

Modeling a Local Geoid: Application in Thies, Senegal

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Abstract

Many applications in geodesy, hydrography and engineering require determining heights linked to the geoid. Direct leveling, which is the traditional method of obtaining these elevations, is slow, time consuming and expensive. The contribution of space techniques can make it possible to overcome these constraints provided that we have a precision geoid model compatible with that obtained by the GNSS method. There are today relatively precise regional geoid models, at least outside of mountain ranges, in all developed countries, *which* is not yet the case in developing countries like Senegal. An alternative is to use local models restricted to a small area. Thus, this study aims to produce a geoid model by combining multi-source data for the city of Thies intended mainly to support leveling operations by GNSS. To achieve this objective, direct precision leveling and GNSS leveling (static mode) were carried out covering the study area. The reference points used are, among others, those of the RRS04 (Reference Network of Senegal 2004) and the NGAO53 (General Leveling of West Africa 1953). Additionally, gravimetric measurements were conducted using the Sensor Play-Data Recorder application. The calculation of the model was carried out by the SRBF (Spherical Radial Basis Function) method using the PAGravf4.5 software. The SRBF method uses EGM08 to first calculate height and gravity anomalies. These are then compared with the raw data in order to determine the residuals which will allow the model to be refined. In order to validate our model, control points (GNSS/levelled) were chosen based on a homogeneous geographical distribution in the area in order to evaluate their *altitude*. An accuracy of less than 2 cm was obtained. Comparing our model with the existing local model GGSV12v1 shows that our model is more accurate.

Keywords

Geoid, Direct Leveling, GNSS, Gravity, PAGrav4.5, SRBF, Thies

1. Introduction

A better understanding of our environment has always been a continuous quest of man. In that eagerness for knowledge, an important place is devoted to the shape and dimensions of Earth. Geodesy is defined as the science that studies and determines the size and shape of Earth as well as its gravity and deformation of the crust [1]. The continuous geodetic studies of the shape and dimensions of the globe as well as its gravity are crucial in several aspects of geographic sciences. Gravity measurements have been, up to the 70s, the only method for determining the shape and dimensions of Earth [2]. Knowing terrestrial gravity is necessary for 2 types of works: for determining altitudes and orbit of artificial satellites of Earth. The implementation of a geoid can be done with several techniques: (astro-geodetic leveling, GNSS coupling and leveling, surface gravimetry, satellite altimetry, etc.). Indeed, the fact that Earth is not a homogenous body makes the geoid present ripples relative to the ellipsoid reflecting different aspects of the heterogeneity of Earth's thickness [3]. Since the appearance of GNSS space positioning techniques, the situation has radically changed: geoid has become an essential tool for converting heights from GPS into altitudes [4]. To convert and combine the different reference systems, the geoid model must be known to an accuracy compatible with that of GNSS. This means that the determination of geoid models with better precision is among the main objectives of the different research groups [5].

To properly carry out our study, we exploited the geodetic urban network of the city of Thies which was established by the Direction of Geographical and Cartographic Works (DTGC) of Senegal in 2011 and the points of the NGAO53 network created in 1953 which still remains a reference for altitude measurements in Senegal and West Africa. A campaign of direct precision leveling, GNSS leveling and determination of the variation of gravity in the city of Thies with Sensor Play Data Recorder was carried out.

At the end of this work, a local geoid model was produced in the city of Thies with a precision comparable to the existing local model GGS12v1 (Senegal Geoid of 2012 Version 1).

2. Materials and Methods

2.1. Study Area

The city of Thies, with its geographical location (70 km from Dakar) has become the receptacle of the capital's overflow and is thus increasingly acquiring numerous large development and infrastructure projects (airport, toll highway, Thies new town...). Furthermore, the region of Thies with an area of 6601 km² (Figure 1), is the third most densely populated region, with 319 inhabitants/km² (ANSD,

2021) [6]. Thus, an accurate local geoid model will guarantee data accuracy in these large works with the integration and use of GNSS for orthometric height determination.

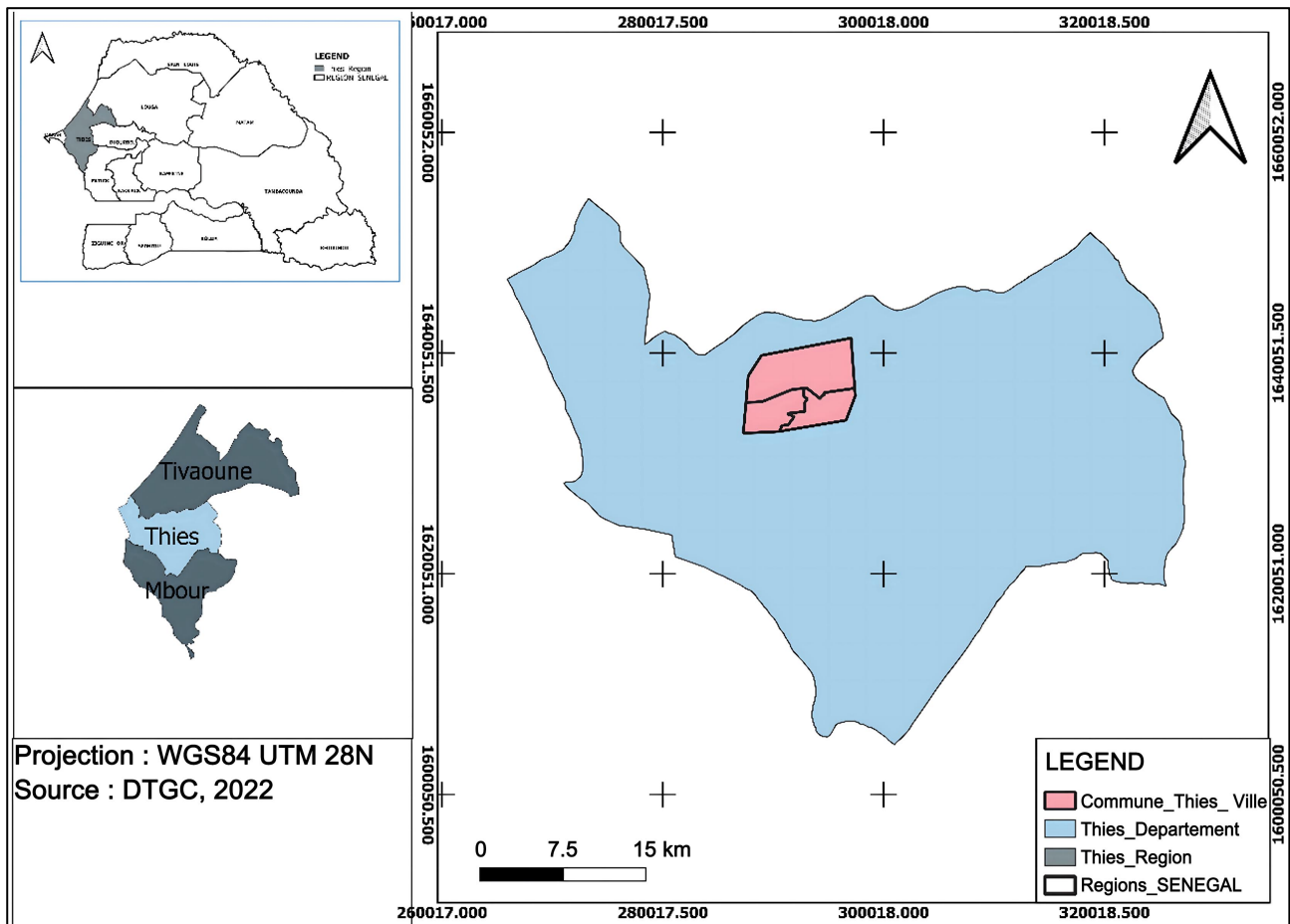


Figure 1. Presentation of the study area.

Located in the department of Thies with an area of 68.82 km², surrounded by the commune of Fandene to the north and southeast, with a small open area to the northwest in the commune of Keur Moussa [7]. From a geodetic point of view, it has a first order point of the RRS04 and twenty-three (23) other second order points of the urban network. In addition to this planar measurement network, there is also a NGAO altimeter network composed of 31 terminals.

2.2. Data Acquisition

The geoid is defined as an equipotential surface of the Earth's gravity field which coincides with the mean level of the oceans at rest, extended under the continents [8]. Its development requires a set of datasets. As part of this study, GNSS leveling data, direct leveling, and gravity measurements were used. The study strategy that was retained is to start first of all with the recognition of the geodetic points of the existing urban network and leveling network through the identification sheets

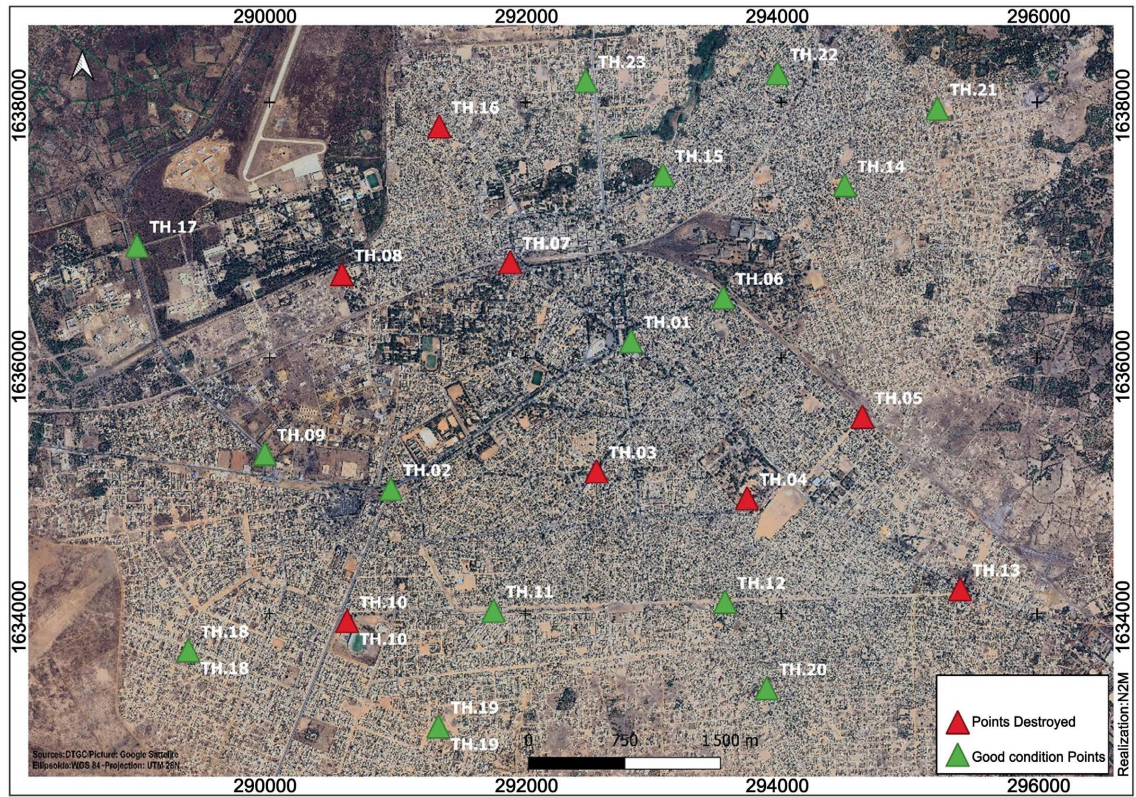


Figure 2. Distribution of geodesic points of the Thies urban network.

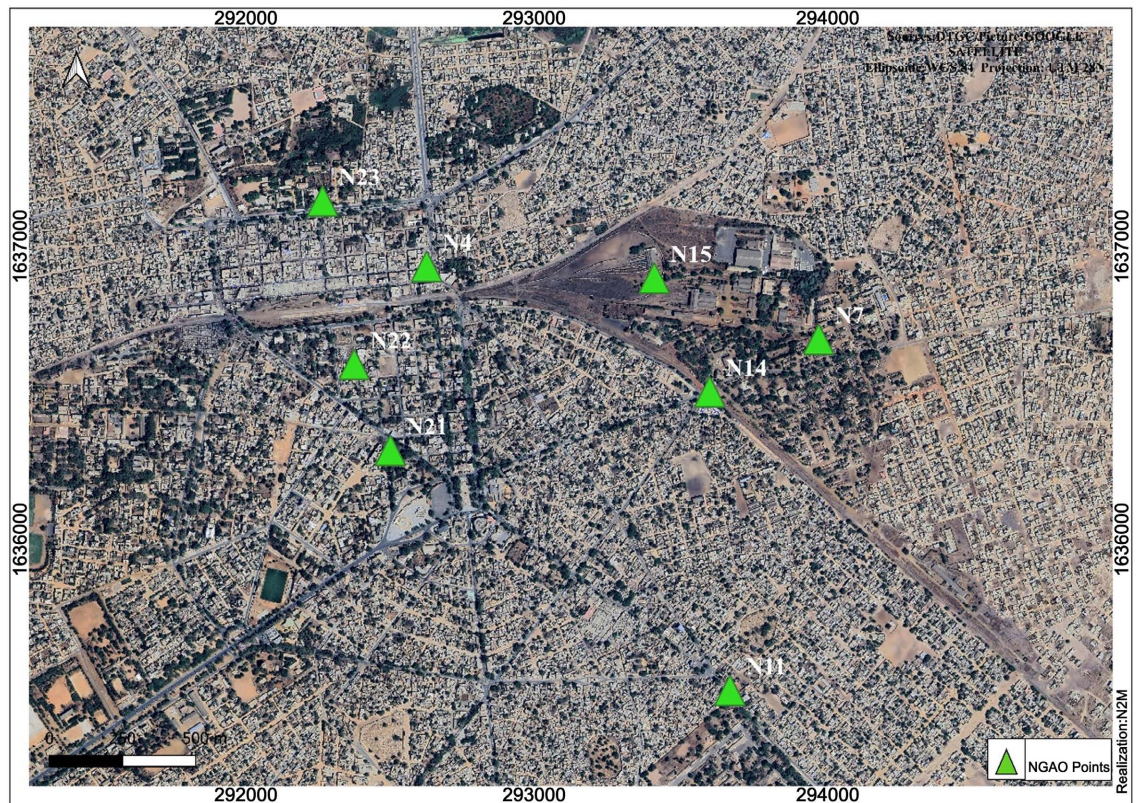


Figure 3. Distribution of NGA053 benchmarks.

which were offered to us by the DTGC (Directorate of Geographical Works and cartographic). At the end of this field visit, we noted that eight (08) points out of the 23 of the urban networks were destroyed (See **Figure 2** above).

Similar work was carried out for the leveling network which constitutes the practical and material realization of a vertical reference system. Information on the location of these markers was provided to us by the Thies land registry office. This network is made up of thirty-one (31) benchmarks which belong to the NGAO53 network and is attached to the tide gauge installed at the port of Dakar. With the information sheets we were able to take stock of the situation. The eight (08) points found are recorded in **Figure 3**. Certainly, the age of the network explains the weakness of the points that were found.

The entire methodological approach used as part of this study is summarized in **Figure 4** below:

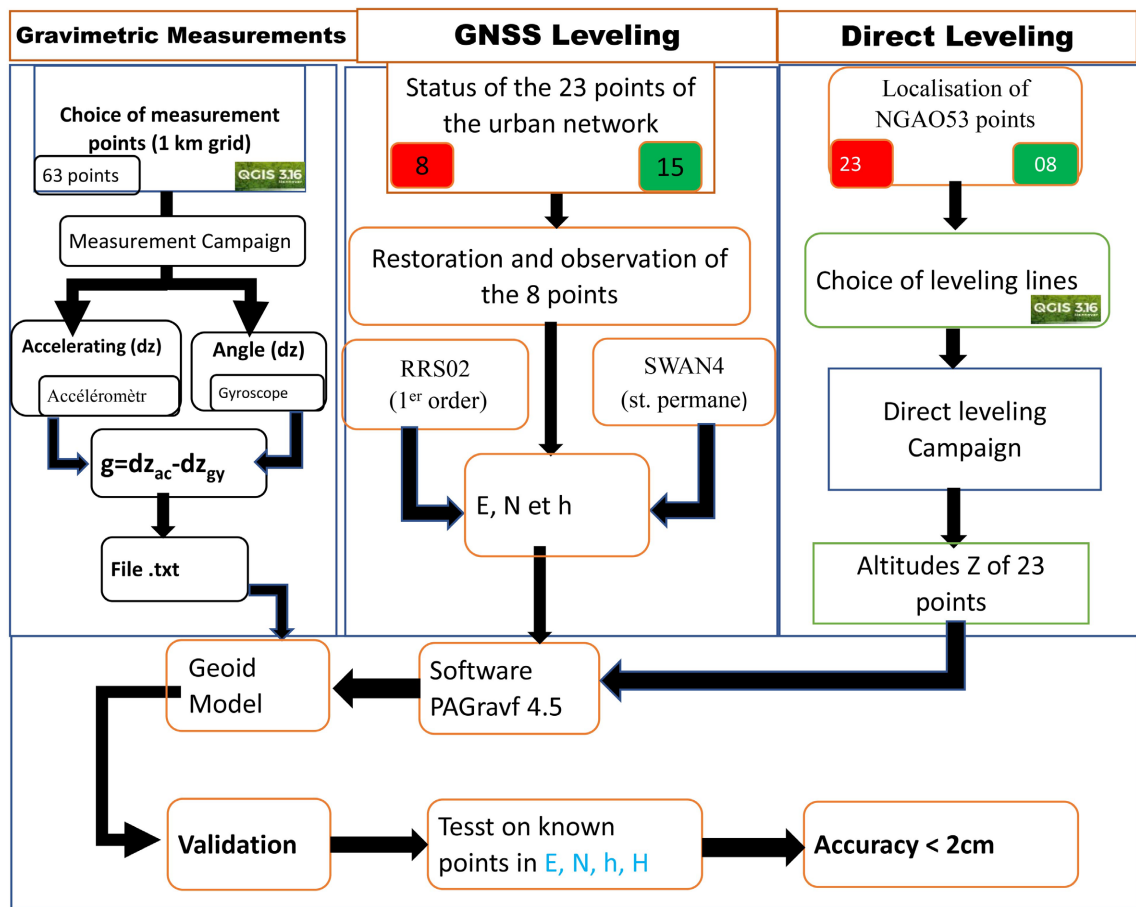


Figure 4. Methodological approach.

2.2.1. GNSS Leveling

Satellite positioning systems have greatly contributed to the precise determination of points on the earth. However, determining the altimeter component was one of the limitations of this system, as it could only measure the height relative to the associated ellipsoid. This did not correspond to the physical quantity (altitude)

that users were interested in. It was necessary to wait for the development of geoid models that could be integrated into GNSS receivers or calculation software to obtain altitudes from these measurements and the undulation provided by the model [9].

Obtaining this undulation requires leveling by GNSS, in this present study, the static mode was adopted to observe the 08 points which were destroyed. Indeed, the static mode is the method of GNSS techniques recommended for the establishment of canvases or networks in geodesy, topometry, photogrammetry or geophysics. This involves observing the phases on at least 4 satellites at each station. The observation time must be sufficient to resolve ambiguities and depends on the length of the baseline, the geometry of the satellite constellation and the atmospheric conditions [10].

The RRS02 point of the first order of RRS04 as well as the permanent station SWAN4 were used as a reference. The baselines are calculated from the Quantum GIS software and the coordinates of the destroyed points are used as approximate coordinates. However, since no baseline does not exceed the distance of 5 km (see **Figure 5**), the observation time was homogenized at 30 minutes for all points.

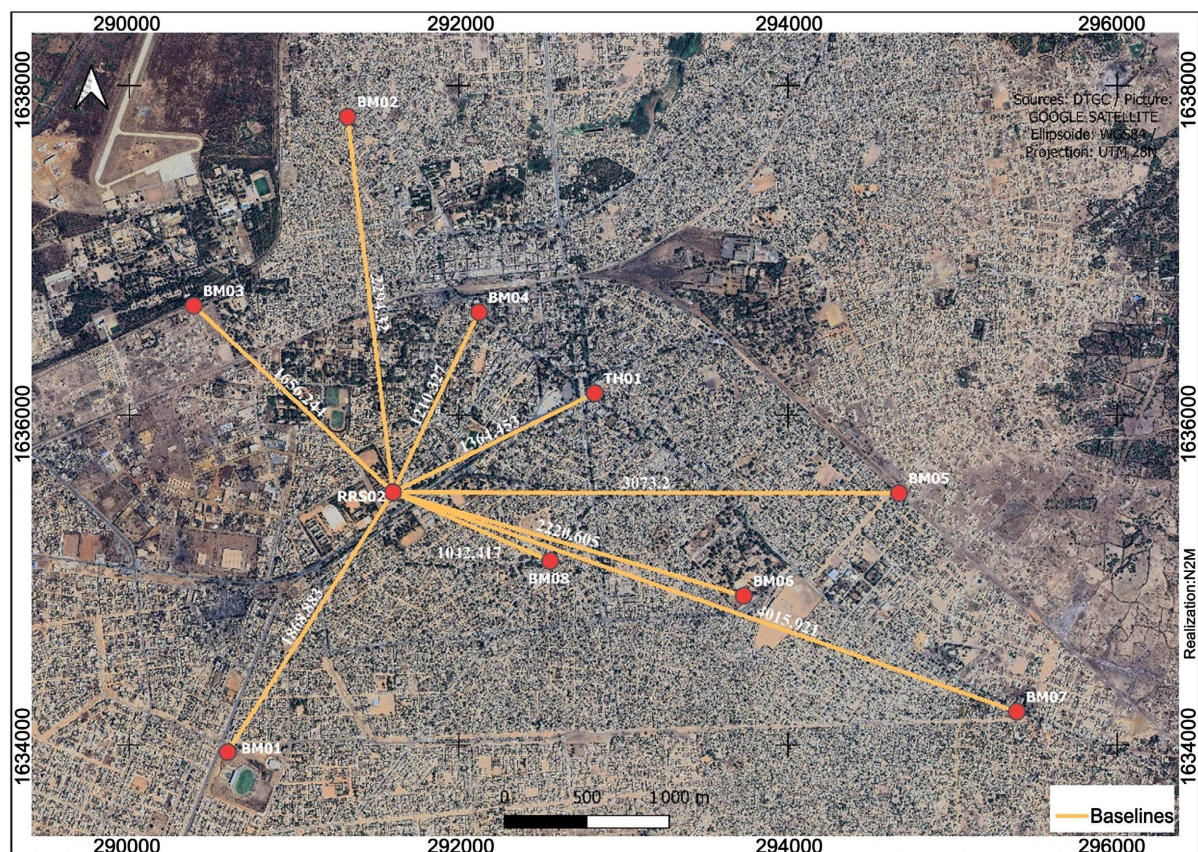


Figure 5. Baselines.

The observations were carried out with 8 CHCNAV i50 receivers and the technical characteristics of the receivers are detailed in the following **Table 1**:

Table 1. Technical characteristics of the CHCNAV i50 GNSS Receiver.

| GNSS receiver accuracy | |
|------------------------|--|
| Post-processing static | Horizontal: 3 mm + 0.5 ppm RMS Vertical: 5 mm + 0.5 ppm RMS |
| Différentiel Code | Horizontal: 0.4 m RMS Vertical: 0.8 m RMS |

This work carried out using the GNSS technique makes an important contribution both to determining the coordinates of the new points and to their restorations in order to regain the homogeneity of the network. However, the altitude obtained by these points is ellipsoidal and the establishment of the geoid, the reference surface of the true altitude, requires the determination of the undulation. Thus, geometric leveling becomes a requirement for the rest of our work.

2.2.2. Direct Leveling

Precision leveling is currently one of the most effective techniques for measuring elevation and its change. This technique consists in measuring the height difference between two points (landmarks) on a horizontal line of sight on the surface of the earth. The sum of the height differences measured along a leveling line gives the height of a benchmark relative to the initial benchmark. A line is defined by sections between two points. The expected precision on the height depends on the equipment used, the operating method in the field and the processing carried out on the raw measurements [11].

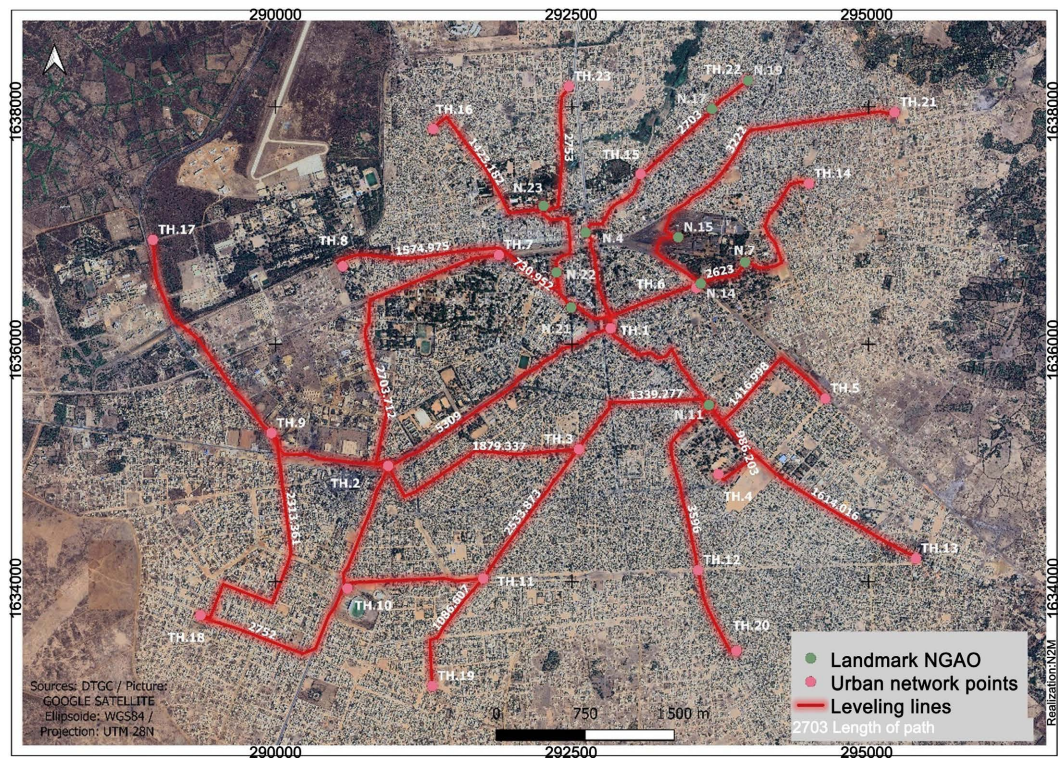


Figure 6. Leveling lines.

Modeling a local geoid requires good knowledge of the orthometric altitudes of a set of well-distributed points. Considering that the fifteen (15) points of the urban network remained intact and had orthometric coordinates, we integrated them into the choice of routes for a good coverage of the study area. Thus, we used an orthoimage in Quantum GIS [12] to determine the best routes by avoiding obstacles, busy areas as much as possible and preferably following asphalt roads. **Figure 6** above illustrates the grading lines that were followed. Some precautions were taken for the smooth running of the work: make equal ranges by counting steps (30 to 40 m) maximum, ensure the verticality of the staff and carry out a progress check by following a path at double stations.

The results obtained with this geometric leveling are certainly important and decisive for the estimation of the orthometric altitude in the city of Thies but remain insufficient. Indeed, the vertical frame of reference adopted by Senegal depends on the values of the force of gravity g along the vertical line and they are generally undefined and inaccessible in reality [13]. Thus, we opted to take gravimetric measurements over the entire study area.

2.2.3. Gravimetric Measurements

In practice, it is impossible to carry out gravity measurements at all points on the earth's surface. However, measurements are made on discrete points and information is predicted on areas without measurements [14]. For a homogeneous

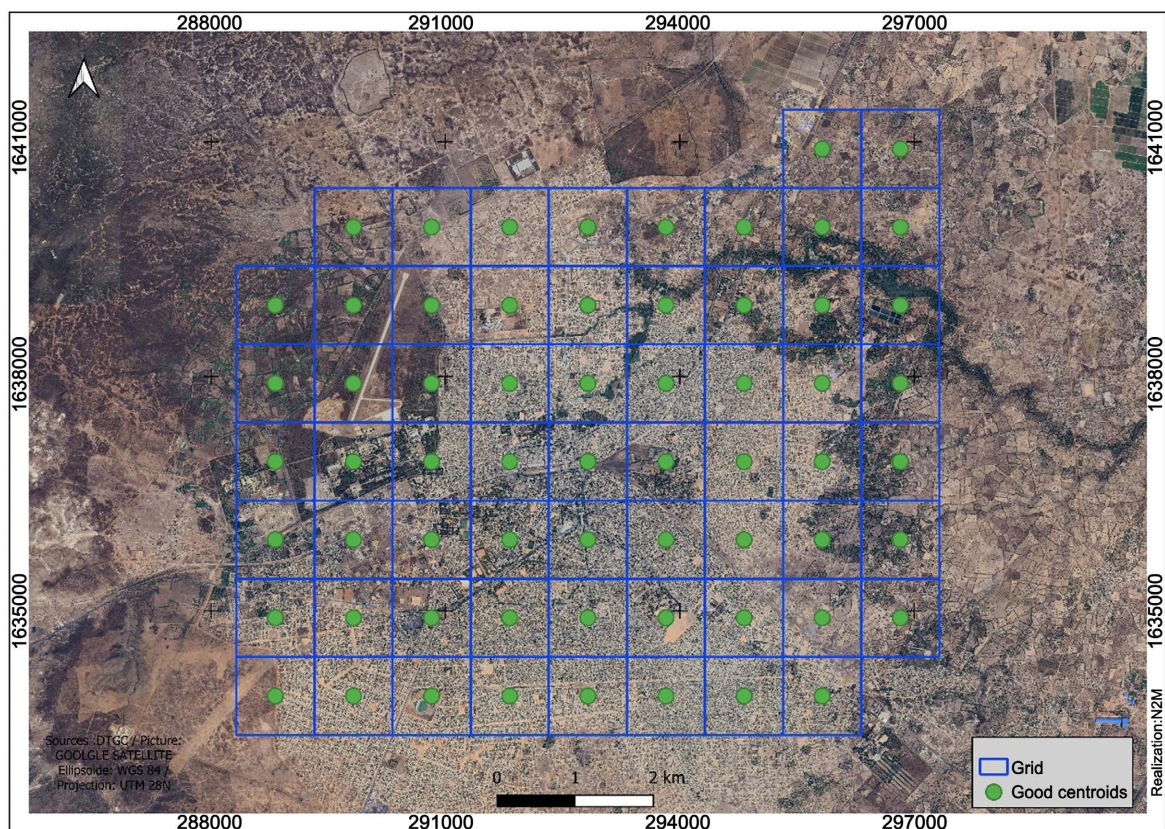


Figure 7. Distribution of planned points for measurements.

distribution and good resolution, we established a regular 1km grid covering the entire study area and the centroids of each cell constitute the planned points for making measurements (**Figure 7**).

Gravimetric data were acquired by the Sensor Play Data Recorder application used on an iPhone Xr type smartphone equipped with an accelerometer and a gyroscope. It is a very useful application for teaching science, technology, engineering and mathematics, in university and industry. The steps for measuring the sixty-three (63) points on the ground are first of all to get as close as possible to the measurement point using the keyhole Markup Language (KML) file viewed on Google Earth, then place and stabilize the smartphone on the point, start recordings from accelerometer, gyroscope and GPS for 10 seconds and finally these data are saved in a CSV format file generated by the application.

3. Processing and Analysis of Results

3.1. GNSS Leveling

At this stage of our work, the first order point RRS02 of the Senegalese reference system located within the grounds of the Thies gendarmerie company was used as a reference point. Following the setting up of the station while respecting all the rules (measuring the antenna height, bubbler, optical plummet, etc.), the recording of the raw data with a frequency of 1 Hz was launched. The same procedure is implemented for the nine (09) points to be calculated and the observations were made simultaneously. At the end of the observation phase, the recorded raw data were recovered and processed with the CGO2 software (CHC Geomatics Office 2) [15]. However, in the calculation process, data from the permanent station SWAN4 are integrated to further improve the precision of the determined coordinates. It should also be noted that TH 01 of the urban network was observed at the same time to serve as a control point at the end of operations. **Table 2** below is a summary of the results obtained.

Table 2. Coordinates calculated in static mode and leveled altitudes.

| Points | East (m) | North (m) | Ellipsoïdal Height (m) | Leveled Height (m) |
|--------|-------------|--------------|------------------------|--------------------|
| RRS02 | 291601.0396 | 1635533.3550 | 110.9760 | --- |
| TH01 | 292826.3194 | 1636134.1213 | 98.4645 | --- |
| BM01 | 290597.2575 | 1633956.6110 | 124.7487 | 94.356 |
| BM02 | 291325.5618 | 1637811.3959 | 100.0515 | 69.705 |
| BM07 | 295390.3169 | 1634201.7726 | 90.9664 | 65.275 |
| BM05 | 294674.6321 | 1635527.8771 | 95.0496 | 60.714 |
| BM06 | 293731.0963 | 1634904.6695 | 103.1675 | 72.838 |
| BM08 | 292556.3836 | 1635115.9302 | 111.5539 | 81.231 |
| BM03 | 290391.1033 | 1636664.7122 | 100.9959 | 69.508 |
| BM04 | 292123.2551 | 1636625.4083 | 95.2635 | 64.859 |

The comparison of our results at the TH 01 control point shows in **Table 3** an accuracy of our calculations in planimetry of ± 1 cm and in altimetry of ± 3 cm.

Table 3. Comparison of DTGC and calculated coordinates.

| Control points | Known coordinates (DTGC) | | | Gab for each component | | |
|----------------|--------------------------|--------------|--------------|------------------------|----------------|----------------|
| | <i>E</i> (m) | <i>N</i> (m) | <i>H</i> (m) | ΔE (m) | ΔN (m) | ΔH (m) |
| TH01 | 292826.336 | 1636134.128 | 98.499 | -0.0166 | -0.0067 | -0.0345 |

The results obtained for the determination of altitude by GNSS technique make it possible to obtain good precision.

3.2. Geometric Leveling

The NGAO 53 network constitutes our reference for determining orthometric altitudes. First of all, the altitudes of the eight (08) restored points were calculated by a precision leveling operation over a length of 18 km. Then the altitudes of the points of the urban geodetic network were verified through supervised paths and for the new points round-trip paths were retained. The results of the leveling work are shown in **Table 4**. The calculation of the tolerance was based on the formula for regulatory precision leveling from 1980 in force in Senegal [16]: For $n \leq 16$

$$T\Delta H = \sqrt{9L + L^2} \quad \text{Pour } n \geq 16: T\Delta H = \sqrt{N + \frac{N^2}{16}}$$

with: L = total length of the route in Km

N = number of elevations

$n = N/L$ (km)

Closure = $Z_{\text{observed}} - Z_{\text{actual}}$

At the end of the calculations, we noted path lengths varying from 1147 km to 5420 km with closures between 2 mm and 8 mm which are well below the tolerances. The calculated altitudes of the new points are recorded in **Table 4**.

3.3. Gravimetric Measurements

Nowadays, the combination of several sources and types of data makes it possible to better probe the long, medium and short wavelengths of the gravity field [17].

A homogeneous and dense spatial distribution of gravity data is of paramount importance to have a high-precision geoid model, since determining the undulation of the geoid at a point requires knowledge of the gravity data around that point. Determining the value of g is based on measuring the acceleration g to which the Smartphone is subjected. Indeed, if the latter is perfectly still, the acceleration corresponds to g (static gravity). If the Smartphone is moving, the acceleration will include gravity (g) and the acceleration linked to the force of inertia (g dynamic). Since it is difficult, if not impossible, to completely immobilize the Smartphone, we considered the second case: the case where the accelerometer measures the sum of g static and g dynamic. We have:

$$g_{ac} = g_s + g_d$$

- ✓ g_{ac} is the total g measured by the accelerometer,
- ✓ g_s is the part of g total related to the earth's gravity,
- ✓ g_d is the part of g total related to the mobility of the device.

To determine the value of g_s , the gyroscope made it possible to determine the value of g_d which constitutes the part linked to the movement of the device and the accelerometer recorded the three components of g_{ac} . Thus the files generated by the device were used to calculate g_s , for each measurement and then the average is determined over all the measurements at each point. All of the average g_s , are then grouped into a single file. The interpolation of these points of g made it possible to obtain the representation of its variation (**Figure 8**).

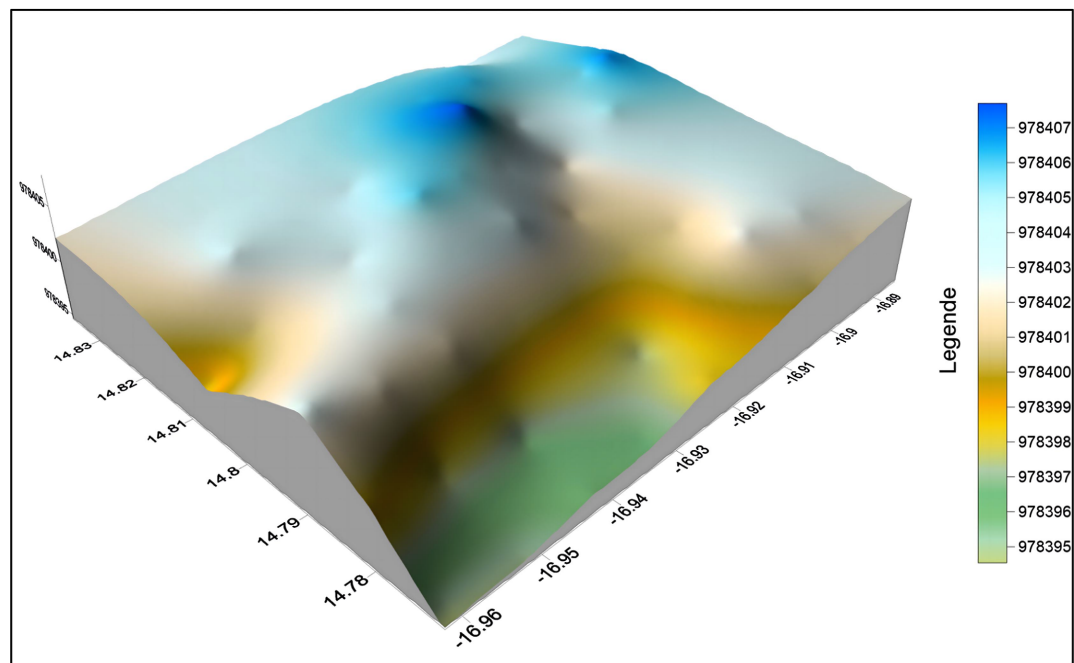


Figure 8. Representative map of the variation of g (mgal).

4. Calculation of the Geoid Model and Validation

4.1. Model Calculation

Creating a regional gravity model involves several steps. The first step is about collecting or acquiring data. It is then necessary to check the quality, consistency and proper integration of the different data sets (pre-existing and measured). The last step consists in calculating the geoid [18].

As part of this study, the PAGrav4.5 software [19] is used to determine the geoid. The calculation process is subdivided into four (04) main steps:

- ✓ Construction of the heterogeneous observation file, from the observed GNSS leveling height anomaly file and the EGM 2008 global geopotential model.
- ✓ Detection and removal of gross errors in GNSS leveling data. It detects and removes the raw GNSS leveling points, the statistical average of the GNSS

leveling residuals, and GNSS leveling residuals, then merges them again according to the agreed format to generate the new heterogeneous observation file. Here, the statistical average represents the difference between the local height reference and the global height reference.

- ✓ Determination of the residual height anomaly using the SRBF (Spherical Radial Basis Function) approach. The SRBF method uses EGM 2008 to first calculate the height anomaly and the gravity anomaly. These are then compared with the raw data in order to determine the residuals which will allow the model to be refined.
- ✓ Restoration of the reference gravity field from the EGM 2008 file, the ellipsoidal height grid of the calculation surface and the residual grid.

Figure 9 below summarizes the different processing phases in the PAGrav4.5 software

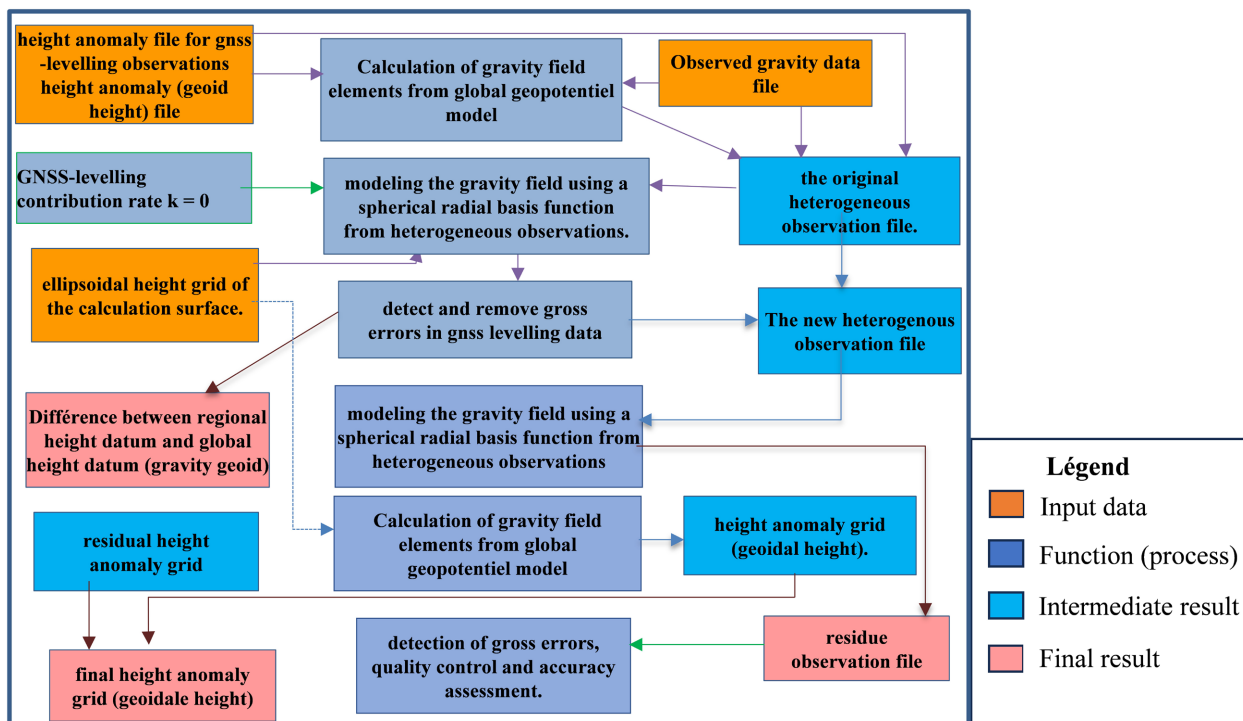


Figure 9. Geoid modeling phases with PAGrav4.5.

4.2. Results and Validation

At the end of the calculations, a grid of undulations covering the city of Thies was generated and we called it Thies Gravimetric Geoid (GGravT). The variation of the undulation in the town of Thies is from 30.36 to 30.48 m (**Figure 10**).

For the evaluation of the precision of geoid models determined gravimetrically generally two methods are used. The first, considered internal, makes it possible to calculate the covariance matrix of the local model of the geoid using the law of propagation of errors on altitudes, local gravity and the harmonic coefficients of the geopotential model integrated into the final solution of the geoid. This matrix

can be used in the combined adjustment of the different types of altitudes as well as an initial covariance matrix. The second approach uses as control values the geoid undulations obtained by GNSS/Leveling colocation to evaluate the external precision of the local geoid model. Gravimetrically calculated geoid heights are compared to geoid undulations derived from GNSS and precision leveling observations.

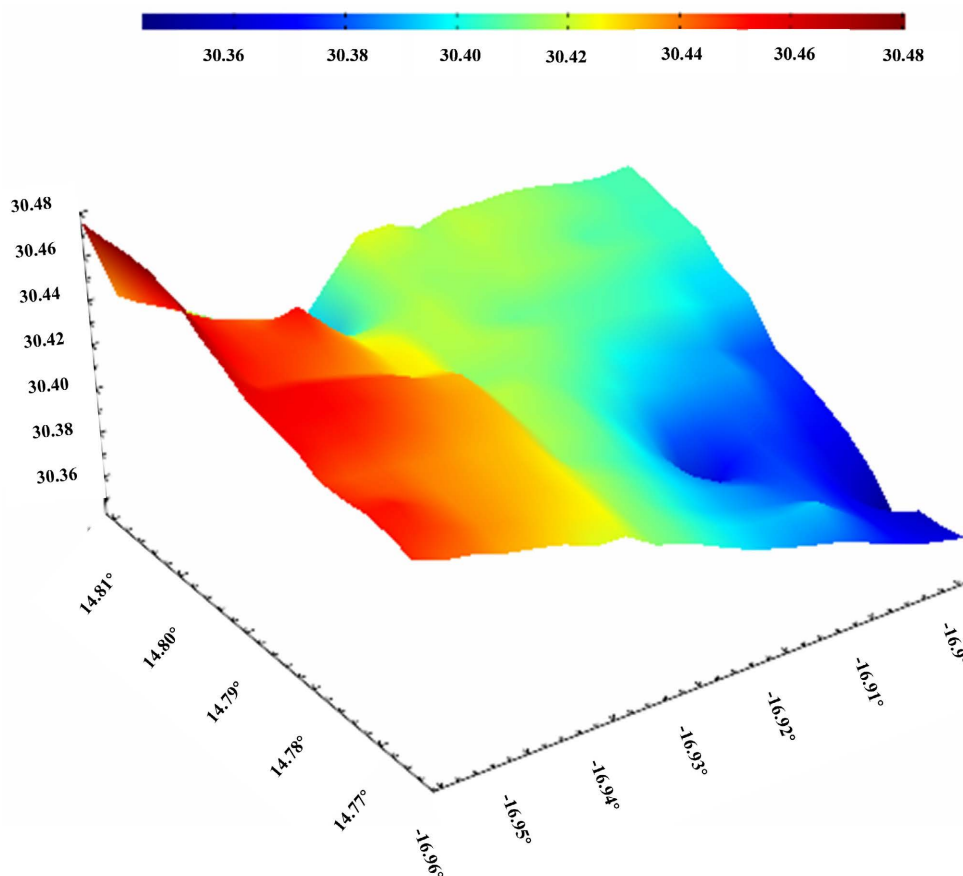


Figure 10. Visualization of the GGravT geoid model in 3D.

Table 4. Comparative results between models.

| Points | Longitude | Latitude | Height | Reference height | Calculated height | Altitude_GGS12V1 | Gap with GGravT (m) | Gap with GGS12V1 |
|--------|-------------|------------|---------|------------------|-------------------|------------------|---------------------|------------------|
| K4 | -16.9500605 | 14.776621 | 120.519 | 90.006 | 90.0171 | 90.067 | 0.0111 | 0.061 |
| TH07 | -16.9337139 | 14.7968278 | 95.915 | 65.481 | 65.4977 | 65.478 | 0.0167 | -0.003 |
| Cr01 | -16.9568991 | 14.7880744 | 106.853 | 76.329 | 76.3319 | 76.366 | 0.0029 | 0.037 |
| RRS02 | -16.9362232 | 14.7857974 | 110.976 | 80.593 | 80.5919 | 80.543 | -0.0011 | -0.05 |
| Cr03 | -16.9292292 | 14.7958608 | 94.152 | 63.766 | 63.7602 | 63.724 | -0.0058 | -0.042 |
| Cr04 | -16.9148484 | 14.7920339 | 98.666 | 68.329 | 68.3184 | 68.264 | -0.0106 | -0.065 |
| Cr05 | -16.9303995 | 14.778122 | 113.268 | 82.905 | 82.9051 | 82.85 | 0.0001 | -0.055 |
| R01 | -16.9134774 | 14.7817485 | 99.768 | 69.417 | 69.4136 | 69.376 | -0.0034 | -0.041 |

Thus, eight (08) points well distributed in the city of Thies were chosen for altitude control with the GGravT model. **Table 4** shows the differences between the reference altitudes and those of our GGravT model.

The results recorded in the table below show a variation in deviations between 0.1 mm to 1.67 cm with a root mean square error of 8.43 mm. We can deduce that our model has an accuracy less than or equal to 2 cm.

The same control points were recalculated with the Geoid Geometric Senegal 2012 Version1 (GGS12v1) model [20] with Circé Senegal to compare the two models. The results obtained are mentioned in **Table 4**.

Following these results, we can note a root mean square error of 47.8 mm with GGS12v1 compared to 8.43 mm for GGravT. This allows us to conclude that our model offers better results than GGSV12.

In addition, a statistical analysis between the two models was carried out to refine the comparison. **Table 5** below and **Figure 11** below represent the results of this analysis.

Table 5. Statistical analysis.

| Model | AME | ERMS | Mean | Standard deviation |
|---------|-----------|------------|-------------|--------------------|
| GGravT | 0.0064625 | 0.00843571 | 0.0012375 | 0.00892059 |
| GGS12V1 | 0.04425 | 0.04781997 | -0.01388889 | 0.0469666 |

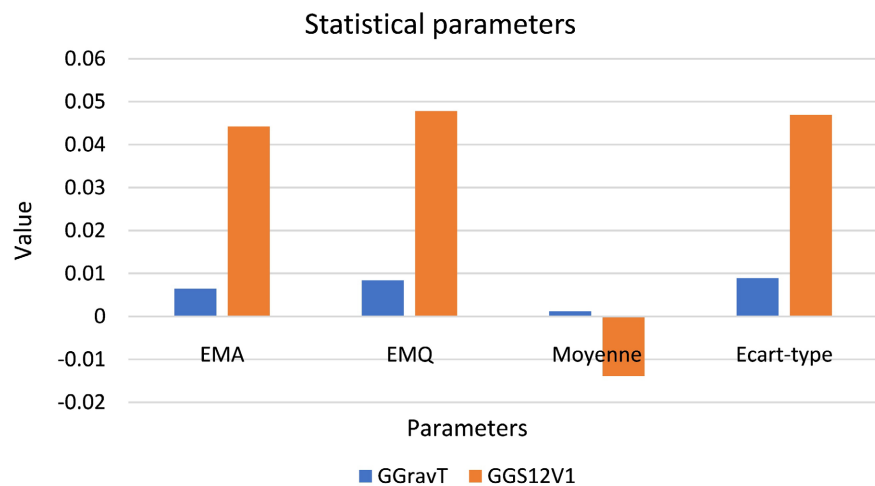


Figure 11. Differences with GGravT and GGS12V1.

For all statistical parameters calculated, the results obtained with our model are lower than those obtained with the GGS12v1. This further allows us to confirm that the GGravT model is more accurate than GGS12v1.

5. Discussions

The result obtained through this study is very satisfactory. It reveals that with the appropriate tools we can significantly improve the knowledge and control of the

local geoid, which constitutes a major asset for all engineering work.

Regarding our specific work, we are not aware of a study to determine a local geoid with the same parameters in our study area. This is what makes the comparative study with other existing studies difficult. Nevertheless, our result can be compared with other work on local determination of the geoid throughout the world even if the same tools were not used. This is the case for the application of the gravimetric method in the study by Rebaï, N. *et al.*, in Tunisia in the Mejez El Bab area [21]. He was able to obtain a local geoid model of ± 3 cm accuracy using the Digital Terrain Model (DTM), direct leveling, GPS leveling and gravity measurements. Jackson, K.P. and Moka, E.C achieved the modeling of orthometric heights with the gravimetric method (RCR) using the EGM2008 geopotential model. They thus obtained an accuracy of ± 3 cm in Rivers State in Nigeria [22].

Other non-gravimetric methods have been developed with satisfactory results. In the Gaza Strip, El Hallaq obtained a local geoid model with an accuracy of 4.28 cm using local GNSS leveling [23]. In Estonia, an accuracy of 1.3 cm is obtained by Jürgenson *et al.* from the adjustment of a gravimetric geoid surface to the geometric heights of the geoid [24]. In Nigeria, still with the geometric method, a precision of 0.511 m was obtained by Tata and Ono in Akure near Akure [25]. Ismail MK *et al.*, were able to obtain 8 mm of accuracy for a local geoid with the geometric method at the Malaysian University of Technology [26]. In Busoga in Uganda, still using the geometric method with direct leveling and GNSS leveling Bruno Kyamulesire *et al.* found an accuracy of 6.7 cm [27].

All these results obtained with different methodologies agree well with ours and tend to confirm the relevance of our approach. The particularity of our approach lies in the use of the Sensor Play Data Recorder application (at lower cost) to determine gravimetry and arrive at this level of result. This study made it possible to improve altitude control in the city of Thies and can be extended to other cities and throughout Senegal.

6. Conclusions

Determining the altitude of a given point using GNSS data requires knowledge of the precise geoid model. Indeed, the geoid appears as an irregular surface which presents undulations in relation to the reference ellipsoidal surface.

The geoid model calculation methods are diverse, we have developed three (03) methods, namely the GNSS leveling method, direct leveling and the gravimetric method.

The gravimetric method was used to calculate a local geoid model for the city of Thies, by applying the SRBF calculation method with the PAGravf 4.5 software. The data used are, among others, the values of terrestrial gravity, altitude data, GNSS data and the global geopotential model EGM 2008. The altitude data were obtained from direct precision leveling, those from GNSS by observation in static mode. Gravity data was obtained using the Sensor Play-Data Recorder application. A heterogeneous file combining these different types of data is created

allowing the implementation of a GGravT altitude conversion grid linked to gravity.

In order to validate the GGravT geoid, it is tested on eight (08) well-distributed known points at altitude. Small differences (<2 cm) were noted between the altitudes obtained by leveling and those calculated with the GGravT.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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