables of the family (29):

$$T_0 = \tilde{T}_0 - 2 * \xi / V_{stk} \tag{30}$$

and creates a new family of functions by substituting (30) into (28):

$$\tilde{T} = \sqrt{\left(\tilde{T}_0 - 2 * \xi / V_{stk}\right)^2 + 4 * \eta^2 / V_{stk}^2} . \tag{31}$$

The envelope of the family (31) is determined based on the Equation (5):

$$\tilde{T}'_{(V_{stk})} = \left(\tilde{T}_0 - 2 * \xi / V_{stk}\right) * \xi - \left(2 * \xi^2\right) / V_{stk} = 0 ; V_{stk} = 2 * \left(\xi^2 + \eta^2\right) / \left(\tilde{T}_0 * \xi\right) ; \tag{32}$$

$$\tilde{T} = \tilde{T}_0 * \eta / \sqrt{\xi^2 + \eta^2} . \tag{33}$$

Consequently, if we sum the recorded CMP traces along (33) for different values of the parameters and variables, we obtain a new CMP record with the traveltimes curves for transformed waves which are linear functions (**Figure 10**).

According to (33), values of V_{stk} is not used directly to perform this procedure but their approximate values can be used to maintain the “touch character” during summing for constant. In this case we use the expansion of functions (28) and (33) in a Taylor series by analogy with (13)-(17) and obtain

$$\Delta \eta_i = \frac{\alpha}{\frac{2 * \left(\tilde{T}_0 - 2 * \xi / V_{stk}\right)^2}{V_{stk}^2 * \left(\sqrt{\left(\tilde{T}_0 - 2 * \xi / V_{stk}\right)^2 + 4 * \eta_i^2 / V_{stk}^2}\right)^3} - \frac{3 * \tilde{T}_0 * \xi^2 * \eta_i}{2 * \left(\sqrt{\xi^2 + \eta_i^2}\right)^5}} . \tag{34}$$

If is no information about V_{stk} , the approximation technique mentioned in the introduction is employed to maintain the “touch character” as a constant.

The result of testing this procedure is shown in **Figure 10**.

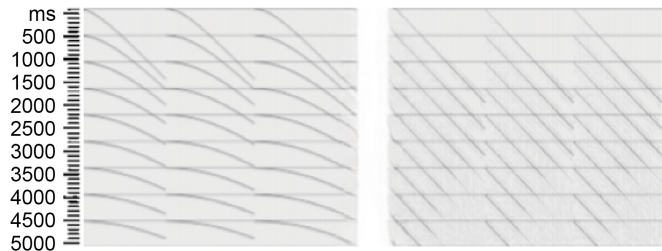


Figure 10. Synthetic CMP seismograms with reflected from flat reflectors waves and CMP seismograms after transformation.

Authors didn't find analogs for this procedure.

3.2. Poststack Time Migration Using Stacking Velocities

It is possible to transform time sections into time migrated sections using the values of V_{stk} acquired in the process of realization of the CMP method (**Figure 11**).

Due to the fact that the cosine of the reflector decline angle is equal $\sqrt{4 - t^2 * V^2} / 2$

$$V_{stk} = 2 * V / \sqrt{4 - t^2 * V^2} \tag{35}$$

and considering that (8) is the equation of the family of traveltimes curves for reflected waves on time migrated sections, we obtain the equation for family of reflection traveltimes curves on time migrated sections when using the stack velocity:

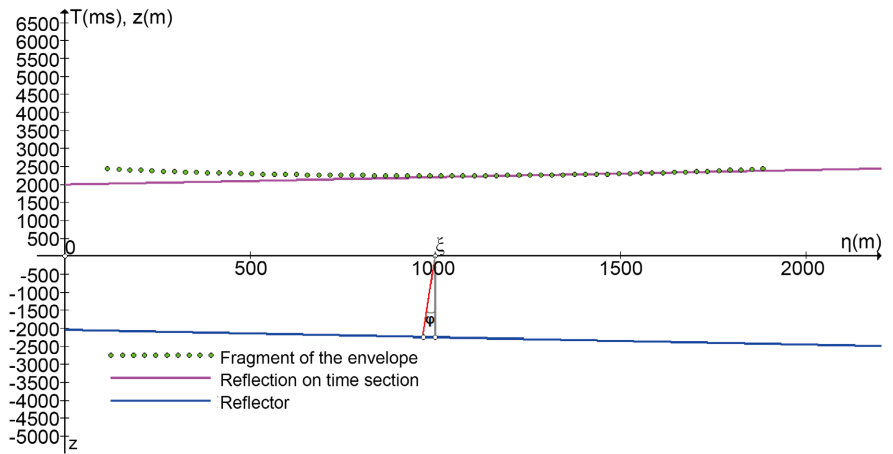


Figure 11. Poststack time migration using stacking velocities.

$$\tilde{T}_0 = (T_0 + t\xi) * \sqrt{4 - t^2 * V_{stk}^2} / 2, \tag{36}$$

where ξ is the location of the migrated section trace.

To obtain the equation of the family that connects the family for reflection traveltimes on time sections with the family for reflection traveltimes on time migrated sections the parameter T_0 from Equation (36) should be written in terms of the other parameters and variables of this equation and substituted into Equation (7):

$$T_0 = 2 * \tilde{T}_0 / \sqrt{4 + t^2 * V_{stk}^2} - t * \xi \tag{37}$$

and

$$\tilde{T} = 2 * \tilde{T}_0 / \sqrt{4 + t^2 * V_{stk}^2} + t * (\eta - \xi). \tag{38}$$

The necessary condition for the existence of an envelope of a family of functions yields the following equation:

$$(\eta - \xi) = 2 * \tilde{T}_0 * t * V_{stk}^2 / \sqrt{4 + t^2 * V_{stk}^2}^3. \tag{39}$$

To solve this equation, almost any numerical method may be utilised. One such strategy is Newton’s approach. In this case,

$$F = (\eta - \xi) - 2 * \tilde{T}_0 * V_{stk}^2 / \sqrt{4 + t^2 * V_{stk}^2}^3 = 0, \tag{40}$$

$$t_{i+1} = t_i - F(t_i) / F'_t(t_i), \tag{41}$$

$$F'_t = -4 * \tilde{T}_0 * t * V_{stk}^2 * (2 - t^2 * V_{stk}^2) / \sqrt{4 + t^2 * V_{stk}^2}^5 = 0, \tag{42}$$

because the equation of the envelope of the family (38) cannot be written analytically, the approximation technique mentioned in the introduction is employed to maintain the “touch character” as a constant.

The result of testing this procedure is shown in **Figure 12**.

Authors didn’t find analogs for this procedure.

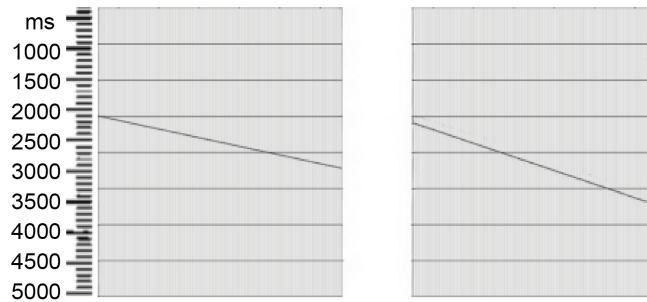


Figure 12. Synthetic time section ($T_0 = 2000$ ms, $t = 0.2$ ms/m, $V_{stk} = 2.041$ m/ms) and poststack time migration using stacking velocity.

3.3. Prestack Time and Depth Migrations Using Average Velocities

Despite its undeniable benefits, the CMP method distorts the recorded wave field to some degree, similar to every other seismic data processing method. In many seismic-geological situations, it is preferable to acquire migrated sections immediately from an observed wave field.

Largely due to the works of [2] [3] [5] [6] [9] [15] [16] [18]-[20] [32] [38]-[40] and others, time and depth migrations have found wide application in the practice of seismic research.

To construct such a procedure using the stationary-phase method, we must first determine the equation of the family of traveltime curves for the waves reflected from the flat reflector for the average velocity medium model.

According to method of stationary-phase transformations of seismic data for implementing desirable transformations first of all we need to have equations of families of traveltime curves of reflected from a flat reflector in an average velocity medium registered on CMP seismograms and families of traveltime curves of reflected waves on time and depth migrated sections (Figure 13).

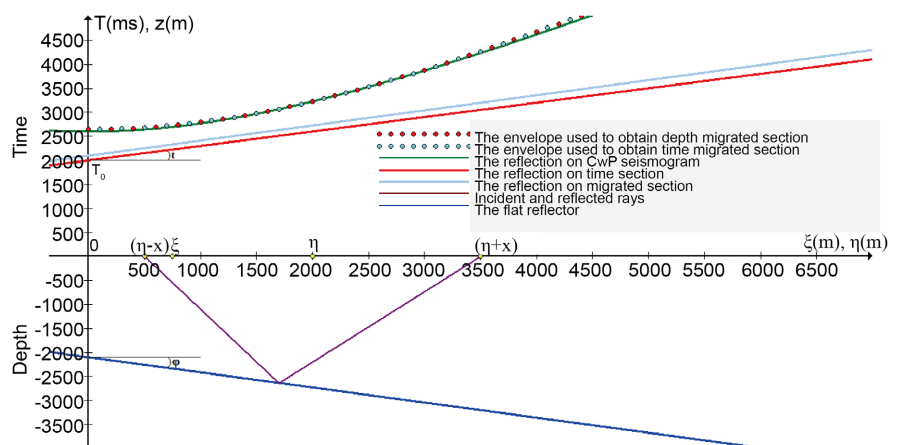


Figure 13. Prestack time and depth migrations using average velocities.

The first of required equation can be obtained from [32] and [37] and so on, and the rest two we got earlier and they equal, respectively, (8) and (21). For the

sake of completeness, we will provide all 3 equations.

$$T = \sqrt{(T_0 + t * \eta)^2 + (4 - t^2 * V^2) * x^2 / V^2}, \tag{43}$$

$$\tilde{T}_0 = 2 * (T_0 + t \xi) / \sqrt{4 - t^2 * V^2}, \tag{44}$$

$$z = (T_0 + t * \xi) * V / \sqrt{4 - t^2 * V^2}. \tag{45}$$

When we express parameter T_0 in terms of the other parameters and variables of Equation (44)

$$T_0 = \tilde{T}_0 * \sqrt{4 - t^2 * V^2} / 2 - t \xi \tag{46}$$

and substitute it into (43) we obtain the family of functions (47) that connects families (43) and (44).

$$\tilde{T} = \sqrt{\left[\tilde{T}_0 * \sqrt{4 - t^2 * V^2} / 2 + t * (\eta - \xi) \right]^2 + (4 - t^2 * V^2) * x^2 / V^2}. \tag{47}$$

We'll use condition $\tilde{T}'_t = 0$ to find the envelope of this family:

$$\begin{aligned} \tilde{T}'_t = & \left[\tilde{T}_0 * \sqrt{4 - t^2 * V^2} / 2 + t * (\eta - \xi) \right] \\ & * \left[(\eta - \xi) - \tilde{T}_0 * t * V^2 / (2 * \sqrt{4 - t^2 * V^2}) \right] - t * x^2 = 0. \end{aligned} \tag{48}$$

$$t = \sqrt{2 * \left(1 - A / \sqrt{A^2 + B} \right)} / V, \tag{49}$$

where

$$A = x^2 - (\eta - \xi)^2 + T_0^2 * V^2 / 4 \text{ and } B = T_0^2 * V^2 * (\eta - \xi)^2.$$

The envelope of the family (47) is equal

$$\tilde{\tilde{T}} = \sqrt{\tilde{T}_0^2 / 4 + [x - (\eta + \xi)]^2 / V^2} + \sqrt{\tilde{T}_0^2 / 4 + [x - (\eta - \xi)]^2 / V^2}. \tag{50}$$

Summing the seismic traces registered on the CMP seismograms along curve (50) results in a time migrated section (Figure 14).

Because Equation (50) is the equation of a diffracted wave with the center of diffraction at point (\tilde{T}_0, ξ) , the stationary-phase method for the average velocity model of the medium in kinematic terms is identical to the Kirchhoff method and differs from it in dynamic features.

The difference between complete differentials in tangent point is used to store “touch character” as a constant. These expressions are too extensive to fit in this study.

To obtain the depth migrated section, we need to use the equation of the family of functions (45).

By analogy (46)-(50) we express T_0 in terms of the other parameters and variables of the Equation (45) and substitute it into (43) and obtain the equation of the family of functions connecting these families:

$$\tilde{\tilde{T}} = \sqrt{\left[z * \sqrt{4 - t^2 * V^2} + t * V * (\eta - \xi) \right]^2 + (4 - t^2 * V^2) * x^2} / V. \tag{51}$$

The envelope of this family is found using condition $\tilde{T}'_t = 0$ and by substituting found value of

$$t = \sqrt{2 * \left\{ 1 - \frac{z^2 + x^2 - (\eta - \xi)^2}{\sqrt{4 * z^2 * (\eta - \xi)^2 + [z^2 + x^2 - (\eta - \xi)^2]^2}} \right\}} / V \tag{52}$$

into (51).

Representing this envelope explicitly is too cumbersome.

Figure 14 illustrates the outcomes of our test.

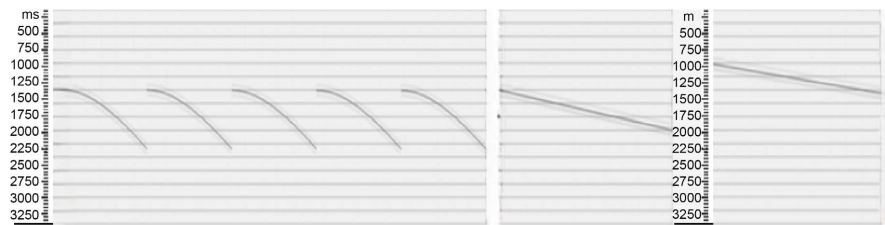


Figure 14. Synthetic CMP seismograms with recorded reflections ($T_0 = 2000$ ms, $t = 0.2$ ms/m, $V = 1.5$ m/ms), prestack time and depth migrations.

The approximation technique mentioned in the introduction is employed to maintain the “touch character” as a constant.

3.4. Prestack Time Migration Using Stacking Velocities

In the absence of information on the propagation average velocities of seismic waves, the use of this procedure to acquire a time migrated section appears sufficiently justifiable.

The equations necessary for the implementation of this procedure were obtained earlier (43) and (36), and they are presented below (**Figure 15**):

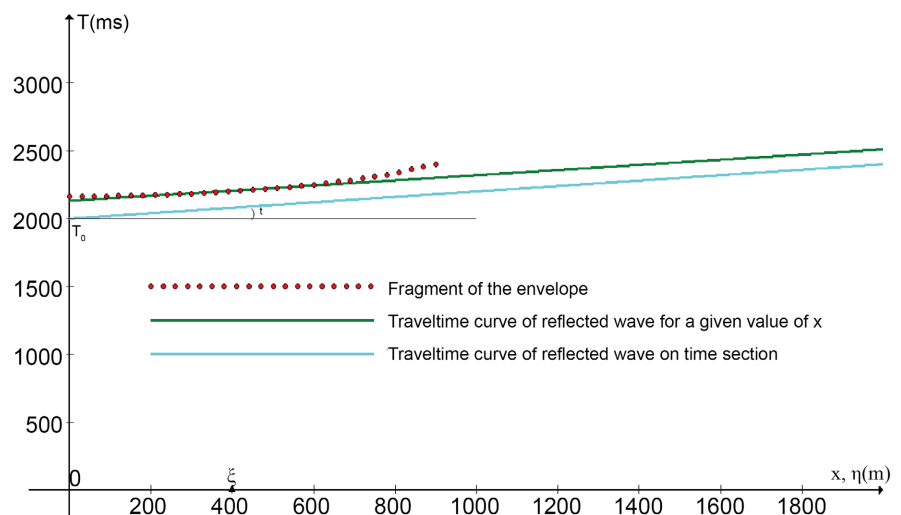


Figure 15. Prestack time migration using stacking velocities.

The equation of families of traveltime curves of reflected from a flat reflector in an average velocity medium registered on CMP seismograms-

$$T = \sqrt{(T_0 + t * \eta)^2 + (4 - t^2 * V^2) * x^2 / V^2}, \tag{53}$$

and the equation of family of reflection traveltime curves on time migrated sections when using the stack velocity:

$$\tilde{T}_0 = (T_0 + t\xi) * \sqrt{4 - t^2 * V_{stk}^2} / 2. \tag{54}$$

If we express parameter T_0 in terms of the other parameters and variables of Equation (54) and substitute it into (53) we obtain the equation of family of functions connecting (53) and (54) that has the following form:

$$\tilde{T} = \sqrt{\left[2 * \tilde{T}_0 / \sqrt{4 + t^2 * V_{stk}^2} + t * (\eta - \xi) \right]^2 + 4 * x^2 / V_{stk}^2}. \tag{55}$$

The envelope of this family will be found using condition $\tilde{T}'_t = 0$:

$$\tilde{T}'_t = \left[2 * \tilde{T}_0 / \sqrt{4 + t^2 * V_{stk}^2} + t * (\eta - \xi) \right] * \left[(\eta - \xi) - \frac{2 * \tilde{T}_0 * t * V_{stk}^2}{\sqrt{4 + t^2 * V_{stk}^2}} \right] = 0. \tag{56}$$

This equation is being decided by using Newton’s method.

Because the equation of the envelope of the family (55) cannot be expressed analytically, the approximate method described in the introduction is utilized to keep the “touch character” as constant.

The result of our test is shown in **Figure 16**.

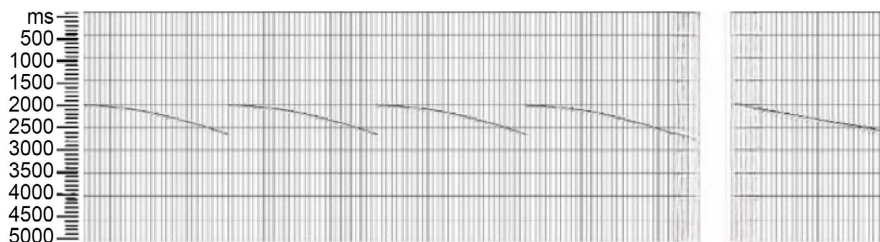


Figure 16. Synthetic CMP seismograms with recorded reflections ($T_0 = 2000$ ms, $t = 0.2$ ms/m, $V_{stk} = 2.041$ m/ms), and prestack time migration using stacking velocities.

Authors didn’t find analogs for this procedure.

3.5. Transformation of Input Data into Timesection Using Average Velocities

The CMP approach for dipping reflectors, as is widely known [38], includes records in the summing process that are unrelated to the common reflecting point. The DMO procedure [41] was created to address this problem to a certain extent.

The following approach is used to produce a time section that includes reflec-

tions from one point.

The equation of the family of traveltime curves of the reflected waves for a medium-velocity model with flat reflectors recorded on CMP seismograms equals (43), and (7) is equation of the family of traveltime curves for the same waves recorded on time sections (Figure 17).

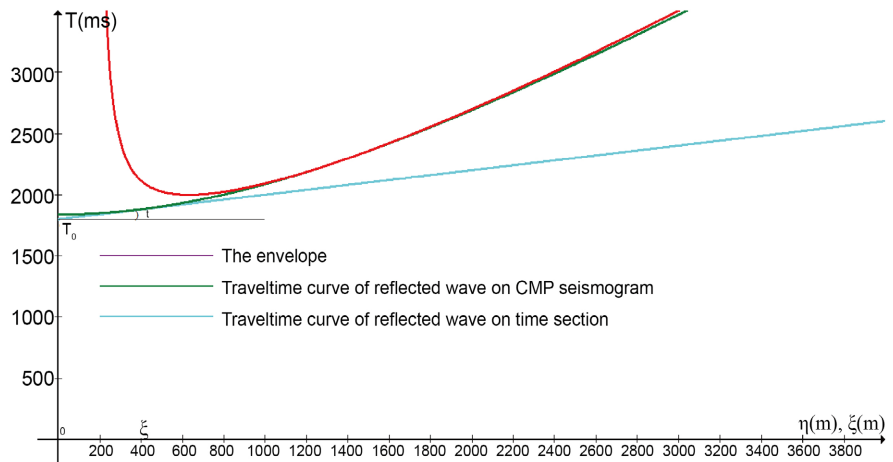


Figure 17. Transformation of input data into time section using average velocities.

When we express parameter T_0 in terms of the other parameters and variables of Equation (7),

$$T_0 = \tilde{T}_0 - t * \xi, \tag{57}$$

where ξ is the location of the resulting time section trace, and putting it into Equation (43) yields:

$$\tilde{T} = \sqrt{[\tilde{T}_0 + t * (\eta - \xi)]^2 + (4 - t^2 * V^2) * x^2 / V^2}. \tag{58}$$

It is equation of the family of functions that connect families (43) and (7).

Condition

$$\tilde{T}' = \tilde{T}_0 * (\eta - \xi) + t * (\eta - \xi)^2 - t * x^2 = 0 \tag{59}$$

is used to find the parameter t :

$$t = \tilde{T}_0 * (\eta - \xi) / [x^2 - (\eta - \xi)^2]. \tag{60}$$

The equation of the envelope used to convert reflections on CMP seismograms into reflections on time sections was obtained by substituting the value of t into the equation of the family of the functions (58) and has the form:

$$\tilde{T} = \sqrt{\tilde{T}_0^2 * x^2 / [x^2 - (\tau - \xi)^2] + 4 * x^2 / V^2}. \tag{61}$$

Figure 18 displays the outcome of this procedure's testing.

Authors didn't find analogs for this procedure.

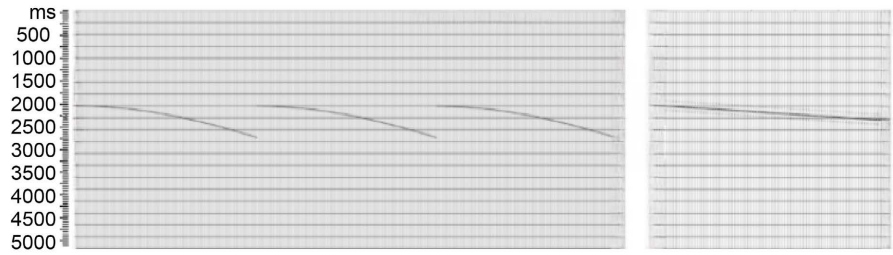


Figure 18. Synthetic CMP seismograms with recorded reflections ($T_0 = 2000$ ms, $t = 0.2$ ms/m, $V = 2.0$ m/ms) and time section obtained using average velocity.

3.6. Poststack Depth Migration Using Velocity Functions Linear with Depth

The following parametric equation describes the propagation of seismic waves in an inhomogeneous medium [42]-[44]:

$$x = s - \int_0^z \frac{m * V(x, z)}{\sqrt{1 - m^2 * V^2(x, z)}} dz, \quad t = \int_0^z \frac{dz}{V(x, z) * \sqrt{1 - m^2 * V^2(x, z)}}, \quad (62)$$

where s is the source point, x is the abscissa of a wavefront point, z is the ordinate of this point, and t is the time of wave propagation from the point $(s, 0)$ to the point (x, z) , $V(x, z)$ is the velocity of wave propagation in the point (x, z) , i_z is the angle between the ray and vertical at a point (x, z) , and $m = \sin(i_z)/V(x, z)$ (Figure 19).

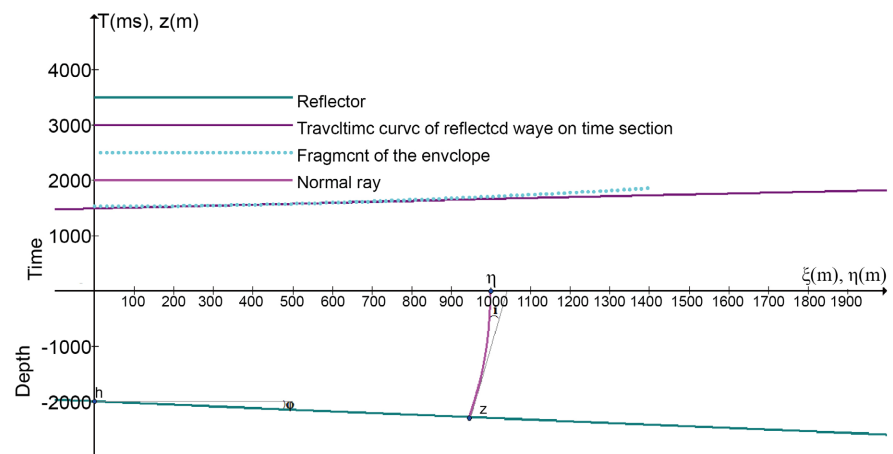


Figure 19. Poststack depth migration using a velocity function linear with depth.

If $V(x, z)$ is the linear function of z , then $V = V_0 * (1 + \beta * z)$ and equation (65) will have the following form [40]:

$$x = s - \frac{1}{m * V_0 * \beta} * \left[\sqrt{1 - m^2 * V_0^2} - \sqrt{1 - m^2 * V_0^2 * (1 + \beta * z)^2} \right],$$

$$t = \frac{1}{V_0 * \beta} * \ln(1 + \beta * z) * \left(1 + \sqrt{1 - m^2 * V_0^2} \right) \quad (63)$$

$$1 + \sqrt{1 - m^2 * V_0^2 * (1 + \beta * z)^2}.$$

For a flat reflector $z = h + kx$ (Figure 19) and a normal ray $m = \sin(\varphi)/V(z)$, where φ is the dip angle of the reflector and $k = \tan(\varphi)$. If t designated as T and s as η , we have the following:

$$x = \eta - \left(\sqrt{(1 + \beta * h + k * \beta * x)^2 * (1 + k^2) - k^2} - (1 + \beta * h + k * \beta * x) \right) / (\beta * k), \tag{64}$$

$$T = \frac{2}{\beta * V_0} * \ln \frac{(1 + \beta * h + k * \beta * x) * \sqrt{1 + k^2} + \sqrt{(1 + \beta * h + k * \beta * x)^2 * (1 + k^2) - k^2}}{1 + \sqrt{1 + k^2}}. \tag{65}$$

Substituting variable x from Equation (63) into Equation (65) gives equation (66), which describes the traveltimes curves for waves propagating at a velocity linearly dependent on depth and reflected from inclined boundaries:

$$T = \frac{2}{\beta * V_0} * \ln \frac{(1 + \beta * h + k * \beta * \eta) + \sqrt{(1 + \beta * h + k * \beta * \eta)^2 * (1 + k^2) - k^2}}{1 + \sqrt{1 + k^2}}. \tag{66}$$

The equation of the family of reflection traveltimes curves on depth sections has the form:

$$z = h + k * \xi, \tag{67}$$

where ξ is the location of the result trace.

By substituting the parameter h from Equation (67) into Equation (66), we obtain the family of functions that connect families (66) and (67), whose envelope is used to accomplish the required transformation. The equation of this family of functions is as follows:

$$\tilde{T} = \frac{2}{\beta * V_0} * \ln \frac{1 + \beta * z + k * \beta * (\eta - \xi) + A}{1 + \sqrt{1 + k^2}}, \tag{68}$$

where

$$A = \sqrt{(1 + \beta * z + k * \beta * (\eta - \xi))^2 + k^2}.$$

$$\tilde{T}'_k = \frac{\left((1 + \beta * z + k * \beta * (\eta - \xi)) * \beta * (\eta - \xi) + k + A \right) * \beta * (\eta - \xi)}{A} - \frac{A + 1 + \beta * z + k * \beta * (\eta - \xi)}{\sqrt{1 + k^2} * (1 + \sqrt{1 + k^2})} = 0. \tag{69}$$

This equation is solved by the numerical Newton method.

Figure 20 depicts the outcome of testing this procedure

3.7. Transformation of VSP Data into Time Sections, Time Migrated Sections, and Depth Migrated Sections

The main advantage of VSP is the absence of surface waves, which makes it possible to obtain additional information in relation to ground-based observations. Therefore, improving the existing procedures for processing VSP data and developing new ones is important.

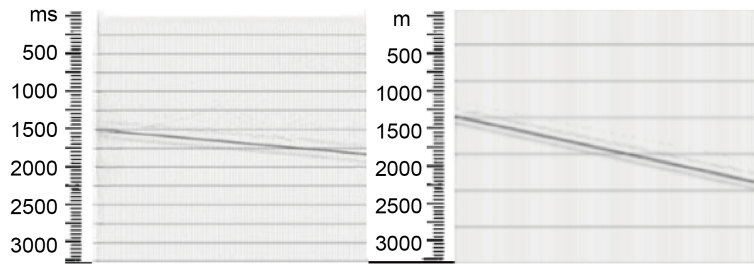


Figure 20. Synthetic time section with recorded reflection and poststack depth migration using a velocity functions linear with depth ($h=1500$ m, $k=0.3$, $V=2.0*(1+1/3000*z)$ m/ms).

Galperin (1974) ([45]) proposed the VSP method. [32] [46] [47] and others have contributed to the development of this method.

First, it is necessary to find the equation of a family of traveltime curves for reflected from the inclined reflectors and recorded in the VSP data waves.

The equation of the family of traveltime curves for waves reflected from flat boundary and recorded on the VSP data were obtained from these works and has the next form

$$T = \sqrt{\left(2 * T_0 * V - h * \sqrt{4 - t^2 * V^2} + t * p * V\right)^2 + \left(p * \sqrt{4 - t^2 * V^2} + h * t * V\right)^2} / (2 * V), \tag{70}$$

where h is location of geophone in borehole and p is the source location.

We assume that the velocity of the wave propagation is constant. As previously demonstrated, the equation for the family of traveltime curves for reflected from an inclined reflectors recorded on the time sections waves has the form (7).

First of all let's transform registered on VSP data reflected waves into time-sections (Figure 21).

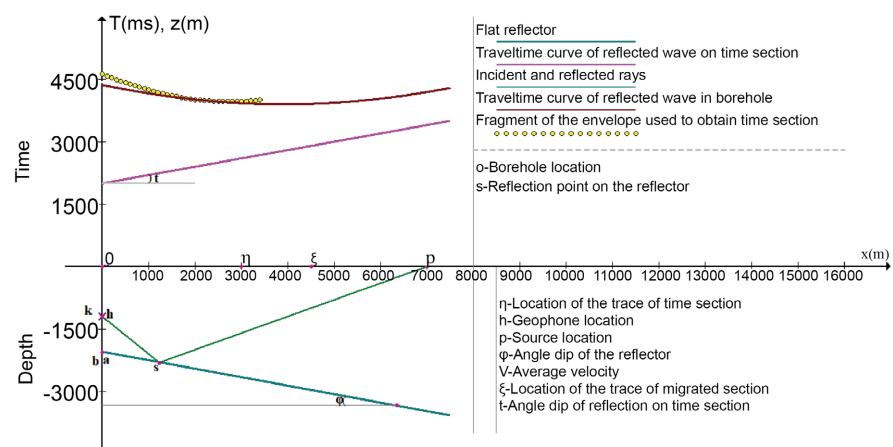


Figure 21. Transformation of VSP data into time sections.

The equation for the family of traveltime curves for waves reflected from flat reflectors on times sections is as follows:

$$\tilde{T}_0 = T_0 + t * \xi, \tag{71}$$

where ξ is the location of the result time section trace. When we express the parameter T_0 through other terms of the expression (71)

$$T_0 = \tilde{T}_0 - t * \xi \tag{72}$$

and substitute it into (71) will be obtain the equation of family of functions that connecting families (70) and (71).

$$\tilde{T} = \sqrt{\left(2 * \tilde{T}_0 * V - h * \sqrt{4 - t^2 * V^2} + t * V * (p - 2 * \xi)\right)^2 + \left(p * \sqrt{4 - t^2 * V^2} - h * t * V\right)^2} / (2 * V). \tag{73}$$

The following equation was used to determine the envelope of the family (73):

$$\begin{aligned} \tilde{T}'_t = & \left(2 * \tilde{T}_0 * V - h * \sqrt{4 - t^2 * V^2} + t * V * (p - 2 * \xi)\right) \\ & * \left(h * t * V + (p - 2 * \xi) * \sqrt{4 - t^2 * V^2}\right) \\ & + \left(p * \sqrt{4 - t^2 * V^2} + h * t * V\right) * \left(h * \sqrt{4 - t^2 * V^2} - p * t * V\right) = 0. \end{aligned} \tag{74}$$

To solve this equation is used the numerical Newton's method.

The method described in the introduction is used to store as a constant the difference between the second derivatives of the envelope and any function of the family (70) at the point of tangency.

The time migrated section may be obtained using Equations (70) and (8) (Figure 22).

We present the Equation (8) again for completeness of presentation:

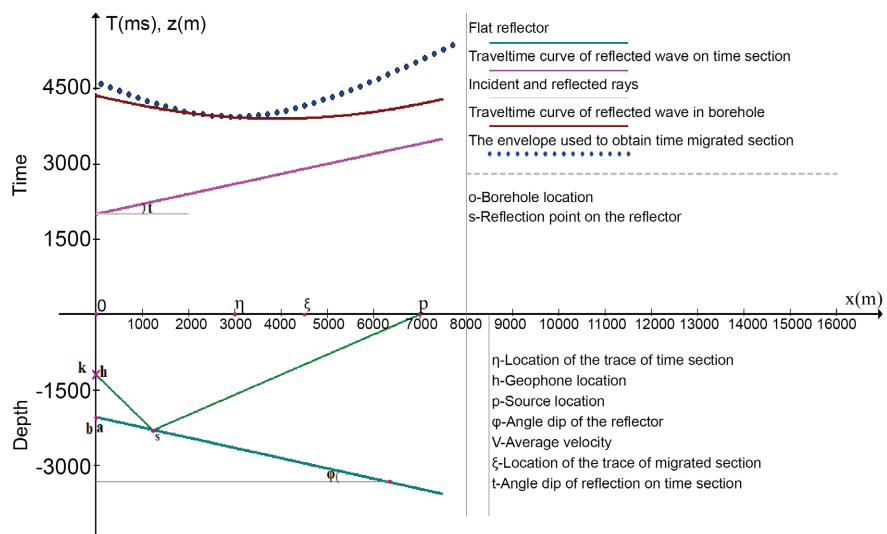


Figure 22. Transformation of VSP data into time migrated sections.

$$\tilde{T}_0 = 2 * (T_0 + t * \xi) / \sqrt{4 - t^2 * V^2}.$$

Let's represent the parameter T_0 through the Equation's (75) other parame-

ters and variables

$$T_0 = \tilde{T}_0 * \sqrt{4 - t^2 * V^2} / 2 - t * \xi. \tag{75}$$

Substituting (75) into (70) we obtain the equation that connects these families:

$$\tilde{T} = \sqrt{\left[\sqrt{4 - t^2 * V^2} * (\tilde{T}_0 * V - h) + t * V * (p - 2 * \xi) \right]^2 + \left(p * \sqrt{4 - t^2 * V^2} + h * t * V \right)^2} / (2 * V). \tag{76}$$

To find the envelope of this family, we must solve the following equation:

$$\begin{aligned} \tilde{T}'_t = & \left[\sqrt{4 - t^2 * V^2} * (\tilde{T}_0 * V - h) + t * V * (p - 2 * \xi) \right] \\ & * \left[-t * V^2 * (\tilde{T}_0 * V - h) / \sqrt{4 - t^2 * V^2} * V * (p - 2 * \xi) \right] \\ & + \left(p * \sqrt{4 - t^2 * V^2} + h * t * V \right) * \left(-p * t * V^2 / \sqrt{4 - t^2 * V^2} + h * V \right) = 0, \end{aligned} \tag{77}$$

$$t = \sqrt{2 * \left(\sqrt{4 * \gamma^2 + \delta^2} - \delta \right) / \left(\sqrt{4 * \gamma^2 + \delta^2} * V^2 \right)}, \tag{78}$$

where

$\gamma = \tilde{T}_0 * V * (p - 2 * \xi) + 2 * z * \xi$ and $\delta = \tilde{T}_0 * V * (\tilde{T}_0 * V - 2 * z) + 4 * \xi * (p - \xi)$, and substitute the value t in (76).

The approach described in the introduction is used to save as a constant the difference between the second derivatives of the envelope of the family (74) and any function of the family (68) at the point of their touching.

To obtain a depth migrated section, we can use the equations of families (21) and (70) (Figure 23).

From Equation (21), let's represent the parameter T_0 through the other parameters and variables of this equation:

$$T_0 = z * \sqrt{4 - t^2 * V^2} / V - t * \xi. \tag{79}$$

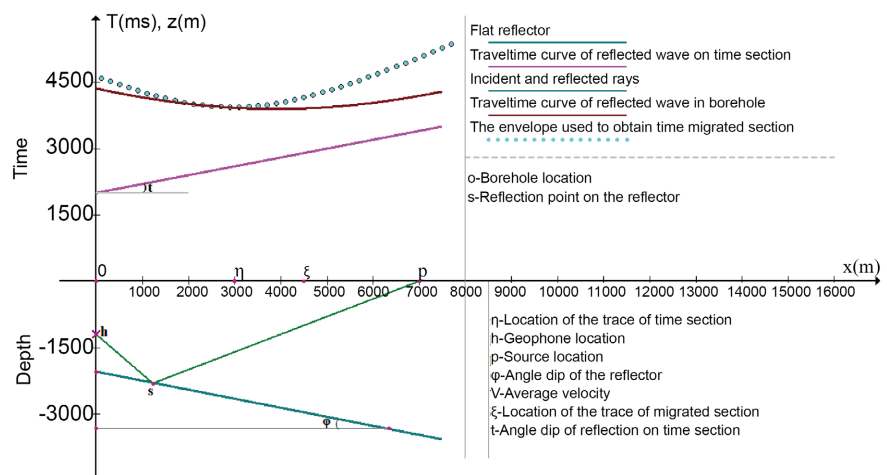


Figure 23. Transformation of VSP data into depth migrated sections.

We can obtain the equation of family of functions that connects these families

by inserting (79) into (70):

$$\tilde{T} = \sqrt{\left[(2 * z - h) * \sqrt{4 - t^2 * V^2} + t * V * (p - 2\xi) \right]^2 + \left(p * \sqrt{4 - t^2 * V^2} + h * t * V \right)^2} / (2 * V). \tag{80}$$

The envelope of this family can be found from condition $\tilde{T}'_t = 0$, that leads to:

$$\begin{aligned} \tilde{T}'_t = & \left[(2 * z - h) * \sqrt{4 - t^2 * V^2} + t * V * (p - 2\xi) \right] \\ & * \left[(p - 2\xi) - (2 * z - h) * t * V / \sqrt{4 - t^2 * V^2} \right] \\ & + \left(p * \sqrt{4 - t^2 * V^2} + 2 * t * V \right) * \left(h - p * t * V / \sqrt{4 - t^2 * V^2} \right) = 0 \end{aligned} \tag{81}$$

and

$$t = 2 * C / \sqrt{4 * A^2 + C^2} / V, \tag{82}$$

where $A = (2 * z - h) * (p - 2 * \xi) + p * z$; $C = \sqrt{4 * A^2 + B^2} - B$;
 $B = (2 * z - h)^2 + p^2 - h^2$.

If t is substituted into (80), we obtain the equation of the envelope of the family (80):

$$\tilde{T} = \sqrt{4 * \left[(2 * z - h)^2 + p^2 \right] - 4 * C^2 * B / D + 16 * A^2 * C / D} / (2 * V), \tag{83}$$

where $D = 4 * A^2 + C^2$.

The VSP data will transform into depth migrated sections by summation along the envelope (83). The approximate approach described in the introduction is used to keep the “touch character” as a constant.

Figure 24 depicts the outcome of testing this procedure.

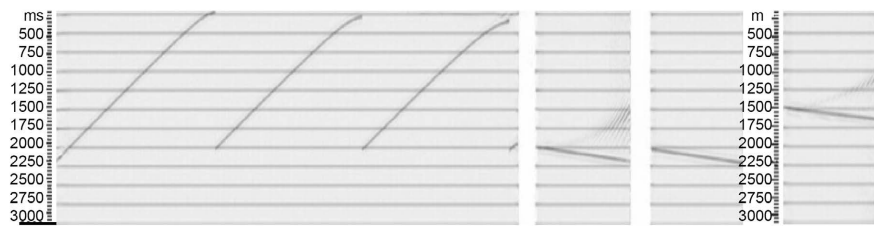


Figure 24. Synthetic VSP data ($T_0 = 2000$ ms, $t = 0.1$ ms/m, $V = 1.5$ m/ms), time section, time migrated section and depth migrated section.

3.8. Transformation Refracted Waves into Time Sections, Time Migrated Sections, and Depth Migrated Sections

The geometry of the refraction path is detailed in [20] [48] and others.

The equation of the family of traveltimes curves for waves refracted from flat boundary were obtained from these works and has the next form (**Figure 25**):

$$T = \frac{T_0 * \sqrt{V_1^2 - V_0^2}}{V_1} + \frac{x * \sqrt{4 - t^2 * V_0^2}}{2 * V_1} + \frac{t * x * \sqrt{V_1^2 - V_0^2}}{2 * V_1}, \tag{84}$$

where V_0 is the velocity of the propagation of reflection waves and V_1 is the

velocity of the propagation of headwaves.

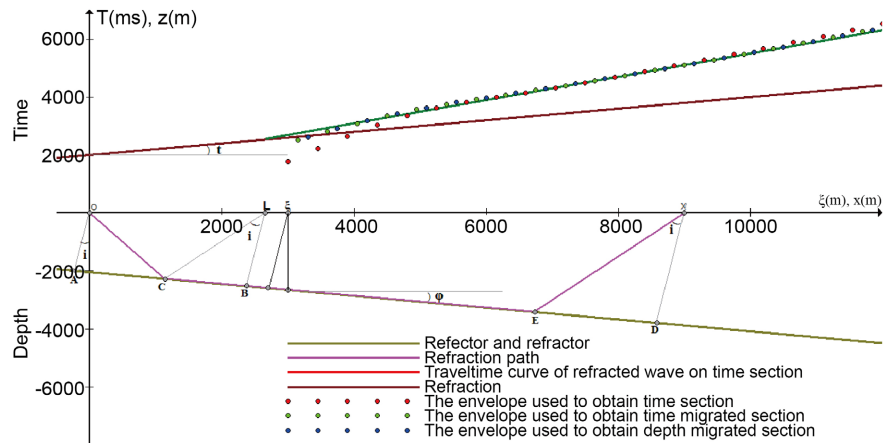


Figure 25. Transformation of refracted waves (headwaves) into time sections, time migrated sections and depth migrated sections.

The refraction arrival point is equal to:

$$L = \frac{2 * T_0 * V_0^2}{\sqrt{4 - t^2 * V_0^2} * \sqrt{V_1^2 - V_0^2} - t * V_0^2} \tag{85}$$

To obtain the time section, we express the parameter T_0 through the remaining parameters and variables of Equation (7) and substitute it into Equation (84):

$$\tilde{T} = \frac{(\tilde{T}_0 - t * \xi) * \sqrt{V_1^2 - V_0^2}}{V_1} + \frac{x * \sqrt{4 - t^2 * V_0^2}}{2 * V_1} + \frac{x * t * \sqrt{V_1^2 - V_0^2}}{2 * V_1} \tag{86}$$

To determine the envelope of Equation (86), condition (5) is used:

$$t = \sqrt{\frac{4 * (x - 2 * \xi)^2 * (V_1^2 - V_0^2)}{V_0^2 * [x^2 * V_0^2 + (x - 2 * \xi)^2 * (V_1^2 - V_0^2)]}} \tag{87}$$

and this value is substituted into (86).

To preserve “touch character” as a constant, the approximation approach given in the introduction is utilized.

To obtain the time migrated section, we used Equation (8).

The parameter T_0 is expressed through the rest of the parameters and variables of this equation, and the result is plugged into (84):

$$\tilde{T} = \frac{(\tilde{T}_0 * \sqrt{4 - t^2 * V_0^2} / 2 - t * \xi) * \sqrt{V_1^2 - V_0^2}}{V_1} + \frac{x * \sqrt{4 - t^2 * V_0^2}}{2 * V_1} + \frac{x * t * \sqrt{V_1^2 - V_0^2}}{2 * V_1} \tag{88}$$

From condition (5), the value of t is found:

$$t = \sqrt{\frac{4 * (x - 2 * \xi)^2 * (V_1^2 - V_0^2)}{V_0^2 * \left[\left(\tilde{T}_0 * \sqrt{V_1^2 - V_0^2} - x \right)^2 * V_0^2 + (x - 2 * \xi)^2 * (V_1^2 - V_0^2) \right]}}. \quad (89)$$

By substituting the values of t into (88), we find the envelope of the family (88) used to obtain the time migrated section.

To transform refracted waves into depth migrated sections, we may use the equations of families (109) and (21).

Let's represent the parameter T_0 from Equation (21) through the other parameters and variables of this equation:

$$T_0 = z * \sqrt{4 - t^2 * V^2} / V - t * \xi. \quad (90)$$

After substituting (90) into (84), we obtain the equation of the family of functions that connect these families.

$$\begin{aligned} \tilde{T} = z * \sqrt{4 - t^2 * V_0^2} * \sqrt{V_1^2 - V_0^2} / (V_0 * V_1) - t * \xi * \sqrt{V_1^2 - V_0^2} / V_1 \\ + x * \sqrt{4 - t^2 * V_0^2} / (2 * V_1) + t * (x - 2 * \xi) * \sqrt{V_1^2 - V_0^2} / (2 * V_1). \end{aligned} \quad (91)$$

From condition (5) let's find the parameter t :

$$t = \frac{2 * (x - 2 * \xi) * \sqrt{V_1^2 - V_0^2}}{V_0 * \sqrt{\left(2 * z * \sqrt{V_1^2 - V_0^2} + V_0 * x \right)^2 + (x - 2 * \xi)^2 * (V_1^2 - V_0^2)}} \quad (92)$$

and substitute it into (91).

Obtained in this way the envelope of the family (91) will be used for transforming recorded refracted waves into depth migrated sections.

The approximate method described in the introduction is used to save "touch character" as a constant.

Figure 26 depicts the outcome of testing these procedures.

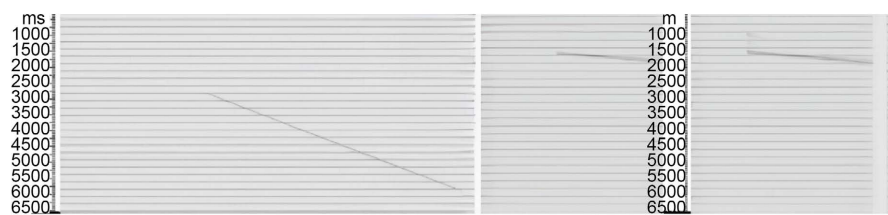


Figure 26. Synthetic refraction survey data ($T_0 = 2000$ ms, $t = 0.1$ ms/m, $V_0 = 1.5$ m/ms, $V_1 = 2.0$ m/ms), time section, time migrated section and depth migrated section.

3.9. Transformation of Input Data into a "Floating" Time Sections without Using Information about Velocities

As an innovative method of transforming seismic data for further analysis, it is proposed to obtain so-called "floating" time sections. These sections differ from CMP time sections primarily in that the velocities used for their realization have a significant effect primarily on the reflection timecurve of the recorded waves,

rather than on the reflection amplitude. If the velocity corresponds to the actual velocity, the position of the reflecting horizon on the “floating” time section is correct. The greater the difference between the accepted and true velocities, the greater the difference between the true and accepted recorded wave times. In this case, the dynamics of the reflecting horizons on the “floating” time sections is practically independent of the velocities used. The value of the difference between the recorded reflections from the same horizon on the CMP sections and “floating” time sections can be used to calculate the real propagation velocity of the reflected wave.

The procedure for obtaining a “floating” time section is divided into two stages. First, the CMP seismograms are transformed into “floating” seismograms, in which the family of the reflection traveltime curves is described by a function depending on parameters such as the estimated \tilde{V} and true wave propagation velocities V . At the next stage, parameter V will be excluded from the process of determining the envelope used to finally obtain a “floating” time section.

As previously proven, (43) is an equation of the family of traveltimes curves for reflected from the flat reflector waves for the average velocity model recorded on the CMP gathers.

Let’s build a procedure for transformation CMP seismograms into new seismograms, the traveltime curves of reflected waves on which are described by the following equation:

$$\tilde{T}_0 = T_0 + (t + 2/V - 2/\tilde{V}) * \xi + \rho * [1 + (t + 2/V - 2/\tilde{V})^2 * V^2] / \tilde{V}. \tag{93}$$

This is the family of functions of two variables (ξ, ρ) that depends on three parameters (t, V, \tilde{V}) , where \tilde{V} is the assumed average velocity, ξ is the location of the result seismogram, and ρ is the location of the result trace on this seismogram (Figure 27).

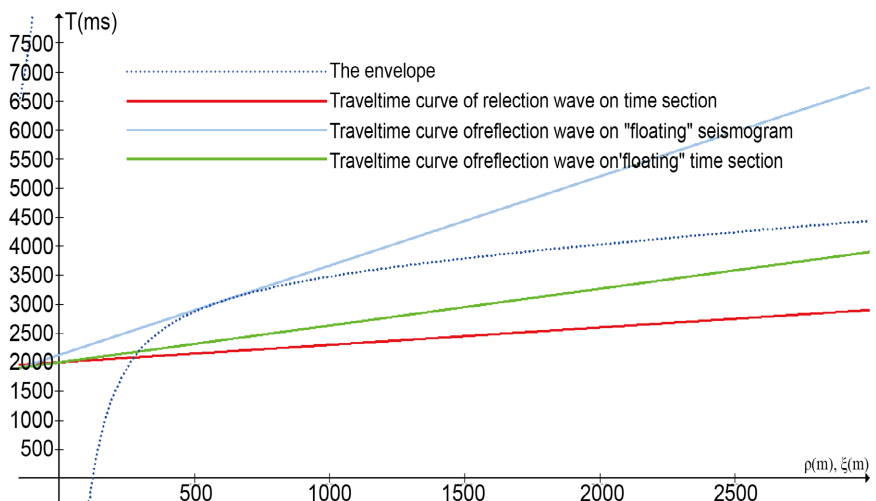


Figure 27. Transformation CMP seismograms into “floating” seismograms.

To find an equation for the family of functions that connect families (51) and (93), we represent parameter T_0 by other parameters and variables from Equation (93) and substitute it into (51):

$$T = \sqrt{\left\{ \tilde{T}_0 - \left(t + \frac{2}{V} - \frac{2}{\tilde{V}} \right) * \xi - \frac{\rho * \left[1 + \left(t + 2/V - \frac{2}{\tilde{V}} \right)^2 * \tilde{V}^2 \right]}{\tilde{V}} + t * \eta \right\}^2 + \frac{(4 - t^2 * V^2) * x^2}{\tilde{V}^2}}. \quad (94)$$

To obtain the envelope of the family (94), two parameters should be excluded using the following conditions:

$$\tilde{T}'_t = 0 \quad \text{and} \quad \tilde{T}'_V = 0. \quad (95)$$

$$\begin{aligned} \tilde{T}'_t = & \left\{ \tilde{T}_0 - \left(t + 2/V - 2/\tilde{V} \right) * \xi - \rho * \left[1 + \left(t + 2/V - 2/\tilde{V} \right)^2 * \tilde{V}^2 \right] / \tilde{V} + t * \eta \right\} \\ & * \left[-\xi - 2 * \rho * \left(t + 2/V - 2/\tilde{V} \right) + \eta \right] - t * x^2 = 0. \end{aligned} \quad (96)$$

$$\begin{aligned} \tilde{T}'_V = & \left\{ \tilde{T}_0 - \left(t + 2/V - 2/\tilde{V} \right) * \xi - \rho * \left[1 + \left(t + 2/V - 2/\tilde{V} \right)^2 * \tilde{V}^2 \right] / \tilde{V} + t * \eta \right\} \\ & * \left[\xi + 2 * \rho * \left(t + 2/V - 2/\tilde{V} \right) * \tilde{V} \right] * V - 2 * x^2 = 0, \end{aligned} \quad (97)$$

From Equations (96) and (97), we find values t and V :

$$t = (D - 2)/V \quad \text{and} \quad V = 2 * \rho * D^2 * \tilde{V} / \left[(4 * \rho - \xi) * D + 2 * \eta \right], \quad (98)$$

where

$$D = (2 + t * V) = \frac{-2 * B + \sqrt{4 * B^2 - A * C}}{A},$$

where

$$A = 4 * \left(\tilde{T}_0 - \rho / \tilde{V} \right) * \tau * \rho * \tilde{V} + \eta * \xi^2 + 2 * \left(\eta^2 - x^2 \right) * (4 * \rho - \xi),$$

$$B = \left[\eta^2 - x^2 - \eta * (4 * \rho - \xi) \right] * \tau,$$

$$A * (2 + t * V)^2 + 4 * B * (2 + t * V) + C = 0,$$

and

$$C = -12 * \tau^3.$$

Substituting (98) into (94) yields the envelope of this family, which is then used to transform CMP seismograms into “floating” seismograms.

In the second stage (Figure 28), the “floating” seismograms are transformed in a “floating” time sections, the family of traveltime curves for reflected waves on which have been described by the next equation:

$$\tilde{\tilde{T}}_0 = T_0 + \left(t + 2/V - 2/\tilde{V} \right) * \gamma, \quad (99)$$

where γ is the location of the “floating” time section trace. It is not difficult to

ensure that if V equals the assumed \tilde{V} , then equation (99) coincides with Equation (7). In the case of their discrepancy, the “floating” time section increasingly differs from the true time section as the difference between these velocities increases.

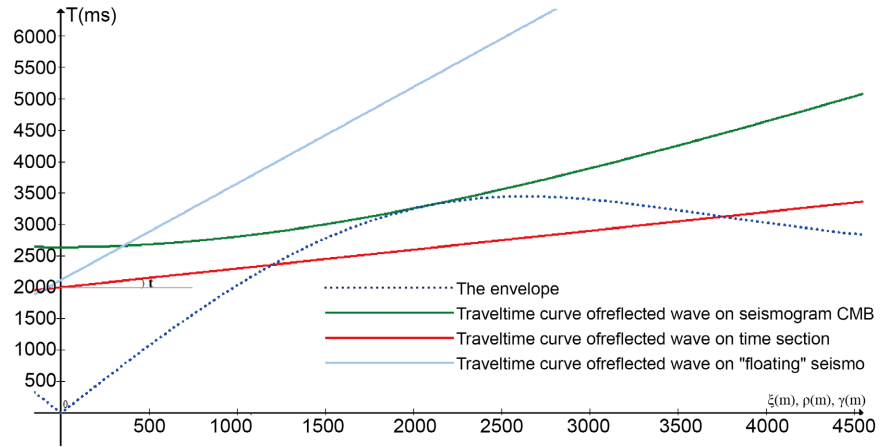


Figure 28. Transformation “floating” seismograms into “floating” time section.

If $\alpha = t + \frac{2}{V} - \frac{2}{\tilde{V}}$, then Equation (93) is represented in the form of (100), and Equation (99) is represented in the form of (101):

$$\tilde{T}_0 = T_0 + \alpha * \xi + \rho * (1 + \alpha^2 * \tilde{V}^2) / \tilde{V}, \tag{100}$$

$$\tilde{T}_0 = T_0 + \alpha * \gamma. \tag{101}$$

Then, we express the parameter T_0 in terms of the remaining parameters and variables of the Equation (101) and substitute it into Equation (100):

$$T_0 = \tilde{T}_0 - \alpha * \gamma \tag{102}$$

$$\tilde{T} = \tilde{T}_0 - \alpha * (\gamma - \xi) + \rho * (1 + \alpha^2 * \tilde{V}^2) / \tilde{V}. \tag{103}$$

After solving the equation $\tilde{T}'_{\alpha} = 0$, we will obtain:

$$\alpha = (\gamma - \xi) / (2 * \rho * \tilde{V}). \tag{104}$$

So the envelope of the family of functions (103) is:

$$\tilde{\tilde{T}} = \tilde{T}_0 + \rho / \tilde{V} - (\gamma - \xi)^2 / (4 * \rho * \tilde{V}). \tag{105}$$

The Equation (105) is used to get the “floating” time section and it does not contain the parameter V .

If the difference in the times of registration of the same reflections on the time section $T = T_0 + t * \eta$ and “floating” time section (101) is denoted as Δt , then:

$$\Delta t = T_0 + (t + 2/V - 2/\tilde{V}) * \gamma - T_0 - t * \gamma. \tag{106}$$

$$\tilde{T}_0 = T_0 - (t + 2/V - 2/\tilde{V}) * \gamma. \quad (107)$$

$$\Delta t = (2/V - 2/\tilde{V}) * \gamma \quad (108)$$

and the value of the real velocity is:

$$V = 2 * \gamma * \tilde{V} / (\Delta t * \tilde{V} + 2 * \gamma). \quad (109)$$

The result of testing this procedure is given in **Figure 29**.

Authors didn't find analogs for this procedure.

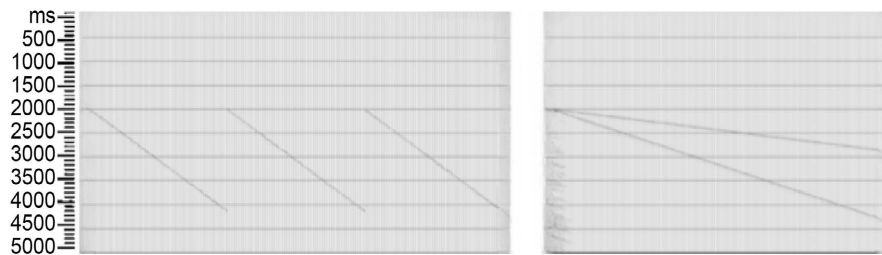


Figure 29. Synthetic “floating” seismograms ($T_0 = 2000$ ms, $t = 0.2$ ms/m, $V = 2.0$ m/ms, $\tilde{V} = 3.0$ m/ms) and “floating” time section with real and “floating” reflections.

4. Conclusions

The stationary-phase transformation method is applicable in the case when the hodographs of recorded waves and the same waves after transformation can be described by families of functions that depend on certain parameters and at least one of these parameters is common to both families. This method is quite general, since it can be applied in almost any situation where the propagation of seismic waves is described analytically.

The main advantage of this method is that the same mathematical apparatus (finding the envelope of a family of functions depended on some parameters) can be used to construct various transformations. Another significant benefit is that we can maintain the difference of the second derivatives at points where the integration line touches the traveltime curve for the recorded seismic wave, what helps us find the best balance between resolution and noise immunity during the transformation process. Despite similar kinematic features, this trait distinguishes Kirchhoff method transformations from their counterparts created by the stationary-phase method.

Many stationary-phase transformations possess inherent uniqueness and don't have of analogs. For example, the transformation of hyperbolic traveltime curves for reflected waves on CMP seismograms into linear reflection traveltime curves without velocities information, prestack and poststack time migrations using stacking velocities, transformation of input data into time sections using average velocities, poststack depth migration using a depth-linear velocity function, and transformation of CMP seismograms into “floating” time sections. They are all

notable innovations.

Given their range of possibilities and ease of application, the stationary-phase method is expected to provide a worthy position within the seismic research processes.

Data Availability Statement

After contacting the corresponding author, it was possible to obtain the results of testing the procedures described in this paper on model data or on a small amount of real materials in SEG-Y format that were sent to the authors by email or mail.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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