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Universe Expansion and Gravitation Unified

Antonio Medina[1,2]

[1]Massachusetts Institute of Technology, Cambridge, MA, USA
[2]NASA Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA, USA
Email: puerta@alum.mit.edu

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Abstract

Aims: The expansion of the Universe and gravitation are considered different fundamental properties of the cosmos. We explore whether they are connected and propose an equation that links and unifies them. This equation describes the expansion of space as related to the mass of matter in that space. An expanding frame of reference, defined as expanding at the same rate, is the natural frame for an observer since the observer is part of the Universe and expands with it. In this frame, the expansion of the Universe by itself accounts for phenomena previously ascribed to gravitation. Gravity is fictitious and unnecessary. Here, we show that universal expansion naturally manifests as "gravity". **Methods:** A theory is presented that postulates a natural expansion of the observer's frame of reference due to matter and a generalized universal principle of equivalence of acceleration and gravitation that extends to any frame. It is consistent with both the general theory of relativity and our knowledge about the expansion of the Universe. **Results:** We tested the theory positively by applying its equation to global and local scales of the Universe with available data. It is applied to a Universe of homogeneous mass and discrete two and three-body systems and other phenomena hitherto thought to be unrelated. The results show that the proposed equation is valid at any scale. The fundamental nature of the Universe, from which gravity and other properties are derived, is its expansion.

Keywords

Gravitation, Universe, Cosmological Theories, Visible Universe, Hubble Expansion, Cosmological Models, Flat Space Cosmology, Dark Energy, Dark Matter, Hubble Parameter, Critical Density

1. Introduction

Isaac Newton was the first to propose a mathematical theory of universal gravitation

[1]. It was the prevailing theory for 300 years. Nevertheless, criticism of the theory started right after it was proposed, mainly because of the absurdity of a force between bodies with nothing in between.

Gravity is just a convenient way to describe how the presence of mass causes an object to change its motion. Einstein generalized his theory of relativity by proposing that mass could distort space and time. The general theory of relativity replaces Newton's gravity with the geometry of space itself. The familiar Newtonian idea of masses placed in smooth and uniform space is replaced with the idea of space that is distorted by the masses it contains. Matter curves space and light and particles follow field lines dictated by their mass [2]. The theory is reminiscent of the field theory used in electromagnetism by Maxwell [3].

Albert Einstein got rid of the mysterious force by postulating in his theory of relativity ("General Relativity") that gravity is not a force, but a fictitious effect created by the bending of space. Unfortunately, despite its great advancement, General Relativity replaced the fictitious force with the fictitious concept of "field", which is implicit in his field equations and spacetime metric. He showed, with a good example in a thought experiment, the Einstein "elevator" or "chest" in free fall [4] [5], how choosing the right frame of reference can drastically change how we view motion. For an observer in a Cartesian frame of reference attached to the Earth, the motion of the elevator and its occupants will appear as accelerated motion caused by a force (gravity). For an observer inside the elevator referencing the motion of the occupants to a frame attached to the elevator, there are no forces, they are at rest, just as if they were weightless in the distant space, and there is no gravity. Using the wrong (Cartesian) frame, although intuitive, leads to grave misconceptions. Gravity and acceleration are equivalent. However, Einstein warned that his principle of equivalence does not imply that a gravitational field is always only an apparent one, he believed that "It is, for instance, impossible to choose a body of reference such that, as judged from it, the gravitational field of the Earth (in its entirety) vanishes" [4]. This paper follows up and explains how choosing an accelerated frame of reference, instead of a Cartesian frame, is the right choice to explain the observations attributed to gravity. Not only does the gravitational field of the Earth vanish in this body of reference, but it defines the accelerated frame that is capable of describing universal expansion and gravity seamlessly.

The above theories have something in common. It is the mass of matter that is the foundation of so-called "gravity". This paper goes further to show that mass is the foundation of universal expansion and "gravity" is fictional and unnecessary in an expanding frame of reference.

The Universe is expanding beyond question. Accepted cosmological theories explain the expansion, assuming its mass density is homogeneous and isotropic. They are therefore inapplicable to small-scale regions with discrete mass. It is an extended belief that although galaxies and clusters are moving farther apart, their internal gravity keeps them from expanding in size. These regions of non-expanding space are well described by Newton's laws. There are problems with this view. The main one is that it considers gravity as a force opposing the expansion, which is not compatible with General Relativity, in which gravitation is a fictitious force.

According to the General Relativity principle of equivalence, results from experiments in a uniformly accelerated reference frame cannot be distinguished from those in a uniform gravitational field. The principle suggests that mass is the reason for the curvature of space and gravity. The fact that a uniformly accelerated reference frame cannot be distinguished from those in a uniform gravitational field is an indication that gravity and an accelerated reference frame are intimately related, if not the same thing.

This report suggests that space and the matter therein are expanding due to the mass of that matter. It shows that Hubble expansion and gravity merge when observed in an accelerated expanding frame proposing a single equation for the acceleration of expansion applicable to large- and small-scale Universe.

2. Methods

The equation for universal expansion is formulated and its predictions are tested with available data. The assumption is that the acceleration of universal expansion $x(r)$ is dependent on mass.

2.1. Proposed Equation for Expansion of Space

We propose that the rate of universal expansion in a volume is related to its mass density.

$$\vec{x}(r) = Gr^n \iiint_V \rho \, \mathrm{d}V \tag{1}$$

where $\vec{x}(r)$, vector acceleration is the rate of expansion of the space in the volume V at any point at distance r from the center of mass in that volume containing mass of density ρ. The acceleration at point r depends on the mass distribution in volume V. Its unit is m/s^2 in the International System of Units (SI).

r is the vector distance from the center of mass to any point in volume V.

G is the universal gravitational constant, equal to 6.67×10^{-11} kg^{-1} m^3 s^{-2} in the International System of Units (SI).

n is the power exponent that will be shown below to be $n = -2$.

ρ is the mass density in volume V, which can be homogeneous or variable with position and time.

$\mathrm{d}V = r^2 \sin\theta \, \mathrm{d}r \, \mathrm{d}\theta \, \mathrm{d}\varphi$ and $0 < r < \infty$, $0 \le \theta \le \pi$, $0 \le \varphi \le 2\pi$, in standard spherical coordinates chosen in this paper.

The integral $\iiint_V \rho \, \mathrm{d}V$ evaluates the mass distribution in volume V. The density is a function of r, θ, φ, and t (time). Therefore, in general, x is also a function of θ, φ, and t but not in the particular cases analyzed here. In the three-body analysis below we avoid the change with time of the mass distribution using a rotating frame. This paper is an introduction to the unifying equation and as explained in the discussion it does not attempt to solve the equation but for simple time invariant cases.

For discrete spherical massive bodies, the integral $\iiint_V \rho \mathrm{d}V$ is simply the mass of the body, and the acceleration x outside the body is determined by the mass and the distance r to the center of mass. To calculate the acceleration inside the mass or in any volume in the Universe we need to calculate the integral and need to know the mass density in that volume, which may be a function of r and the angular coordinates. An alternative to calculating the integral in some cases is to determine the enclosed mass in a radius r with some simpler means. For example, we will calculate the expansion of the Universe at a distance r from us from the Universe's mass density assuming the Universe is homogeneous and isotropic.

For simplicity and to build a spherical comoving frame, we will choose here the volume to be a sphere of radius r and will place the center of our spherical coordinate system at the center of mass, making r our radial coordinate.

The choice of n is not arbitrary, it must be −2 to satisfy the dimensional formula for acceleration $x(r)$ in (1) and also for the acceleration of the Universe to be proportional to r as shown in **Appendix B** and below. A homogeneous isotropic Universe requires $x \propto r$. Therefore, (1) becomes:

$$\vec{x}(r) = Gr^{-2} \iiint_V \rho \mathrm{d}V \tag{2}$$

This equation links for the first time the expansion of space to the mass of matter in that space and describes how matter, including all celestial bodies, moves. A "gravitational force" or "gravitational field" is superfluous, fictional, and unnecessary. We will see that the equation defines a natural metric and expanding frame of reference in which distances do not expand, but change based on laws of motion. We will also refer to this Equation (2) as the expansion integral, the law of universal expansion, or universal expansion.

For a discrete single mass M or a homogeneous mass density ρ the integral of the density in a spherical volume of radius r containing all the mass is M and therefore (2), reduces to Newton's acceleration of gravity:

$$\vec{x}(r) = GM/r^2 \tag{3}$$

A second body at distance R_0 from the center of the body of mass M has a local expansion acceleration equal to the tidal acceleration at distance $\mathrm{d}r$ from R_0. Because axial and horizontal tidal acceleration, calculated as $GM\mathrm{d}r/R_0^3$ and $-2GM\mathrm{d}r/R_0^3$ below, are linear with $\mathrm{d}r$ the expansion is isotropic.

2.2. Frame of Reference

According to the expansion integral, all points in a massive body expand according to their location in the frame from its center of mass and the expansion is a scaling factor. Our choice of frame of reference suits how we explain observation and defines the metric by which distances are measured. A comoving frame of reference that expands according to this equation is most suitable to describe observation since the observer, attached to the frame, also expands according to the equation. Since the frame expands at the same rate the mass and mass density in the volume integral do not change their amount or location with expansion in this comoving frame. The expanding frame defined here, as any other reference frame, does not drag other matter in the frame with it. Their location is determined by laws of motion. For example, we will see that the Moon is in inertial motion in a near-circular orbit around Earth, a geodesic in our expanding frame. The distance from Earth to the Moon and their radii as well as an observer will expand as seen in a Cartesian frame. However, there is no expansion in a comoving frame.

We will define the comoving frame of reference in which coordinate r is comoving distance. The expansion of the frame at distance r from the center or origin of coordinates is determined by Equation (2). An observer expands, as everything else attached to the frame including his measuring rod, so she cannot measure or see expansion. For this reason, such a comoving frame is most appropriate to describe observation. Nothing expands because the metric expands with the matter therein, comoving coordinates do not change with time or expansion. A better way to state that it is impossible to observe the expansion is that it simply does not exist in that frame of reference, making it an artifact in other frames. It is implicit that the expansion of bodies here is referred to a Euclidian space with Cartesian metric only.

We can place the frame at any place where there is an observer and a chosen mass and volume. The frame can be centered anywhere because the expansion is isotropic. For convenience and simplicity, we will place the center of expansion in the examples here at the location of the center of mass and assume it is homogeneous except where indicated.

Each point in this frame is determined by four coordinates, t and (r, θ, φ).

The expansion of the visible Universe can be calculated with (2) by integrating its mass. Below we will test whether the calculated value is confirmed by observation.

2.3. Application of the Expansion Integral to the Universe and Solar System

We next apply the expansion integral to several representative cases.

2.4. Earth's Surface Acceleration

Let's start with Earth's apparent gravity selecting V to be a volume containing Earth, say any sphere of radius r equal or greater than Earth's radius, the volume integral is equal to M_E, the mass of Earth.

An object on Earth's surface is not "pulled" to it by gravity, but our expanding frame at the Earth's surface is accelerating together with the Earth which is attached to it. The acceleration is g which is a function of Earth's mass M_E and radius R_E. The expansion integral reduces to $g = GM_E / R_E^2 = 9.8\ \text{m/s}^2$. Any object on the surface of Earth has the same acceleration and a force (weight) corresponding to that acceleration and the mass of the object.

In our comoving frame, an object on the Earth's surface has coordinates (R_E, θ, φ). The object is accelerated because it is attached to Earth and to the frame that accelerates at a rate of g at its location.

Any object on the surface of Earth has the same acceleration g because that is the frame acceleration, their weight is determined by their inertial mass with Newton's equation $F = mg$. There is no gravitational mass, only inertial. The mystery of why inertial mass is equal to gravitational mass disappears.

2.5. Body Falling to Earth

A body at distance $r_e(t)$ from the center of Earth will have comoving frame coordinates (r_e, θ, φ), the body is at rest in an inertial frame, but in our comoving frame, as it expands, it will move in a straight line towards the center of mass, or "fall" with acceleration $g_t = GM_E/r_e^2$ or $g = GM_E/R_E^2 = 9.8\ \text{m/s}^2$ if $r_e \approx R_E$. See Figure 1. No acceleration can be measured with accelerometers attached to the body; the proper acceleration is zero. However, the body is seen as falling with acceleration g from observers in the comoving frame. Its mass or any other property is irrelevant since its motion is dictated by the expansion of the frame alone.

Figure 1. Falling from the tower of Pisa demonstrates the radial expansion. An observer in the expanding frame will see the girl falling with acceleration g. The "falling" girl can verify that she is at rest, weightless with no acceleration in a frame attached to her, and see that the Earth approaches her with acceleration g. The girl's view is simple and correct. See the discussion section for details.

The straight lines in Figure 2 illustrate his motion. Since the frame is expanding with acceleration, a body at inertial rest moves along the radial straight lines with the opposite acceleration of the frame at the body's location.

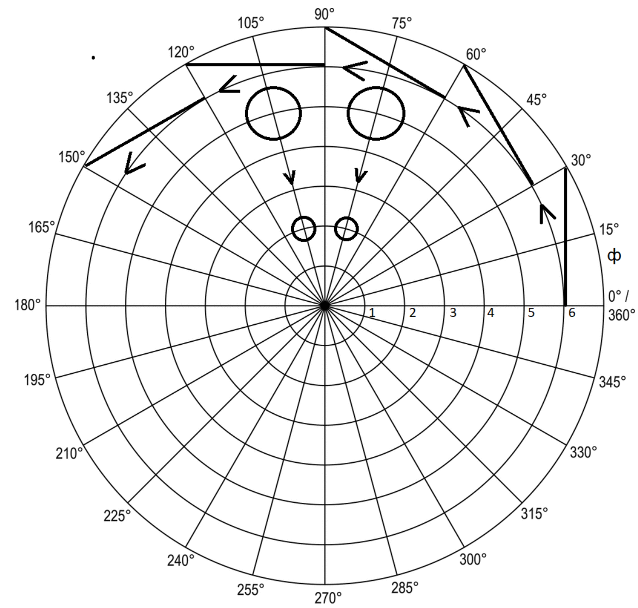

Figure 2. Spherical coordinate system used in our reference frame for central body and satellite, illustrating infinitesimal incremental motion of a satellite. This frame uses comoving coordinates (r, θ, φ) that are unaffected by expansion. The circles of radius r also denote constant radial acceleration x, as $x \propto 1/r^2$. Radii and concentric circles are geodesics representing "free fall" or circular orbits. Polar view section at $\theta = 90^\circ$. Two horizontal balls are also depicted falling towards the central body, e.g. Earth.

2.6. Einstein's Elevator in Free Fall

We can evaluate motion as seen in a comoving frame as defined here and as seen in the frame that Einstein placed in his free-falling elevator. The free-falling frame is inertial because it has no acceleration. The observer in a frame attached to Earth is not in an inertial frame since it has acceleration g and must resort to a fictitious gravitational force to explain observation. The elevator frame is preferred to describe observation because it moves canceling the acceleration seen in the Earth frame and obviates gravitation. It is quite the same in the comoving expanding frame. It describes motion more simply without gravitational forces or fields.

The experimenters inside Einstein's elevator had two limitations. First, they could not see outside the elevator, and second, they were limited to a very small space where tidal forces could not be detected. We next remove those limitations in a free fall from the tower of Pisa experiment.

2.7. Tower of Pisa Free Fall Experiment

A simple thought experiment will generalize the results of Einstein's elevator experiment. We will call it the tower of Pisa experiment in honor of Galileo who is believed to have conducted his free fall experiments there. It is not only a thought experiment; anybody can do it. See Figure 1. If we fall from the top of the tower, we will notice during our fall that in our inertial frame, we are at rest, there is no gravity or acceleration measurable by any means, as in Einstein's falling elevator.

But unlike the elevator where we could not see the outside, we can now see the Earth approaching our rest location with the acceleration $x = g$ given in (3). The Earth is expanding since the same experiment could be done simultaneously at the antipode or any other location with the same result. From this inertial frame, we can detect the radial acceleration of the Earth's surface.

This report makes the case that we are attached to the expanding frame, we do not need gravitation, falling bodies are seen accelerating at the opposite rate of the expanding frame and, with that knowledge, we can conclude that they are at rest with zero net, or proper, acceleration. In the expanding frame, the Earth does not expand.

With a second experiment, we can test for tidal effects by dropping with us a body consisting of two balls connected by a vertical rod. While on the tower, according to classic gravitation, the vertical rod experiences a distending tidal force in the expanding frame because the balls are located at different radial distances. Two balls connected by a horizontal rod can also be used, the tidal force would be inwards instead. When "falling" the rod has no tension since both balls are at rest without acceleration and motionless in the inertial frame attached to them, removing the rod will not alter the distance between the balls since both have zero acceleration and zero velocity relative to each other. The fall is truly inertial, there are no tidal forces at any scale. How we see the fall in the expanding frame is elaborated in the next section "Tidal effects".

This theory is consistent with the principle of equivalence and the realization that the acceleration and gravitational force in a falling elevator observed in a non-inertial frame disappear when we attach the frame to the elevator. However, Einstein warned that his principle of equivalence does not necessarily imply that a gravitational field is always only an apparent one [4]. Einstein believed that the principle of equivalence would only hold in a local, or small region of space where the acceleration due to gravity is substantially constant. This is so because gravitational tidal effects could be used to differentiate a gravitational field from a uniform acceleration caused by an external force propelling the elevator in space. In the frame of reference defined by Equation (2) gravity is always apparent, there is no gravitational field. Furthermore, the principle of equivalence in a free-falling frame is not restricted locally. Equation (2) is in part based on inspiration from Einstein's equivalence principle, *i.e.* "there is no observable distinction between the local effects of gravity and acceleration" but goes a step further showing that free fall is truly inertial and it is possible to have a frame of reference such that the gravitational field of the Earth (or any other body) vanishes not only locally, but at a global scale. It shows that acceleration and gravitation are fully equivalent and leads to the universal principle of equivalence, defined below.

2.8. Tidal Effects

In classical and General Relativity there are tidal effects if the gravitational field is not uniform. Only in small regions of space, we can approximate the field of a massive body as being uniform and having constant acceleration, such as a small volume on the surface of Earth where acceleration is *g*. The expansion integral eliminates tidal effects in homogeneous matter because the local acceleration of the expansion is the same as the tidal acceleration. This is similar to the length contraction of special relativity, which is not visible to the moving, and contracting, observer.

An observer in free fall will not experience any tidal effects in her inertial frame. She, and everything around her in free fall, is at rest and gravity does not exist. Free-falling experimenters cannot detect any proper acceleration, gravity, or tidal effects as demonstrated in the second experiment at the tower of Pisa above. An observer attached to the expanding frame, *i.e.* any observer on Earth, will observe tidal effects in non-homogeneous matter such as at the interface of the Earth's continental crust with ocean water, orbiting bodies, and falling bodies.

If we have two points in a body separated by distance dr, falling to a massive body, say Earth as in the second tower of the Pisa experiment, the tidal acceleration dx between the two points along the radial axis in the comoving frame for a region around R_E is the gradient of the radial acceleration and calculated by differentiation of (3) with respect to r at $r = R_E$, $\mathrm{d}x/\mathrm{d}r = -2M_E G/R_E^3$, or $\mathrm{d}x = -2M_E G\mathrm{d}r/R_E^3$. The tidal acceleration perpendicular to the radial axis can be found to be half, or $M_E G\mathrm{d}r/R_E^3$. This is the tidal acceleration that we observe in our Earth frame. Tidal acceleration is a linear function of dr and the expansion is therefore also linear, or a scaling factor.

2.9. Relativity of Expansion

We have seen above that the local acceleration of expansion defined by (3) is the same as the tidal acceleration, $\mathrm{d}x = K\mathrm{d}r$ where K is a constant equal to g/R_E for our horizontal balls. Since the acceleration is proportional to the distance dr between local points these points will keep their relative position and will simply scale. Therefore, an observer on Earth will see the center of the balls approach as they fall and also the radii of the balls shrink proportionally. The falling inertial observer will not see any change because she carries her metric in her inertial frame. If the balls are 1 m in diameter and spaced by 1 m she can use the diameter of the balls as her measuring rod to measure distances. In that metric, the Earth expands and the balls, their distance, and spacing do not. The Earth observer has the illusion that the Earth does not expand because her metric expands with it, and sees the distance between balls contracting, explaining it as a gravitational tide. Figure 2 depicts the relative size of the balls as seen falling towards Earth. Of course, the tidal acceleration is minuscule, 1.2×10^{-8} m/s^2 for balls 1m apart, which results in an approach and contraction of 6.7×10^{-8} m after a fall of 3 seconds from the tower of Pisa.

2.10. Orbiting Bodies

As an illustrative two-body motion example we analyze the Earth-Moon motion according to Equation (2). A satellite or Moon orbiting Earth in a circular orbit of radius r_S will not fall straight to Earth as in the previous case because it has a velocity perpendicular to the vector r. As the satellite moves, free of gravitational forces, it distances from Earth's center in a non-accelerated frame. Incrementally the satellite after Δt has moved Δr away from the center of Earth in such a frame. For a given velocity at an angle with respect to the r vector, say 90 degrees, the distance it moves radially in an infinitesimal increment of time is the same as the expansion of the accelerated frame defined here. See **Figure 2**. This Δr is the distance between the two outer circles in **Figure 2**. The analysis is equivalent to Newton's description of the falling orbiting Moon. He showed in p. 453 of his Principia Mathematica [1] that in a given time the Moon falls due to gravity a distance towards Earth that is the same as the deviation of the orbit from a straight line. For the same reason, in our comoving frame, the satellite or Moon remains at the same distance from Earth because the frame expands. The satellite's orbit obeys the familiar laws (Kepler/Newton) for this equality to hold. The satellite, as seen by an observer in the expanding frame is moving in a circle around Earth. Elliptical orbits are also possible as discussed below.

A satellite in a circular orbit at a distance R_0 from the center of Earth, or more exactly the barycenter, will have comoving frame coordinates (R_0, θ, φ) and will follow a geodesic circle.

Since we are not restricted to the Earth as the central body, we will refer to M_E simply as variable M. If the angular velocity of a satellite of period T satisfies $\omega^2 = GM/R_0^3$ where $\omega = 2\pi/T$, then the time it takes for the satellite to recede a distance Δr in the non-expanding frame is the same as it takes the expanding frame to expand the same distance. As seen in the comoving frame, at the incremental position $\Delta\varphi$, exaggerated as 30 degrees in **Figure 2**, the direction from where the satellite moves, is tangential because while it moves $\Delta\varphi$, r remains constant. The trajectory indicated by arrows is exactly correct and the orbit is circular. If the velocity is greater than required by the above equation (but perpendicular to r) the orbit is elliptical and the starting point is the perigee. If it is smaller and perpendicular to r the orbit is elliptical and the starting point is the apogee. Otherwise, the orbit is elliptical and the starting point is arbitrary.

The lengths of the straight segments in **Figure 2** (tangents to the circle) represent space traversed by the satellite in an incremental time Δt, equal to the "fall" time. If it is not equal, then the trajectory does not follow a circle. For example, if the time segment is greater than the fall time then the next segment will start at a further distance from the center of mass M, and the velocity will decrease because the space traveled in that time is less as seen on the radial frame. The velocity will slow till it reaches the apogee, then increase to reach the starting point (perigee) with the same original velocity in magnitude and direction.

An alternative way to see that circles and ellipses are geodesics, without resorting to Newton's falling Moon parallel is as follows. In the general case, at a given time the satellite has an arbitrary velocity at an angle with the tangent of a circle of our comoving coordinates. This velocity vector can be decomposed into tangential and radial components. The radial component will make the satellite move towards or away from the orbited body. As this happens, say it falls, it will reach a radial distance R_0 where the tangential velocity is the appropriate one for circular orbit because it satisfies the equation above. Passed this middle point R_0 a distance r' (r' is the vector distance from R_0 and $r = R_0 + r'$) the satellite will tend to increase its radial distance r since the velocity is too high for a circular orbit of that radius. It is easy to see that the radial acceleration at position r' is proportional to r' because $\mathrm{d}^2 r'/\mathrm{d}t^2 = -GMr'/(R_0 + r')^3 \approx -GMr'/R_0^3$. See **Appendix A**. The solution of this differential equation is a sine function, which we can decompose in an x and y parametric equation for an ellipse. See Equation (A) in **Appendix A**.

Circular and elliptical orbits are geodesics because motion along them continues unchanged indefinitely if undisturbed. No forces are applied to maintain the motion and no external forces nor acceleration can be detected in bodies moving along them. Centrifugal and gravitational forces do not cancel each other; they simply do not exist. The horizontal and vertical components of orbital motion are sine and cosine functions, indicating simple harmonic motion and the parametric equation of an ellipse.

2.11. Calculation of Acceleration in Three-Body System

Only the basic idea underlying this theory is presented here. To test that it is correct we used basic examples of classical two-body problem, where the expansion integral is trivial to evaluate as there are no variables other than r. The time variable has also been largely ignored for the same reason. We will nevertheless venture into the three-body problem for particular cases. If the position of the third body with respect to the other two does not change, we do not need to solve the integral accounting for the change of enclosed mass with position and time. It is easy to derive the position of the five Lagrange points using (2) as we can choose a volume for the integral where the distribution of mass enclosed is constant.

To solve the three-body problem in general we need to evaluate the expansion integral for two bodies, say the Earth and the Moon. This integral has an analytical solution when the third body has negligible mass. The acceleration is now a function of r, θ, φ, and t. We can also solve numerically for $\theta = 90^\circ$ (coplanar orbit) and for the instant $t = 0$ or in a frame of reference affixed to or rotating with the Moon around the barycenter, since the time variation is simple rotation around the central body.

A numerical solution is easy to obtain by calculating the superimposed acceleration due to each body and the radial acceleration of the rotating frame. In the plane of the orbits for any position (radial vector distance r from the barycenter) the acceleration due to both bodies is the vector sum of the acceleration due to each body.

We can use superposition,

$$\vec{x}(r) = GM_1/r_1^2 + GM_2/r_2^2$$

where the masses of the two bodies and the radial vector distance r to their center of mass are designated with sub-indices 1 and 2, all terms are vectors, and + denotes vector sum.

The solution for the bodies Earth and Moon is plotted in **Figure 3**. The axes are in km × 10^5. The color of the equal acceleration lines indicates the acceleration value, decreasing from yellow to violet (zero acceleration). These lines are the geodesics of a third body satellite and indicate its possible orbits around the other two, used as reference.

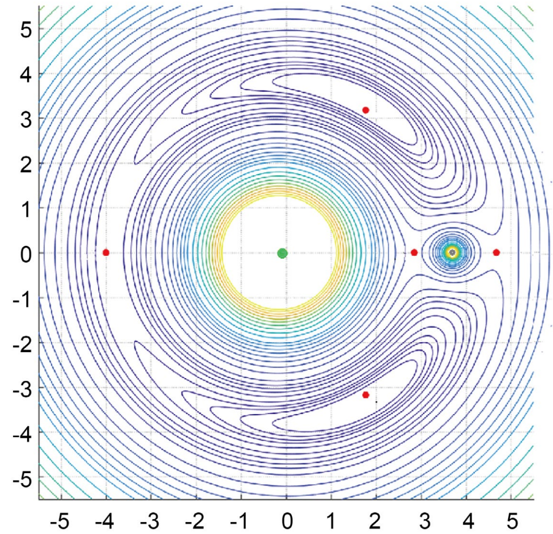

Figure 3. The solution of a centrally restricted three-body problem in the Earth-Moon system is obtained by numerical evaluation of the expansion integral (2) for these two bodies. The curves are the possible geodesics or orbits of the third body. Axes are km × 10^5, centered at the barycenter. Each color indicates a constant magnitude acceleration, decreasing in a yellow-red-green-blue-violet scale, the direction of the acceleration is perpendicular to the curve's tangent. The green dot is Earth and the grey one is the Moon. The red points, of zero acceleration, are the Lagrange points.

2.12. Acceleration of the Universe

Equation (2) says that the expansion of the Universe depends on its mass. Hubble's law says that it depends on distance. Here we show that both are compatible. Hubble law is a particular case in this theory.

We derive here the acceleration x of the Universe at a distance r from Earth. The comoving frame extends now to a comoving coordinate system for any volume of the visible Universe.

From Equation (2):

$$\vec{x}(r) = Gr^{-2} \iiint_V \rho \mathrm{d}V = Gr^{-2} \rho V$$

where V is the volume of the Universe at radius r from Earth and ρ is its density.

Since the volume of a sphere of radius r is $V = 4/3\pi r^3$,

$$\vec{x}(r) = Gr^{-2} \rho 4/3\pi r^3 = 4/3\pi G \rho r \tag{4}$$

If the present density of the Universe is the critical density $\rho = \rho_c = 3H^2/8\pi G$, where H is the present Hubble constant, also known as H_0, substituting ρ_c in (4):

$$\ddot{x}(r) = 1/2\,H^2 r \qquad (5)$$

Equation (5) derived from this theory is identical to Equation (B) in **Appendix B**, which is derived from Friedmann's equations. It shows that $x \propto r$, as it is required in a homogeneous isotropic Universe.

We can also see, equating (4) and (5), that:

$$H^2 = 8/3\,\pi G \rho_c \qquad (6)$$

Equation (6) is the same as Friedmann's acceleration equation for a flat Universe [6]. It shows that H and G are related, and we can calculate H from this equation.

2.13. Behavior of Body of Same Density as the Surrounded Volume: Archimedes' Principle and Hydrostatics

This situation represents the motion of a body that has the same density as the volume we are integrating. This could be an object in hydrostatic equilibrium or motion with the Hubble flow in the large-scale Universe.

Let's consider a planet of liquid composition, or with a large amount of water like Earth, in which an object of the same density, like a fish, is immersed and therefore in hydrostatic equilibrium. The fish, even if not swimming, has the acceleration of the expanding frame because it is part of the integrated mass to which the frame is attached. If a frame of reference is attached to the fish it will be equally accelerated, and therefore no acceleration could be detected in this frame as in the free fall or Einstein elevator case. This frame is inertial. The fish is therefore subjected to no forces; it has no weight. The classical explanation, as formulated by Archimedes, that its weight is canceled with an equal "flotation" force is unnecessary and fictitious. Any frame that moves with the Hubble flow in an expanding homogeneous Universe is inertial for the same reason.

Archimedes' principle was found empirically. Inherent in Archimedes' principle is the assumption that fluids have a pressure gradient linear with depth. Some efforts to prove it using hydrostatic laws are circular arguments since those laws are equivalent to the principle as they also assume a pressure gradient and in effect, the principle. We derive Archimedes' principle for an arbitrary body of any density in **Appendix C** where the only assumption is the universal principle of equivalence defined below. This derivation requires minimal mathematics and does not require either gravity or empiric assumptions. Not only gravity is unnecessary, but the law of universal expansion explains it in simpler terms.

3. Discussion

Expansion of Space

Friedmann showed mathematically for the first time that the Universe may be expanding on a large scale [6]. From Equation (2), we show with considerably less mathematical complexity the same equation derived by Friedmann for the expanding Universe. We have derived the equation showing that H and G are related by $H^2 = KG$ where $K = 8/3\pi\rho_c$. The value of H can be calculated using this equation, matching the observed value. The Friedmann equation and cosmology models assume that gravitation is counteracting the expansion of the Universe, which our model does not, therefore the assumption of critical density in (5) and those in **Appendix B** are unnecessary. We simply showed that with the same assumptions, we get the same results for the purpose of testing this theory.

Hubble [7] and others have observed the expansion of the Universe at a large scale. Hubble's empiric law confirms the Friedmann equation as shown in **Appendix B**. It is a particular case of the law of universal expansion when the volume is the visible Universe.

It may appear to some that such expansion, as described here, will make bodies expand to enormous sizes and reach huge speeds. The same criticism applies to the known expansion of the Universe, and Einstein's falling elevator which will also reach huge speed. But it is only an artifact of the choice of frame. Only observers that are detached from the expanding frame can detect the expansion. In the comoving frame, to which we are attached, bodies do not expand. For the particular case of a single massive body illustrated in **Figure 2**, the radial acceleration on concentric spheres is constant but the radius does not change, it is also constant. The equation for acceleration in General Relativity similarly includes a term related to the metric, making acceleration without expansion possible.

Any frame of reference, accelerated or not, can be used to describe observation accurately. This is the "general principle of relativity" [8]. If we account for the acceleration of the frame, a non-inertial frame, such as a rotating or expanding frame, is the preferred frame, as demonstrated by examples of both types in this paper. The frame of reference is only a way for an observer in the frame to describe the motion of objects in space accurately according to his observation.

All laws of physics apply in our comoving frame of reference because its acceleration is precisely defined. The Galilean concept of rest does not apply; instead, an object is at rest if it has null acceleration when falling, when in orbital motion, or when flowing with the expanding frame. No gravitational force or field is needed.

4. Conclusions

We have seen that the simple Equation (2) can correctly predict the weight of an object on a massive body like Earth, the fall motion, the orbits, universal expansion, and other phenomena, all without resorting to gravitation. In the expanding Universe without gravity, free fall is simply the inertial motion of objects that do not flow with the expansion.

According to this equation, the mass of matter is the suggested cause of universal expansion and also the apparent gravitational attraction of matter. Gravity is a fictitious manifestation of the expanding Universe. Classically, gravitation is an attractive force and the expansion of the Universe and the observed galactic recession are considered unrelated to gravity but caused by dark energy. Furthermore, many consider them opposing forces and believe that the future of the Universe depends on which is greater. There is no known cause for gravity or expansion. Equation (2) suggests that they are both the same manifestation of matter and shows that gravity is an apparent, non-existing force. It may be that the existence of dark energy is also unnecessary since the expansion of the universe is not opposed by gravity.

4.1. Universal Principle of Equivalence

We can now postulate the *universal principle of equivalence* of gravity and acceleration. *In any volume, the acceleration attributed to a gravitational field is indistinguishable from the acceleration of universal expansion.* Universal expansion is defined by Equation (2), and we have seen that it coincides with Hubble expansion in any large-scale volume of the Universe.

We only need to provide the correct acceleration to a frame of reference to explain gravity. Orbiting bodies are shown to be following geodesics that are conic sections as classically known. The geodesics are local trajectories determined by matter. The Universe expands at a different rate in local frames because the distribution of matter can be concentrated and discrete while it is considered uniform density in large-scale space.

4.2. Connection with General Relativity

We have presented only the outline of a theory and application to particular situations that reduce the expansion integral to simple equations. The intent is to show that it is a valid concept with a minimum mathematic deployment. We avoided terminology and equations from specific theories, including General Relativity, and do not attempt here to make elaborate connections to current theories. The purpose of this paper is to demonstrate the plausibility and validity of a theory that unifies universal expansion and gravity using simple tools. For this reason, we have described only a few situations that are correctly predicted by the theory. However, many other phenomena can be equally explained.

Einstein developed his General Theory of Relativity based on two assumptions: 1) The laws of nature are the same in different frames of reference (where acceleration is taken into consideration) and 2) Gravitational mass is equivalent to inertial mass. As we have seen above, we only need to account for the acceleration of the frame to apply physics laws in our expanding frame. The second assumption is unnecessary; it is a consequence rather than an assumption. What has been called gravitational mass is in fact inertial mass, so it is no surprise that they were found experimentally to be equal. This theory shows for the first time that what has been called gravitational mass is necessarily identical to inertial mass.

We have avoided on purpose the variation with time of the mass density distribution in the examples presented. The dependence on time would complicate obtaining a solution to the expansion integral (2), and it is beyond the scope of this paper. Whether relativistic phenomena are explained with the expansion integral when time is considered is a question that is not reached here. It is, however, straightforward to see that there is time dilation qualitatively in the expanding frame. If we send a photon from different circles in Figure 2, it is evident that the one from the higher acceleration circle (shorter radius or closer to the mass) will take longer to reach the same distance compared to one on a circle of lower acceleration because the distance traversed, or frame has expanded more. Therefore, time runs more slowly with acceleration or equivalently, near massive matter.

Einstein wrote, "One now has to remember that by our knowledge 'matter' is not to be perceived as something primitively given or physically plain". We can now add that by our knowledge, matter is the source of the expansion of the Universe and the fiction of gravity.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A

Derivation of Elliptical Orbits

We show that the orthogonal components of the radial distance of orbits allowed by (3) represent harmonic simple motion conforming to elliptical orbits.

The magnitude acceleration at the radial distance r (where $r = R_0 + r'$) is the acceleration due to a circular angular velocity, $R_0^4 \omega^2 / r^3 = GMR_0 / r^3$ (angular velocity ω *satisfies* $\omega^2 = GM / R_0^3$ as discussed in the text) minus the acceleration of the expanding frame:

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = \frac{GMR_0}{r^3} - \frac{GM}{r^2} = \frac{GM\left(R_0 - r\right)}{r^3}$$

Substituting $r = R_0 + r'$,

$$\frac{\mathrm{d}^2\left(R_0 + r'\right)}{\mathrm{d}t^2} = \frac{-GMr'}{\left(R_0 + r'\right)^3}$$

$$\frac{\mathrm{d}^2 r'}{\mathrm{d}t^2} = \frac{-GMr'}{\left(R_0 + r'\right)^3} \approx \frac{-GMr'}{R_0^3} = -\omega^2 r'$$

Since $\omega^2 = GM / R_0^3$ for nearly circular orbits.

We can see that a sine function satisfies this differential equation.

As an alternative to solving this differential equation, we just need to recall that a motion where the acceleration is proportional to displacement is a harmonic simple motion. We then have:

$$r' = A \sin \omega t$$

$$r = R_0 + \sin \omega t$$

A is a constant that represents the amplitude of the satellite's oscillation around the circular radius R_0 along r. If we decompose this radial oscillation of magnitude r, and R_0 into their orthogonal components x and y, recalling from the text that the components of R_0 and r' are in opposite phase:

$$x = R_0 \sin \omega t + A \sin \omega t, \quad y = R_0 \cos \omega t - A \cos \omega t$$

$$x = \left(R_0 + A\right) \sin \omega t, \quad y = \left(R_0 - A\right) \cos \omega t \qquad \text{(A)}$$

Equation (A) is the parametric equation of an ellipse.

Appendix B

Conversion of the Hubble Parameter *H* to Acceleration *h*

Derivation of $h = 0.5 H^2 r$, where h is Hubble acceleration, H in this appendix is Hubble parameter, a function of time, and r is radial distance.

Hubble law states that a galaxy at distance r from Earth recedes with velocity v:

$$v = Hr$$

Acceleration h is the derivative of velocity, so:

$$h = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}\left(Hr\right)}{\mathrm{d}t}$$

$$h = \frac{r\mathrm{d}H}{\mathrm{d}t} + \frac{H\mathrm{d}r}{\mathrm{d}t}$$

In the Standard Model of Cosmology, Hubble parameter change over time is expressed as:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = -H^2(1+q)$$

where q is the deceleration parameter.

Substituting it into the acceleration expression gives:

$$h = -rH^2(1+q) + \frac{H\mathrm{d}r}{\mathrm{d}t}$$

Since $\mathrm{d}r/\mathrm{d}t$ is velocity:

$$h = -rH^2(1+q) + Hv$$

Substituting $v = Hr$, gives:

$$h = H^2 r - H^2 r(1+q)$$

The term $H^2 r$ cancels by operation, resulting:

$$h = -qH^2 r$$

The current estimate is $q \approx -0.5$ [9], so the equation above becomes:

$$h = 0.5H^2 r \tag{B}$$

Appendix C

Derivation of Archimedes' Principle with the Expansion Integral

If a submerged body of volume V is of density ρ_1 different from the density ρ_2 of the surrounding volume, then:

$$\rho_1 = \rho_2 + \Delta\rho = \rho_2 + (\rho_1 - \rho_2)$$

We have seen above that the volume of density ρ_2 confers no weight, so only a volume of density $\rho_1 - \rho_2$ needs to be considered. The force f on such volume according to the expansion integral is:

$$f = (\rho_1 - \rho_2)Vx$$

where x is the acceleration of the expanding frame at that location. At the Earth's surface $x = g$, and force f_E is:

$$f_E = (\rho_1 - \rho_2)Vg = \rho_1 Vg - \rho_2 Vg \tag{C}$$

which is Archimedes' principle.

Since according to Equation (4) the acceleration in matter of uniform density varies linearly with r, it is straightforward to conclude that hydrostatic pressure must vary also linearly with depth.