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Comparative Study of Slope Soils Analysis by Using Limit Equilibrium and Finite Element Methods

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Abstract

The slope soil analysis remains a corporate concern in construction activities. Because of its significance, the evaluation of slope soil stability has called widespread attention to several researchers all over the world. Many methods have been technologically advanced to evaluate the stability of slopes soils founded on distinct expectations and circumstances. Every method has specific benefits and limits. This work makes a comparison among safety factors and slip surfaces of slopes soils based on using Limit Equilibrium and Finite Element methods. Therefore, SLIDE 6.0 and PLAXIS 8.0 software were used for Limit Equilibrium and Finite Element methods, respectively. The computations of safety factors were performed for diverse shapes of slopes including different types of soils. Failure surfaces and values of safety factors obtained were compared for both methods used. It was noticed that the safety factors obtained from Limit Equilibrium methods were larger than of which is obtained by the finite element code. Moreover, an important change is noticed between the slip surfaces obtained by using both approaches.

Keywords

Slopes Soils, Slip Surface, Limit Equilibrium, Finite Element, Factor of Safety

1. Introduction

Landslides as natural adversity are often observed in Republic of Congo predominantly in the northern region of Brazzaville City. Constructions are devastated by landslides practically each year in the rainy seasons. About 70% of the total rainfall in Republic of Congo happens in rainy seasons. Consequently, landslides incidence is the extreme in this rainy period [1]. Construction works escorted by deforestation of delicate soils and located on abrupt slopes cause instability of slopes soils as well. Among all the damages produced by landslides in Brazzaville City, 45% of losses remain in mountain areas [2].

The stability analyses of slopes soils, which are vulnerable to rotational failures of soil mass, can be analysed by old-style limit equilibrium approaches for example Bishop's method [3], Janbu's simplified method [4], Fellenius's method [5] or the advanced methods founded on conventions concerning the inclination and location of the forces among slices [6] [7] [8]. For various suitcases, the Limit Equilibrium approaches were confirmed to give considerable results even though its limits. However, they tend to offer conventional values of safety factors, then the full shear strength is expected to be mobilised instantaneously beside the slip surface.

The usage of Limit Equilibrium methods for slope stability analysis has been expressively accompanied by using several vertical slices approaches. Five approaches of limit equilibrium for the safety factor computations are acknowledged, such as Method of Simplified Bishop [3], Method of Simplified Janbu [4], Method of Fellenius [5], Method of Spencer [6], and Method of Morgenstern Price [7]. Limit Equilibrium methods are based on determining the shear resistance beside the failure surface by using the criterion of Mohr-Coulomb failure.

Fellenius presented for the first time a limit equilibrium approach for a circular failure surface. After that, Bishop performed its research based on a revised method of circular failure surface. In the intervening time, Janbu made a difference focused on a technique for non-circular failure surfaces putting out away from a limited vertical slice, a potential sliding mass of soils. Far along methods were carried out by Morgenstern-Price, Spencer [7], Sarma [8] and others to make additional advances about forces among slices. In the current work, for particular reasons, the limit equilibrium methods engaged are Simplified Method of Bishop, Simplified method of Janbu and Fellenius's Method [3] [4] [5].

Numerical modelling was established and has converted progressively into a popular method for stability study of complex slopes in situations where the failure mechanism is not measured entirely by distinct geological structures. The engineering for slope instability problems needs a good appreciation of systematic methods, analytical tools and stabilisation measures [9]. Consistent with Nash [10], a quantitative evaluation of the factor of safety is significant when choices are made. Similarly, it is noticed that the primary objective of slope stability analyses is to subsidise the safe and economic design of excavation, embankment and earth dams [11].

In earlier years, the factor of safety was computed by using a traditional method of kinematic analysis. But different numerical methods evaluated landslide occurrence by neo-tectonic movement [12] [13] [14] [15] [16]. While slope failures can be considered as worries for a man. The satisfactory slope evaluation represents an engineering challenge related to slope instability. Academics have made it possible to develop many methods for the safety factor studies of slopes. One of the first studies appeared since 1970 and was going on progressively [17]-[22]. However, recently the progress found in the field of computational approaches, made possible the improvement of numerous software packages for a magnificent analysis of slopes stability [23] [24] [25] [26]. The methods of slope stability can be classified into Limit Equilibrium Methods and Numerical Analysis. The Limit Equilibrium Method is basic for establishing the equilibrium situations of the destabilising and stabilising forces by using slope stimulations. The stable state for the slope remains when destabilising forces are equal to stabilising forces [27] [28] [29] [30]. Whereas, numerical analysis treats to divide the slopes into finite quantities of elements and areas. Then, computations of forces and displacements are carried out from the slope constitutive laws. The methods of numerical analysis include various approaches such as Discrete Element, Finite Element and Boundary Element methods [31] [32] [33] [34].

In recent years, numerical analysis has become progressively a powerful tool in geotechnical analysis to resolve complex engineering problems. Finite Element and Finite Different approaches are the two most common of numerical methods. In the current work, the slope modelling by using finite element code considered the stress-strain behaviour of soils. The main factor considered as beneficial into the analysis by using finite element code does not involve assumptions for the sliding surface and the slope form. The use of Finite Element code for the slope modelling needs to split slope model into an amount of mesh elements. The constitutive law for including materials of the slope model is applied to determine stresses or strains. Areas in which the soil shear resistance is incapable to sustain the appeared shear strains obviously cause failure occurrence. In the end, from the Finite Element method, the Reduction Factor can be determined by using Reduction method of Cohesion and Friction angle (c-φ). This method needs increasingly the reduction of soil resistance parameters till the failure is produced. The shear strength method allows the Finite Element code to compute slope safety factor [34].

The present study tries to analyse the slopes soils stability in the northern area of Brazzaville City, Republic of Congo. The slopes soils in the area are highly vulnerable and experienced two major landslides in 2013 and 2017 along with negligible slides reported almost every year, especially during rainy season. A comparative study was carried out by using limit equilibrium and finite element methods. The comparison is largely based on simplified slope geometry and input parameters such as material properties and geotechnical properties of soils. Among the Limit Equilibrium Methos (LEM), the Bishop simplified (BS), Janbu simplified (JS), Fellenius method (FM) and Finite Element Method (FEM) are compared from values of slope safety factors.

2. Material and Methods

2.1. Study Area

The study area is located in the Northern region of Brazzaville City and the embankment slope under study is shown in **Figure 1**. The study area has a flat relief, valleys and hills. Years 80's were less warm than the years 90's. Before and after 1970 two periods plainly characterized the recent evolution of temperatures in Congo. A net variation of temperature from 1932 to 2010 was predicted in Brazzaville with an increase of +0.5˚C to 1˚C as average temperatures in the preceding two decades. Then, the maximum and minimum average temperatures are retained for 1990s showing an increase for the two recent decades with 1100 m and 360 m of maximum and minimum altitudes, respectively. The climate is tropical with a rainy season from October to May and dry seasons between January-February and June-September. The annual rainfall wavers between 1250 mm and 1350 mm/year [35]. The hydrogeology is represented by 270 km^2 Bateke's water table [36]. The soils are settled on diverse materials of three sedimentary sequences from the base to the top, with the presence of Inkisi's sandstone series, Stanley-Pool's sandstone series and the Bateke's plateau series. In general, the soils have a very low percentage of clay [37]. The Central Basin, includes the Central Africa's intracratonic depression with sediments accumulation, tectonic activity and erosion process during an extended history. So, the geological background is constituted of a Precambrian to Paleozoic age formation as basement and Mesozoic to Cenozoic sedimentary cover that reposes unconformably on this basement. Consequently, the Precambrian to Paleozoic basement which outcrops downstream from the Stanley Pool and the sedimentary cover is composed basically by sandy materials which appear upstream from the Stanley-Pool's series [38].

2.2. Geometry and Input Parameters

The geometry of the slope under study was separated into four layers founded on mechanical properties of soils such as Soft Silty Sand (SM), Hard Silty Sand (SM), loose Sand (SP) and Dense Sand (SP). The strength criteria of Mohr–Coulomb was used for the material properties. The geometry of slope, soil layers' location and mechanical properties were submitted to simulations. The input properties of different types of soils are shown in **Table 1**.

Table 1. Types of soils used in the present study and presumed elastic/plastic parameters.

Parameters	Soils types				Unit
	1-Soft Silty Sand (SM)	2-Hard Silty Sand (SM)	3-loose Sand (SP)	4-Dense Sand (SP)	
Poisson's ratio (μ)	0.43	0.33	0.35	0.25	-
Elasticity modulus (E)	11	28	30	80	MPa
Unit weight (γ)	17	20.50	17	19.50	kN/m^3
Friction angle (φ)	27	34	29	36	Degree

Figure 1. Slope under study.

2.3. Slope Geometry

It is firstly important to acknowledge that the slope stability depends directly on various parameters, such as the height and slope angle (gradient) of the slope and soil properties. Therefore, the combination of different heights and gradients together with different types of soils is the key for modelling a slope. Then, a modelling of four (4) distinctive slope angles such as 1H:1V; 1.5H:1V; 1.75H:1V and 2H:1V was carried out for three different heights (H) of 6 m, 11m and 16 m, obtaining a total of twelve (12) arrangements for slope shapes. Then, for a well procedure of slope stability study, the slope model was supposed to be rigid at the base.

2.4. Soil Properties

There were considered four different types of soils with unique constitutive material law using Limit Equilibrium and Finite Element methods for slope stability assessment. The soils types are supposed to provide extensively the strength appearances from cohesive soils as shown in Table 1.

2.5. Slope Soils Stability Analysis

2.5.1. Limit Equilibrium Evaluation

Software based on Limit equilibrium approach such as SLIDE is making it possible to analyse ever-increasing complexity in the slope soils stability evaluation. Using Limit Equilibrium approach, SLIDE can solve many problems related to heterogeneous soils by modelling slopes with complex stratigraphy and sliding surfaces geometries, and environment conditions with variation of pore-water pressure using a large selection of soils models. The conventional Limit Equilibrium method is used to investigate the embankment slope stability. The SLIDE Program is articulated by terms of moments and forces equilibrium from equations of safety factor. Then, slope soils analysis offers a safety factor, defined as a ratio of available shear strength to that is essential for equilibrium [22]. The limit equilibrium method for computing the safety factor includes the comparison of available shear resistance along the failure surface with the forces required to preserve the slope in equilibrium state. So as part of this analyse, Bishop's simplified method uses the slope stability model to calculate the safety factor from an equation by summing slice forces in the vertical direction. The importance of this method is that the slope stability becomes a function of the safety factor. This in turn makes the equation of safety factor nonlinear and an iterative procedure is therefore necessary to calculate the safety factor [30].

The SLIDE software as a Limit Equilibrium Software, becomes extensively useful in the geotechnical industry with capacity to study quickly different slope shapes. The criterion of Mohr-Coulomb failure governs calculations of safety factors in SLIDE excluding failure strain. Strength parameters such as the angle of internal friction (φ) and cohesion (c) are essential for modelling. Slide distinctive grid for determining critical failure surfaces is illustrated in **Figure 2(a)**.

2.5.2. Finite Element Method

Duncan's review of Finite Element analysis of slopes focussed largely on deformation rather than slopes stability analysis; though, care was taken to some significant premature works in which elasto-plastic soil models were recycled to evaluate slope soils stability. Smith & Hobbs [39] described results of slopes and obtained judicious agreement with Taylor's diagrams [40]. Zienkiewicz [41] achieved good agreement with sliding surfaces solutions. Griffiths [13] prolonged this work to demonstrate consistent slope stability outcomes above a varied range of soils properties and slopes geometries as compared with diagrams of Bishop [3]. Following use of the Finite Element method in slope soils stability investigation added more importance in the finite element method [13] [14] [15]. Duncan [23] mentions the possibility for enhanced graphical outcomes and reporting by using Finite Element Method, but attentions against artificial precision being presumed when the input parameters themselves are so inconstant. Wong [42] gives a potential source of error in the slope modelling from Finite Element, although current findings, including those offered in this work, specify that improved precision is now probable.

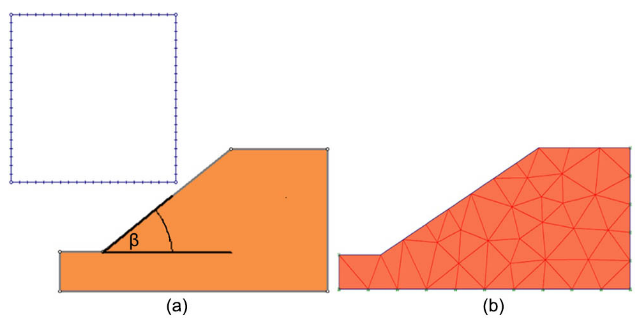

Figure 2. (a) Distinctive auto-grid generated by Slide (b) Distinctive mesh generated by PLAXIS 2D.

Practising engineers are often cynical for such complexity, particularly in view of the poor quality of soils properties from data frequently accessible based on repetitive site surveys. While this cynicism is frequently acceptable, there are some kinds of geotechnical problems for which the Finite Element Method provides actual benefits. The challenge for a qualified engineer is to recognise which kind of problem would profit from a Finite Element management and which would not. Generally, linear problems such as the prediction of settlements and deformations, the computation of low quantities because of stable seepage or the study of passing effects because of consolidation is all extremely willing to solution by finite element usage.

The PLAXIS software uses Finite Element method for slope stability study. The strength reduction approach is the method in which the factor of safety is computed in PLAXIS. This method reduces consecutively the shear strength parameters of soils such as the angle of internal friction (φ) and cohesion (c), till the failure occurrence. The shear strength values of soils parameters for safety factors computations are obtained from parameter of a total multiplier (ΣMsf). **Figure 2(b)** illustrates the distinctive mesh produced by PLAXIS software (**Figure 2**).

3. Results

Four (4) different soils, four (4) characteristic slope angles, and three (3) different slope heights were integrated into SLIDE and PLAXIS for slope stability analysis. The factor of safety assessment was carried out by using Fellenius, simplified Bishop and simplified Janbu methods as Limit Equilibrium approaches. Whereas the strength reduction technique was approved for safety factor computations as a Finite Element Method. The findings are available from **Tables 2-5**. The critical slip surfaces found from slide and PLAXIS analyses are exemplified in **Figure 2**. While, **Figure 3**, **Figure 4** and 4 make comparison among safety factors calculated from various slope models and different soils parameters. The safety factors overestimation was noticed by Limit Equilibrium Method as compared to Finite Element Code. In all conditions studied, safety factor estimation by Limit Equilibrium Method is superior to which is found by using the Finite Element code. Moreover, among Limit Equilibrium methods, the simplified method of Bishop gave the highest safety factor values in the majority of cases studied. This inconsistency observed into safety factors findings from Limit Equilibrium Methods, remains in the analysis mechanism in each Limit Equilibrium Method. Then, each Limit Equilibrium Method has its particular assumptions, for example, in simplified method of Bishop the shear forces have no influence among slices, whereas, theatrically in simplified method of Janbu, safety factors values are grander, due to the influence of vertical forces among slices and every single slice moment. Moreover, experience assumed that the hard or dense soils have a habit of offering more strength to the destabilising forces and therefore, comparing to the loose or soft soils, produced larger safety factors values. The same can be noticed in the hard and dense soils when these soils are characterised by higher strength, with tendency to resist more to the external applied forces, offering more resistance to failure. The hard silty sand (S2) evidenced the highest resistance while, in all slope geometries the loose sand (S3) produced the lowest safety factor values. Besides, in all Limit Equilibrium and Finite Element approaches, the safety factor has tendency to decrease for abrupt slopes. It was also perceived that the slope height increased with the significant decrease observed in the safety factor values by using both methods.

Table 2. Safety factors for critical surface of slope = 1H:1V when using different methods.

Slope Height = 6 m					Slope Height = 11 m					Slope Height = 16 m				
	Soil types					Soil types					Soil types			
	S1	S2	S3	S4		S1	S2	S3	S4		S1	S2	S3	S4
FM	1.56	3.70	0.46	1.32	FM	1.34	2.87	0.21	0.97	FM	1.60	3.91	0.53	1.36
JSM	1.56	3.66	0.47	1.32	JSM	1.23	2.81	0.30	0.96	JSM	1.69	3.94	0.53	1.38
BSM	1.68	3.80	0.47	1.42	BSM	1.29	2.94	0.30	0.99	BSM	1.74	3.98	0.51	1.36
FEM	1.38	3.47	0.06	0.86	FEM	0.91	2.72	0.08	0.61	FEM	1.06	3.40	0.21	0.68

Memory S1 = Soft Silty Sand (SM); S2 = Hard Silty Sand (SM); S3: Loose Sand (SP); S4: Dense Sand (SP); FM: Fellenius's Method; JSM: Simplified Janbu's Method; BSM: Simplified Bishop's Method; FEM: Finite Element Method.

Table 3. Safety factors for critical surface for slope = 1.5H:1V when using different methods.

Slope Height = 6 m					Slope Height =11 m					Slope Height =16 m				
	Soil types					Soil types					Soil types			
	S1	S2	S3	S4		S1	S2	S3	S4		S1	S2	S3	S4
FM	1.49	3.92	0.59	0.84	FM	1.64	3.89	0.50	1.54	FM	3.63	3.78	0.65	1.23
JSM	1.49	3.94	0.56	0.85	JSM	1.67	3.80	0.48	1.59	JSM	3.64	3.80	0.61	1.29
BSM	1.40	3.92	0.59	0.89	BSM	1.64	3.89	0.48	1.44	BSM	3.63	3.80	0.63	1.26
FEM	1.10	3.90	0.35	0.38	FEM	1.31	3.47	0.21	1.20	FEM	3.33	3.10	0.26	1.10

Memory S1 = Soft Silty Sand (SM); S2 = Hard Silty Sand (SM); S3: Loose Sand (SP); S4: Dense Sand (SP); FM: Fellenius's Method; JSM: Simplified Janbu's Method; BSM: Simplified Bishop's Method; FEM: Finite Element Method.

Table 4. Safety factors for critical surface of slope = 1.75H:1V when using different methods.

Slope Height = 6m					Slope Height = 11 m					Slope Height = 16 m				
	Soil types					Soil types					Soil types			
	S1	S2	S3	S4		S1	S2	S3	S4		S1	S2	S3	S4
FM	1.90	3.35	0.44	1.49	FM	1.94	3.63	0.43	0.76	FM	1.20	1.95	0.18	0.62
JSM	1.95	3.36	0.40	1.45	JSM	1.94	3.51	0.40	0.78	JSM	1.12	1.89	0.21	0.68
BSM	1.90	3.35	0.44	1.49	BSM	1.99	3.61	0.41	0.66	BSM	1.18	1.97	0.21	0.70
FEM	1.94	3.15	0.16	1.06	FEM	1.55	3.82	0.27	0.47	FEM	1.18	1.67	0.21	0.50

Memory S1 = Soft Silty Sand (SM); S2 = Hard Silty Sand (SM); S3: Loose Sand (SP); S4: Dense Sand (SP); FM: Fellenius's Method; JSM: Simplified Janbu's Method; BSM: Simplified Bishop's Method; FEM: Finite Element Method.

Table 5. Safety factors for critical surface of slope = 2H:1V when using different methods.

	Slope Height = 16 m					Slope Height = 16 m					Slope Height =16 m			
	Soil types					Soil types					Soil types			
	S1	S2	S3	S4		S1	S2	S3	S4		A	B	C	D
FM	1.33	1.94	0.26	0.70	FM	1.47	1.93	0.27	0.81	FM	1.41	1.75	0.24	0.59
JSM	1.27	1.91	0.28	0.80	JSM	1.35	1.91	0.31	0.89	JSM	1.43	1.85	0.28	0.67
BSM	1.36	1.96	0.28	0.78	BSM	1.45	1.96	0.30	0.90	BSM	1.54	1.94	0.32	0.89
FEM	0.86	1.78	0.15	0.50	FEM	0.66	1.67	0.18	0.41	FEM	0.75	1.32	0.15	0.50

Memory S1 = Soft Silty Sand (SM); S2 = Hard Silty Sand (SM); S3: Loose Sand (SP); S4: Dense Sand (SP); FM: Fellenius's Method; JSM: Simplified Janbu's Method; BSM: Simplified Bishop's Method; FEM: Finite Element Method.

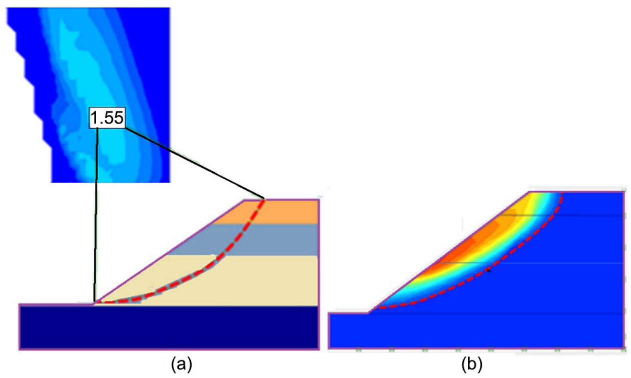

Figure 3. Different sliding surfaces settled by using (a) Limit Equilibrium approach (b) Finite Element code.

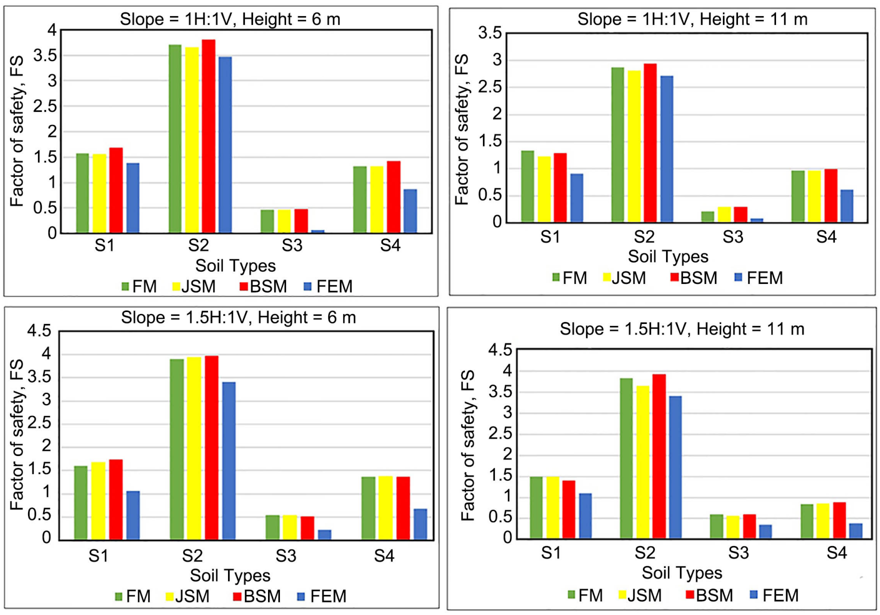

Figure 4. Safety factors for critical surface by using different methods.

4. Discussion

When observing the factors of safety (FS) in Figures 4-6, there is unexpected increase in the dimensionless displacement. Figure 6 specifies close agreement between the results of Finite Element Method and the factors of safety obtained for the same problem by using Bishop method. Figure 5 shows the influence of slope angle increase in the factor of safety. With a failure, factor of safety with value of 3.8 applied to the soil's properties, the graphs obtained correspond to the maximum stability state of the slope when the slope angle variation was applied in a single increase as compared with that obtained with both two methods. Figures 4-6 demonstrated that the values of safety factors obtained are affected by an increase in slope angle. Using the Finite Element Method, the deformed mesh corresponding to unconverged solution provided a more diffuse

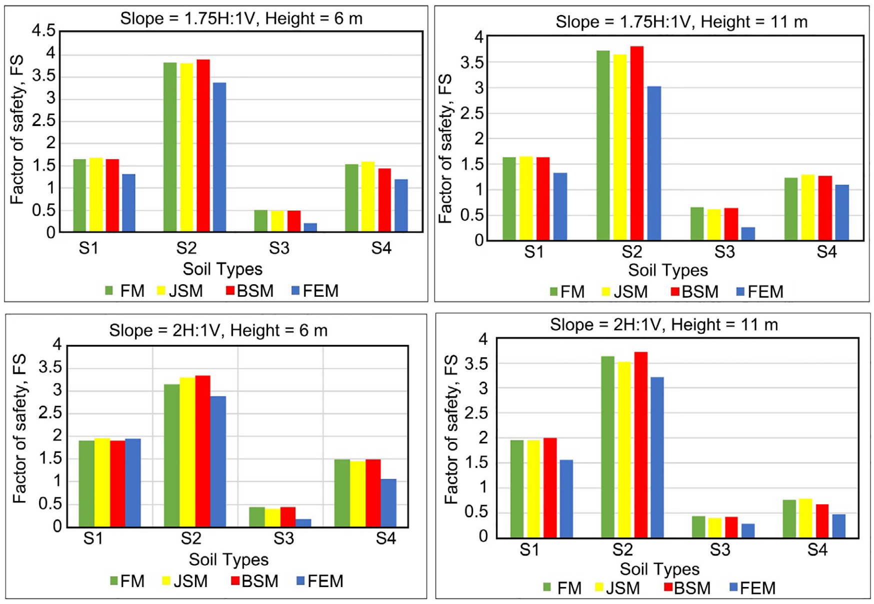

Figure 5. Safety factors for critical surface using different methods.

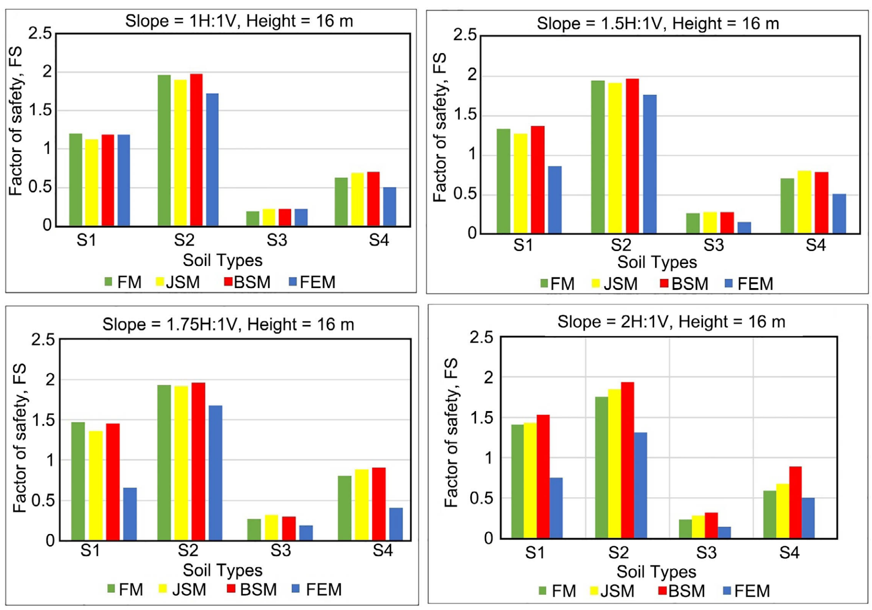

Figure 6. Safety factors for critical surface using different methods.

indication of the mechanism of slope failure. Conventional Finite Element analysis is incapable to model overweight breaks along possible failure surfaces, while methods were defined for improving the failure surfaces visualisation [14]. The results obtained in this work are in agreement with Whitman [43] observations to the automation of limit equilibrium methods, based on emerging numerical analysis which can be suitably accurate for confidence in its usage for the input parameters.

5. Conclusions

At large, despite judiciously safety factors values provided by Limit Equilibrium Methods, these methods are not accurate for complex slopes study, due to their non-existence in the behaviour of stress and strain. While the Finite Element Method makes possible the slope study founded on theories of stress and strain behaviour. Despite some benefits and limits presented by both methods, they can determine the failure surfaces and factors of safety for slope stability analysis. Though, it is important to distinguish that all over the world, findings from similar studies have approved robust nature of the finite Element Method. From the current research, conclusions are withdrawn such as:

The Limit Equilibrium and Finite Element Methods produced consistent Factors of safety. Comparing the both methods, it is noticed that the critical failure surfaces produced by Finite Element method have tendency to move higher plastic strain to the slope toe.

The critical sliding surfaces do not have the same location on the slope. The interface strength has an influence on the safety factors when using Finite Element Method, while in the Limit Equilibrium Method, this interface strength is not considered, and therefore it is finally perceived its effect on the computed safety factors values.

Excepting a significant role played by the slope shape in the safety factors findings, safety factors and slip surfaces obtained from computations make the soil shear strength the most dominant of materials properties.

In scarcer cohesive soils, the slope stability has a tendency to be controlled by surficial failures. While the slope angle (β) can be considered as the key factor when safety factors computation is related to non-cohesive soils. Limit Equilibrium and Finite Element Methods found critical slip surfaces moderately comparable to sand soils. Critical slip surfaces are moved toward the profundity for the presence of cohesive soils. Slope height (H) was considered as the primary characteristic of slope form with effects on the slope stability of silty sands with Safety factors particularly superior than the corresponding Reduction Factor method. Considering both methods advantageous, the critical slip surface consists in a relative verdict of Limit Equilibrium approach, while the Finite Element code extends its analysis capacity towards the critical slip broader area.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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