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Linear Stability of Non-Axial Libration Points in the Kite Configuration of First Kind

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Abstract

This paper deals with the linear stability of non-axial libration points in the kite configuration of first kind. It is to be noted that the kite configuration of the first kind exists for the mass parameter $\mu \in]0,1/3[$, where μ is the ratio of the smallest mass of the kite to the whole mass of the system. To check the linear stability of the libration points, the variational equation was derived in the neighbourhood of the libration points, and then the characteristic equation was formed. If all the roots of the characteristic equation are purely imaginary, then all criteria for stability will be satisfied and hence, the libration points will be stable; otherwise, unstable. For this, the stability criteria given in Equation (15) have been followed, and it was found that L_4 and L_5 are stable for $\mu = 0.10$ only and for all values of $\mu \in]0,1/3[$, $L_j\left(j = 1,2,3,6,7,8,9,10,11,12,13\right)$ are unstable.

Keywords

Kite Configuration, Cyclic Kite Configuration, Rotating Frame, Mass Parameter, Axial and Non-Axial Libration Points, Characteristic Roots, Stability Criteria

1. Introduction

In space science, the two-body, the three-body and the restricted three-body problems have been studied by different authors starting from Newton to date. MacMillon *et al.* [1] provided detailed proof of two theorems for the existence of a quadrilateral configuration in the field of four-body configurations. Brumberg [2] invented a permanent solution for a four-body configuration. For the first time, Albouy *et al.* [3] [4] examined the symmetric central configuration of equal masses. Cors *et al.* [5] and Corbera *et al.* [6] examined the cyclic configuration in the Newtonian four-body problem, in which they used the six mutual distances of the particles as their coordinates and demonstrated that the four-point masses constitute a kite lying on a two-dimensional plane.

They have also demonstrated that a line of symmetry must exist in any central configuration with two equal masses on opposite sides. By describing the masses of the central configuration in terms of angle coordinates, Balint *et al.* [7] extended the work of Cors *et al.* [5] in three cases (two concave cases and one convex case). They further asserted that the exact analytical solutions of the four-body configuration are represented by the obtained formulas. Deng *et al.* [8] demonstrated that the diagonals of a cyclic quadrilateral can't be perpendicular unless the configuration is a kite by using mutual distances as the coordinates. Further, they verified the same theorem in the four-vortex central configuration.

Hassan [9] [10] classified the kite configuration into two categories and proved three theorems on the existence of cyclic kite configurations. In the first theorem, he uniquely expressed the masses of the four particles in terms of a mass parameter μ and the total mass M of the system as $m_1 = M(1-\mu)/2$, $m_2 = M\mu$, $m_3 = M(1-3\mu)/2$, $m_4 = M\mu$ and their coordinates as $(R,0)$, $\left(-R/2, \sqrt{3}R/2\right)$, $(-R,0)$, $\left(-R/2, -\sqrt{3}R/2\right)$ where R is the radius of the common circular orbit of the kite. Hassan [9] [10] calculated the mean motion of the rotating frame lying on the kite's plane as a function of μ.

Khatun *et al.* [11] extended the work of Hassan [9] [10] by taking the first body of mass m_1 as an oblate spheroid and showed the effects of oblateness on the mean motion of the system. It is found that the mean motion increases with the increase of oblateness. Further, showed that the axial libration points move away from the origin due to an increase in the oblateness parameter A. As the analytical existence of non-axial libration points wasn't possible so, we presently propose Python programming for finding the location of non-axial libration points with the help of points of intersection of contour plots of zero partial derivatives of the potential function of the satellite.

2. Equation of Motions

Let at any time t, $P(x,y)$ be the position of the satellite moving in the gravitational field of the four-point masses of the kite configuration, and $P_k(x_k, y_k), k = 1,2,3,4$ be the positions of the four vertices of the kite configuration at which the four bodies of respective masses $(1-\mu)/2, \mu, (1-3\mu)/2, \mu$ be located on the rotating frame (O, XY) as shown in **Figure 1**.

Let

$\boldsymbol{OP} = \boldsymbol{\rho}$, then $\boldsymbol{\rho} = x\boldsymbol{i} + y\boldsymbol{j} \Rightarrow \rho^2 = x^2 + y^2$ and

$$\boldsymbol{P_k P} = (x - x_k)\hat{i} + (y - y_k)\hat{j} = \boldsymbol{\rho}_k,$$

$$\Rightarrow \rho_k^2 = (x - x_k)^2 + (y - y_k)^2.$$

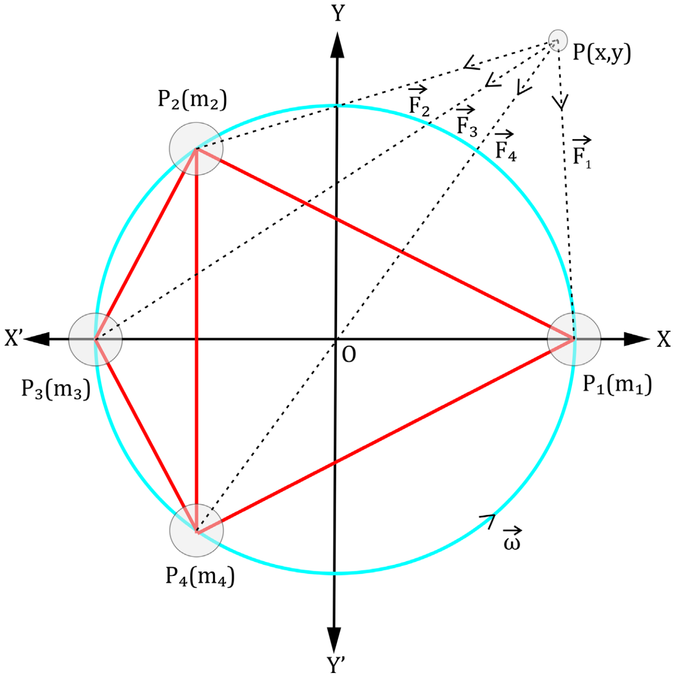

Figure 1. Cyclic kite configuration.

As $P_k \equiv (x_k, y_k)$ then

$$P_1 \equiv \left(\frac{1}{2}, 0\right), \quad P_2 \equiv \left(-\frac{1}{4}, \frac{\sqrt{3}}{4}\right), \quad P_3 \equiv \left(-\frac{1}{2}, 0\right), \quad P_4 \equiv \left(-\frac{1}{4}, -\frac{\sqrt{3}}{4}\right),$$

$$\left.\begin{aligned} \rho_1^2 &= \left(x-\frac{1}{2}\right)^2 + y^2, \quad \rho_2^2 = \left(x+\frac{1}{4}\right)^2 + \left(y-\frac{\sqrt{3}}{4}\right)^2, \\ \rho_3^2 &= \left(x+\frac{1}{2}\right)^2 + y^2, \quad \rho_4^2 = \left(x+\frac{1}{4}\right)^2 + \left(y+\frac{\sqrt{3}}{4}\right)^2. \end{aligned}\right\} \tag{1}$$

In a rotating frame, the equations of motion of the satellite at $P(x,y)$ can be written as

$$\left.\begin{aligned} \ddot{x} - 2n\dot{y} &= \frac{\partial \Omega}{\partial x} = \Omega_x \\ \ddot{y} + 2n\dot{x} &= \frac{\partial \Omega}{\partial y} = \Omega_y \end{aligned}\right\}, \tag{2}$$

where is n the mean motion of the rotating frame and Ω is the kinetic potential given by

$$\Omega = \frac{n^2}{2}\left(x^2 + y^2\right) + \frac{1-\mu}{2\rho_1} + \frac{\mu}{\rho_2} + \frac{1-3\mu}{2\rho_3} + \frac{\mu}{\rho_4}. \tag{3}$$

Let us write the System (2) as

$$\left.\begin{aligned} \ddot{x} - 2n\dot{y} &= \frac{\partial \Omega}{\partial x} = \Omega_x = f(x,y), \\ \ddot{y} + 2n\dot{x} &= \frac{\partial \Omega}{\partial y} = \Omega_y = g(x,y). \end{aligned}\right\} \tag{4}$$

3. Location of Libration Points

For libration points $\dot{x}=\dot{y}=\ddot{x}=\ddot{y}=0$ *i.e.*, $\Omega_x=0$ and $\Omega_y=0$.

The points of intersection of $\Omega_x=0$ and $\Omega_y=0$ give the locations of libration points of the kite configuration. The contour plots of these equations are given below:

The symbols $L_1, L_2, \cdots, L_7$ in **Figure 2** are the positions of seven libration points indicated through the intersection of $\Omega_x=0$ and $\Omega_y=0$ for $\mu=0.03$, $n=1.901802$ and those are in **Figure 3** for $\mu=0.07$, $n=1.888594$. Here black dots $P_i\left(i=1,2,3,4\right)$ represent four bodies forming a kite and four pink spots $L_j\left(j=4,5,6,7\right)$ represent non-axial libration points.

The symbols $L_1, L_2, \cdots, L_7$ in **Figure 4** are the positions of seven libration points indicated through the intersection of $\Omega_x=0$ and $\Omega_y=0$ for $\mu=0.09$, $n=1.882361$ and those are in **Figure 5** for $\mu=0.11$, $n=1.876288$. Here, black dots $P_i\left(i=1,2,3,4\right)$ represent four bodies forming a kite and four pink spots $L_j\left(j=4,5,6,7\right)$ represent nonaxial libration points.

The symbols $L_1, L_2, \cdots, L_{13}$ in **Figure 6** are the positions of thirteen libration points indicated through the intersection of $\Omega_x=0$ and $\Omega_y=0$ for $\mu=0.15$, $n=1.821047$ and those are in **Figure 7** for $\mu=0.22$, $n=1.624152$. Here black dots $P_i\left(i=1,2,3,4\right)$ represent four bodies forming a kite and ten pink spots $L_j\left(j=4,5,\cdots,13\right)$ represent non-axial libration points.

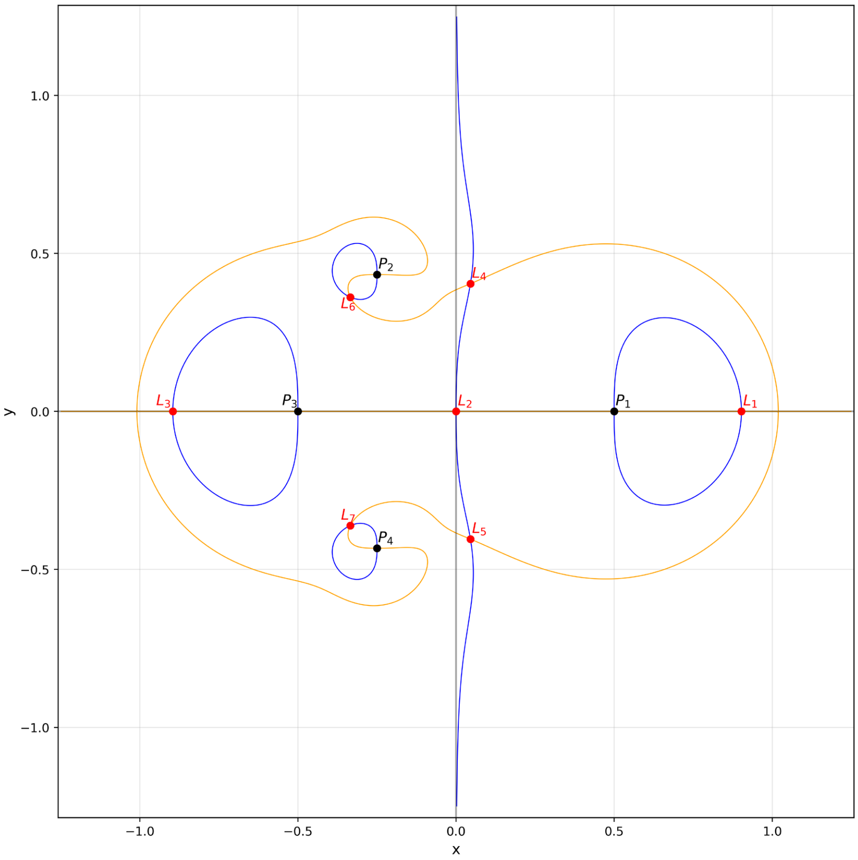

Figure 2. Positions of libration points for $\mu=0.03$.

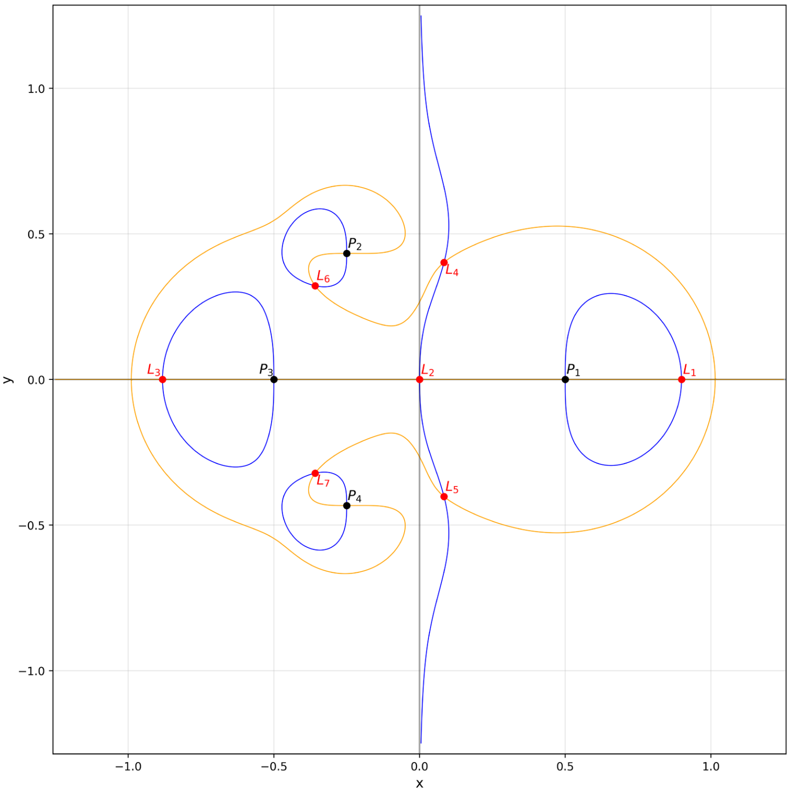

Figure 3. Positions of libration points for $\mu = 0.07$.

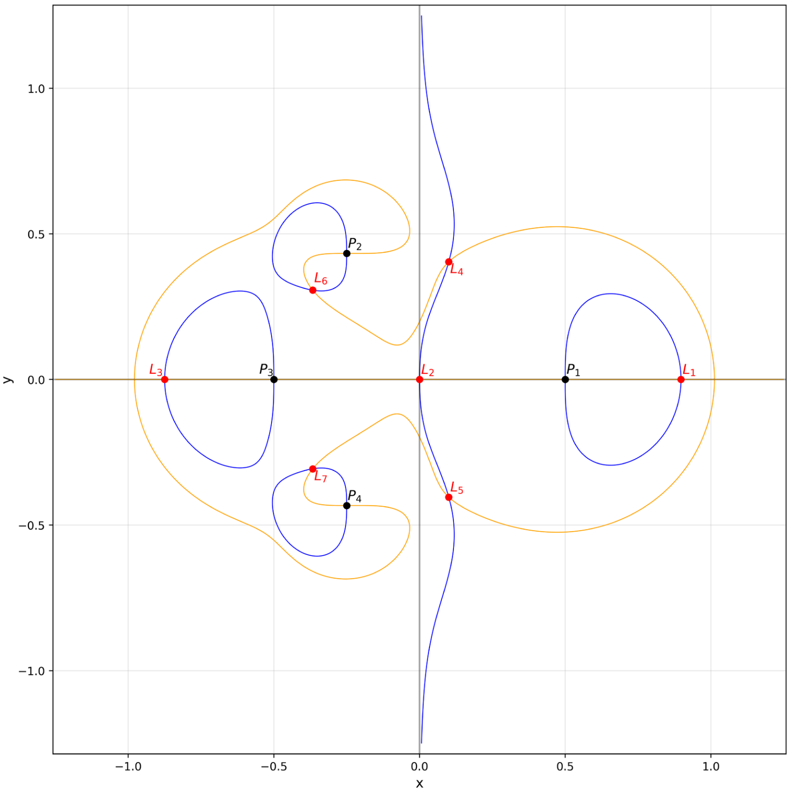

Figure 4. Positions of libration points for $\mu = 0.09$.

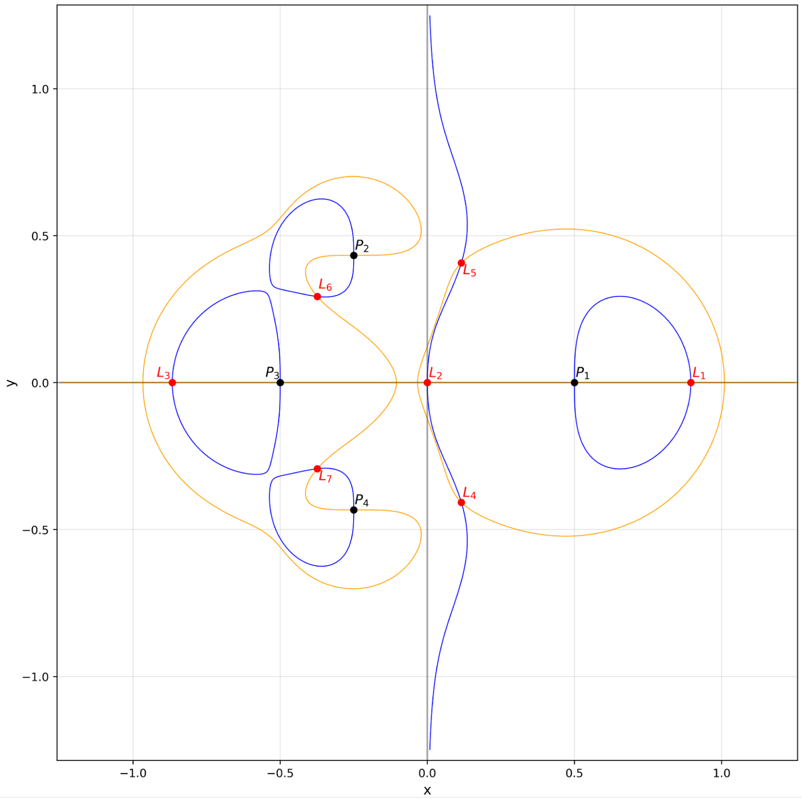

Figure 5. Positions of libration points for $\mu = 0.11$.

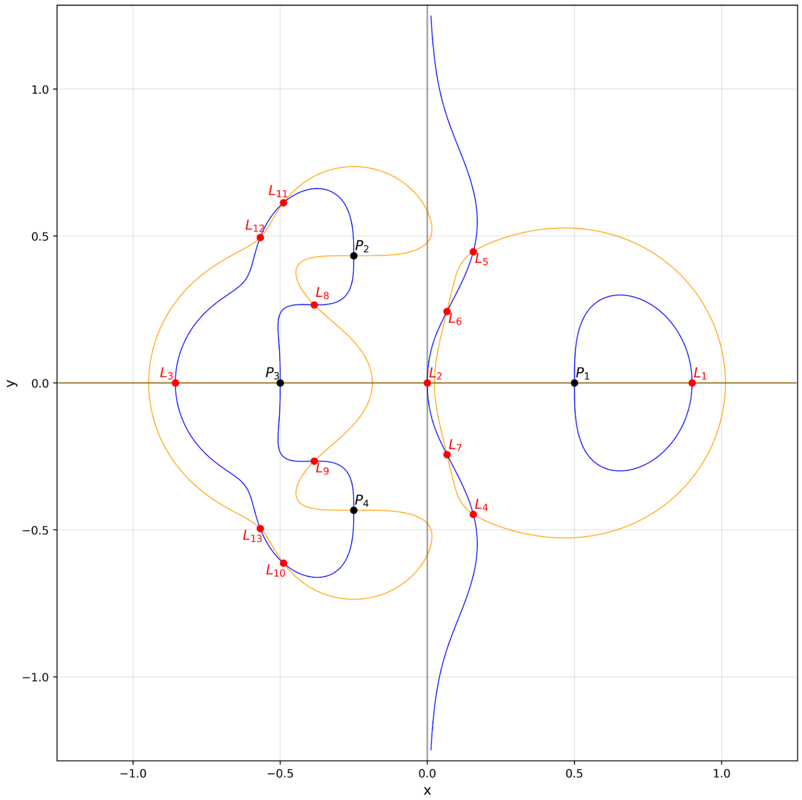

Figure 6. Positions of libration points for $\mu = 0.15$.

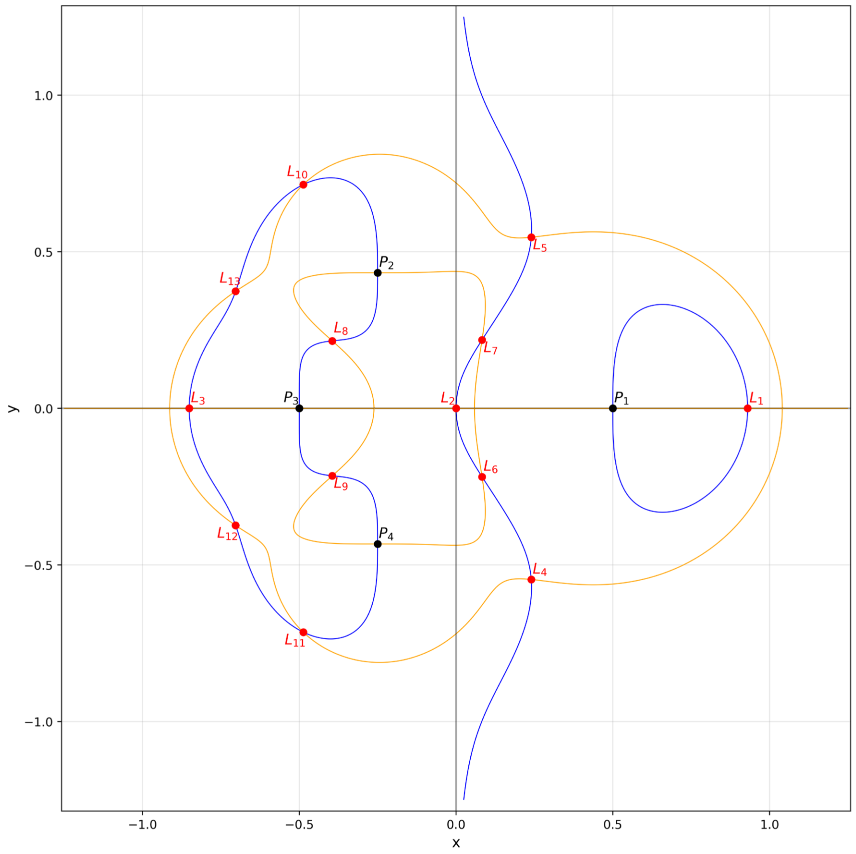

Figure 7. Positions of libration points for $\mu = 0.22$.

The symbols $L_1, L_2, \cdots, L_{11}$ in **Figure 8** are the positions of eleven libration points indicated through the intersection of $\Omega_x = 0$ and $\Omega_y = 0$ for $\mu = 0.30$, $n = 1.364779$ and those are in **Figure 9** for $\mu = 0.31$, $n = 1.328802$. Here black dots $P_i\left(i = 1, 2, 3, 4\right)$ represent four bodies forming a kite and eight pink spots $L_j\left(j = 4, 5, \cdots, 11\right)$ represent non-axial libration points.

For different values of $\mu \in \left(0, 3^{-1}\right)$ and corresponding values of n, the coordinates $\left(x, y\right)$ of libration points $L_j\left(j = 4, 5, \cdots, 13\right)$ are given in the 2nd, 3rd, 4th and 5th columns respectively of the Stability **Tables 1-10**.

4. Stability Criteria

The motion of a satellite is said to be stable near the libration points when given a very small displacement and small velocity, the satellite oscillates for a considerable time around the points.

Let ξ, η denote the small displacement of the infinitesimal body (artificial satellite) from the libration points $\left(x_0, y_0\right)$, then the variational equations of motion can be easily obtained by substituting $x = x_0 + \xi,\ y = y_0 + \eta$ in Equation (4). Thus, Equation (4) becomes

$$\left.\begin{aligned} \ddot{\xi} - 2n\dot{\eta} &= f\left(x_0 + \xi, y_0 + \eta\right), \\ \ddot{\eta} + 2n\dot{\xi} &= g\left(x_0 + \xi, y_0 + \eta\right). \end{aligned}\right\} \tag{5}$$

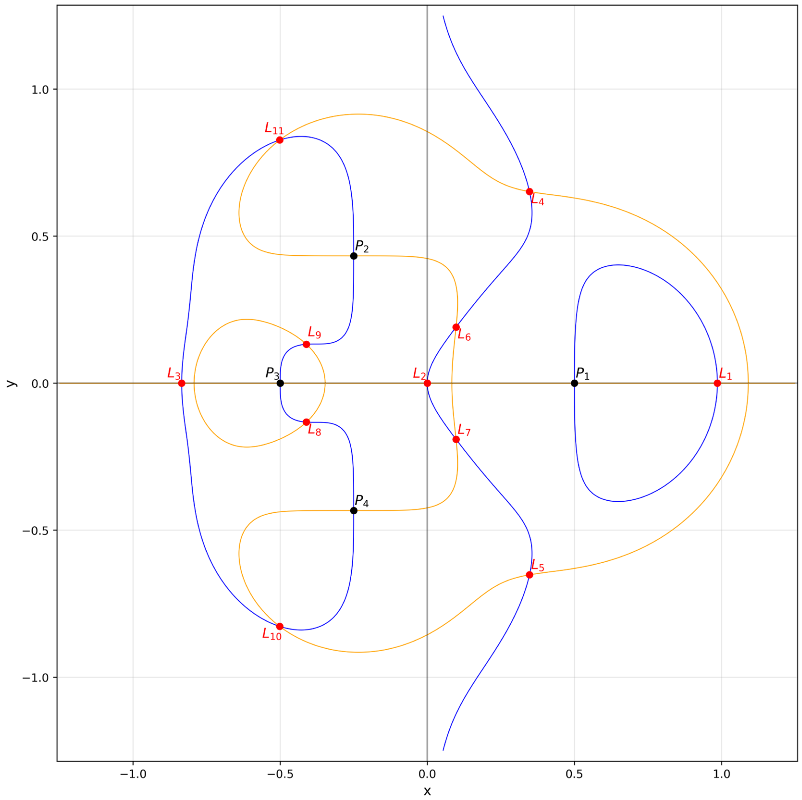

Figure 8. Positions of libration points for $\mu = 0.30$.

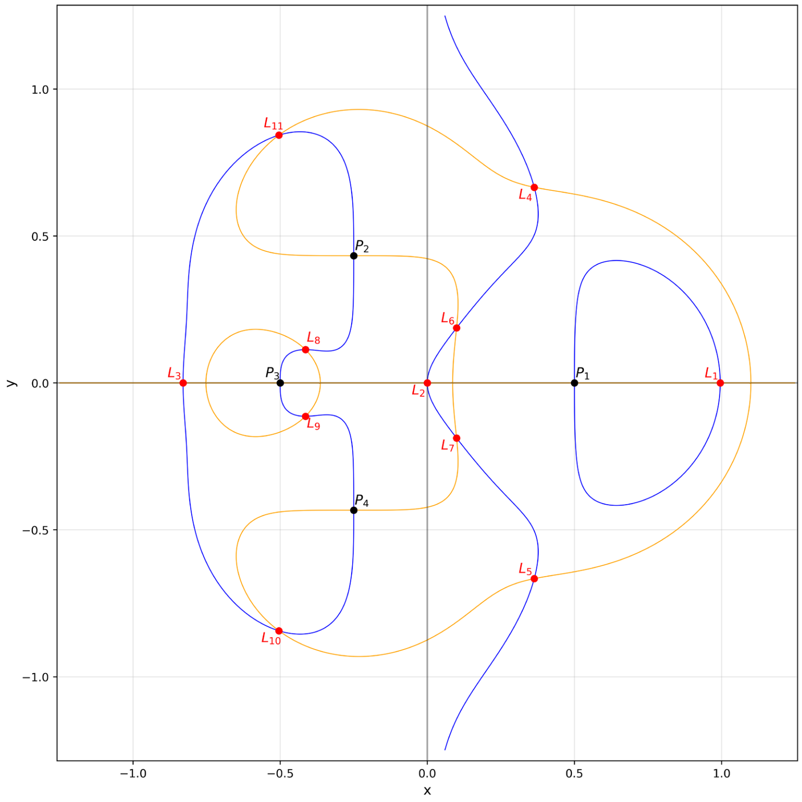

Figure 9. Positions of libration points for $\mu = 0.31$.

Now applying Taylor's theorem in the neighbourhood of $\left(x_0, y_0\right)$ in the right-hand side of the above equations, we get

$$\ddot{\xi} - 2n\dot{\eta} = f\left(x_0, y_0\right) + \left(\xi \frac{\partial}{\partial x} + \eta \frac{\partial}{\partial y}\right) f\left(x_0, y_0\right) + \cdots + \text{higher order term of } \xi \,\&\, \eta$$

$$\text{and } \ddot{\eta} + 2n\dot{\xi} = g\left(x_0, y_0\right) + \left(\xi \frac{\partial}{\partial x} + \eta \frac{\partial}{\partial y}\right) g\left(x_0, y_0\right) + \cdots + \text{higher order term of } \xi \,\&\, \eta$$

$$\left.\begin{aligned} &\Rightarrow \ddot{\xi} - 2n\dot{\eta} = f\left(x_0, y_0\right) + \xi\left(\frac{\partial f}{\partial x}\right)_0 + \eta\left(\frac{\partial f}{\partial y}\right)_0 + \cdots + \text{higher order term of } \xi \,\&\, \eta \\ &\text{and } \ddot{\eta} + 2n\dot{\xi} = g\left(x_0, y_0\right) + \xi\left(\frac{\partial g}{\partial x}\right)_0 + \eta\left(\frac{\partial g}{\partial y}\right)_0 + \cdots + \text{higher order term of } \xi \,\&\, \eta \end{aligned}\right\} \tag{6}$$

But at the libration points $\Omega_x^0 = f\left(x_0, y_0\right) = \Omega_y^0 = g\left(x_0, y_0\right) = 0$ So, from Equation (6), we get

$$\ddot{\xi} - 2n\dot{\eta} = \xi \frac{\partial}{\partial x}\left(\frac{\partial \Omega}{\partial x}\right)_0 + \eta \frac{\partial}{\partial y}\left(\frac{\partial \Omega}{\partial x}\right)_0 + \cdots + \text{higher order term of } \xi \,\&\, \eta$$

$$\text{and } \ddot{\eta} + 2n\dot{\xi} = \xi \frac{\partial}{\partial x}\left(\frac{\partial \Omega}{\partial y}\right)_0 + \eta \frac{\partial}{\partial y}\left(\frac{\partial \Omega}{\partial y}\right)_0 + \cdots + \text{higher order term of } \xi \,\&\, \eta.$$

For linear stability, neglecting the higher-order terms of ξ and η, the variational equations are reduced to

$$\left.\begin{aligned} \ddot{\xi} - 2n\dot{\eta} &= \xi\Omega_{xx}^0 + \eta\Omega_{yx}^0, \\ \ddot{\eta} + 2n\dot{\xi} &= \xi\Omega_{xy}^0 + \eta\Omega_{yy}^0. \end{aligned}\right\} \tag{7}$$

where $\Omega_{xx}^0, \Omega_{xy}^0, \Omega_{yx}^0, \Omega_{yy}^0$ represent the second-order derivatives of Ω at the libration points $\left(x_0, y_0\right)$. The above system of equations can be extended as

$$\left.\begin{aligned} \dot{\xi} &= \xi \cdot 0 + \eta \cdot 0 + \dot{\xi} \cdot 1 + \dot{\eta} \cdot 0, \\ \dot{\eta} &= \xi \cdot 0 + \eta \cdot 0 + \dot{\xi} \cdot 0 + \dot{\eta} \cdot 1, \\ \ddot{\xi} &= \xi\Omega_{xx}^0 + \eta\Omega_{xy}^0 + \dot{\xi} \cdot 0 + \dot{\eta} \cdot 2n, \\ \ddot{\eta} &= \xi\Omega_{xy}^0 + \eta\Omega_{yy}^0 + \dot{\xi}\left(-2n\right) + \dot{\eta} \cdot 0. \end{aligned}\right\} \tag{8}$$

The system of Equation (8) can be written in the form of a single matrix equation as

$$\dot{X} = TX, \tag{9}$$

$$\text{where } T = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \Omega_{xx}^0 & \Omega_{yx}^0 & 0 & 2n \\ \Omega_{xy}^0 & \Omega_{yx}^0 & -2n & 0 \end{bmatrix} \text{ and } X = \begin{bmatrix} \xi \\ \eta \\ \dot{\xi} \\ \dot{\eta} \end{bmatrix}.$$

If any matrix X satisfy the equation

$$TX = \lambda X, \tag{10}$$

then X is said to be an eigenvector of the coefficient matrix T and scalar λ is its corresponding eigenvalue. If T is thought of as a transformation matrix, then the result of applying T to the particular vector X satisfying Equation (10) is to produce a vector in the same direction as X but of a different magnitude.

Now, Equation (10) can be written as $\left(T-\lambda I\right)X=0$.

The set of four simultaneous linear equations in four unknowns $\xi,\eta,\dot{\xi},\dot{\eta}$ will have non-trivial solutions provided the determinant of the characteristic matrix $\left(T-\lambda I\right)$ vanishes.

$$\text{i.e., } \left|A-\lambda I\right|=0, \tag{11}$$

$$\Rightarrow \lambda^4-\left(\Omega^0_{xx}+\Omega^0_{yy}-4n^2\right)\lambda^2+2n\left(\Omega^0_{yx}-\Omega^0_{xy}\right)\lambda+\left(\Omega^0_{xx}\Omega^0_{yy}-\Omega^0_{xy}\Omega^0_{yx}\right)=0,$$

$$\Rightarrow \lambda^4-\left(\Omega^0_{xx}+\Omega^0_{yy}-4n^2\right)\lambda^2+\left\{\Omega^0_{xx}\Omega^0_{yy}-\left(\Omega^0_{xy}\right)^2\right\}=0. \quad \left(\text{as } \Omega^0_{xy}=\Omega^0_{yx}\right) \tag{12}$$

This equation is called a characteristic equation corresponding to the equations of a matrix A. Therefore, Equation (12) can be written as

$$\lambda^4-A\lambda^2+B=0, \tag{13}$$

where $A=\Omega^0_{xx}+\Omega^0_{yy}-4n^2$ & $B=\Omega^0_{xx}\Omega^0_{yy}-\left(\Omega^0_{xy}\right)^2$.

Equation (13) is biquadratic in λ, so taking $\lambda^2=\wedge$, Equation (13) is reduced to a quadratic equation

$$\wedge^2-A\wedge+B=0. \tag{14}$$

Let $\wedge_1$ and $\wedge_2$ be the two roots of the characteristic Equation (14), then

$$\wedge_1+\wedge_2=A \text{ and } \wedge_1\wedge_2=B.$$

But from Equation (13), $\wedge=\lambda^2=\dfrac{A\pm\sqrt{A^2-4B}}{2}$.

Let

$$\wedge_1=\frac{A+\sqrt{A^2-4B}}{2},\quad \wedge_2=\frac{A-\sqrt{A^2-4B}}{2}.$$

As $\lambda^2=\wedge$, so let $\lambda_1^2=\wedge_1$ and $\lambda_2^2=\wedge_2$, then

$$\lambda_1^2=\frac{A+\sqrt{A^2-4B}}{2} \text{ and } \lambda_2^2=\frac{A-\sqrt{A^2-4B}}{2}.$$

$$\Rightarrow \lambda_1=\pm\left(\frac{A+\sqrt{A^2-4B}}{2}\right)^{\frac{1}{2}} \text{ and } \lambda_2=\pm\left(\frac{A-\sqrt{A^2-4B}}{2}\right)^{\frac{1}{2}}.$$

Let $\lambda_{11}=+\left(\dfrac{A+\sqrt{A^2-4B}}{2}\right)^{\frac{1}{2}}$ and $\lambda_{12}=-\left(\dfrac{A+\sqrt{A^2-4B}}{2}\right)^{\frac{1}{2}}$.

$$\lambda_{21}=+\left(\frac{A-\sqrt{A^2-4B}}{2}\right)^{\frac{1}{2}} \text{ and } \lambda_{22}=-\left(\frac{A-\sqrt{A^2-4B}}{2}\right)^{\frac{1}{2}}.$$

For simplicity, let us write

$$\lambda_1^*=+\left(\frac{A+\sqrt{A^2-4B}}{2}\right)^{\frac{1}{2}} \text{ and } \lambda_2^*=-\left(\frac{A+\sqrt{A^2-4B}}{2}\right)^{\frac{1}{2}},$$

$$\lambda_3^* = +\left(\frac{A-\sqrt{A^2-4B}}{2}\right)^{\frac{1}{2}} \quad \text{and} \quad \lambda_4^* = -\left(\frac{A-\sqrt{A^2-4B}}{2}\right)^{\frac{1}{2}}.$$

as the four roots of the Characteristic Equation (13).

The criteria for linear stability of non-axial libration points are as follows:

Any libration point $L_j\left(x_j, y_j\right), j = 4,5,6,\cdots,13$ is said to be stable if λ_1^2, λ_2^2 are negative real and

$$\left.\begin{array}{l} \text{(i)}\ \ A = \lambda_1^2 + \lambda_2^2 < 0 \\ \text{(ii)}\ \ B = \lambda_1^2 \lambda_2^2 > 0 \\ \text{(iii)}\ D = A^2 - 4B \geq 0 \end{array}\right\} \text{Refer Szebehely [12]} \qquad (15)$$

are satisfied together.

To satisfy the above conditions of stability, we need $\Omega_{xx}^0, \Omega_{xy}^0$ and Ω_{xy}^0. So, differentiating the potential function Ω partially twice with respect to x and y, we get

$$\begin{aligned} \Omega_{xx} = n^2 &- \left[\frac{1-\mu}{2\rho_1^3} + \frac{\mu}{\rho_2^3} + \frac{1-3\mu}{2\rho_3^3} + \frac{\mu}{\rho_4^3}\right] \\ &+ 3\left[\frac{(1-\mu)(x-1/2)^2}{2\rho_1^5} + \frac{\mu(x+1/4)^2}{\rho_2^5} + \frac{(1-3\mu)(x+1/2)^2}{2\rho_3^5} + \frac{\mu(x+1/4)^2}{\rho_4^5}\right], \end{aligned}$$

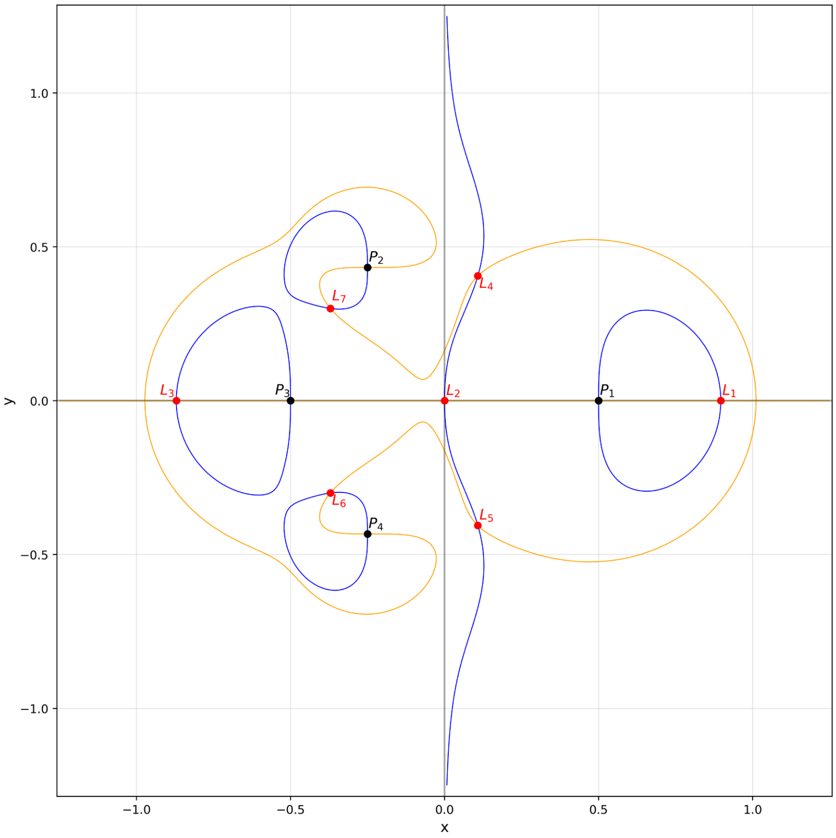

Figure 10. Positions of libration points $\mu = 0.10$ (Stable).

$$\begin{aligned}\Omega_{yx} = {} & \frac{3(1-\mu)(x-1/2)y}{2\rho_1^5} + \frac{3\mu(x+1/4)\left(y-\sqrt{3}/4\right)}{\rho_2^5} \\ & - \frac{3(1-3\mu)(x+1/2)y}{2\rho_3^5} + \frac{3\mu(x+1/4)\left(y+\sqrt{3}/4\right)}{\rho_4^5} = \Omega_{xy},\end{aligned}$$

$$\begin{aligned}\Omega_{yy} = {} & n^2 - \left[\frac{1-\mu}{2\rho_1^3} + \frac{\mu}{\rho_2^3} + \frac{1-3\mu}{2\rho_3^3} + \frac{\mu}{\rho_4^3}\right] \\ & + 3\left[\frac{(1-\mu)y^2}{2\rho_1^5} + \frac{\mu\left(y-\sqrt{3}/4\right)^2}{\rho_2^5} + \frac{(1-3\mu)y^2}{2\rho_3^5} + \frac{\mu\left(y+\sqrt{3}/4\right)^2}{\rho_4^5}\right].\end{aligned}$$

The symbols $L_1, L_2, \cdots, L_7$ in **Figure 10** are the positions of seven libration points indicated through the intersection of $\Omega_x = 0$ and $\Omega_y = 0$ for $\mu = 0.10$ & $n = 1.879308$. Here black dots $P_i\left(i = 1,2,3,4\right)$ represent four bodies forming a kite and four pink spots $L_j\left(j = 4,5,6,7\right)$ represent non-axial libration points.

5. Stability Tables

The values of $\Omega_{xx}^0, \Omega_{xy}^0, \Omega_{yy}^0, A, B, D$ are given in the 6th, 7th, 8th, 9th, 10th and 11th columns respectively in stability tables from **Table 1-10**. The nature of the stability of each non-axial libration point satisfying the conditions of Equation (15), is mentioned in the last column of each Stability table.

Table 1. Stability table of $L_4\left(x_4, y_4\right)$ for $\mu \in \left(0, 3^{-1}\right)$ and corresponding values of n.

No.	μ	n	x	y	Ω_{xx}^0	Ω_{yy}^0	Ω_{xy}^0	A	B	D	Nature
1	0.01	1.908937	0.018715	0.409329	7.521447	3.934277	−5.47942	−3.12044	−0.43256	11.46739	Unstable
2	0.02	1.905315	0.033261	−0.406280	8.186627	3.598856	5.545818	−2.73542	−1.29361	12.65693	Unstable
3	0.03	1.901802	0.045526	−0.404160	8.665476	3.366872	5.590232	−2.43505	−2.07515	14.23005	Unstable
4	0.04	1.898384	0.056352	0.402755	9.023149	3.201589	−5.61368	−2.19071	−2.62498	15.29915	Unstable
5	0.05	1.895050	0.066202	0.401977	9.293269	3.084013	−5.61688	−1.98758	−2.88878	15.50559	Unstable
6	0.06	1.891790	0.075363	−0.401760	9.495481	3.003024	5.600356	−1.81698	−2.84883	14.69673	Unstable
7	0.07	1.888594	0.084026	−0.402060	9.642363	2.951490	5.564665	−1.67330	−2.50616	12.82457	Unstable
8	0.08	1.885454	0.092319	0.402822	9.742692	2.924443	−5.51062	−1.55261	−1.87503	9.910718	Unstable
9	0.09	1.882361	0.100333	−0.404010	9.803166	2.918111	5.439474	−1.45185	−0.98115	6.032460	Unstable
10	0.10	1.879308	0.108129	0.405551	9.829358	2.929383	−5.35295	−1.36845	0.139929	1.312932	Stable
11	0.11	1.876288	0.115746	0.407389	9.826224	2.955519	−5.25321	−1.30009	1.445367	−4.09124	Unstable
12	0.12	1.873296	0.123207	−0.409450	9.798319	2.994022	5.142731	−1.24461	2.888699	−10.0057	Unstable
13	0.13	1.870326	0.130525	0.411663	9.749836	3.042603	−5.024050	−1.20004	4.423827	−16.2552	Unstable
14	0.14	1.847462	0.143400	0.429000	9.165740	3.228103	−4.40491	−1.25862	10.18476	−39.1549	Unstable
15	0.15	1.821047	0.156395	0.446782	8.542373	3.408772	−3.79294	−1.31370	14.73263	−57.2047	Unstable
16	0.16	1.794242	0.168838	0.463086	7.962130	3.561745	−3.25559	−1.35334	17.76020	−69.2093	Unstable
17	0.17	1.767031	0.180929	0.478337	7.420572	3.688016	−2.77883	−1.38101	19.64528	−76.6740	Unstable

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18	0.18	1.739394	0.192808	0.492816	6.914108	3.788885	−2.35248	−1.39898	20.66262	−80.6933	Unstable
19	0.19	1.711311	0.204579	0.506725	6.439730	3.865749	−1.96889	−1.40887	21.01786	−82.0865	Unstable
20	0.20	1.682760	0.216329	−0.520210	5.994857	3.920004	1.622202	−1.41186	20.86832	−81.4799	Unstable
21	0.21	1.653715	0.228131	−0.533400	5.577249	3.952989	1.307826	−1.40886	20.33640	−79.3607	Unstable
22	0.22	1.624152	0.240051	0.546385	5.184944	3.965967	−1.02211	−1.40056	19.51860	−76.1128	Unstable
23	0.23	1.594040	0.252152	−0.559250	4.816208	3.960111	0.762127	−1.38753	18.49188	−72.0423	Unstable
24	0.24	1.563348	0.264494	0.572065	4.469504	3.936510	−0.52550	−1.37021	17.31810	−67.3949	Unstable
25	0.25	1.532041	0.277140	−0.584900	4.143455	3.896170	0.310274	−1.34898	16.04733	−62.3696	Unstable
26	0.26	1.500082	0.290155	0.597822	3.836823	3.840022	−0.11487	−1.32413	14.72029	−57.1278	Unstable
27	0.27	1.467426	0.303607	−0.610890	3.548490	3.768930	−0.06202	−1.29593	13.37017	−51.8012	Unstable
28	0.28	1.434027	0.317575	0.624162	3.277437	3.683698	0.221467	−1.26460	12.02404	−46.4970	Unstable
29	0.29	1.399831	0.332143	0.637710	3.022729	3.585076	0.364358	−1.23030	10.70396	−41.3022	Unstable
30	0.30	1.364779	0.347409	−0.651600	2.783505	3.473774	−0.49140	−1.19320	9.427794	−36.2874	Unstable
31	0.31	1.328802	0.363486	−0.665910	2.558963	3.350464	−0.60316	−1.15343	8.209905	−31.5092	Unstable
32	0.32	1.291824	0.380507	0.680710	2.348350	3.215788	0.700094	−1.11110	7.061664	−27.0121	Unstable

Table 2. Stability table of $L_5(x_5, y_5)$ for $\mu \in (0, 3^{-1})$ and corresponding values of n.

No.	μ	n	x	y	Ω^0_{xx}	Ω^0_{yy}	Ω^0_{xy}	A	B	D	Nature
1	0.01	1.908937	0.018715	−0.40933	7.521447	3.934277	5.479417	−3.12044	−0.43256	11.46739	Unstable
2	0.02	1.905315	0.033261	0.406282	8.186627	3.598856	−5.54582	−2.73542	−1.29361	12.65693	Unstable
3	0.03	1.901802	0.045526	0.404156	8.665476	3.366872	−5.59023	−2.43505	−2.07515	14.23005	Unstable
4	0.04	1.898384	0.056352	−0.40276	9.023149	3.201589	5.613680	−2.19071	−2.62498	15.29915	Unstable
5	0.05	1.895050	0.066202	−0.40198	9.293269	3.084013	5.616880	−1.98758	−2.88878	15.50559	Unstable
6	0.06	1.891790	0.075363	0.40176	9.495481	3.003024	−5.60036	−1.81698	−2.84883	14.69673	Unstable
7	0.07	1.888594	0.084026	0.402056	9.642363	2.951490	−5.56466	−1.67330	−2.50616	12.82457	Unstable
8	0.08	1.885454	0.092319	−0.40282	9.742692	2.924443	5.510624	−1.55261	−1.87503	9.910718	Unstable
9	0.09	1.882361	0.100333	0.404006	9.803166	2.918111	−5.43947	−1.45185	−0.98115	6.032460	Unstable
10	0.10	1.879308	0.108129	−0.40555	9.829358	2.929383	5.352945	−1.36845	0.139929	1.312932	Stable
11	0.11	1.876288	0.115746	−0.40739	9.826224	2.955519	5.253211	−1.30009	1.445367	−4.09124	Unstable
12	0.12	1.873296	0.123207	0.409449	9.798319	2.994022	−5.14273	−1.24461	2.888699	−10.0057	Unstable
13	0.13	1.870326	0.130525	−0.41166	9.749836	3.042603	5.024047	−1.20004	4.423827	−16.2552	Unstable
14	0.14	1.847462	0.143400	−0.42900	9.165740	3.228103	4.404906	−1.25862	10.18476	−39.1549	Unstable
15	0.15	1.821047	0.156395	−0.44678	8.542373	3.408772	3.792937	−1.31370	14.73263	−57.2047	Unstable
16	0.16	1.794242	0.168838	−0.46309	7.962130	3.561745	3.255591	−1.35334	17.76020	−69.2093	Unstable
17	0.17	1.767031	0.180929	−0.47834	7.420572	3.688016	2.778831	−1.38101	19.64528	−76.6740	Unstable
18	0.18	1.739394	0.192808	−0.49282	6.914108	3.788885	2.352475	−1.39898	20.66262	−80.6933	Unstable
19	0.19	1.711311	0.204579	−0.50672	6.439730	3.865749	1.968888	−1.40887	21.01786	−82.0865	Unstable

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20	0.20	1.682760	0.216329	0.520214	5.994857	3.920004	−1.62220	−1.41186	20.86832	−81.4799	Unstable
21	0.21	1.653715	0.228131	0.533401	5.577249	3.952989	−1.30783	−1.40886	20.33640	−79.3607	Unstable
22	0.22	1.624152	0.240051	−0.54639	5.184944	3.965967	1.022112	−1.40056	19.51860	−76.1128	Unstable
23	0.23	1.594040	0.252152	0.559249	4.816208	3.960111	−0.76213	−1.38753	18.49188	−72.0423	Unstable
24	0.24	1.563348	0.264494	−0.57207	4.469504	3.936510	0.525495	−1.37021	17.31810	−67.3949	Unstable
25	0.25	1.532041	0.277140	0.584902	4.143455	3.896170	−0.31027	−1.34898	16.04733	−62.3696	Unstable
26	0.26	1.500082	0.290155	−0.59782	3.836823	3.840022	0.114873	−1.32413	14.72029	−57.1278	Unstable
27	0.27	1.467426	0.303607	0.610888	3.548490	3.768930	0.062016	−1.29593	13.37017	−51.8012	Unstable
28	0.28	1.434027	0.317575	−0.62416	3.277437	3.683698	−0.22147	−1.26460	12.02404	−46.4970	Unstable
29	0.29	1.399831	0.332143	−0.63771	3.022729	3.585076	−0.36436	−1.23030	10.70396	−41.3022	Unstable
30	0.30	1.364779	0.347409	0.651600	2.783505	3.473774	0.491400	−1.19320	9.427794	−36.2874	Unstable
31	0.31	1.328802	0.363486	0.665906	2.558963	3.350464	0.603164	−1.15343	8.209905	−31.5092	Unstable
32	0.32	1.291824	0.380507	−0.68071	2.348350	3.215788	−0.70009	−1.11110	7.061664	−27.0121	Unstable

Table 3. Stability table of $L_6\left(x_6,y_6\right)$ for $\mu\in\left(0,3^{-1}\right)$ and corresponding values of n.

No.	μ	n	x	y	Ω^0_{xx}	Ω^0_{yy}	Ω^0_{xy}	A	B	D	Nature
1	0.01	1.908937	−0.30853	0.393918	32.94522	9.486996	32.11377	27.85605	−718.743	3650.931	Unstable
2	0.02	1.905315	−0.32414	−0.37538	22.54720	16.16054	−26.7476	24.18684	−351.060	1989.242	Unstable
3	0.03	1.901802	−0.33419	0.361271	17.44210	19.90883	23.86824	22.88353	−222.441	1413.421	Unstable
4	0.04	1.898384	−0.34178	−0.34955	14.15295	22.51164	−21.8060	22.24914	−156.895	1122.603	Unstable
5	0.05	1.895050	−0.34800	0.339360	11.75889	24.49184	20.14218	21.88586	−117.710	949.8328	Unstable
6	0.06	1.891790	−0.35335	0.330221	9.889723	26.07636	18.71288	21.65060	−92.2837	837.8833	Unstable
7	0.07	1.888594	−0.35811	0.321854	8.362198	27.38453	17.43774	21.47957	−75.0800	761.6920	Unstable
8	0.08	1.885454	−0.36245	0.314069	7.073176	28.48690	16.27159	21.34033	−63.2719	708.4974	Unstable
9	0.09	1.882361	−0.36647	−0.30673	5.959291	29.42860	−15.1865	21.21476	−55.2568	671.0935	Unstable
10	0.10	1.879308	−0.37026	−0.29975	4.979040	30.24019	−14.1641	21.09204	−50.0540	645.0898	Unstable
11	0.11	1.876288	−0.37387	0.293042	4.103867	30.94325	13.19141	20.96529	−47.0265	627.6490	Unstable
12	0.12	1.873296	0.044160	0.221381	13.12205	1.702497	−8.86067	0.787590	−56.1711	225.3049	Unstable
13	0.13	1.870326	0.060263	0.248422	13.00582	1.764334	−9.10249	0.777683	−59.9087	240.2398	Unstable
14	0.14	1.847462	0.064304	−0.24742	12.99683	1.698913	9.172979	1.043281	−62.0631	249.3407	Unstable
15	0.15	1.821047	0.066922	−0.24362	12.98263	1.624584	9.237542	1.342370	−64.2408	258.7652	Unstable
16	0.16	1.794242	0.069440	0.239884	12.95645	1.555328	−9.29301	1.634556	−66.2085	267.5059	Unstable
17	0.17	1.767031	0.071866	−0.23621	12.91935	1.490625	9.339860	1.920382	−67.9751	275.5882	Unstable
18	0.18	1.739394	0.074210	0.232592	12.87226	1.430024	−9.37849	2.200315	−69.5484	283.0350	Unstable
19	0.19	1.711311	0.076479	0.229014	12.81597	1.373133	−9.40923	2.474756	−70.9355	289.8666	Unstable
20	0.20	1.682760	0.078678	−0.22547	12.75117	1.319609	9.432363	2.744058	−72.1429	296.1015	Unstable
21	0.21	1.653715	0.080814	0.221964	12.67848	1.269148	−9.44813	3.008532	−73.1763	301.7563	Unstable

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22	0.22	1.624152	0.082891	−0.21848	12.59845	1.221477	9.456718	3.268452	−74.0408	306.8460	Unstable
23	0.23	1.594040	0.084915	−0.21501	12.51156	1.176353	9.458293	3.524062	−74.7413	311.3842	Unstable
24	0.24	1.563348	0.086889	−0.21156	12.41825	1.133554	9.452978	3.775580	−75.2820	315.3831	Unstable
25	0.25	1.532041	0.088817	0.208119	12.31892	1.092881	−9.44087	4.023202	−75.6669	318.8539	Unstable
26	0.26	1.500082	0.090702	−0.20468	12.21393	1.054150	9.422043	4.267104	−75.8996	321.8065	Unstable
27	0.27	1.467426	0.092547	0.201245	12.10361	1.017191	−9.39654	4.507445	−75.9832	324.2499	Unstable
28	0.28	1.434027	0.094355	0.197805	11.98825	0.981848	−9.36437	4.744369	−75.9208	326.1922	Unstable
29	0.29	1.399831	0.096129	−0.19436	11.86814	0.947976	9.325539	4.978008	−75.7150	327.6405	Unstable
30	0.30	1.364779	0.097871	−0.19090	11.74352	0.915438	9.280013	5.208480	−75.3682	328.6009	Unstable
31	0.31	1.328802	0.099583	0.187420	11.61465	0.884106	−9.22774	5.435894	−74.8825	329.0791	Unstable
32	0.32	1.291824	0.101267	−0.18392	11.48172	0.853859	9.168631	5.660348	−74.2600	329.0797	Unstable

Table 4. Stability table of $L_7\left(x_7, y_7\right)$ for $\mu \in \left(0, 3^{-1}\right)$ and corresponding values of n.

No.	μ	n	x	y	Ω^0_{xx}	Ω^0_{yy}	Ω^0_{xy}	A	B	D	Nature
1	0.01	1.908937	−0.30853	−0.39392	32.94522	9.486996	−32.1138	27.85605	−718.743	3650.931	Unstable
2	0.02	1.905315	−0.32414	0.375377	22.5472	16.16054	26.74761	24.18684	−351.060	1989.242	Unstable
3	0.03	1.901802	−0.33419	−0.36127	17.44210	19.90883	−23.8682	22.88353	−222.441	1413.421	Unstable
4	0.04	1.898384	−0.34178	0.349555	14.15295	22.51164	21.80598	22.24914	−156.895	1122.603	Unstable
5	0.05	1.895050	−0.34800	−0.33936	11.75889	24.49184	−20.1422	21.88586	−117.710	949.8328	Unstable
6	0.06	1.891790	−0.35335	−0.33022	9.889723	26.07636	−18.7129	21.65060	−92.2837	837.8833	Unstable
7	0.07	1.888594	−0.35811	−0.32185	8.362198	27.38453	−17.4377	21.47957	−75.0800	761.6920	Unstable
8	0.08	1.885454	−0.36245	−0.31407	7.073176	28.48690	−16.2716	21.34033	−63.2719	708.4974	Unstable
9	0.09	1.882361	−0.36647	0.306735	5.959291	29.42860	15.18652	21.21476	−55.2568	671.0935	Unstable
10	0.10	1.879308	−0.37026	0.299751	4.979040	30.24019	14.16408	21.09204	−50.0540	645.0898	Unstable
11	0.11	1.876288	−0.37387	−0.29304	4.103867	30.94325	−13.1914	20.96529	−47.0265	627.6490	Unstable
12	0.12	1.873296	0.044160	−0.22138	13.12205	1.702497	8.860666	0.787590	−56.1711	225.3049	Unstable
13	0.13	1.870326	0.060263	−0.24842	13.00582	1.764334	9.102492	0.777683	−59.9087	240.2398	Unstable
14	0.14	1.847462	0.064304	0.247420	12.99683	1.698913	−9.17298	1.043281	−62.0631	249.3407	Unstable
15	0.15	1.821047	0.066922	0.243616	12.98263	1.624584	−9.23754	1.342370	−64.2408	258.7652	Unstable
16	0.16	1.794242	0.069440	−0.23988	12.95645	1.555328	9.293011	1.634556	−66.2085	267.5059	Unstable
17	0.17	1.767031	0.071866	0.236212	12.91935	1.490625	−9.33986	1.920382	−67.9751	275.5882	Unstable
18	0.18	1.739394	0.074210	−0.23259	12.87226	1.430024	9.378488	2.200315	−69.5484	283.0350	Unstable
19	0.19	1.711311	0.076479	−0.22901	12.81597	1.373133	9.409228	2.474756	−70.9355	289.8666	Unstable
20	0.20	1.682760	0.078678	0.225474	12.75117	1.319609	−9.43236	2.744058	−72.1429	296.1015	Unstable
21	0.21	1.653715	0.080814	−0.22196	12.67848	1.269148	9.448128	3.008532	−73.1763	301.7563	Unstable

Continued

22	0.22	1.624152	0.082891	0.218478	12.59845	1.221477	−9.45672	3.268452	−74.0408	306.8460	Unstable
23	0.23	1.594040	0.084915	0.215012	12.51156	1.176353	−9.45829	3.524062	−74.7413	311.3842	Unstable
24	0.24	1.563348	0.086889	0.211561	12.41825	1.133554	−9.45298	3.775580	−75.2820	315.3831	Unstable
25	0.25	1.532041	0.088817	−0.20812	12.31892	1.092881	9.440871	4.023202	−75.6669	318.8539	Unstable
26	0.26	1.500082	0.090702	0.204682	12.21393	1.054150	−9.42204	4.267104	−75.8996	321.8065	Unstable
27	0.27	1.467426	0.092547	−0.20125	12.10361	1.017191	9.396537	4.507445	−75.9832	324.2499	Unstable
28	0.28	1.434027	0.094355	−0.19781	11.98825	0.981848	9.364371	4.744369	−75.9208	326.1922	Unstable
29	0.29	1.399831	0.096129	0.194357	11.86814	0.947976	−9.32554	4.978008	−75.7150	327.6405	Unstable
30	0.30	1.364779	0.097871	0.190897	11.74352	0.915438	−9.28001	5.208480	−75.3682	328.6009	Unstable
31	0.31	1.328802	0.099583	−0.18742	11.61465	0.884106	9.227737	5.435894	−74.8825	329.0791	Unstable
32	0.32	1.291824	0.101267	0.183923	11.48172	0.853859	−9.16863	5.660348	−74.2600	329.0797	Unstable

Table 5. Stability table of $L_8(x_8, y_8)$ for $\mu \in (0, 3^{-1})$ and corresponding values of n.

No.	μ	n	x	y	Ω_{xx}^0	Ω_{yy}^0	Ω_{xy}^0	A	B	D	Nature
1	0.12	1.873296	0.007779	−0.10043	14.34202	0.642134	5.018361	0.947195	−15.9745	64.79499	Unstable
2	0.13	1.870326	−0.38071	0.280212	2.592424	32.08279	11.36054	20.68274	−45.8896	611.3342	Unstable
3	0.14	1.847462	−0.38256	0.273095	1.616787	32.79207	10.33920	20.75639	−53.8812	646.3526	Unstable
4	0.15	1.821047	−0.38413	0.265947	0.698534	33.40784	9.286233	20.84154	−62.8976	685.9600	Unstable
5	0.16	1.794242	−0.38565	0.258869	−0.11916	33.89554	8.224326	20.89917	−71.6784	723.4888	Unstable
6	0.17	1.767031	−0.38715	0.251818	−0.84529	34.26381	7.153661	20.92892	−80.1378	758.5708	Unstable
7	0.18	1.739394	−0.38864	0.244750	−1.48659	34.51889	6.073773	20.93033	−88.2060	790.9029	Unstable
8	0.19	1.711311	−0.39012	−0.23762	−2.04782	34.66493	−4.98361	20.90276	−95.8238	820.2205	Unstable
9	0.20	1.682760	−0.39162	0.230390	−2.53200	34.70409	3.881531	20.84537	−102.937	846.2777	Unstable
10	0.21	1.653715	−0.39314	0.223001	−2.94045	34.63657	2.765284	20.75703	−109.494	868.8298	Unstable
11	0.22	1.624152	−0.39469	0.215401	−3.27266	34.46041	1.631855	20.63628	−115.440	887.6165	Unstable
12	0.23	1.594040	−0.39628	0.207521	−3.52602	34.17110	0.477288	20.48123	−120.716	902.3434	Unstable
13	0.24	1.563348	−0.39794	0.199280	−3.69523	33.76086	−0.70364	20.28941	−125.249	912.6570	Unstable
14	0.25	1.532041	−0.39968	0.190574	−3.77128	33.21745	−1.91792	20.05757	−128.951	918.1093	Unstable
15	0.26	1.500082	−0.40152	−0.18127	−3.73956	32.52194	3.175151	19.78140	−131.699	918.1009	Unstable
16	0.27	1.467426	−0.40349	0.171163	−3.57631	31.64468	−4.48885	19.45502	−133.321	911.7814	Unstable
17	0.28	1.434027	−0.40564	−0.15999	−3.24148	30.53736	5.878835	19.07015	−133.547	897.8579	Unstable
18	0.29	1.399831	−0.40802	0.147290	−2.66268	29.11531	−7.37537	18.61452	−131.921	874.1844	Unstable
19	0.30	1.364779	−0.41074	0.132304	−1.69354	27.21263	−9.02657	18.06860	−127.565	836.7332	Unstable
20	0.31	1.328802	−0.41397	−0.11344	0.021976	24.43853	10.90685	17.39764	−118.422	776.3676	Unstable
21	0.32	1.291824	−0.41812	0.086355	3.705489	19.49560	−13.0352	16.52585	−97.6748	663.8030	Unstable

Table 6. Stability table of $L_9\left(x_9, y_9\right)$ for $\mu \in \left(0, 3^{-1}\right)$ and corresponding values of n.

No.	μ	n	x	y	Ω_{xx}^0	Ω_{yy}^0	Ω_{xy}^0	A	B	D	Nature
1	0.12	1.873296	0.007779	0.100428	14.34202	0.642134	−5.01836	0.947195	−15.9745	64.79499	Unstable
2	0.13	1.870326	−0.38071	−0.28021	2.592424	32.08279	−11.3605	20.68274	−45.8896	611.3342	Unstable
3	0.14	1.847462	−0.38256	−0.27309	1.616787	32.79207	−10.3392	20.75639	−53.8812	646.3526	Unstable
4	0.15	1.821047	−0.38413	−0.26595	0.698534	33.40784	−9.28623	20.84154	−62.8976	685.9600	Unstable
5	0.16	1.794242	−0.38565	−0.25887	−0.11916	33.89554	−8.22433	20.89917	−71.6784	723.4888	Unstable
6	0.17	1.767031	−0.38715	−0.25182	−0.84529	34.26381	−7.15366	20.92892	−80.1378	758.5708	Unstable
7	0.18	1.739394	−0.38864	−0.24475	−1.48659	34.51889	−6.07377	20.93033	−88.2060	790.9029	Unstable
8	0.19	1.711311	−0.39012	0.237623	−2.04782	34.66493	4.983607	20.90276	−95.8238	820.2205	Unstable
9	0.20	1.682760	−0.39162	−0.23039	−2.53200	34.70409	−3.88153	20.84537	−102.937	846.2777	Unstable
10	0.21	1.653715	−0.39314	−0.22300	−2.94045	34.63657	−2.76528	20.75703	−109.494	868.8298	Unstable
11	0.22	1.624152	−0.39469	−0.21540	−3.27266	34.46041	−1.63185	20.63628	−115.440	887.6165	Unstable
12	0.23	1.594040	−0.39628	−0.20752	−3.52602	34.17110	−0.47729	20.48123	−120.716	902.3434	Unstable
13	0.24	1.563348	−0.39794	−0.19928	−3.69523	33.76086	0.703639	20.28941	−125.249	912.6570	Unstable
14	0.25	1.532041	−0.39968	−0.19057	−3.77128	33.21745	1.917924	20.05757	−128.951	918.1093	Unstable
15	0.26	1.500082	−0.40152	0.181266	−3.73956	32.52194	−3.17515	19.78140	−131.699	918.1009	Unstable
16	0.27	1.467426	−0.40349	−0.17116	−3.57631	31.64468	4.488850	19.45502	−133.321	911.7814	Unstable
17	0.28	1.434027	−0.40564	0.159985	−3.24148	30.53736	−5.87883	19.07015	−133.547	897.8579	Unstable
18	0.29	1.399831	−0.40802	−0.14729	−2.66268	29.11531	7.375375	18.61452	−131.921	874.1844	Unstable
19	0.30	1.364779	−0.41074	−0.13230	−1.69354	27.21263	9.026568	18.06860	−127.565	836.7332	Unstable
20	0.31	1.328802	−0.41397	0.113442	0.021976	24.43853	−10.9069	17.39764	−118.422	776.3676	Unstable
21	0.32	1.291824	−0.41812	−0.08636	3.705489	19.49560	13.03516	16.52585	−97.6748	663.8030	Unstable

Table 7. Stability table of $L_{10}\left(x_{10}, y_{10}\right)$ for $\mu \in \left(0, 3^{-1}\right)$ and corresponding values of n.

No.	μ	n	x	y	Ω_{xx}^0	Ω_{yy}^0	Ω_{xy}^0	A	B	D	Nature
1	0.12	1.873296	−0.37734	0.286545	3.313352	31.55353	12.25923	20.82993	−45.7407	616.8488	Unstable
2	0.14	1.847462	−0.49372	−0.59153	8.562976	5.758836	8.303592	0.669346	−19.6369	78.99551	Unstable
3	0.15	1.821047	−0.48877	−0.61316	7.447650	6.373874	8.592281	0.556683	−26.3569	105.7375	Unstable
4	0.16	1.794242	−0.48648	0.630625	6.564147	6.762434	−8.57096	0.449363	−29.0718	116.4889	Unstable
5	0.17	1.767031	−0.48539	0.646212	5.825240	7.013568	−8.41090	0.349213	−29.8875	119.6721	Unstable
6	0.18	1.739394	−0.48501	0.660737	5.191923	7.166377	−8.17508	0.256330	−29.6246	118.5642	Unstable
7	0.19	1.711311	−0.48512	0.674614	4.641029	7.243760	−7.89461	0.170442	−28.7063	114.8544	Unstable
8	0.20	1.682760	−0.48557	−0.68809	4.156924	7.260992	7.587171	0.091194	−27.3818	109.5354	Unstable
9	0.21	1.653715	−0.48631	0.701348	3.728285	7.229043	−7.26368	0.018230	−25.8090	103.2365	Unstable

Continued

10	0.22	1.624152	−0.48728	0.714505	3.346525	7.156169	−6.93122	−0.04878	−24.0936	96.37662	Unstable
11	0.23	1.594040	−0.48844	0.727669	3.004927	7.048790	−6.59461	−0.11013	−22.3078	89.24341	Unstable
12	0.24	1.563348	−0.48980	−0.74093	2.698111	6.912022	6.257176	−0.16609	−20.5028	82.03896	Unstable
13	0.25	1.532041	−0.49132	−0.75437	2.421692	6.750024	5.921275	−0.21689	−18.7150	74.90710	Unstable
14	0.26	1.500082	−0.49301	−0.76807	2.172038	6.566228	5.588611	−0.26271	−16.9705	67.95091	Unstable
15	0.27	1.467426	−0.49486	−0.78210	1.946107	6.363505	5.260424	−0.30374	−15.2880	61.24424	Unstable
16	0.28	1.434027	−0.49688	0.796563	1.741324	6.144286	−4.93763	−0.34012	−13.6810	54.83956	Unstable
17	0.29	1.399831	−0.49908	−0.81153	1.555487	5.910653	4.620900	−0.37197	−12.1588	48.77346	Unstable
18	0.30	1.364779	−0.50146	−0.82711	1.386698	5.664403	4.310752	−0.39938	−10.7278	43.07058	Unstable
19	0.31	1.328802	−0.50404	0.843406	1.233309	5.407107	−4.00757	−0.42244	−9.39197	37.74636	Unstable
20	0.32	1.291824	−0.50684	−0.86055	1.093881	5.140147	3.71165	−0.44121	−8.15363	32.80920	Unstable

Table 8. Stability table of $L_{11}\left(x_{11}, y_{11}\right)$ for $\mu \in \left(0, 3^{-1}\right)$ and corresponding values of n.

No.	μ	n	x	y	Ω_{xx}^0	Ω_{yy}^0	Ω_{xy}^0	A	B	D	Nature
1	0.12	1.873296	−0.37734	−0.28655	3.313352	31.55353	−12.2592	20.82993	−45.7407	616.8488	Unstable
2	0.14	1.847462	−0.49372	0.591532	8.562976	5.758836	−8.30359	0.669346	−19.6369	78.99551	Unstable
3	0.15	1.821047	−0.48877	0.613157	7.447650	6.373874	−8.59228	0.556683	−26.3569	105.7375	Unstable
4	0.16	1.794242	−0.48648	−0.63063	6.564147	6.762434	8.570961	0.449363	−29.0718	116.4889	Unstable
5	0.17	1.767031	−0.48539	−0.64621	5.825240	7.013568	8.410901	0.349213	−29.8875	119.6721	Unstable
6	0.18	1.739394	−0.48501	−0.66074	5.191923	7.166377	8.175079	0.256330	−29.6246	118.5642	Unstable
7	0.19	1.711311	−0.48512	−0.67461	4.641029	7.243760	7.894608	0.170442	−28.7063	114.8544	Unstable
8	0.20	1.682760	−0.48557	0.688094	4.156924	7.260992	−7.58717	0.091194	−27.3818	109.5354	Unstable
9	0.21	1.653715	−0.48631	−0.70135	3.728285	7.229043	7.263676	0.018230	−25.8090	103.2365	Unstable
10	0.22	1.624152	−0.48728	−0.71451	3.346525	7.156169	6.931224	−0.04878	−24.0936	96.37662	Unstable
11	0.23	1.594040	−0.48844	−0.72767	3.004927	7.048790	6.594613	−0.11013	−22.3078	89.24341	Unstable
12	0.24	1.563348	−0.48980	0.740930	2.698111	6.912022	−6.25718	−0.16609	−20.5028	82.03896	Unstable
13	0.25	1.532041	−0.49132	0.754370	2.421692	6.750024	−5.92127	−0.21689	−18.7150	74.90710	Unstable
14	0.26	1.500082	−0.49301	0.768068	2.172038	6.566228	−5.58861	−0.26271	−16.9705	67.95091	Unstable
15	0.27	1.467426	−0.49486	0.782105	1.946107	6.363505	−5.26042	−0.30374	−15.2880	61.24424	Unstable
16	0.28	1.434027	−0.49688	−0.79656	1.741324	6.144286	4.937627	−0.34012	−13.6810	54.83956	Unstable
17	0.29	1.399831	−0.49908	0.811531	1.555487	5.910653	−4.62090	−0.37197	−12.1588	48.77346	Unstable
18	0.30	1.364779	−0.50146	0.827108	1.386698	5.664403	−4.31075	−0.39938	−10.7278	43.07058	Unstable
19	0.31	1.328802	−0.50404	−0.84341	1.233309	5.407107	4.007569	−0.42244	−9.39197	37.74636	Unstable
20	0.32	1.291824	−0.50684	0.860552	1.093881	5.140147	−3.71165	−0.44121	−8.15363	32.80920	Unstable

Table 9. Stability table of $L_{12}\left(x_{12}, y_{12}\right)$ for $\mu \in \left(0, 3^{-1}\right)$ and corresponding values of n.

No.	μ	n	x	y	Ω_{xx}^{0}	Ω_{yy}^{0}	Ω_{xy}^{0}	A	B	D	Nature
1	0.14	1.847462	−0.54522	0.514416	10.35123	3.870755	−3.64157	0.569523	26.80606	−106.900	Unstable
2	0.15	1.821047	−0.56769	0.495212	9.862545	3.813450	−2.05915	0.411153	33.37021	−133.312	Unstable
3	0.16	1.794242	−0.58778	−0.47914	9.310829	3.839356	0.874584	0.272967	34.98269	−139.856	Unstable
4	0.17	1.767031	−0.60700	0.463839	8.764180	3.875943	0.090941	0.150529	33.96119	−135.822	Unstable
5	0.18	1.739394	−0.62590	−0.44836	8.247443	3.896096	−0.90576	0.041568	31.31243	−125.248	Unstable
6	0.19	1.711311	−0.64478	0.432119	7.773699	3.885179	1.604297	−0.05547	27.62844	−110.511	Unstable
7	0.20	1.682760	−0.66382	0.414628	7.352043	3.832910	2.205408	−0.14177	23.31589	−93.2435	Unstable
8	0.21	1.653715	−0.68316	0.395397	6.990623	3.730188	2.718614	−0.21829	18.68548	−74.6943	Unstable
9	0.22	1.624152	−0.70293	−0.37385	6.698345	3.567318	−3.14617	−0.28581	13.99671	−55.9052	Unstable
10	0.23	1.594040	−0.72323	0.349221	6.486286	3.332564	3.482815	−0.34500	9.485964	−37.8248	Unstable
11	0.24	1.563348	−0.74417	0.320443	6.369348	3.010470	3.712919	−0.39641	5.388962	−21.3987	Unstable
12	0.25	1.532041	−0.76585	0.285803	6.368697	2.579407	3.802980	−0.44050	1.964805	−7.66518	Unstable
13	0.26	1.500082	−0.78838	0.242186	6.515836	2.007490	3.681213	−0.47765	−0.47086	2.111582	Unstable
14	0.27	1.467426	−0.81185	0.182335	6.860132	1.245050	3.167237	−0.50817	−1.49019	6.218982	Unstable
15	0.28	1.434027	−0.83637	−0.07098	7.484022	0.209439	−1.41863	−0.53227	−0.44507	2.063593	Unstable

Table 10. Stability table of $L_{13}\left(x_{13}, y_{13}\right)$ for $\mu \in \left(0, 3^{-1}\right)$ and corresponding values of n.

No.	μ	n	x	y	Ω_{xx}^{0}	Ω_{yy}^{0}	Ω_{xy}^{0}	A	B	D	Nature
1	0.14	1.847462	−0.54522	−0.51442	10.35123	3.870755	3.641569	0.569523	26.80606	−106.900	Unstable
2	0.15	1.821047	−0.56769	−0.49521	9.862545	3.813450	2.059154	0.411153	33.37021	−133.312	Unstable
3	0.16	1.794242	−0.58778	0.479141	9.310829	3.839356	−0.87458	0.272967	34.98269	−139.856	Unstable
4	0.17	1.767031	−0.60700	−0.46384	8.764180	3.875943	−0.09094	0.150529	33.96119	−135.822	Unstable
5	0.18	1.739394	−0.62590	0.448360	8.247443	3.896096	0.905758	0.041568	31.31243	−125.248	Unstable
6	0.19	1.711311	−0.64478	−0.43212	7.773699	3.885179	−1.60430	−0.05547	27.62844	−110.511	Unstable
7	0.20	1.682760	−0.66382	−0.41463	7.352043	3.832910	−2.20541	−0.14177	23.31589	−93.2435	Unstable
8	0.21	1.653715	−0.68316	−0.39540	6.990623	3.730188	−2.71861	−0.21829	18.68548	−74.6943	Unstable
9	0.22	1.624152	−0.70293	0.373846	6.698345	3.567318	3.146175	−0.28581	13.99671	−55.9052	Unstable
10	0.23	1.594040	−0.72323	−0.34922	6.486286	3.332564	−3.48281	−0.34500	9.485964	−37.8248	Unstable
11	0.24	1.563348	−0.74417	−0.32044	6.369348	3.010470	−3.71292	−0.39641	5.388962	−21.3987	Unstable
12	0.25	1.532041	−0.76585	−0.28580	6.368697	2.579407	−3.80298	−0.44050	1.964805	−7.66518	Unstable
13	0.26	1.500082	−0.78838	−0.24219	6.515836	2.007490	−3.68121	−0.47765	−0.47086	2.111582	Unstable
14	0.27	1.467426	−0.81185	−0.18233	6.860132	1.245050	−3.16724	−0.50817	−1.49019	6.218982	Unstable
15	0.28	1.434027	−0.83637	0.070982	7.484022	0.209439	1.418632	−0.53227	−0.44507	2.063593	Unstable

6. Conclusion

The paper concludes the study of stability of libration points in the kite configuration of first kind. In Section 1, previous works have been reviewed starting from MacMillon *et al.* [1] to Khatun *et. al.* [11]. In Section 2, the equations of motion of the satellite moving in the gravitational field of the kite have been derived. In Section 3, we discussed the locations of non-axial libration points that have been exhibited with the intersection of contour plots of $\Omega_x = 0$ and $\Omega_y = 0$. In Section 4, only the value of $\mu = 0.10$, the libration points L_4 and L_5 are found stable, but for all other values of μ all libration points are unstable. The stable case is shown in **Figure 10**. In Section 5, the stability criteria are discussed through stability tables.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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