

# Relativistic Motion with Viscosity III: Stokes's Law of Resistance in Alternative Relativity

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## Abstract

An equation of motion in the presence of a drag force proportional to the velocity is derived in the framework of alternative relativity. The results allow modeling the trajectory of SN 1993J and its light curve in four different astronomical bands.

## Keywords

Supernovae, General Supernovae, Individual (SN 1993J) Ism, Supernova Remnants

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## 1. Introduction

Newton's second law of motion, that force is equal to mass times acceleration, was stated by Newton in 1687 [1]. The rise of special relativity, due to Einstein and Poincaré in 1905 [2] [3], modified this law in such a way that force is equal to the time derivative of the vector that represents the relativistic momentum. This relativistic equation of motion is widely used and apparently, there is no need for further improvement. Starting in 1991, Huang introduced an alternative special relativity which progressively covered different topics: relativistic kinematics [4], the electromagnetic force law [5], a relativistic modification of Newton's gravitational force law [6], and the compatibility of the differential Lorentz transformation and Heisenberg's uncertainty principle [7]. A careful analysis of the arguments in the references listed above allows saying that the applications of the alternative relativity to astrophysics have not yet been explored. Here, we focus on the relativistic motion with friction in 1D and we apply the results to SN 1993J. In Section 2, we review special relativity and introduce the alternative relativity. Section 3 applies the results of alternative relativity to the trajectory and to the light curve of SN 1993J.

## 2. The Two Relativities

We present the widely used equation of motion of special relativity and a different equation of motion from the alternative relativity.

### 2.1. Special Relativity

In classical mechanics, motion is modeled by the second law of Newton

$$\mathbf{F} = m\mathbf{a}, \quad (1)$$

where  $\mathbf{F}$  is the force,  $m$  is the mass, and  $\mathbf{a}$  is the acceleration. As an example, the trajectory,  $x(t)$ , in the presence of a constant force along the  $X$ -axis is

$$x(t) = \frac{Ft^2}{2m}. \quad (2)$$

In special relativity, the equation of motion is

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}, \quad (3)$$

where  $\mathbf{F}$  is the force acting on the point with momentum  $\mathbf{p}$ , which is

$$\mathbf{p} = \gamma m\mathbf{v}, \quad (4)$$

where  $m$  is the rest mass and

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \quad (5)$$

We now analyze the case of a particle in the presence of a constant force  $F$  in 1D. We start from the relativistic equation of motion (3) and we derive the velocity when  $v(0) = 0$

$$v(t) = \frac{dx}{dt} = \frac{Ft}{m} \frac{1}{\sqrt{1 + \frac{F^2 t^2}{c^2 m^2}}}. \quad (6)$$

We then find the trajectory when  $x = 0$  at  $t = 0$

$$x(t) = \frac{1}{F} mc^2 \left( \sqrt{1 + \frac{F^2 t^2}{c^2 m^2}} - 1 \right). \quad (7)$$

### 2.2. The Alternative Relativity

The alternative theory of relativity is characterized by an equation of motion, which is

$$\frac{d^2 \mathbf{x}}{dt^2} = \mathbf{a} = \left( \frac{\mathbf{F}}{m} \right) \left( 1 - \left( \frac{v}{c} \right)^2 \right), \quad (8)$$

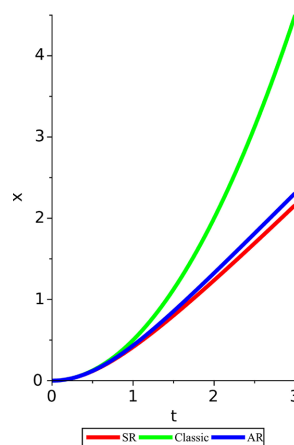
where  $\mathbf{x}$  is the position,  $\mathbf{a}$  is the acceleration,  $\mathbf{F}$  is the force, and  $m$  is the mass, see Equation (55) in [8]. In this case, the acceleration is in the same direction as the force. We now analyze the case of a particle in the presence of a constant force  $F$  in 1D. We start from the above alternative relativistic equation of motion and we derive the velocity when  $v(0) = 0$

$$v(t) = c \tanh\left(\frac{tF}{mc}\right), \tag{9}$$

where  $\tanh$  is the hyperbolic tangent. The 1D trajectory in alternative relativity when  $x = 0$  at  $t = 0$  is

$$x(t) = \frac{c^2 m \ln\left(\cosh\left(\frac{tF}{mc}\right)\right)}{F}, \tag{10}$$

where  $\cosh$  is the hyperbolic cosine. The three solutions for the 1D motion in the presence of a constant force are presented in **Figure 1** when  $F = 1$ ,  $c = 1$  and  $m = 1$ .



**Figure 1.** Plot of the classical solution, Equation (2), (green line), relativistic solution, Equation (7), (red line) and solution in alternative relativity, Equation (10), (blue line).

We now analyze one-dimensional motion with a resistive force of Stokes’s type [9],  $F_{res} = -Amv(t)$ , where  $A$  is a constant. The equation that governs the motion, according to Equation (8), is

$$\frac{d}{dt} v(t) = -\frac{Av(t)\left(1 - \frac{v(t)^2}{c^2}\right)}{m}, \tag{11}$$

and the solution is

$$v(t) = \frac{e^{\frac{At_0}{m}} v_0 c}{\sqrt{v_0^2 e^{\frac{2At_0}{m}} + (c^2 - v_0^2) e^{\frac{2At}{m}}}}, \tag{12}$$

where  $v_0$  is the velocity at  $t = t_0$ . The 1D trajectory in alternative relativity can be derived from the indefinite integral,  $I(t)$ , of the previous equation, which is

$$I(t) = -\frac{e^{\frac{At_0}{m}} cm \operatorname{arctanh}\left(\frac{\sqrt{v_0^2 e^{\frac{2At_0}{m}} + (c^2 - v_0^2) e^{\frac{2At}{m}}}}{v_0 \sqrt{e^{\frac{2At_0}{m}}}}\right)}{A \sqrt{e^{\frac{2At_0}{m}}}}, \tag{13}$$

and therefore

$$x(t) = I(t) - I(t_0) + x_0, \tag{14}$$

where  $x = x_0$  at  $t = t_0$  and  $\operatorname{arctanh}$  is the inverse of the hyperbolic tangent function. We now derive the time as a function of position from the above equation

$$t = \frac{\left( \ln \left( \frac{v_0^2 \operatorname{sech} \left( \frac{-c \ln \left( \frac{v_0 - c}{v_0} \right) + c \ln \left( \frac{c + v_0}{v_0} \right) - \frac{2A(x - r_0)}{m}}{2c} \right)^2}{c^2 - v_0^2} \right) + \frac{At_0}{m} \right) m}{A}. \tag{15}$$

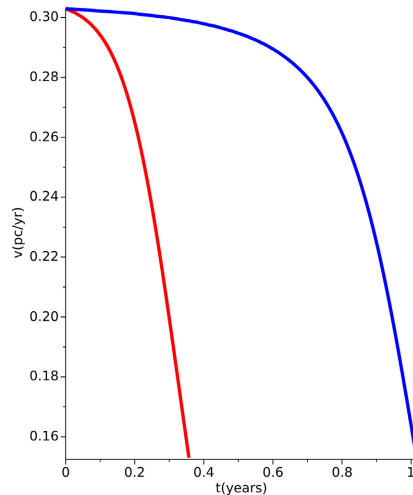
The velocity as a function of the spatial variables can be derived as

$$v(x) = \frac{e^{\frac{At_0}{m}} v_0 c}{\sqrt{R}}, \tag{16}$$

where

$$R = v_0^2 e^{\frac{2At_0}{m}} \left( \tanh^2 \left( \frac{-cm \ln \left( \frac{v_0 - c}{v_0} \right) + cm \ln \left( \frac{c + v_0}{v_0} \right) - 2A(x - x_0)}{2cm} \right) \right), \tag{17}$$

with  $\tanh$  being the hyperbolic tangent function. In the case of a resistive force of Stokes's type in the framework of special relativity, a solution for the velocity exists in an implicit form, see Equation (7) in [10]. A comparison between velocity as a function of time in alternative and special relativity is presented in **Figure 2**: the velocity in special relativity decreases more quickly than in the alternative relativity.



**Figure 2.** Plot of the velocity as a function of time for a resistive force of Stokes's type, see Equation (12), (red line) and in special relativity (blue line). The parameters are  $A = 7$ ,  $m = 1$ ,  $\beta_0 = \frac{v_0}{c} = 0.99$ ,  $t_0 = 10^{-4}$  yr and  $r_0 = 10^{-4}$  pc.

### 3. Astrophysical Applications of SN 1993J

This section reviews the theoretical formula for the luminosity in a classical and relativistic framework, presents a simple approach to the absorption of light, applies the new equation of motion for a resistive force of Stokes's type in alternative relativity to SN 1993J and models the light curve of SN 1993J in the various bands.

#### 3.1. The Road to Luminosity

In the *classical* case, the rate  $L_m$  of the transfer of mechanical energy is

$$L_m(t) = \frac{1}{2} \rho(t) 4\pi r(t)^2 v(t)^3, \quad (18)$$

where  $\rho(t)$ ,  $r(t)$  and  $v(t)$  are the temporary density, radius and velocity of the SN. We assume that the density in front of the advancing expansion scales as

$$\rho(t) = \rho_0 \left( \frac{r_0}{r(t)} \right)^d, \quad (19)$$

where  $r_0$  is the radius at  $t_0$  and  $d$  is a parameter which allows matching the observations; as an example, a value of  $d = 3$  is reported in [11]. With the above assumption, the mechanical luminosity is

$$L_m(t) = \frac{1}{2} \rho_0 \left( \frac{r_0}{r(t)} \right)^d 4\pi r(t)^2 v(t)^3. \quad (20)$$

The energy fraction,  $L_\nu$ , of the mechanical luminosity deposited in the frequency  $\nu$  is assumed to be proportional to the mechanical luminosity through a constant  $\epsilon_\nu$

$$L_\nu = \epsilon_\nu L_m. \quad (21)$$

The flux at frequency  $\nu$  and distance  $D$  is

$$F_\nu = \frac{\epsilon_\nu L_m}{4\pi D^2}. \quad (22)$$

For practical purposes, we impose a match between the observed luminosity,  $L_{obs}$ , and the theoretical luminosity,  $L_m$ ,

$$L_{obs} = C_{obs} L_m, \quad (23)$$

where  $C_{obs}$  is a constant which equalizes the observed and the theoretical luminosity and varies on the basis of the selected astronomical band. In an analogous way, the observed absolute magnitude is

$$M_{obs} = -\log_{10}(L_m) + \Delta m, \quad (24)$$

where  $\Delta m$  is a constant. In the *relativistic* case, the rate  $L_{m,r}$  of transfer of mechanical energy, assuming the same scaling for the density in the advancing layer, is

$$L_{m,r}(t) = 4 \frac{\pi r(t)^2 \rho_0 c^3 \beta(t) \left( \frac{r_0}{r} \right)^d}{1 - \beta(t)^2}, \quad (25)$$

where  $\beta(t) = \frac{v(t)}{c}$ , for more details, see [12]. The conversion in magnitude of the above formula is the same as for Equation (24).

### 3.2. Absorption

The presence of the absorption can be parametrized by introducing a slab of optical thickness  $\tau_v$ . The emergent intensity  $I_v$  after the entire slab is

$$I_v = \int_0^{\tau_v} S_v e^{-t} dt, \quad (26)$$

where  $S_v$  is a uniform source function. Integration gives

$$I_v = S_v (1 - e^{-\tau_v}), \quad (27)$$

see Formula (1.30) in [13]. In the case of an optically thin medium,  $\tau_v = \infty$ , the observed luminosity can be derived from Equation (23), but otherwise, the following equation should be used:

$$L_{obs} = C_{obs} L_m (1 - e^{-\tau_v}), \quad (28)$$

where  $\tau_v$  is a function of time. For the case of the apparent magnitude, we have

$$m_{obs} = -\log_{10}(L_m) - \log_{10}(1 - e^{-\tau_v}) + \Delta m. \quad (29)$$

The value of  $\tau_v$  can be derived with the following equation:

$$\tau_v = -\ln\left(1 - e^{-(m_{obs} - m_{theo}) \ln(10)}\right). \quad (30)$$

where  $m_{theo}$  and  $m_{obs}$  represent the theoretical and the observed apparent magnitude. Due to the complexity of the time dependence of  $\tau_v$ , a polynomial approximation of degree  $M$  is used:

$$\tau_v(t) = a_0 + a_1 t + a_2 t^2 + \dots + a_M t^M, \quad (31)$$

with more details in [14]. In some cases, we apply the logarithms to the pair of data, *i.e.*,  $\log_{10}(x_i)$  and  $\log_{10}(y_i)$ ; we call this the *logarithmic polynomial approximation*.

The absorption in the relativistic case is assumed to be the same once the classical luminosity,  $L_m$ , is replaced by the relativistic luminosity  $L_{m,r}$

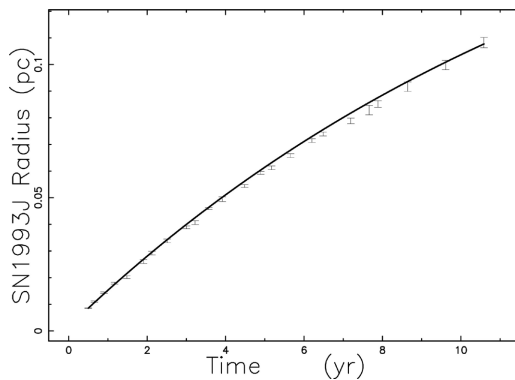
$$L_{obs} = C_{obs} L_{m,r} (1 - e^{-\tau_v}), \quad (32)$$

and

$$m_{obs} = -\log_{10}(L_{m,r}) - \log_{10}(1 - e^{-\tau_v}) + \Delta m. \quad (33)$$

### 3.3. The Trajectory

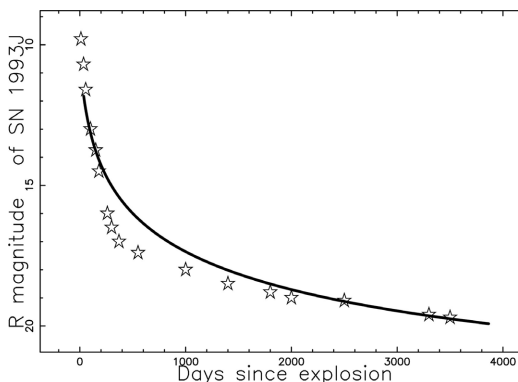
The SN here analyzed is SN 1993J, for which the temporary radius of expansion has been measured for  $\approx 10$  yr in the radio band [15] [16]. A comparison between the observed trajectory and theoretical behavior in the framework of the alternative relativity is presented in **Figure 3**.



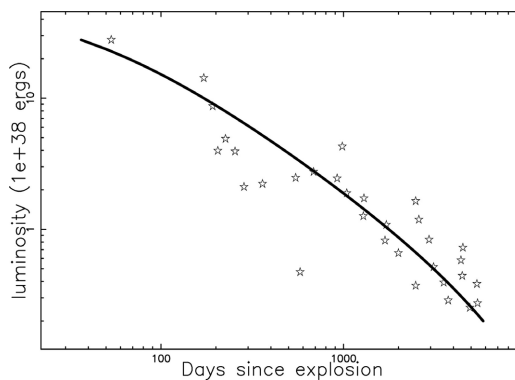
**Figure 3.** Theoretical radius as given by Equation (14),  $v_0 = 13800$  km/s,  $t_0 = 0.1$  yr,  $x_0 = 0.003$  pc, and  $\frac{A}{m} = 0.07$ .

### 3.4. The Light Curve

**Figure 4** presents the decay of the  $R$  magnitude of SN 1993J, which is type II, as well as our theoretical curve.



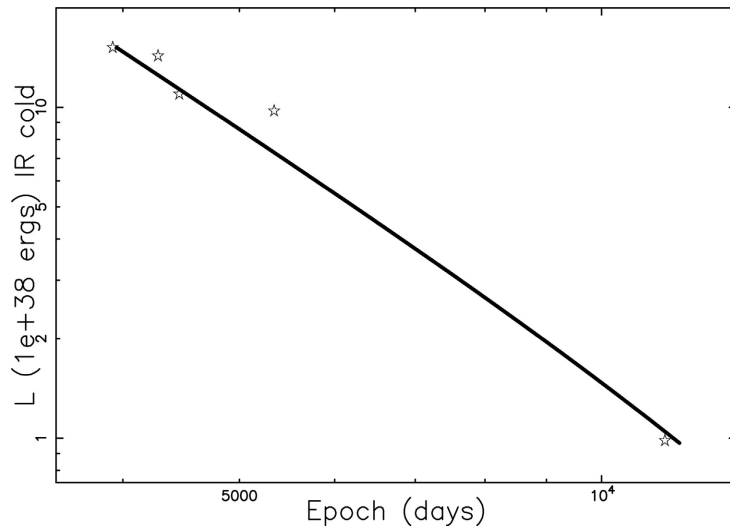
**Figure 4.** The  $R$  light curve of SN 1993J over 10 yr (empty stars) and theoretical curve as given by Equation (25) (full line). Parameters of the trajectory as in **Figure 3**,  $d = 7$ ,  $\Delta m = 5$  and  $\rho_0 = 1$ . The data were extracted by the author from Figure 5 in [17].



**Figure 5.** The  $H - \alpha$  and the 2.0 - 8.0 eV luminosities in units of  $10^{38}$  ergs of SN 1993J over 10 yr (empty stars) and theoretical curve as given by Equation (25) (full line). Parameters of the trajectory as in **Figure 3**,  $d = 3$  and  $\rho_0 = 1$ . Data extracted by the author from Figure 5 of [18].

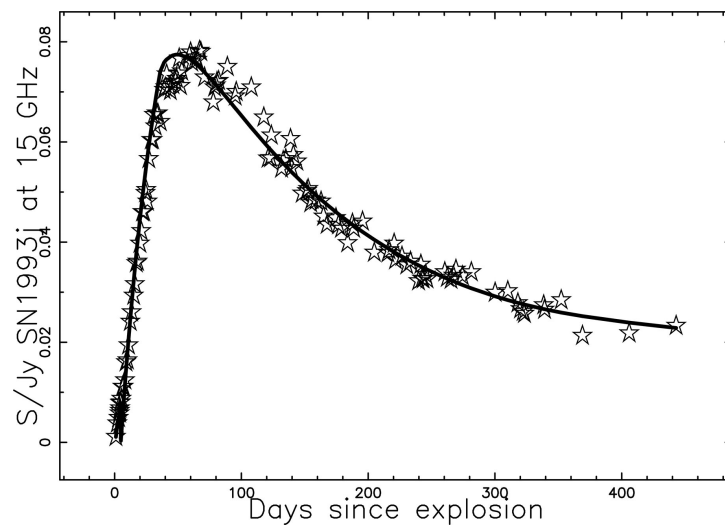
We present the  $H - \alpha$  with soft and hard band  $X$ -ray luminosities as well as the theoretical luminosity in **Figure 5**; the two luminosities are matched in such a way that the maximum of both is at  $123 \times 10^{38}$  ergs.

In the last years, the mid-infrared (mid-IR) wavelengths have also become detectable [19], and this allows comparing the theoretical light curve with the observed one in the so-called IR cold region, see **Figure 6**.

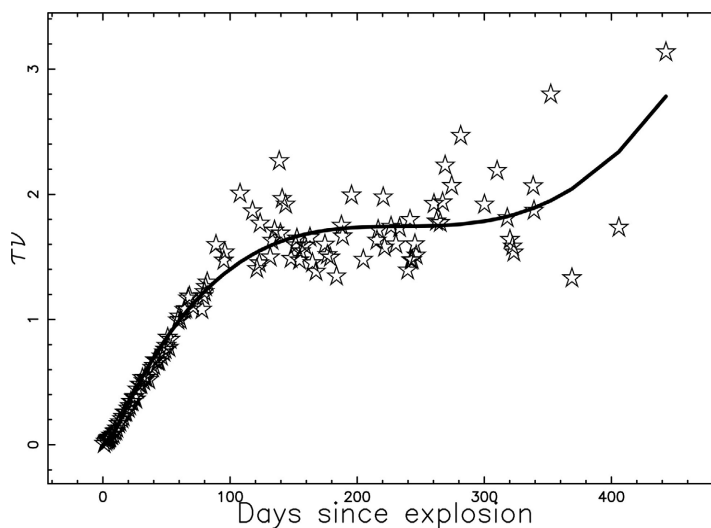


**Figure 6.** The IR cold luminosities in units of  $10^{38}$  ergs of SN 1993J over 10 yr (empty stars) and theoretical curve as given by Equation (25) (full line). Parameters of the trajectory as in **Figure 3**,  $d = 4.5$  and  $\rho_0 = 1$ . Data extracted by the author from Figure 5 of [19].

**Figure 7** presents the radio flux density of SN 1993J at 15.2 GHz observed by the Lyle Telescope, as well as the theoretical flux, which requires a time-dependent evaluation of the optical depth  $\tau_\nu$ , see **Figure 8**.



**Figure 7.** The radio flux density of SN 1993J over 443 days (empty stars) and theoretical curve as given by Equation (25) (full line). Parameters of the trajectory as in **Figure 3**,  $d = 3$  and  $\rho_0 = 1$ .



**Figure 8.** The time dependence of  $\tau_v$  (empty stars) and a polynomial approximation of degree 4 (full line). Parameters as in **Figure 7**.

## 4. Conclusion

We analyzed the one-dimensional relativistic motion in the presence of a resistive force proportional to the velocity in the framework of the alternative relativity. Analytical solutions for the velocity and position as functions of time were derived, see Equation (12) and Equation (14). The results allow modeling the trajectory and the various light curves for SN 1993J.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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