International Journal of Astronomy and Astrophysics, 2025, 15(1), 43-63
https://www.scirp.org/journal/ijaa
ISSN Online: 2161-4725
ISSN Print: 2161-4717

Update to the "New Minimal Standard Model"

Bruce Hoeneisen

Universidad San Francisco de Quito, Quito, Ecuador
Email: bhoeneisen@usfq.edu.ec

How to cite this paper: Hoeneisen, B. (2025) Update to the "New Minimal Standard Model". *International Journal of Astronomy and Astrophysics*, **15**, 43-63.
https://doi.org/10.4236/ijaa.2025.151004

Received: January 8, 2025
Accepted: March 21, 2025
Published: March 24, 2025

Abstract

The Standard Model of quarks and leptons is a great achievement. However, this model is incomplete: it does not include the observed dark matter, neutrino masses and mixings, the matter-antimatter asymmetry of the universe, the current acceleration of the expansion of the universe, nor inflation. We seek a minimal extension of the Standard Model to account for these observations. We constrain its parameters and discuss potential tests for this updated "New Minimal Standard Model".

Keywords

New Minimal Standard Model, Dark Matter, Neutrino Oscillations, Leptogenesis, Inflation

1. Introduction

In 2004 the authors of "The New Minimal Standard Model" [1] set out to "include the minimal number of new degrees of freedom to accommodate convincing (e.g., $> 5\sigma$) evidence for physics beyond the Minimal Standard Model", "based on the principle of minimal particle content and the most general renormalizable Lagrangian.". To this end, the following fields were added to the Standard Model (SM):

- A real scalar field S as dark matter. To make S stable, the Z_2 symmetry $S \leftrightarrow -S$ is imposed. The most general renormalizable Lagrangian from [1] is reproduced in Section 3 below.
- The simplest way to include the observed present acceleration of the universe expansion is to add Einstein's cosmological constant Λ.
- To account for neutrino oscillations, two right handed neutrinos are added. These neutrinos are assumed to be of the Majorana type to allow Type I baryogenesis via leptogenesis in the CP-violating and lepton number non-conserving decay of the heavy neutrino mass eigenstates. "The lighter right-handed

neutrino should be heavier than 10^{10} GeV to have enough CP asymmetry. [1]" A Majorana neutrino has no additive conserved charges, and hence allows a "Majorana" mass term, see Section 3 below.

- Finally, to account for the observed nearly scale-invariant, adiabatic, and Gaussian density fluctuations in the early universe, a real scalar field Φ is added to drive inflation.

The virtue of the "New Minimal Standard Model" is that it is data driven, constraining, allows predictions, and hence is testable.

The present study attempts to update the "New Minimal Standard Model" (NMSM) with the evidence gathered in the intervening twenty years, namely the discovery of the top quark and Higgs boson, new measured properties of dark matter (to be discussed in Section 2), and new ideas on leptogenesis (to be discussed in Section 4). We take cosmological parameter values and notation from [2].

2. Dark Matter

Let us consider non-relativistic warm dark matter when the universe is nearly homogeneous, and let us assume that this dark matter is a gas of particles of mass m_S. These particles may be collisionless, or have elastic dark matter-dark matter collisions. The root-mean-square thermal velocity of the dark matter particles is $v_{h\text{rms}}(a)$, where $a(t)$ is the expansion parameter of the universe, normalized to $a(t_0)=1$ at the present time t_0. As the universe expands, $v_{h\text{rms}}(a)$ varies in proportion to $1/a$ (assuming the dark matter particles have no excited states), so

$$v_{h\text{rms}}(1) = v_{h\text{rms}}(a)a \tag{1}$$

is an adiabatic invariant. This adiabatic invariant defines the "warmness" of the dark matter, and has been *measured* with dwarf galaxy rotation curves [3] [4]:

$$v_{h\text{rms}}(1) = 406 \pm 69 \text{ m/s}. \tag{2}$$

The transition from ultra-relativistic to non-relativistic dark matter occurs at an expansion parameter

$$a_{h\text{NR}} = \frac{v_{h\text{rms}}(1)}{c} = (1.35 \pm 0.23) \times 10^{-6}, \tag{3}$$

when the universe density is dominated by radiation.

Let $P_{\text{CDM}}(k)$ be the comoving *linear* power spectrum of density fluctuations in the cold dark matter ΛCDM cosmology. Due to warm dark matter free-streaming, or to dark matter-dark matter particle elastic collisions, the linear power spectrum becomes attenuated at high k by a cut-off factor $\tau^2(k)$, *i.e.*, $P(k) = \tau^2(k) P_{\text{CDM}}(k)$. We define the comoving cut-off wavevector k_c so $\tau^2(k_c) = 1/e$ at expansion parameter $a = 0.01$ (other definitions in the literature are $\tau^2(k_c) = 1/2$ or 1/4). We have chosen $a = 0.01$ because numerical integrations to account for non-linear density evolution and baryonic feedback often start at $a \approx 0.01$, and because free-streaming is almost complete by $a \approx 0.01$.

For $v_{h\mathrm{rms}}(1) = 406$ m/s the comoving free-streaming cut-off wavevector is

$$k_{\mathrm{fs}} \approx 0.28 \frac{2\pi}{l_{\mathrm{fs}}} = 0.32\ \mathrm{Mpc}^{-1}, \tag{4}$$

l_{fs} is the comoving dark matter free-streaming length.

The dark matter self interaction cross-section per unit mass σ_{SS}/m_S is observed to be less than 0.47 cm^2/g at the present time [2]. The corresponding mean distance between collisions is greater than $1/\left(\left(\sigma_{SS}/m_S\right)\Omega_c \rho_{\mathrm{crit}}\right) = 3\times10^5$ Mpc. In conclusion, dark matter-dark matter collisions have a negligible contribution to k_c.

k_c has been estimated from galaxy stellar mass distributions (1 to 3 Mpc^{-1}), galaxy ultra-violet luminosity distributions (2 to 4 Mpc^{-1}), first galaxies (1.6 to 3 Mpc^{-1}), and reionization (1 to 4 Mpc^{-1}), see [4] and references therein. For reionization see also [5]. Studies of the Lyman-α forest of quasar light place lower bounds on the "standard thermal relic" mass m_X in the range 550 eV [6] up to 5.7 keV [7]. These limits, due to assumed warm dark matter free-streaming, are really limits on k_c, *i.e.*, $k_c \gtrsim 3$ Mpc^{-1} up to $k_c \gtrsim 39$ Mpc^{-1}. These measurements of k_c are in disagreement with each other, and with (2), and are sensitive to the *non-linear* regenerated power spectrum that is large and uncertain, see figure 4 of [8], and figure 18 of [9]. Future Cosmic Microwave Background (CMB) measurements may reach and test k_{fs} of (4). A direct measurement of the *linear* power spectrum of density perturbations will become possible with observations of weak gravitational lensing of the CMB radiation [8].

Every measurement has its issues. When measurements disagree at most one is correct, and all need to be revised. The purpose of the present study is to explore the consequences of the assumption that $v_{h\mathrm{rms}}(1)$ of (2) is correct and of cosmological origin [4] [10]-[12], in spite of the tensions mentioned above that need to be resolved.

Galaxy observations alone can not obtain the dark matter particle mass, see Section 5 of [12]. To make progress, we need to assume a cosmological model of dark matter. To break the degeneracy between dark matter temperature and mass, we assume that ultra-relativistic dark matter has zero chemical potential. With this assumption, for spin zero dark matter, with the measured $v_{h\mathrm{rms}}(1)$ of (2), we obtain the dark matter particle mass [4]

$$m_S = 177 \pm 23\ \mathrm{eV}, \tag{5}$$

and the dark matter-to-photon temperature ratio, after e^-e^+ annihilation, while dark matter is still ultra-relativistic, to be

$$\frac{T_h}{T_\gamma} = 0.343 \pm 0.015, \tag{6}$$

sufficiently cold to not spoil the success of big-bang nucleosynthesis.

These measurements are consistent with spin zero dark matter in thermal and diffusive equilibrium with the early Standard Model sector, *i.e.* no "freeze-in", that decouples from the Standard Model sector while still ultra-relativistic, and does not decay or annihilate when it becomes non-relativistic, *i.e.* no "freeze-out" [4].

Our assumption is self-consistent because early equilibrium with the Standard Model sector implies that ultra-relativistic dark matter has zero chemical potential.

To the dark matter we assign a scalar field S with Z_2 symmetry, *i.e.* $S \leftrightarrow -S$, so that dark matter is stable. Since dark matter has not been observed in direct or indirect searches, we will assume that dark matter has no $SU(3)_c$, $SU(2)_L$ or $U(1)_Y$ gauge interactions. We therefore assume that the only interaction of S with the Standard Model sector is a contact coupling $-\frac{1}{2}\lambda_{\phi S}(\phi^\dagger\phi)S^2$ with the Higgs boson ϕ. The coupling $|\lambda_{\phi S}|$ is sufficiently strong to bring S and the Standard Model sector into thermal and diffusive equilibrium by the Electro-Weak Symmetry Breaking (EWSB) temperature $T_{\text{EWSB}} = 159 \pm 1$ GeV [13]. When the Higgs boson becomes non-relativistic and decays or annihilates, dark matter decouples from the Standard Model sector. In this scenario, for spin zero dark matter, the *predicted* dark matter mass and temperature ratio (after e^+e^- annihilation) are $m_S = 150 \pm 2$ eV and $T_h/T_\gamma = 0.345$ [4], in agreement with the *measurements*. In this scenario, spin $\frac{1}{2}$ Dirac or Majorana dark matter, and spin 1 vector dark matter, disagree with the observations by more than 5σ [4]. See **Figure 1**.

Slightly above the temperature T_{EWSB} the NMSM is composed of the dark matter scalar S, the Higgs boson ϕ, the gauge bosons $\vec{G}_i$, $\vec{W}$ and $\vec{B}$, and mass-less fields that carry the Weyl-L or Weyl-R 2-dimensional irreducible representations of the proper Lorentz group, see **Figure 2**. The Weyl-L representation is also denoted "left-handed", or $\left(\frac{1}{2},0\right)$ [14], or $\left(\frac{1}{2},-\frac{3}{2}\right)$ [15]. The Weyl-R representation is also denoted "right-handed", or $\left(0,\frac{1}{2}\right)$ [14], or $\left(\frac{1}{2},\frac{3}{2}\right)$ [15]. Below the temperature T_{EWSB}, the Higgs boson acquires a vacuum expectation value increasing to $v/\sqrt{2} \approx 246/\sqrt{2}$ GeV that ties together Weyl-L and Weyl-R fields obtaining massive 4-dimensional Dirac fields, as shown with dashed lines in **Figure 2**. The fields in **Figure 2** have wavelengths much shorter than their mean free path.

Consider $T \gg T_{M_H}$. The $SS \leftrightarrow hh$ cross-section, with $\phi \to h/\sqrt{2}$, is

$$\sigma(SS \leftrightarrow hh) = \frac{|\lambda_{\phi S}|^2}{16\pi s}, \tag{7}$$

where s is the Mandelstam variable, *i.e.*, the square of the total center of mass energy. We are interested in the mean time to the next collision $t_c = 1/(\sigma n c)$. The mean value of s, obtained with the Maxwell distribution, is $\langle s \rangle = (2kT)^2$. The cross-section (7) scales as a^2. The number density of dark matter particles n scales as $T^3 \propto a^{-3}$, so the S particle mean-free-path $1/(\sigma n)$ scales as a, slower than the proper distance to the horizon $\propto a^2$. So S and ϕ approach equilibrium towards the future. To agree with the observed "no freeze-in and no

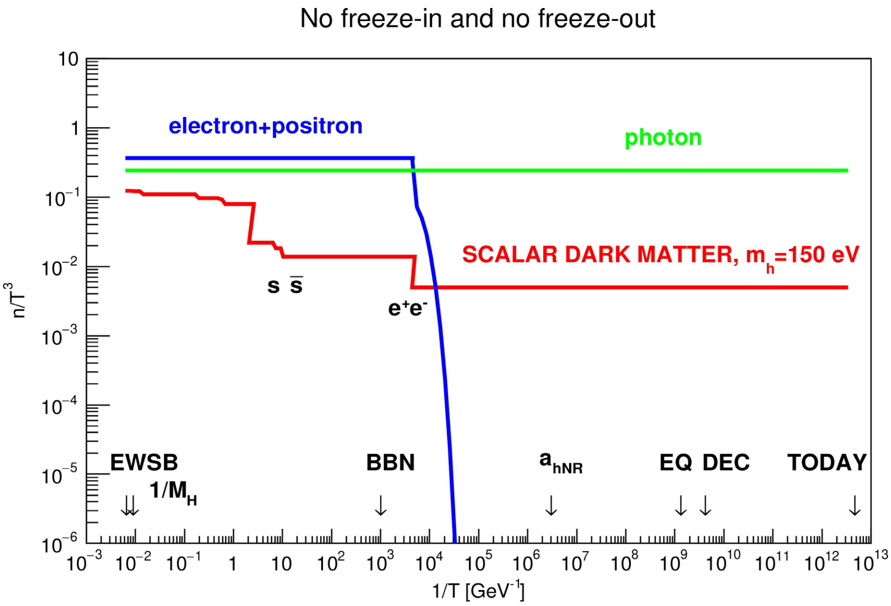

Figure 1. The "no freeze-in and no freeze-out" dark matter scenario is illustrated for spin zero warm dark matter particles coupled to the Higgs boson. T is the *photon* temperature, and the n 's are particle number densities. The abbreviations stand for "Electro-Weak Symmetry Breaking", "Big Bang Nucleosynthesis", "EQuivalence" of matter and radiation densities, and "DECoupling" of photons from the proton-electron plasma when it recombines to neutral hydrogen. Dark matter particles become non-relativistic at $a_{h\mathrm{NR}}$. Time advances towards the right. Figure from [4].

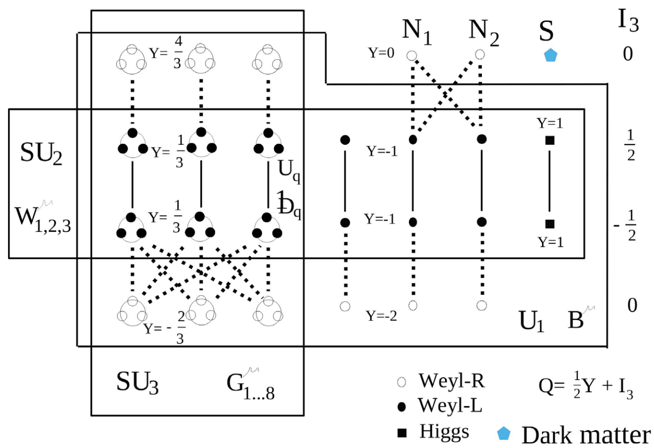

Figure 2. The elementary particles of the "New Minimal Standard Model" are shown. Before EWSB the fields carry 2-dimensional Weyl-L (filled dots) and Weyl-R (empty dots) representations of the proper Lorentz group. After EWSB, the vacuum expectation value v of the Higgs boson couples Weyl-L and Weyl-R fields into 4-dimensional Dirac fields, as indicated by the dashed lines. Circles and vertical lines illustrate, respectively, $SU(3)_c$ and $SU(2)_L$ transformations. These transformations are mediated, respectively, by gluons G_i^μ and weak W_j^μ gauge bosons. Y is the charge corresponding to the $U(1)_Y$ symmetry with gauge boson B^μ. I_3 is the iso-spin component of $SU(2)_L$. The "New Minimal Standard Model" adds to the Standard Model the sterile particles S, N_1 and N_2 with $Y = I = I_3 = 0$, and Einstein's cosmological constant (not shown).

freeze-out" scenario, we need S and ϕ to come into thermal and diffusive equilibrium by $T = M_H$, where M_H is the Higgs boson mass. This requirement implies $|\lambda_{hS}| \gtrsim 6\times10^{-7}$. For the leptogenesis scenario to be discussed in Section 4 we require thermal and diffusive equilibrium by T_{EWSB}, so $|\lambda_{\phi S}| \gtrsim 7\times10^{-7}$. To not exceed the Higgs boson invisible decay width 0.37 MeV [2] we need $|\lambda_{\phi S}| < 6\times10^{-3}$.

The non-relativistic dark matter elastic scattering cross-sections in two channels are

$$\sigma\left(SS \leftrightarrow h^* \leftrightarrow SS\right) = \frac{9}{2^8 \pi s}\left(\frac{\lambda_{\phi S} v}{M_H}\right)^4, \tag{8}$$

$$\sigma\left(SS \leftrightarrow SS\right) = \frac{|\Lambda_S|^2}{16\pi s}, \tag{9}$$

with $s = \left(2m_S\right)^2$. The observed limit on the dark matter elastic scattering cross-section per unit mass 0.47 cm^2/g [2] requires $|\Lambda_{\phi S}| \lesssim 10^{-4}$ and $|\Lambda_S| \lesssim 4\times10^{-8}$. From the preceeding estimates, we take

$$7\times10^{-7} \lesssim |\lambda_{hS}| \lesssim 10^{-4}. \tag{10}$$

3. Lagrangian

To account for the observed neutrino flavor oscillations, and to allow lepton number violation needed for leptogenesis, we add to the Standard Model Lagrangian, $m \ge 2$ sterile, *i.e.* $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge singlet, Majorana neutrinos with the Weyl-R fields N_j with $j = 1,2,\cdots,m$. The case $m = 2$ leaves one of the light neutrinos mass-less. We also add to the Standard Model the scalar dark matter field S with Z_2 symmetry, *i.e.* $S \leftrightarrow -S$. To simplify the notation, and without loss of generality, we work in a weak flavor basis with diagonal $SU(2)_L$ gauge interactions, diagonal *charged* lepton Yukawa coupling matrices Y^l, and diagonal sterile neutrino Majorana mass matrix M_N, see **Figure 2**. Then, the action of the "New Minimal Standard Model" is

$$\mathcal{S}_J = \int \mathrm{d}^4x\sqrt{-g}\left\{\mathcal{L}_{SM} + \mathcal{L}_N + \mathcal{L}_S + \mathcal{L}_{S\phi} + \mathcal{L}_{\text{grav}}\right\}, \tag{11}$$

$$\mathcal{L}_N = +\bar{N}_k \gamma^\mu i\partial_\mu N_k \tag{12}$$

$$-\frac{1}{2}\bar{N}_j^c M_{N,jj} N_j - \frac{1}{2}\bar{N}_j M_{N,jj} N_j^c \tag{13}$$

$$-Y_{k\alpha}^{\nu*}\bar{L}_\alpha \phi^c N_k - Y_{k\alpha}^{\nu}\bar{N}_k \phi^{c\dagger} L_\alpha, \tag{14}$$

$$\mathcal{L}_S = +\frac{1}{2}\partial_\mu S\cdot\partial^\mu S - \frac{1}{2}\bar{m}_S^2 S^2 - \frac{\lambda_S}{4!}S^4, \tag{15}$$

$$\mathcal{L}_{S\phi} = -\frac{1}{2}\lambda_{\phi S}\left(\phi^\dagger\phi\right)S^2, \tag{16}$$

$$\mathcal{L}_{\text{grav}} = -\Omega_\Lambda \rho_{\text{crit}} - \left(\frac{M_P^2}{2(8\pi)} + \xi_\phi \phi^\dagger\phi + \xi_S S^2\right)R. \tag{17}$$

Here, $\alpha = (e,\mu,\tau)$ denotes a flavor index, and $j,k = 1,2,\cdots,m$ are indices for the singlet Majorana neutrinos. Sums over repeated indices are understood. To use Dirac notation, the Weyl-L (Weyl-R) 2-dimensional fields are embedded into 4-dimensional Dirac fields by adding null Weyl-R (Weyl-L) components.

The dark matter particle mass is

$$m_S = \sqrt{\overline{m}_S^2 + \frac{1}{2}\lambda_{\phi S} v^2} = 150 \pm 2\ \text{eV}, \tag{18}$$

with $\overline{m}_S^2$ assumed positive, while $\lambda_{\phi S}$ is negative. Note that there is fine-tuning because $\sqrt{|\lambda_{\phi S}|}\, v \gg m_S$.

Comments on notation follow. The lepton $SU(2)_L$ doublets of the Standard Model (SM) are

$$L_\alpha \equiv \begin{pmatrix} \nu_\alpha \\ \alpha^- \end{pmatrix}_L \equiv \begin{pmatrix} \nu_{\alpha L} \\ \alpha_L^- \end{pmatrix} = \frac{1-\gamma_5}{2} \begin{pmatrix} \nu_\alpha \\ \alpha^- \end{pmatrix}, \tag{19}$$

with $\alpha = e, \mu, \tau$. Note that we work in a flavor basis with diagonal $SU(2)_L$ doublets. The fields $\nu_{\alpha L}$ and α_L^- carry the Weyl-L irreducible representation of the proper Lorentz group.

N_j, with $j = 1, 2, \cdots, m$, are $SU(2)_L$ singlets that carry the Weyl-R irreducible representation of the proper Lorentz group. The conjugate fields $N_j^c \equiv -i\sigma_2 N_j^*$ carry the Weyl-L irreducible representation.

After EWSB, a Weyl-L field couples to a Weyl-R field obtaining a 4-dimensional Dirac field for a massive quark, a massive charged lepton or a massive neutrino. The Dirac field ψ_α and its charge-conjugate $(\psi_\alpha)^c$, in the basis of the direct sum Weyl-L $\oplus$ Weyl-R, are

$$\psi_\alpha = \begin{pmatrix} \psi_{\alpha L} \\ \psi_{\alpha R} \end{pmatrix}, \quad (\psi_\alpha)^c \equiv \begin{pmatrix} (\psi_{\alpha R})^c \\ (\psi_{\alpha L})^c \end{pmatrix} = -i\gamma^2 \psi_\alpha^* = \begin{pmatrix} -i\sigma_2 \psi_{\alpha R}^* \\ i\sigma_2 \psi_{\alpha L}^* \end{pmatrix}. \tag{20}$$

Note the convention that the Weyl-L component of a Dirac field goes on top. We work in a basis with γ-matrices

$$\gamma^0 = \begin{pmatrix} 0 & \sigma_0 \\ \sigma_0 & 0 \end{pmatrix}, \quad \gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -\sigma_0 & 0 \\ 0 & \sigma_0 \end{pmatrix}. \tag{21}$$

Also, $\overline{\psi} \equiv \psi^\dagger \gamma^0$. $\overline{\psi}\psi$ is a Lorentz scalar, and $\overline{\psi}\gamma^\mu\psi$ is a Lorentz 4-vector. R is the Ricci curvature scalar, and M_P is the Planck mass. We assume, tentatively, that the dark matter scalar S is also the inflaton, hence the quartic potential $\frac{1}{4!}\lambda_S S^4$ in (15), see Section 6.

The Higgs boson field and its conjugate are

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \phi^c \equiv i\sigma_2 \phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^{+*} \end{pmatrix}. \tag{22}$$

After EWSB,

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}, \quad \phi^c = \frac{1}{\sqrt{2}} \begin{pmatrix} v+h \\ 0 \end{pmatrix}. \tag{23}$$

Let

$$M_{D,k\alpha} \equiv Y^{\nu}_{k\alpha} \frac{v}{\sqrt{2}}. \tag{24}$$

The mass terms in (13) and (14), for $T < T_{\text{EWSB}}$, can be written as [2]

$$\frac{1}{2}\left(\overline{\vec{\nu}^c_L}, \overline{\vec{N}}\right)\begin{pmatrix} 0 & M_D^{\text{T}} \\ M_D & M_N \end{pmatrix}\begin{pmatrix} \vec{\nu}_L \\ \vec{N}^c \end{pmatrix} + h.c. \equiv \overline{\vec{\nu}^c} M_\nu \vec{\nu} + h.c.. \tag{25}$$

$\vec{\nu}_L$ has components $\left(\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}\right)^{\text{T}}$ and $\vec{N}^c$ has components $\left(\vec{N}^c_1, \vec{N}^c_2, \cdots, \vec{N}^c_m\right)^{\text{T}}$. $\left(N_j\right)^c = -i\sigma_2 N^*_j$ carries the representation Weyl-L, while $\left(\nu_{L,\alpha}\right)^c = i\sigma_2 \nu^*_{L,\alpha}$ carries the representation Weyl-R. The matrix M_ν can be diagonalized with a $(3+m)\times(3+m)$ unitary matrix V^ν:

$$\left(V^\nu\right)^{\text{T}} M_\nu V^\nu = \text{diag}\left(m_1, m_2, \cdots, m_{3+m}\right). \tag{26}$$

The neutrino mass eigenstates are

$$\vec{\nu}_{\text{mass}} = \left(V^\nu\right)^\dagger \vec{\nu}. \tag{27}$$

For $M_{N,jj}$ much greater than $M_{D,k\alpha}$, and after EWSB, the light neutrino masses are given by the "see-saw" equation [2] [16]

$$M_l \equiv \text{diag}\left(m_1, m_2, m_3\right) \equiv \text{diag}\left(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}\right) \approx -V_l^{\text{T}} M_D^{\text{T}} M_N^{-1} M_D V_l, \tag{28}$$

while the heavy neutrino masses are $M^h \approx M_N$. M_l is of dimension 3×3, real and diagonal.

The charge current interaction in the mass basis now reads [2]

$$-\mathcal{L}_{CC} = \frac{g}{\sqrt{2}}\left[\left(\overline{e}_L, \overline{\mu}_L, \overline{\tau}_L\right)\gamma^\mu U \vec{\nu}_{\text{mass}} W^-_\mu + \overline{\vec{\nu}}_{\text{mass}} U^\dagger \gamma^\mu \left(e_L, \mu_L, \tau_L\right)^{\text{T}} W^+_\mu\right] \tag{29}$$

with U a $3\times(3+m)$ matrix that satisfies $UU^\dagger = 1_{3\times 3}$. The matrix U is equal to the first three rows of the matrix V^ν (once nonphysical phases are removed by re-phasing the neutrino fields):

$$U \approx \left[\left(1 - \frac{1}{2} M_D^\dagger M_N^{*-1} M_N^{-1} M_D\right) V_l, \quad M_D^\dagger M_N^{*-1}\right], \tag{30}$$

valid for $\left[M_D^\dagger M_N^{*-1}\right]_{\alpha j} \ll 1$. These equations assume the basis with diagonal charged lepton Yukawa matrices Y^l, and diagonal sterile neutrino Majorana mass matrix M_N. This $3\times(3+m)$ matrix has elements $U_{\alpha j}$ with indices $\alpha = e, \mu, \tau$ and $j = 1, 2, \cdots, (3+m)$ ($j = 1, 2, 3$ correspond to the mass eigenstates ν_e, ν_μ, ν_τ, while $j = 4, 5, 6$ correspond to the mass eigenstates N_1, N_2, N_3 for the case $m = 3$). U has two physical phases for the case $m = 2$ [17], or 3 physical phases for the case $m = 3$ [2]. The unitary 3×3 matrix V_l is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix. See current values of its elements in [2]. Multiplying (28) on the left and right by $\left(M_l\right)^{-1/2}$, assuming all $m_{\nu_\alpha} \neq 0$, obtains

$$1 = R^{\text{T}} R, \quad R = i\left(M_N\right)^{-1/2} M_D V_l \left(M_l\right)^{-1/2}, \quad M_D = -i\sqrt{M_N} R \sqrt{M_l} V_l^\dagger. \tag{31}$$

Notation: the square root refers to the diagonal elements m_α of the diagonal matrix M_l. The Casas-Ibarra matrix R [18], with elements $R_{j\alpha}$ with $j = 1, 2, \cdots, m$ and $\alpha = e, \mu, \tau$, is a complex matrix satisfying $R^{\mathrm{T}} R = 1_{3\times 3}$. In detail, for the case $m = 2$,

$$M_D^\dagger M_N^{*-1} = iV_l \begin{pmatrix} \sqrt{\frac{m_{\nu_e}}{M_{N1}^*}} R_{1e}^* & \sqrt{\frac{m_{\nu_e}}{M_{N2}^*}} R_{2e}^* \\ \sqrt{\frac{m_{\nu_\mu}}{M_{N1}^*}} R_{1\mu}^* & \sqrt{\frac{m_{\nu_\mu}}{M_{N2}^*}} R_{2\mu}^* \\ \sqrt{\frac{m_{\nu_\tau}}{M_{N1}^*}} R_{1\tau}^* & \sqrt{\frac{m_{\nu_\tau}}{M_{N2}^*}} R_{2\tau}^* \end{pmatrix}. \tag{32}$$

An explicit form of $R^\dagger$ [17] for normal neutrino mass ordering and $m_{\nu_e} = 0$, is

$$R^\dagger = \begin{pmatrix} 0 & 0 \\ \cos z & -\sin z \\ \sin z & \cos z \end{pmatrix}, \tag{33}$$

while for inverted mass ordering and $m_{\nu_\tau} = 0$,

$$R^\dagger = \begin{pmatrix} \cos z & -\sin z \\ \sin z & \cos z \\ 0 & 0 \end{pmatrix}. \tag{34}$$

z is a complex parameter. $\cos^2 z + \sin^2 z = 1$ holds.

4. Leptogenesis

The observed baryon-to-photon ratio of the universe, at the present time, is $n_B/n_\gamma = (6.04 \pm 0.12) \times 10^{-10}$ [2]. The corresponding baryon asymmetry before baryon-anti-baryon annihilation is $(B - \bar{B})/(B + \bar{B}) \approx 6.04 \times 10^{-10}/(3 \times 0.399^3) = 3.2 \times 10^{-9}$, where, at this temperature, B is the sum of protons and neutrons (the factor 3 is $(3/4) N_f / N_b$, with $N_f = 8$ and $N_b = 2$, and the factor $0.399 = (43/(11 \times g_{\mathrm{dec}}))^{1/3}$). Assuming "initial" baryon-anti-baryon symmetry, how did the universe acquire this excess of baryons? We consider the following baryogenesis scenario: out of thermal equilibrium charge conjugation and parity (CP) violating and lepton number violating interactions produce a lepton asymmetry that is partially converted to a baryon asymmetry by non-perturbative sphaleron transitions [13] [19] [20]. These transitions conserve the baryon-minus-lepton number $B - L$, and violate $B + L$. Sphaleron transitions freeze-out at $T_{\mathrm{sph}} = 131.7 \pm 2.3$ GeV [13]. At lower temperature the sphaleron rate drops below the universe expansion rate, and the lepton and baryon asymmetries become frozen while ultra-relativistic. Thus, an excess of anti-leptons is partially converted to an exess of baryons (three left-handed anti-leptons, one of each generation, are converted to nine left-handed quarks, three of each generation). The required lepton asymmetry, assuming no sphaleron transitions, at a temperature approxiately T_{sph} is obtained dividing the baryon asymmetry by ≈ 0.343 [21]:

$$\frac{L-\bar{L}}{L+\bar{L}} \approx -9.3\times10^{-9}, \tag{35}$$

where the lepton number is $L \equiv \sum_\alpha \left(n_{\alpha_L^-} + n_{\nu_{\alpha L}} \right)$, and anti-lepton number is $\bar{L} = \sum_\alpha \left(n_{\alpha_L^+} + n_{\bar{\nu}_{\alpha L}} \right)$, with $\alpha = e, \mu, \tau$.

Current measurements and observations under-constrain baryogenesis, so that several models are viable. Example reviews are [17] [22] [23]. An often considered scenario is "Type I Leptogenesis" with 2 [17] or more Majorana sterile neutrinos with masses 10^9 to $>10^{12}$ GeV decaying out of thermal equilibrium and violating the CP symmetry and lepton number conservation. The high masses are needed to obtain sufficient lepton asymmetry. Such models are inspired by Grand Unified Theories. The Higgs boson was discovered at the Large Hadron Collider (LHC) and, surprisingly, no new particles were observed! The measured Higgs boson mass turns out to be exactly the mass needed so that the Standard Model may be valid up to very high energies, with no Grand Unified Theory in sight. How are these high mass neutrinos created? In the spirit of the New Minimal Standard Model, we will consider, as a proof of principle, a specific viable leptogenesis scenario [24] [25] with the lepton asymmetry being generated between the temperatures T_{EWSB} and T_{sph} [16], knowing that nature might have chosen otherwise.

Leptogenesis requires that the rate of transitions $L \to \bar{L}$ be greater than the rate of $\bar{L} \to L$. Therefore, out of equilibrium CP violation and lepton number violation is needed. To obtain CP violation, two amplitudes with different "weak" phases and different "strong" phases need to interfere (as in (38) below). By definition, a "weak" phase changes sign under CP-conjugation, while a "strong" phase remains invariant. "Weak" phases come from complex Yukawa couplings $Y^\nu_{k\alpha}$. "Strong" phases come from interference between (nearly) on-shell intermediate massive particle propagation. This is the motivation for considering leptogenesis at temperatures below T_{EWSB} at which Standard Model particles acquire mass. In "Type I Leptogenesis" the needed "strong" phases come from on-shell massless particles in diagrams with loops.

As an example, consider a neutrino oscillation experiment with a base-line l. The weak phases are the phases in the unitary PMNS matrix V_l, or its extension U with added sterile neutrinos (these phases come from complex Yukawa couplings) [2], while the strong phases are the propagation phase differences $2\chi_{ij} = \left(m_i^2 - m_j^2\right) l / (2p)$. The active and sterile neutrinos are ultra-relativistic in the model under consideration. The probability of a lepton conserving event, e.g. $e^- W^+ \to \nu_i \to \mu^- W^+$, is [2]

$$\begin{aligned} P_{\alpha\beta} &= P_{\alpha\beta\text{CPC}} + P_{\alpha\beta\text{CPV}}, \\ P_{\alpha\beta\text{CPC}} &= \delta_{\alpha\beta} - 4\sum_{i<j}^{3+m} \text{Re}\left[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} \right] \sin^2 X_{ij}, \\ P_{\alpha\beta\text{CPV}} &= +2\sum_{i<j}^{3+m} \text{Im}\left[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} \right] \sin\left(2X_{ij}\right). \end{aligned} \tag{36}$$

The sums in (36) are over coherent [16] [26] mass eigenstates $i < j$ with masses m_i and m_j that can not be discriminated in the experiment [26] [27]. Under CP conjugation, $U \to U^*$. The term $P_{\alpha\beta\text{CPC}}$ is CP conserving, while $P_{\alpha\beta\text{CPV}}$ may be CP violating. Note that to obtain CP violation, at least one physical phase in U is needed, in addition to the propagation phase difference $2X_{ij}$. Note that $X_{ij} \neq 0$ only if two neutrinos acquire different masses at $T < T_{\text{EWSB}}$. A massive neutrino needs a Weyl_R component added to the Standard Model. Equations (36) are the standard equations used to describe neutrino oscillations in vacuum [2]. There are non-trivial subtleties described in [26] and [27].

To obtain lepton number violation, and also to account for the observed neutrino flavor oscillations, we add $m = 2$ (or more) sterile Majorana neutrinos N_1 and N_2 to the Standard Model. Let us consider the universe in the temperature range of interest between T_{EWSB} and T_{sph}. The probability to observe a lepton violating event, e.g. $e^- W^+ \to \nu_i \to \mu^+ W^-$, is [16]

$$P_{\alpha\bar{\beta}} = P_{\alpha\bar{\beta}\text{CPC}} + P_{\alpha\bar{\beta}\text{CPV}} \ll 1,$$
$$P_{\alpha\bar{\beta}\text{CPC}} = \frac{1}{2E^2}\left\{\left|\sum_i^{3+m} m_i U^*_{\alpha i} U^*_{\beta i}\right|^2 - 4\sum_{i<j}^{3+m} m_i m_j \,\text{Re}\left[U_{\alpha i} U_{\beta i} U^*_{\alpha j} U^*_{\beta j}\right]\sin^2 X_{ij}\right\}, \quad (37)$$
$$P_{\alpha\bar{\beta}\text{CPV}} = \frac{1}{2E^2}\left\{2\sum_{i<j}^{3+m} m_i m_j \,\text{Im}\left[U_{\alpha i} U_{\beta i} U^*_{\alpha j} U^*_{\beta j}\right]\sin\left(2X_{ij}\right)\right\},$$

where we have included the angle averaged polarization miss-match factors $\sqrt{2\left[(E-p)/(E+p)\right]} \approx m/\left(\sqrt{2}E\right)$ for an ultra-relativistic neutrino [16]. The probability $P_{\bar{\alpha}\beta}$ for the CP-conjugate event, e.g. $e^+ W^- \to \bar{\nu}_i \to \mu^- W^+$, is obtained by $U \to U^*$. Note that the term $P_{\alpha\bar{\beta}\text{CPC}}$ violates lepton number but conserves the CP symmetry. Note that the term $P_{\alpha\bar{\beta}\text{CPV}}$ is lepton number violating and may be CP violating, and is the source of the leptogenesis studied in this article. Note that CPT-invariance holds.

Let us outline the derivation of (37) in order to discuss coherence issues. The cross section $\sigma_{\alpha\bar{\beta}}$ includes the interference of two amplitudes

$$\propto \left|U^*_{\beta i}\frac{m_i}{\sqrt{2}E}\mathrm{e}^{-iA_i}U^*_{\alpha i} + U^*_{\beta j}\frac{m_j}{\sqrt{2}E}\mathrm{e}^{-iA_j}U^*_{\alpha j}\right|^2. \quad (38)$$

with

$$A_{i(j)} = (E \pm \Delta E)t - \sqrt{(E \pm \Delta E)^2 - (m \mp \Delta m)^2}\,(l \pm \Delta l), \quad (39)$$

where ΔE is the neutrino energy uncertainty, Δl is the base-line uncertainty, and $m \mp \Delta m$ are the masses of ν_i and ν_j. We consider the case of ultra-relativistic neutrinos, so $E \gg m_{i(j)}$. Equation (38) obtains (37). Consider the universe at the temperature $T_{\text{sph}} = 132$ GeV. The age of the universe is $t_u = 1.4 \times 10^{-11}$ s. The mean time of an electron to the next collision $e^- W^+ \to \nu_e \to e^- W^+$, or of an anti-neutrino to the next collision $\bar{\nu}_e W^+ \to e^+ \to \bar{\nu}_e W^+$, is $t_c \approx 6 \times 10^{-23}$ s. The corresponding electron energy quantum uncertainty in this case is $\Delta E \approx \hbar/t_c \approx 0.01$ GeV.

The build-up of the lepton asymmetry is described by these equations, where

j is the time step of duration t_c, *i.e.*, the lifetime of the neutrinos:

$$n_{\beta,j+1} = n_{\beta,j} + \sum_{\alpha} \left[n_{\alpha,j} \left(P_{\alpha\beta\mathrm{CPC}} + P_{\alpha\beta\mathrm{CPV}} \right) + n_{\bar{\alpha},j} \left(P_{\alpha\bar{\beta}\mathrm{CPC}} - P_{\alpha\bar{\beta}\mathrm{CPV}} \right) \right], \tag{40}$$

$$n_{\bar{\beta},j+1} = n_{\bar{\beta},j} + \sum_{\alpha} \left[n_{\alpha,j} \left(P_{\alpha\bar{\beta}\mathrm{CPC}} + P_{\alpha\bar{\beta}\mathrm{CPV}} \right) + n_{\bar{\alpha},j} \left(P_{\alpha\beta\mathrm{CPC}} - P_{\alpha\beta\mathrm{CPV}} \right) \right]. \tag{41}$$

Now subtract (41) from (40), sum over β, and divide by $L+\bar{L}$. The terms with $P_{\alpha\beta\mathrm{CPC}}$ and $P_{\alpha\beta\mathrm{CPV}}$ do not change $L-\bar{L}$ and are neglected. The terms with $P_{\alpha\bar{\beta}\mathrm{CPV}} > 0$ build the lepton asymmetry $L-\bar{L} < 0$, until the terms with $P_{\alpha\bar{\beta}\mathrm{CPC}} > 0$ saturate the build up. The lepton asymmetry (without sphaleron transitions) becomes [16]

$$\frac{L-\bar{L}}{L+\bar{L}} = -t_u \sum_{\alpha\beta} \frac{P_{\alpha\bar{\beta}\mathrm{CPV}}}{t_c}. \tag{42}$$

This asymmetry needs to reach the value (35). Equation (42) is valid while wash-out from $P_{\alpha\bar{\beta}\mathrm{CPC}}$ does not dominate, *i.e.* while

$$\left(n_{\alpha} - n_{\bar{\alpha}} \right) P_{\alpha\bar{\beta}\mathrm{CPC}} + \left(n_{\alpha} + n_{\bar{\alpha}} \right) P_{\alpha\bar{\beta}\mathrm{CPV}} > 0. \tag{43}$$

$P_{\alpha\bar{\beta}\mathrm{CPV}}$ includes a factor $\sin\left(2\chi_{ij}\right)$ with $2\chi_{ij} \approx \left(m_i^2 - m_j^2\right) t_c / (2E)$. We only include coherent events with small t_c with $2\chi_{ij} \lesssim \pi/2$. Events with $2\chi_{ij} \gtrsim \pi/2$ are incoherent and cancel on average. The coherent events have $P_{\alpha\bar{\beta}CPV} \propto t_c$, so t_c drops out of (42).

The leptogenesis problem at hand is under-constrained. As a proof-of-principle, let us consider examples with $m=2$ sterile neutrinos and normal neutrino mass ordering, with $m_{\nu_e} = 0$, $m_{\nu_\mu} = 0.009$ eV and $m_{\nu_\tau} = 0.050$ eV. We write z in (33) in the form $z = \alpha + iB$ with real α and B. Leptogenesis requires $e^B \gg 1$,

so $\cos z \approx -i \sin z \approx \frac{1}{2} \mathrm{e}^B \mathrm{e}^{-i\alpha}$. Let us estimate the cross-section of

$W^+ e^- \to \nu_e \to W^+ e^-$ scattering at T_{sph} [28]:

$$\sigma \approx \frac{1}{16\pi s} \frac{g^4}{4} \left| V_{l e \nu_e} \right|^4 K \tag{44}$$

with

$$K \equiv \frac{1}{4\pi} \int \mathrm{d}\Omega \cdot 2 \left(-\frac{u}{s} - \frac{s}{u} \right) \tag{45}$$

$$= \int_{-1+\epsilon}^{1} \mathrm{d}\cos\theta \left(\frac{1+\cos\theta}{2} + \frac{2}{1+\cos\theta} \right) \tag{46}$$

$$\approx 1 + 2\ln\left(\frac{2}{\epsilon}\right) \approx 1 + 2\ln\left(\frac{16 \cdot \left(2T_{\mathrm{sph}}\right)^4}{M_W^4} \right) \approx 8.5. \tag{47}$$

θ is the angle between the outgoing and incoming electron. The value of $m_i^2 - m_j^2$ fixes the maximum coherent neutrino mean free path l_c corresponding to $2\chi_{ij} = \pi/2$. We present calculations at T_{sph}. The age of the universe at T_{sph} is 1.4×10^{-11} s. The mean free path and lifetime of ν_e due to the reaction $W^+ e^- \to \nu_e \to W^+ e^-$ is $l_c \approx 1/(\sigma n_{eL}) \approx 2 \times 10^{-14}$ m and $t_c = l_c / c \approx 6 \times 10^{-23}$ s, respectively, and similarly for ν_μ and ν_τ. n_{eL} is the density of left-handed electrons. $2\chi_{ij} = \pi/2$ at this l_c if $m^2_{\nu_i} - m^2_{\nu_j} \approx 9$ GeV^2.

- **Example 1:** $m = 2$. **Active coherent neutrinos**, e.g. $\tau^- W^+ \to \nu_\tau \nu_\mu \nu_e \to \mu^+ W^-$. In this example $2\chi_{\tau\mu} \approx 10^{-21}$ is too small to have a viable leptogenesis solution.
- **Example 2:** $m = 2$. **Three active and one sterile coherent neutrinos**, e.g. $\tau^- W^+ \to \nu_e \nu_\mu \nu_\tau$, $N_1 \to \mu^+ W^-$. For $2\chi_{e4} \approx \pi/2$ the needed coherent sterile neutrino mass is $M_{N_1} \approx 3$ GeV. We choose the second sterile neutrino to be incoherent, e.g. $M_{N_2} = 20$ GeV. We obtain

$$\frac{P_{\tau\bar{\mu}\mathrm{CPV}}}{t_c} = \frac{M_{N_1}}{E^2} \mathrm{Im}\left\{ \sum_{i=1}^{3} \left(m_{\nu_i} V_{l\tau i} V_{l\mu i} \right) \right. \tag{48}$$

$$\cdot \left(iV_{l\tau\nu_\mu} \sqrt{\frac{m_{\nu_\mu}}{M_{N_1}}} \cos z + iV_{l\tau\nu_\tau} \sqrt{\frac{m_{\nu_\tau}}{M_{N_1}}} \sin z \right)^*$$

$$\left. \cdot \left(iV_{l\mu\nu_\mu} \sqrt{\frac{m_{\nu_\mu}}{M_{N_1}}} \cos z + iV_{l\mu\nu_\tau} \sqrt{\frac{m_{\nu_\tau}}{M_{N_1}}} \sin z \right)^* \right\} \frac{M^2_{N_1}}{2E} \tag{49}$$

$$\approx 0.06 \cdot \frac{m^2_{\nu_\tau}}{E^2} \mathrm{e}^{i2\alpha} \mathrm{e}^{2B} \frac{M^2_{N_1}}{2E}. \tag{50}$$

It is possible to choose α so that the product of the four U 's in (37) is purely imaginary. This choice maximizes $P_{\tau\bar{\mu}\mathrm{CPV}}$ while obtaining zero wash-out from $P_{\tau\bar{\mu}\mathrm{CPC}}$. From (35), (42) and (50) we obtain $\mathrm{e}^B \approx 3500$. We veryfy that the largest element of $M_D^\dagger M_N^{-1}$ is ≈0.003 ≪ 1, leaving some room for larger baryon-to-photon ratio than observed. In conclusion, with just two sterile Majorana neutrinos it is possible to obtain successful leptogenesis in this channel.

- **Example 3:** $m = 2$. **Two sterile coherent neutrinos**, e.g. $\tau^- W^+ \to N_1$, $N_2 \to \mu^+ W^-$. There is a single pair of coherent neutrinos: the two heavy sterile neutrinos N_1 and N_2, which in this example have masses $M_{N_1} = 10 - 0.2$ GeV and $M_{N_2} = 10 + 0.2$ GeV to still be coherent with $2\chi_{ij} \lesssim \pi/2$, and much less than E to be ultra-relativistic. We obtain

$$P_{\tau\bar{\mu}\mathrm{CPV}} = \frac{M_{N_1} M_{N_2}}{E^2} \mathrm{Im}\left\{ \left(iV_{l\tau\nu_\mu} \sqrt{\frac{m_{\nu_\mu}}{M_{N_1}}} \cos z + iV_{l\tau\nu_\tau} \sqrt{\frac{m_{\nu_\tau}}{M_{N_1}}} \sin z \right) \right.$$

$$\cdot \left(iV_{l\mu\nu_\mu} \sqrt{\frac{m_{\nu_\mu}}{M_{N_1}}} \cos z + iV_{l\mu\nu_\tau} \sqrt{\frac{m_{\nu_\tau}}{M_{N_1}}} \sin z \right)$$

$$\cdot \left(-iV_{l\tau\nu_\mu} \sqrt{\frac{m_{\nu_\mu}}{M_{N_2}}} \sin z + iV_{l\tau\nu_\tau} \sqrt{\frac{m_{\nu_\tau}}{M_{N_2}}} \cos z \right)^* \tag{51}$$

$$\left. \cdot \left(-iV_{l\mu\nu_\mu} \sqrt{\frac{m_{\nu_\mu}}{M_{N_2}}} \sin z + iV_{l\mu\nu_\tau} \sqrt{\frac{m_{\nu_\tau}}{M_{N_2}}} \cos z \right)^* \right\} 2\chi_{ij}$$

$$= 0,$$

because the product $U_{\tau 4}U_{\mu 4}U_{\tau 5}^{*}U_{\mu 5}^{*}$ is real, so there is no lepton asymmetry in this channel.

Therefore, instead of adding two sterile neutrinos, let us consider three.

- **Example 4:** $m = 3$ **. Two sterile coherent neutrinos**, e.g. $\tau^{-}W^{+} \to N_1$, $N_2 \to \mu^{+}W^{-}$. The Casas-Ibarra matrix $\left(R^{\dagger}\right)_{\alpha j}$, satisfying $R^{\mathrm{T}}R = 1_{3\times 3}$, can be written as

$$R^{\dagger} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{52}$$

where $c_{ij} \equiv \cos z_{ij}$, $s_{ij} \equiv \sin z_{ij}$, and the z's have the form $z_{ij} \equiv \alpha_{ij} + iB_{ij}$, with α_{ij} and B_{ij} real. The problem is under-constrained, so, as a proof of principle, we may set $z_{12} = 0$, and $B_{13} = 1$. Then, for $\exp(B_{23}) \gg 1$,

$$\begin{aligned} U_{\tau 4}U_{\mu 4}U_{\tau 5}^{*}U_{\mu 5}^{*} \approx & \left(-iV_{l\tau\nu_\mu}\sqrt{\frac{m_{\nu_\mu}}{M_{N_1}}}s_{13}s_{23} - iV_{l\tau\nu_\tau}\sqrt{\frac{m_{\nu_\tau}}{M_{N_1}}}s_{13}c_{23} \right) \\ & \cdot \left(-iV_{l\mu\nu_\mu}\sqrt{\frac{m_{\nu_\mu}}{M_{N_1}}}s_{13}s_{23} - iV_{l\mu\nu_\tau}\sqrt{\frac{m_{\nu_\tau}}{M_{N_1}}}s_{13}c_{23} \right) \\ & \cdot \left(iV_{l\tau\nu_\mu}\sqrt{\frac{m_{\nu_\mu}}{M_{N_2}}}c_{23} - iV_{l\tau\nu_\tau}\sqrt{\frac{m_{\nu_\tau}}{M_{N_2}}}s_{23} \right)^{*} \\ & \cdot \left(iV_{l\mu\nu_\mu}\sqrt{\frac{m_{\nu_\mu}}{M_{N_2}}}c_{23} - iV_{l\mu\nu_\tau}\sqrt{\frac{m_{\nu_\tau}}{M_{N_2}}}s_{23} \right)^{*} \end{aligned} \tag{53}$$

$$\approx 0.03 \cdot \frac{m_{\nu_\tau}^{2}}{M_{N_1}M_{N_2}} \mathrm{e}^{-i2\alpha_{13}} \mathrm{e}^{4B}. \tag{54}$$

It is possible to choose α_{13} so $U_{\tau 4}U_{\mu 4}U_{\tau 5}^{*}U_{\mu 5}^{*}$ is purely imaginary. This choice maximizes $P_{\tau\bar{\mu}\mathrm{CPV}}$, while the wash-out from $P_{\tau\bar{\mu}\mathrm{CPC}}$ is zero. As an example, we take $M_{N_1} = 10 - 0.2$ GeV and $M_{N_2} = 10 + 0.2$ GeV, corresponding to $2\chi_{45} \approx \pi/2$. N_3 needs to be incoherent, so we choose, for example, $M_{N_3} = 20$ GeV. From (35), (37) and (42) we obtain $\mathrm{e}^{B} \approx 72$. We verify that the largest element of $M_D^{\dagger}M_N^{-1}$ is $\approx 7\times 10^{-5} \ll 1$, leaving room for larger baryon-to-photon ratio than observed.

In summary, successful baryogenesis can be achieved by coherent flavor-violating and CP-violating oscillations of Majorana neutrinos in the temperature range from T_{EWSB} to T_{sph} [24] [25]. For coherent interference between one sterile neutrino and the active neutrinos, successful leptogenesis is obtained with $m \geq 2$ sterile neutrinos added to the Standard Model. One of these sterile neutrinos has a mass of order 3 GeV. For coherent interference between two sterile neutrinos, at least $m = 3$ sterile neutrinos need to be added to the Standard Model. The $m \geq 3$ cases are under-constrained. We note that the dark matter field S plays no role in these leptogenesis scenarios.

5. Sterile Neutrino Production and Decay

Let us list some sterile neutrino production cross-sections and decy rates (assumed to be above threshold):

$$\sigma\left(\nu_\tau W^- \leftrightarrow \tau^- \leftrightarrow N_1 W^-\right) \approx \frac{K}{16\cdot\pi s}\left|V_{l\tau\nu_\tau}\right|^2 \frac{g^4}{4}\left|V_l\sqrt{M_l}R^\dagger M_N^{-1/2}\right|^2_{\tau 4}, \tag{55}$$

$$\sigma\left(\tau^- W^+ \leftrightarrow \nu_\tau \leftrightarrow N_1 h\right) \approx \frac{g^2}{32\cdot\pi s v^2}\left|V_{l\tau\nu_\tau}\right|^2 \left|\sqrt{M_N}R\sqrt{M_l}V_l^\dagger\right|^2_{4\tau}. \tag{56}$$

Once N_1 becomes non-relativistic at $T \approx M_{N_1}$:

$$\sigma\left(N_1 N_1 \to \nu_\tau \nu_\tau\right) \approx \frac{M_{N_1}^4}{12\pi s v^4 m_H^4}\left|\sqrt{M_N}R\sqrt{M_l}V_l^\dagger\right|^4_{\tau 4}. \tag{57}$$

As an example, consider sterile neutrino production in the channel of (55) at the temperature T_{sph}. The age of the universe is 1.4×10^{-11} s, while the mean time for a ν_τ to convert to a N_1 is $\approx 10^{-17}$ s (with the parameters of Example 2, e.g. $\mathrm{e}^B = 3500$), or $\approx 10^{-14}$ s (with the parameters of Example 4, e.g. $\mathrm{e}^B = 72$). Note that N_1 comes into thermal and diffusive equilibrium with the Standard Model sector as soon as the Higgs boson comes into equilibrium with S, and N_1 becomes depleted from the universe as soon as the temperature drops below the threshold $M_W + M_{N_1}$.

On the other hand, the depletion of N_1 in the channel (57), when N_1 becomes non-relativistic, is negligible.

6. Inflation

The original "New Minimal Standard Model" [1] adds a real scalar field Φ that drives inflation. The Lagrangian of Section 3 includes two scalar fields: the warm dark matter field S, and the Higgs boson field ϕ. S or ϕ may drive inflation [29]-[33] so we tentatively do not add the field Φ to the updated "New Minimal Standard Model". The field Φ allows, and requires, more interacting terms with the Standard Model sector [1], which may become necessary when pre-heating and re-heating are better understood.

Higgs inflation is nearly critical: the Higgs quartic coupling λ_ϕ may run negative at energies approaching the Planck scale (within the experimental uncertainties of the top quark mass, the Higgs boson mass, and the $SU(3)_c$ coupling constant α_3) [33] [34]. Inflation driven by the dark matter field S is not critical. We will assume that the dark matter field S drives inflation.

A detailed study of inflation can be found in [33], see **Figure 3**. Here we present a few comments on the Lagrangian (11):

- For $\lambda_S = 0$ the dark matter particles have a large mass $\approx 10^{13}$ GeV corresponding to cold dark matter. For the observed warm dark matter we need the quartic potential with $\lambda_S > 0$.
- The non-minimal couplings ξ_ϕ and ξ_S to the curvature Ricci scalar R are needed for inflation and also for renormalization [31].

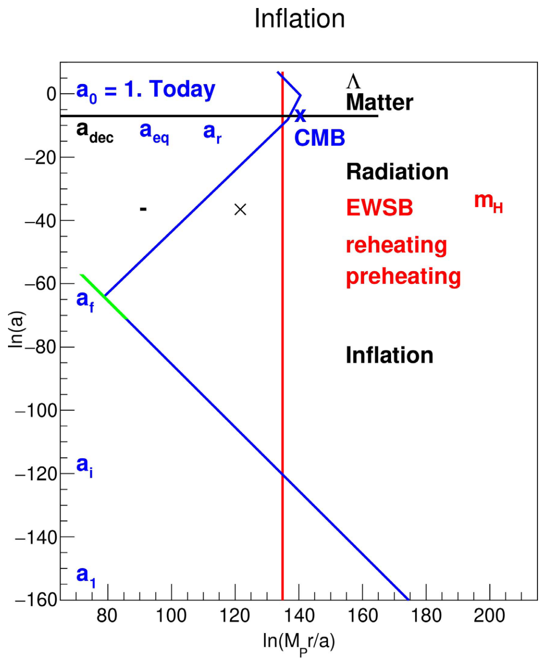

Figure 3. Past and future history of the universe. Vertical axis: natural logarithm of the expansion parameter a, *i.e.,* "e-folds". Horizontal axis: natural logarithm of comoving distances $M_P r/a$ in Planck units. The red line corresponds to a fixed point in space at a reference comoving distance $1/k = 20$ Mpc. The blue line is the comoving distance to the horizon $M_P/(aH)$ in Planck units. The thin green box (with a width too small to be resolved on this scale) is the *prediction* $\left(M_P/(a_f H_f), a_f\right)$ for the end of inflation with the observed values of the comoving power spectrum amplitude $P_\zeta(k)$ and tilt n_s, at 68% confidence, *with no free parameter*! The Cosmic Microwave Background radiation (CMB) that we see today propagates to us from point "X". Symbols "-" and "×" illustrate segments of "light" rays inside and outside of the horizon. Figure from [33].

- The measured comoving power spectrum $P_\zeta(k_{\text{ref}})$ at the reference wave-vector $k_{\text{ref}} = 0.05\ \text{Mpc}^{-1}$, and its tilt n_s [2], pass a closure test with no free parameters [33], *i.e.,* the end of inflation coincides, within observational uncertainties, with the beginning of pre-heating, followed by re-heating and the hot Big Bang cosmology. This coincidence is quite amazing!

Some numerical results are summarized in **Table 1**.

7. A Brief History of the Universe

The problem at hand is under-constrained, so several alternatives are possible. One viable sequence of events follows [4] [16] [33], see **Figure 1** and **Figure 3**. Note that some of the numbers are model-dependent.

1) The dark matter field S is the only field in the early universe. S drives inflation. The reference comoving distance $1/k_{\text{ref}} = 20$ Mpc enters the horizon at expansion parameter $a_i = \exp(-121.28 \pm 0.12)$.

2) Inflation ends at $a_f = \exp(-64.1 \pm 6.8)$.

3) Pre-heating is followed by re-heating, and the hot Big Bang cosmology follows.

Table 1. Inflation parameters obtained from the measured comoving power spectrum $P_\zeta(k_{\text{ref}}) \equiv \Delta_R^2 = \exp(3.044 \pm 0.014) \times 10^{-10}$ at the reference wave-vector $k_{\text{ref}} = 0.05\ \text{Mpc}^{-1}$, and its tilt $n_s = 0.965 \pm 0.004$ [2]. a_i and S_i are the expansion parameter a and amplitude S when the comoving distance $1/k_{\text{ref}}$ enters the horizon. $N = \ln(a_f/a_i)$ is the number of e-folds of inflation, a_f is the expansion parameter at the end of inflation, and r is the tensor-to-scalar ratio. Here we assume that the dark matter field S drives inflation. For ϕ driving inflation replace $S \to \phi$. For details see [33].

Parameter	Value
$N \equiv \ln(a_f/a_i)$	57.1 ± 6.6
$\ln(a_i)$	-121.28 ± 0.12
$\ln(a_f)$	-64.1 ± 6.8
r	$(3.7 \pm 0.9) \times 10^{-3}$
$\sqrt{\xi_S} S_i$	$(1.74 \pm 0.10)\ \text{M}_\text{P}$
$\lambda_S^{1/4} S_i$	$(8.05 \pm 0.03) \times 10^{-3}\ \text{M}_\text{P}$
λ_S / ξ_S^2	$(4.6 \pm 1.1) \times 10^{-10}$

4) S and the Higgs boson ϕ, and hence the gauge bosons $\vec{G}_i$, $\vec{W}$ and $\vec{B}$, and Weyl-L and Weyl-R fields, come into thermal and diffusive equilibrium before $T = T_{\text{EWSB}}$ if $|\lambda_{\phi S}| \gtrsim 7 \times 10^{-7}$. This is the "Weyl" epoch.

5) The Higgs boson ϕ acquires a vaccuum expectation value at the temperature $T_{\text{EWSB}} = 159 \pm 1$ GeV. Weyl_L and Weyl_R fields pair up into massive Dirac fields. This is the "Dirac" epoch. Linear superpositions of $\vec{W}_\mu$ and B_μ become massive W^+, W^- and Z bosons, leaving the photon γ mass-less.

6) All particles of the NMSM (including the sterile neutrinos) reach equilibrium.

7) Coherent CP-violating and lepton number-violating oscillations between a sterile neutrino (with mass of order 3 GeV) and the three active neutrinos (or between two sterile neutrinos if $m \geq 3$) produce a lepton asymmetry. Due to sphaleron transitions [13] [19] [20], part of the lepton asymmetry is converted to a baryon asymmetry.

8) At $T_{\text{sph}} = 131.7 \pm 2.3$ GeV, sphalerons freeze-out, *i.e.,* the conversion rate of anti-leptons to three quarks (for each generation), become less than the universe expansion rate. The lepton and baryon asymmetries freeze-out.

9) At $T = m_H = 125.20 \pm 0.11$ GeV the Higgs boson becomes non-relativistic and decays. S decouples from the remaining fields of the Standard Model sector.

10) Sterile neutrinos N_i are depleted from the universe as soon at the temperature drops below $M_W + M_{N_i}$.

11) Particles and anti-particles annihilate as they become non-relativistic. Baryon-anti-baryon annihilations result in the present day baryon-to-photon ratio $n_B/n_\gamma = 6.12 \pm 0.04 \times 10^{-10}$.

12) The first three minutes: Big Bang Nucleosynthesis forms helium, and other light, nuclei.

13) The light neutrinos decouple. Electrons and positrons annihilate.

14) The warm dark matter particles S, of mass $m_S = 150 \pm 2$ eV, become non-relativistic at expansion parameter $a_{h\mathrm{NR}} = (1.35 \pm 0.23) \times 10^{-6}$.

15) Matter density begins to dominate at $a \approx 1/3403$, and the universe becomes transparent to photons as neutral hydrogen atoms form at $a \approx 1/1091$.

8. Tests

We present a list of observations that may test the "New Minimal Standard Model" or constrain its parameters:

- High resolution weak lensing measurements of the Cosmic Microwave Background (CMB) radiation can measure precisely the *linear* power spectrum of density fluctuations $P(k)$ up to a wave-vector $\approx 40h\ \mathrm{Mpc}^{-1}$ [8]. Such a measurement would definitely rule out or confirm the interpretation that $v_{h\mathrm{rms}}(1)$ of (2) is of cosmological origin.
- Next generation measurements of the CMB power spectrum may reach and constrain k_{fs} of (4).
- Observations and simulations of the Lyman-α forest of quasar light set limits to the warm dark matter free-streaming cut-off wavevector k_{fs}, see, e.g. [6]. Measurements of $v_{h\mathrm{rms}}(1)$ and of k_{fs}, and perhaps also **Figure 2** of [8], are in disagreement with these Lyman-α limits (see Section 10 of [4]). Resolving this discrepancy with new observations and studies will constrain possible extensions of the Standard Model.
- The cores of some dwarf galaxies are dominated by dark matter. Dedicated studies of the cores of these galaxies can further constrain the dark matter adiabatic invariant $v_{h\mathrm{rms}}(1)$, which is of cosmological origin [4] [12]. An example is [35] (that obtains $v_{h\mathrm{rms}}(1) = 515 \pm 15$ (stat) m/s [4]).
- If nature has chosen only two sterile neutrinos, then one of them has a mass of order 3 GeV. Neutrino oscillation experiments, focusing on lepton number violating events, can constrain the parameters in Example 2 of Section 4.
- Constrain or measure the dark matter-dark matter elastic scattering cross-section per unit mass $\sigma(SS \to SS)/m_S$. The current upper limit is 0.47 cm^2/g. From the limit on $\lambda_{\phi S} \gtrsim 7 \times 10^{-7}$ obtained in Section 2, the lower limit is $\approx 10^{-9}$ cm^2/g. Galaxy properties imply that dark matter has self-elastic interactions [12].
- Constrain or measure the power spectrum tensor-to-scalar ratio. The predicted value is $r = (3.7 \pm 0.9) \times 10^{-3}$ at $k_{\mathrm{ref}} = 0.05\ \mathrm{Mpc}^{-1}$ (see **Table 1**), while the current limit is $r < 0.036$ at 95% confidence [2].
- The Higgs boson quartic coupling λ_ϕ is critical. Within the experimental uncertainties of the top quark mass, the Higgs boson mass, and the $SU(3)_c$ coupling constant α_3, λ_ϕ may run negative at high energy [34]. Understanding this near criticality, and tighter experimental constraints, may shed light on the early universe.
- Measurements or tighter limits on the $0\nu 2\beta$ effective Majorana electron-neutrino mass $\langle m \rangle_{ee} = \left| m_1 V_{l e \nu_e}^2 + m_2 V_{l e \nu_\mu}^2 + m_3 V_{l e \nu_\tau}^2 \right|$ may shed light on the Dirac

or Majorana nature of neutrinos [2] [17].

- Re-heating following inflation needs new theoretical ideas.

9. Conclusions

The Standard Model of quarks and leptons is a great achievement. However, this model is incomplete: it does not include the observed dark matter, neutrino masses and mixings, the matter-antimatter asymmetry of the universe, the current acceleration of the expansion of the universe, nor inflation. In 2004, Hooman Davoudiasl, Ryuichiro Kitano, Tianjun Li, and Hitoshi Murayama proposed the "New Minimal Standard Model" to account for these observations [1]. This minimal extension of the Standard Model is valid to this day! The minimal extension of the Standard Model to account for all current confirmed observations is the addition of a scalar warm dark matter field S with Z_2 symmetry $S \leftrightarrow -S$, two sterile neutrinos with both Yukawa and Majorana mass terms to give mass to two of the active neutrinos, and allow lepton number violation needed for baryogenesis via leptogenesis, and Einstein's cosmological constant. See **Figure 2**.

In the intervening twenty years, the top quark and Higgs boson were discovered, and great advances in neutrino physics, cosmology, dark matter, baryogenesis *via* leptogenesis, and inflation have been achieved. As a result, we have been able to explore details of the "New Minimal Standard Model", constrain some of the new parameters, and propose tests.

Acknowledgements

I thank Karsten Müller for many useful discussions, and an anonymous reviewer for detailed and constructive feedback that has improved this article.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Davoudiasl, H., Kitano, R., Li, T. and Murayama, H. (2005) The New Minimal Standard Model. *Physics Letters B*, **609**, 117-123. https://doi.org/10.1016/j.physletb.2005.01.026

[2] Navas, S., *et al*. (2024) Review of Particle Physics. *Physical Review D*, **110**, Article ID: 030001.

[3] Hoeneisen, B. (2022) Measurement of the Dark Matter Velocity Dispersion with Dwarf Galaxy Rotation Curves. *International Journal of Astronomy and Astrophysics*, **12**, 363-381. https://doi.org/10.4236/ijaa.2022.124021

[4] Hoeneisen, B. (2024) Measurements of the Dark Matter Mass, Temperature and Spin. *International Journal of Astronomy and Astrophysics*, **14**, 184-202. https://doi.org/10.4236/ijaa.2024.143012

[5] Lapi, A., Salucci, P. and Danese, L. (2018) Precision Scaling Relations for Disk Galaxies in the Local Universe. *The Astrophysical Journal*, **859**, Article 2. https://doi.org/10.3847/1538-4357/aabf35

[6] Viel, M., Lesgourgues, J., Haehnelt, M.G., Matarrese, S. and Riotto, A. (2005) Constraining Warm Dark Matter Candidates Including Sterile Neutrinos and Light Gravitinos with WMAP and the Lyman-α Forest. *Physical Review D*, **71**, Article ID: 063534. https://doi.org/10.1103/physrevd.71.063534

[7] Iršič, V., Viel, M., Haehnelt, M.G., Bolton, J.S., Molaro, M., Puchwein, E., *et al.* (2024) Unveiling Dark Matter Free Streaming at the Smallest Scales with the High Redshift Lyman-α Forest. *Physical Review D*, **109**, Article ID: 043511. https://doi.org/10.1103/physrevd.109.043511

[8] MacInnis, A. and Sehgal, N. (2024) CMB-HD as a Probe of Dark Matter on Sub-Galactic Scales. arXiv: 2405.12220.

[9] Despali, G., *et al.* (2025) Introducing the AIDA-TNG Project: Galaxy Formation in Alternative Dark Matter Models. arXiv: 2501.12439.

[10] Hoeneisen, B. (2023) Understanding the Formation of Galaxies with Warm Dark Matter. *Journal of Modern Physics*, **14**, 1741-1754. https://doi.org/10.4236/jmp.2023.1413103

[11] Hogan, C.J. and Dalcanton, J.J. (2000) New Dark Matter Physics: Clues from Halo Structure. *Physical Review D*, **62**, Article ID: 063511. https://doi.org/10.1103/physrevd.62.063511

[12] Hoeneisen, B. (2025) Why Do Galaxies Have Extended Flat Rotation Curves? *International Journal of Astronomy and Astrophysics*, **15**, 1-10. https://doi.org/10.4236/ijaa.2025.151001

[13] D'Onofrio, M., Rummukainen, K. and Tranberg, A. (2014) Sphaleron Rate in the Minimal Standard Model. *Physical Review Letters*, **113**, Article ID: 141602. https://doi.org/10.1103/physrevlett.113.141602

[14] Schwartz, M.D. (2013) Quantum Field Theory and the Standard Model. Cambridge University Press. https://doi.org/10.1017/9781139540940

[15] Gel'fand, I.M., Minlos, R.A. and Shapiro, Z.Y.A. (1963) Representations of the Rotation and Lorentz Groups and their Applications. Pergamon Press.

[16] Hoeneisen, B. (2021) Active-Sterile Neutrino Oscillations and Leptogenesis. *Journal of Modern Physics*, **12**, 1248-1266. https://doi.org/10.4236/jmp.2021.129077

[17] Xing, Z. and Zhao, Z. (2020) The Minimal Seesaw and Leptogenesis Models. arXiv: 2008.12090.

[18] Casas, J.A. and Ibarra, A. (2001) Oscillating Neutrinos and $\mu \to e, \gamma$. *Nuclear Physics B*, **618**, 171-204. https://doi.org/10.1016/s0550-3213(01)00475-8

[19] Manton, N.S. (2019) The Inevitability of Sphalerons in Field Theory. *Philosophical Transactions of the Royal Society A*: *Mathematical*, *Physical and Engineering Sciences*, **377**, Article ID: 20180327. https://doi.org/10.1098/rsta.2018.0327

[20] Rubakov, V.A. and Shaposhnikov, M.E. (1996) Electroweak Baryon Number Non-Conservation in the Early Universe and in High-Energy Collisions. *Physics-Uspekhi*, **39**, 461-502. https://doi.org/10.1070/pu1996v039n05abeh000145

[21] Khlebnikov, S.Y. and Shaposhnikov, M.E. (1996) Melting of the Higgs Vacuum: Conserved Numbers at High Temperature. *Physics Letters B*, **387**, 817-822. https://doi.org/10.1016/0370-2693(96)01116-1

[22] Davidson, S., Nardi, E. and Nir, Y. (2008) Leptogenesis. *Physics Reports*, **466**, 105-177. https://doi.org/10.1016/j.physrep.2008.06.002

[23] Klaric, J., Shaposhnikov, M. and Timiryasov, I. (2021) Reconciling Resonant Leptogenesis and Baryogenesis via Neutrino Oscillations. *Physical Review D*, **104**, Article ID: 055010. https://doi.org/10.1103/physrevd.104.055010

[24] Asaka, T. and Shaposhnikov, M. (2005) The νMSM, Dark Matter and Baryon Asymmetry. arXiv: 0505013.

[25] Asaka, T., Blanchet, S. and Shaposhnikov, M. (2005) The νMSM, Dark Matter and Neutrino Masses. arXiv: 0503065.

[26] Akhmedov, E. (2019) Quantum Mechanics Aspects and Subtleties of Neutrino Oscillations. arXiv: 1901.05232.

[27] Beuthe, M. (2002) Towards a Unique Formula for Neutrino Oscillations in Vacuum. *Physical Review D*, **66**, Article ID: 013003. https://doi.org/10.1103/physrevd.66.013003

[28] Halzen, F., Martin, A.D. and Mitra, N. (1985) Quarks and Leptons: An Introductory Course in Modern Particle physics. *American Journal of Physics*, **53**, 287-287. https://doi.org/10.1119/1.14146

[29] Green, D. (2014) Inflation and the Higgs Scalar (Lecture Notes). arXiv: 1412.2107

[30] Baumann, D. (2012) TASI Lectures on Inflation. arXiv: 0907.5424

[31] Bezrukov, F. and Shaposhnikov, M. (2008) The Standard Model Higgs Boson as the Inflaton. *Physics Letters B*, **659**, 703-706. https://doi.org/10.1016/j.physletb.2007.11.072

[32] Shaposhnikov, M., Shkerin, A. and Zell, S. (2021) Standard Model Meets Gravity: Electroweak Symmetry Breaking and Inflation. *Physical Review D*, **103**, Article ID: 033006. https://doi.org/10.1103/physrevd.103.033006

[33] Hoeneisen, B. (2023) Exploring Inflation Options for Warm Dark Matter Coupled to the Higgs Boson. *International Journal of Astronomy and Astrophysics*, **13**, 217-235. https://doi.org/10.4236/ijaa.2023.133013

[34] Buttazzo, D., Degrassi, G., Giardino, P.P., Giudice, G.F., Sala, F., Salvio, A., *et al*. (2013) Investigating the Near-Criticality of the Higgs Boson. *Journal of High Energy Physics*, **2013**, Article No. 89. https://doi.org/10.1007/jhep12(2013)089

[35] Karukes, E.V. and Salucci, P. (2016) The Universal Rotation Curve of Dwarf Disc Galaxies. *Monthly Notices of the Royal Astronomical Society*, **465**, 4703-4722. https://doi.org/10.1093/mnras/stw3055