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Macroscopic Traversable Wormholes: Minimum Requirements

Peter K. F. Kuhfittig

Department of Mathematics, Milwaukee School of Engineering, Milwaukee, USA
Email: kuhfitti@msoe.edu

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Abstract

While wormholes are just as good a prediction of Einstein's theory as black holes, they are subject to severe restrictions from quantum field theory. To allow for the possibility of interstellar travel, a macroscopic wormhole would need to maintain sufficiently low radial tidal forces. It is proposed in this paper that the assumption of zero tidal forces, *i.e.*, the limiting case, is sufficient for overcoming the restrictions from quantum field theory. The feasibility of this approach is subsequently discussed by 1) introducing the additional conditions needed to ensure that the radial tidal forces can indeed be sufficiently low and 2) by viewing traversable wormholes as emergent phenomena, thereby increasing the likelihood of their existence.

Keywords

Morris-Thorne Wormholes, Traversability, Minimum Requirements, Stability, Compatibility with Quantum Field Theory

1. Introduction

Wormholes have been a subject of interest ever since it was realized that the Schwarzschild solution and therefore black holes can be viewed as wormholes, albeit nontraversable. (The word "wormhole" was coined by John Archibald Wheeler in the 1950's.) More recently, the subject of entanglement has resulted in a renewed interest in a special type of wormhole, the Einstein-Rosen bridge, to explain how two particles can remain in contact even if they are widely separated [1]. A discussion of entangled black holes can be found in Ref. [2]. We will, therefore, assume that a basic wormhole structure can be hypothesized.

In this paper, we are more concerned with macroscopic wormholes suitable for interstellar travel, first proposed by Morris and Thorne in 1988 [3], also discussed in Ref. [4]. While wormholes may be just as good a prediction of Einstein's theory as black holes, they are subject to severe restrictions from quantum field theory. Here, one of the biggest obstacles is the possible existence of large radial tidal forces, just as they are for black holes, leading to what is commonly referred to as "spaghettification". It is proposed in this paper that the tools required to reduce the tidal forces to manageable levels can essentially eliminate the other obstacles, suggesting that traversable wormholes are indeed theoretically possible. The seemingly highly restrictive low-tidal-force assumption can be justified by the charged-wormhole model due to Kim and Lee [5]. The same assumption is used to study the stability of the wormhole by employing an equilibrium condition obtained from the Tolman-Oppenheimer-Volkoff equation.

Finally, the zero-tidal-force assumption enables us to invoke $f(R)$ modified gravity to eliminate the need for exotic matter near the throat. The reason is that $f(R)$ modified gravity can be kept arbitrarily close to Einstein gravity.

This paper is organized as follows: Sec. 2 reviews the basic structure of a Morris-Thorne wormhole, while Sec. 3 discusses the properties of the redshift and shape functions. The zero-tidal-force assumption in Sec. 4 leads to the question of stability in Sec. 5, followed by a discussion of $f(R)$ modified gravity and its consequences in Sec. 6. Sec. 7 considers the compatibility of the zero-tidal-force assumption with quantum field theory. Sec. 8 deals with the problem of throat size. Sec. 9 then returns to the question of the feasibility of the low- or zero-tidal-force assumption, while Sec. 10 discusses wormholes as emergent phenomena.

2. Wormhole Structure

Wormholes are handles or tunnels that could connect even widely separated regions of our Universe or different universes altogether. Wormholes seem to be as good a prediction of Einstein's theory as black holes, but, unlike the latter, they are subject to severe restrictions from quantum field theory. For example, holding a wormhole open requires a violation of the null energy condition, which, in turn, calls for the existence of "exotic matter" in classical general relativity [3], a requirement that many researchers consider to be completely unphysical.

The line element for a Morris-Thorne wormhole is given by

$$\mathrm{d}s^2 = -\mathrm{e}^{2\Phi(r)}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-\frac{b(r)}{r}} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right), \tag{1}$$

using units in which $c = G = 1$. The motivation for this line element comes from Ref. [6]:

$$\begin{aligned}\mathrm{d}s^2 &= -\mathrm{e}^{2\Phi(r)}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-\frac{2m(r)}{r}} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right),\ r \le R \\ &= -\left(1-\frac{2M}{r}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-\frac{2M}{r}} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right),\ r > R.\end{aligned} \tag{2}$$

Here $m(r)$ is the effective mass inside radius r, while M is the mass of a star of radius R as seen by a distant observer. If $\rho(r)$ is the energy density, then the total mass-energy inside radius r is given by

$$m(r)=\int_0^r 4\pi(r')^2\rho(r')\mathrm{d}r',\ m(0)=0. \tag{3}$$

In line element (1), $\Phi=\Phi(r)$ is called the *redshift function*, which must be everywhere finite to prevent the occurrence of an event horizon. The function $b=b(r)$ is called the *shape function* since it determines the spatial shape of the wormhole when viewed, for example, in an embedding diagram [3]. The spherical surface $r=r_0$ is called the *throat* of the wormhole, where $b(r_0)=r_0$. Additional requirements are $b'(r_0)<1$, called the *flare-out condition*, $b(r)<r$ for $r>r_0$, and $b'(r_0)>0$. Another requirement is asymptotic flatness: $\lim_{r\to\infty}\Phi(r)=0$ and $\lim_{r\to\infty}b(r)/r=0$.

It is noted in Ref. [3] that the flare-out condition can only be met by violating the null energy condition (NEC), which states that

$$T_{\alpha\beta}k^\alpha k^\beta\geq 0 \tag{4}$$

for all null vectors k^α, where $T_{\alpha\beta}$ is the energy-momentum tensor. As mentioned above, matter that violates the NEC is called "exotic". In particular, for the outgoing null vector $(1,1,0,0)$, the violation has the form

$$T_{\alpha\beta}k^\alpha k^\beta=\rho+p_r<0. \tag{5}$$

Here $T^t{}_t=-\rho$ is the energy density, $T^r{}_r=p_r$ is the radial pressure, and $T^\theta{}_\theta=T^\phi{}_\phi=p_t$ is the lateral (transverse) pressure. Our final task in this section is to list the Einstein field equations:

$$\rho(r)=\frac{b'}{8\pi r^2}, \tag{6}$$

$$p_r(r)=\frac{1}{8\pi}\left[-\frac{b}{r^3}+2\left(1-\frac{b}{r}\right)\frac{\Phi'}{r}\right], \tag{7}$$

and

$$p_t(r)=\frac{1}{8\pi}\left(1-\frac{b}{r}\right)\left[\Phi''-\frac{b'r-b}{2r(r-b)}\Phi'+(\Phi')^2+\frac{\Phi'}{r}-\frac{b'r-b}{2r^2(r-b)}\right]. \tag{8}$$

3. The Redshift and Shape Functions

When discussing Morris-Thorne wormholes, it is generally assumed that $\Phi=\Phi(r)$ and $b=b(r)$ can be freely assigned and still retain a wormhole structure. This assumption does not, however, address the practical problems of traversability, such as the tidal gravitational forces mentioned in the Introduction. It is shown in Ref. [3] that the radial tidal constraint is given by

$$\left|\left(1-\frac{b}{r}\right)\left(-\Phi''+\frac{b'r-b}{2r(r-b)}\Phi'-(\Phi')^2\right)\right|\leq A, \tag{9}$$

where A is a constant [3]. This constraint is most easily met if $\Phi'(r)\equiv 0$ for all r, called the *zero-tidal-force solution* in Ref. [3]. While a small value of $|\Phi'(r)|$ would suffice, we will retain the assumption $\Phi'(r)\equiv 0$ to keep the analysis tractable. The implication of this assumption constitutes a major goal in this paper.

It was noted in the Introduction that we can hypothesize a basic wormhole structure—and this would include the existence of a shape function. This shape function must meet certain requirements, as discussed in the previous section. To address this issue, it is proposed in Ref. [7] that a generic shape function can be defined by starting with the following family:

$$b_\eta(r) = r_0\left(\frac{r}{r_0}\right)^{1-\eta}, \quad 0 < \eta < 1. \tag{10}$$

Evidently, $b_\eta(r_0) = r_0$, while

$$0 < b'_\eta(r_0) = 1 - \eta < 1. \tag{11}$$

For this program to work, we need to assume that a typical shape function $b = b(r)$ is concave down in the immediate vicinity of $r = r_0$ with $b(r) < r$. More precisely, we require that 1) $b(r_0) = r_0$, 2) $0 < b'(r_0) < 1$, and 3) $b(r)$ is concave down near $r = r_0$. Properties 1) and 2) are clearly met. Regarding Property 3), for any $\eta \in (0,1)$,

$$b''_\eta(r_0) = \frac{1}{r_0}(1-\eta)(-\eta) < 0,$$

showing that $b_\eta(r)$ is indeed concave down near $r = r_0$. We conclude that any shape function that meets Properties 1), 2), and 3) can be approximated by some $b_\eta(r)$ in the vicinity of the throat. For example, the special case $\eta = 1/2$ leads to the parabola $b(r) = \sqrt{r_0 r}$, which is concave down to the right of $r = r_0$ with $b(r) < r$ for all *r*. The same behavior is exhibited by all members of the family in Equation (10).

In summary, for every value *a* between 0 and 1, there exists a member $b_\eta(r)$ such that $b'_\eta(r_0) = a$.

4. The Zero-Tidal-Force Assumption

The assumption $\Phi'(r) \equiv 0$ was introduced as a necessary traversability condition. The rest of this paper will be devoted to showing that the purpose of this assumption goes far beyond these considerations. In particular, Sec. 5, 6, and 7 discuss, respectively, the question of stability, the feasibility of invoking $f(R)$ modified gravity to eliminate the need for exotic matter, and the compatibility of traversable wormholes with quantum field theory. We are also going to conclude that Morris-Thorne wormholes can only exist for relatively large throat sizes.

5. Stability

Stability is an important topic in wormhole physics. The special case of zero-tidal-force wormholes is taken up in Ref. [8] by employing an equilibrium condition obtained from the Tolman-Oppenheimer-Volkoff (TOV) equation [9] [10].

$$\frac{\mathrm{d}p_r}{\mathrm{d}r} + \Phi'(\rho + p_r) + \frac{2}{r}(p_r - p_t) = 0. \tag{12}$$

The equilibrium state of a structure is determined from the three terms in this equation, defined as follows: the gravitational force

$$F_g = -\Phi'(\rho + p_r), \tag{13}$$

the hydrostatic force

$$F_h = -\frac{\mathrm{d}p_r}{\mathrm{d}r}, \tag{14}$$

and the anisotropic force

$$F_a = \frac{2(p_t - p_r)}{r} \tag{15}$$

due to the anisotropic pressure in a Morris-Thorne wormhole. Equation (12) then yields the following equilibrium condition: $F_g + F_h + F_a = 0$. Since $\Phi' \equiv 0$, the equilibrium condition becomes

$$F_h + F_a = 0. \tag{16}$$

To show that this condition is met in this paper, we first need to consider $f(R)$ modified gravity.

6. $f(R)$ Modified Gravity

The purpose of this section is two-fold, to introduce $f(R)$ modified gravity and to show that we can remain arbitrarily close to Einstein gravity.

Here we need to retain our assumption $\Phi'(r) \equiv 0$. Otherwise, according to Lobo and Oliveira [11], the analysis becomes intractable. Fortunately, this requirement is in line with our overall goals, as we saw in Sec. 4.

Next, we list the gravitational field equations in the form given in Ref. [11]:

$$\rho(r) = F(r)\frac{b'(r)}{r^2}, \tag{17}$$

$$p_r(r) = -F(r)\frac{b(r)}{r^3} + F'(r)\frac{rb'(r) - b(r)}{2r^2} - F''(r)\left(1 - \frac{b(r)}{r}\right), \tag{18}$$

and

$$p_t(r) = -\frac{F'(r)}{r}\left(1 - \frac{b(r)}{r}\right) + \frac{F(r)}{2r^3}\left[b(r) - rb'(r)\right], \tag{19}$$

where $F = \frac{\mathrm{d}f}{\mathrm{d}R}$. The Ricci curvature scalar is given by

$$R(r) = \frac{2b'(r)}{r^2}. \tag{20}$$

To see the connection to the flare-out condition at the throat, observe that from Equations (5), (6), and (7), we have

$$8\pi\left[\rho(r_0) + p_r(r_0)\right] = \frac{r_0 b'(r_0) - b(r_0)}{r_0^3} < 0$$

since $b(r_0) = r_0$. Given that the radial tension $\tau(r)$ is the negative of $p_r(r)$, Equation (5) implies that $\tau-\rho c^2>0$, temporarily reintroducing c. The last inequality has given rise to the designation "exotic matter" since $\tau>\rho c^2$ implies that there is an enormous radial tension at the throat.

In this paper, we are going to choose

$$f(R)=aR^{1\pm\epsilon},\ \epsilon\ll 1, \tag{21}$$

where a is a constant. The reason is that since ϵ can be arbitrarily close to zero, the resulting $f(R)$ modified gravity can be arbitrarily close to Einstein gravity. Since $F=\frac{\mathrm{d}f}{\mathrm{d}R}$, we get from Equation (20)

$$F=a(1\pm\epsilon)R^{\pm\epsilon}=a(1\pm\epsilon)\left(\frac{2b'(r)}{r^2}\right)^{\pm\epsilon}. \tag{22}$$

This is enough to show that $F_h+F_a=0$:

$$F_h+F_a$$
$$=a(1\pm\epsilon)\left[2^{\pm\epsilon}\left(\frac{\rho}{2^{\pm\epsilon}a(1\pm\epsilon)}\right)^{\frac{\pm\epsilon}{1\pm\epsilon}}\right]\left(\frac{rb'(r)-3b(r)}{r^4}+\frac{b(r)-rb'(r)}{r^4}+\frac{2b(r)}{r^4}\right)=0 \tag{23}$$

(see Ref. [8] for details). We now see that the equilibrium condition is satisfied, thereby yielding a stable wormhole. This outcome leads to one of our most important conclusions: since our modified theory, based on Equation (21), can be arbitrarily close to Einstein's theory, the stability criterion carries over to Morris-Thorne wormholes.

Returning to Ref. [11], it is shown that for the material threading the wormhole, the NEC can be met, thereby allowing the use of ordinary (nonexotic) matter in the modified theory, even if it is arbitrarily close to Einstein's theory. As a first step, imposing the conditions $\rho+p_r\geq 0$ and $\rho\geq 0$, it now follows from Equations (17) and (18) that the function F must be positive and that it must satisfy the following conditions at the throat:

$$\frac{Fb'}{r^2}\geq 0 \tag{24}$$

and

$$\frac{(2F+rF')(b'r-b)}{2r^3}-F''\left(1-\frac{b}{r}\right)\geq 0. \tag{25}$$

From Equation (19), we also have $\rho+p_t\geq 0$ at the throat, F being positive. To complete the proof of the above assertion, it is shown in Ref. [12] that the NEC is met for all null vectors $(1,a,b,c)$, where $0\leq a,b,c\leq 1$ and $a^2+b^2+c^2=1$.

7. Compatibility with Quantum Field Theory

We have seen in this paper that the zero-tidal-force assumption plays a critical role. This section continues the theme by examining how this assumption affects the compatibility of classical wormhole theory with quantum field theory, which places some severe constraints on Morris-Thorne wormholes [13]. More precisely, the wormhole spacetime must satisfy a certain quantum inequality in an inertial Minkowski spacetime without boundary: if u^μ is the observer's four-velocity and $\langle T_{\mu\nu}u^\mu u^\nu \rangle$ is the expected value of the local energy density in the observer's frame of reference, then

$$\frac{\tau_0}{\pi}\int_{-\infty}^{\infty}\frac{\langle T_{\mu\nu}u^\mu u^\nu \rangle \mathrm{d}\tau}{\tau^2+\tau_0^2} \geq -\frac{3}{32\pi^2\tau_0^4}, \tag{26}$$

where τ is the observer's proper time and τ_0 the duration of the sampling time (see Ref. [13] for details). The inequality can be applied in a curved spacetime as long as τ_0 is small compared to the local proper radius of curvature. The desired estimates of the local curvature are obtained from the components of the Riemann curvature tensor in classical general relativity. It follows that the exotic matter must be confined to a narrow band around the throat [13]. Now, according to Refs. [14] [15], this can only be accomplished by fine-tuning the metric coefficients. In other words, to satisfy the quantum inequality, one must strike a balance between reducing the size of the exotic region and the degree of fine-tuning of the metric coefficients required to achieve this reduction. As a result, $\Phi'(r)$ must be fine-tuned to remain in a narrow range. The most important conclusion for our purposes is that $\Phi'(r)\equiv 0$ is outside this range, so that the resulting wormhole solution cannot be compatible with quantum field theory. This also applies to the wormhole solutions in Ref. [3].

Now the reason for invoking $f(R)$ modified gravity becomes apparent: the estimates of the local curvature needed to apply Inequality (26) come from Einstein's theory, not from the modified theory. So the previous objections do not apply. More precisely, in the equation $f(R)=aR^{1\pm\epsilon}$, ϵ is always nonzero. So even if the modified theory is arbitrarily close to Einstein's theory, it remains an $f(R)$ theory, thereby avoiding a direct conflict with quantum field theory.

8. The Throat Size $r = r_0$

We saw in Sec. 6 that $f(R)$ modified gravity can remain arbitrarily close to Einstein gravity. In this section a slight shift in emphasis yields a slightly modified theory that enables us to estimate the throat size $r=r_0$ of a Morris-Thorne wormhole. To that end, observe that the field Equations (17)-(19) reduce to the Einstein equations for $\Phi'(r)\equiv 0$ whenever $F(r)\equiv 1$. Next, from Equations (17) and (20), we have

$$F(r)=\frac{2\rho(r)}{2\frac{b'(r)}{r^2}}=\frac{2\rho(r)}{R(r)}. \tag{27}$$

So a slight change in F results in a slight change in R, enough to quantify the notion of slightly modified gravity: assume that $F(r)$ remains close to unity and relatively "flat," *i.e.*, both $F'(r)$ and $F''(r)$ remain relatively small in absolute value.

We already saw in Sec. 6 the $\rho+p_r\geq 0$ in $f(R)$ modified gravity. So from

Equations (17) and (18)

$$\rho + p_r = \left(rb' - b\right)\left(\frac{F}{r^3} + \frac{F'}{2r^2}\right) - F''\left(1 - \frac{b}{r}\right) \geq 0. \tag{28}$$

To draw our conclusion, it is convenient to use a simple example: suppose $F = 2 - \mathrm{e}^{a(r-r_0)}$; then $F' = -\mathrm{e}^{a(r-r_0)}a$, and $F'' = -\mathrm{e}^{a(r-r_0)}a^2$, where a is a small constant. At $r = r_0$, $F(r_0) = 1$, $F'(r_0) = -a$, and $F''(r_0) = -a^2$. Substituting in Equation (28), we get

$$\left(rb' - b\right)\frac{2 - ar_0}{2r_0^3} + a^2\left(1 - \frac{b}{r}\right) \geq 0 \tag{29}$$

at $r = r_0$, provided that

$$r_0 \geq \frac{2}{a}. \tag{30}$$

Given that a can be extremely small, we get a valid wormhole solution only for sufficiently large throat sizes (for further details, see Ref. [16]).

This result is consistent with a problem already discussed in Ref. [3], the radial tension at the throat. First we need to recall that the radial tension $\tau(r)$ is the negative of the radial pressure $p_r(r)$. According to Ref. [3], the Einstein field equations can be rearranged to yield $\tau(r)$. Temporarily reintroducing c and G, we obtain

$$\tau(r) = \frac{b(r)/r - 2\left[r - b(r)\right]\Phi'(r)}{8\pi G c^{-4} r^2}. \tag{31}$$

The radial tension at the throat therefore becomes

$$\tau(r_0) = \frac{1}{8\pi G c^{-4} r_0^2} \approx 5 \times 10^{41} \frac{\mathrm{dyn}}{\mathrm{cm}^2}\left(\frac{10\ \mathrm{m}}{r_0}\right)^2. \tag{32}$$

As noted in Ref. [3], for a throat size of $r_0 = 3$ km, $\tau(r)$ has the same magnitude as the pressure at the center of a massive neutron star. Equation (32) shows that Morris-Thorne wormholes could only exist on very large scales.

Remark: The above discussion suggests that moderately-sized wormholes are actually compact stellar objects [17] and are thereby beyond the scope of the present study.

9. Feasibility: Wormholes with Low Tidal Forces

Our conclusions so far depended on the zero-tidal-force assumption $\Phi'(r) \equiv 0$. It was noted in Sec. 3, however, that a small value of $\left|\Phi'(r)\right|$ would have been sufficient, thereby remaining the single most important condition. In this section we will examine the circumstances under which this condition can be satisfied. This will naturally call for some requirements beyond those already considered.

We start by assuming a noncommutative-geometry background: an important outcome of string theory is the realization that coordinates may become noncommutative operators on a *D*-brane [18] [19]. The result is a fundamental discretization of spacetime due to the commutator $\left[x^\mu, x^\nu\right] = i\theta^{\mu\nu}$, where $\theta^{\mu\nu}$ is an antisymmetric matrix. According to Refs. [20]-[22], noncommutativity replaces point-like objects by smeared objects, a procedure that is consistent with the Heisenberg uncertainty principle. The purpose is to eliminate the divergences that normally occur in general relativity.

A common way to model the smearing is by means of a Gaussian distribution of minimal length $\sqrt{\alpha}$ instead of the obvious alternative, the Dirac delta function [23]-[26]. A simpler but equally effective way is to assume that the energy density of a static and spherically symmetric and particle-like gravitational source has the form

$$\rho(r) = \frac{m\sqrt{\alpha}}{\pi^2\left(r^2+\alpha\right)^2} \tag{33}$$

(see Refs. [23] and [24]). The basic idea is that the mass m is diffused throughout the region of linear dimension $\sqrt{\alpha}$ due to the uncertainty. It is emphasized in Ref. [21] that noncommutative geometry is an intrinsic property of spacetime and does not depend on any particular feature such as curvature. Furthermore, to make use of Equation (33), we can keep the standard form of the Einstein field equations in the sense that the Einstein tensor retains its original form, but the stress-energy tensor is modified [21]. It follows that the length scales can be macroscopic.

Next, given that black holes can carry an electric charge, it is natural to assume that wormholes can do likewise. It is proposed by Kim and Lee [5] that for a wormhole with constant electric charge Q, the Einstein field equations are given by

$$G_{\mu\nu}^{(0)} + G_{\mu\nu}^{(1)} = 8\pi\left[T_{\mu\nu}^{(0)} + T_{\mu\nu}^{(1)}\right]. \tag{34}$$

According to Ref. [5], since the usual form is $G_{\mu\nu}^{(0)} = 8\pi T_{\mu\nu}^{(0)}$, the modified form, Equation (34), is obtained by adding the matter term $T_{\mu\nu}^{(1)}$ to the right side and the corresponding back reaction term $G_{\mu\nu}^{(1)}$ to the left side. The proposed metric then becomes

$$\mathrm{d}s^2 = -\left(1+\frac{Q^2}{r^2}\right)\mathrm{d}t^2 + \left(1-\frac{b(r)}{r}+\frac{Q^2}{r^2}\right)^{-1}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right). \tag{35}$$

It is shown in Ref. [5] that this metric is a self-consistent solution to the Einstein field equations. It also follows that the (effective) shape function is given by

$$b_{\text{eff}}(r) = b(r) - \frac{Q^2}{r}. \tag{36}$$

In the Kim-Lee model, the total charge is given by $\iiint_V \rho_q(r)\mathrm{d}V$, where ρ_q is the charge density. To adapt this to our noncommutative-geometry background, it is proposed in Ref. [27] that

$$\rho_q(r) = \frac{Q^2\sqrt{\alpha}}{\pi^2\left(r^2+\alpha\right)^2}, \tag{37}$$

where Q^2 refers to the Kim-Lee model. So the smeared charge $Q_\alpha^2(r)$ is

$$Q_\alpha^2(r)=\int_0^r 4\pi(r')^2\frac{Q^2\sqrt{\alpha}}{\pi^2\left[(r')^2+\alpha\right]^2}\mathrm{d}r'=\frac{2Q^2\sqrt{\alpha}}{\pi}\left(\frac{1}{\sqrt{\alpha}}\tan^{-1}\frac{r}{\sqrt{\alpha}}-\frac{r}{r^2+\alpha}\right). \quad (38)$$

It is shown in Ref. [27] that the shape function is

$$\begin{aligned} b_{\text{eff}}(r)=&\frac{4m\sqrt{\alpha}}{\pi}\left(\frac{1}{\sqrt{\alpha}}\tan^{-1}\frac{r}{\sqrt{\alpha}}-\frac{r}{r^2+\alpha}\right)\\ &-\frac{1}{r}\frac{2Q^2\sqrt{\alpha}}{\pi}\left(\frac{1}{\sqrt{\alpha}}\tan^{-1}\frac{r}{\sqrt{\alpha}}-\frac{r}{r^2+\alpha}\right)\\ &-\frac{4m\sqrt{\alpha}}{\pi}\left(\frac{1}{\sqrt{\alpha}}\tan^{-1}\frac{r_0}{\sqrt{\alpha}}-\frac{r_0}{r_0^2+\alpha}\right)+r_0. \end{aligned} \quad (39)$$

Moreover, $b_{\text{eff}}(r)$ satisfies all the requirements of a shape function.

We can now turn to our main goal, estimating $|\Phi'(r)|$. From line element (35),

$$e^{2\Phi}=1+\frac{Q^2}{r^2}, \quad (40)$$

whence

$$\Phi(r)=\frac{1}{2}\ln\left[1+\frac{1}{r^2}\frac{2Q^2\sqrt{\alpha}}{\pi}\left(\frac{1}{\sqrt{\alpha}}\tan^{-1}\frac{r}{\sqrt{\alpha}}-\frac{r}{r^2+\alpha}\right)\right] \quad (41)$$

and

$$\Phi'(r)=\frac{-\frac{1}{r^3}\frac{2Q^2\sqrt{\alpha}}{\pi}\left(\frac{1}{\sqrt{\alpha}}\tan^{-1}\frac{r}{\sqrt{\alpha}}-\frac{r}{r^2+\alpha}\right)+\frac{2Q^2\sqrt{\alpha}}{\pi\left(r^2+\alpha\right)^2}}{1+\frac{1}{r^2}\frac{2Q^2\sqrt{\alpha}}{\pi}\left(\frac{1}{\sqrt{\alpha}}\tan^{-1}\frac{r}{\sqrt{\alpha}}-\frac{r}{r^2+\alpha}\right)}. \quad (42)$$

Since α is a small parameter, the easiest way to estimate $|\Phi'(r)|$ is to let $\alpha\to 0$:

$$|\Phi'(r)|\approx\left|\frac{-\frac{1}{r^3}Q^2}{1+\frac{1}{r^2}Q^2}\right|. \quad (43)$$

We saw in Sec. 8 that we get a valid wormhole solution only for sufficiently large throat sizes, showing that $|\Phi'(r)|$ is indeed arbitrarily small, thereby overcoming the restrictions from quantum field theory.

10. Feasibility: Wormholes as Emergent Phenomena

It was noted in the Introduction that we can assume that a basic wormhole structure may be hypothesized. We will now strengthen this assumption by showing that wormholes can be viewed as emergent phenomena. Some aspects of this problem have already been discussed in Ref. [28].

The basic idea of emergence is that certain high-level phenomena cannot be deduced even in principle from a lower-level domain. For example, ant colonies are capable of building extremely complex structures, but this outcome cannot be explained by examining the behavior of individual ants. This behavior is an example of a *fundamental property* and the structure an example of an *emergent property*. As another example, life emerges from objects that are themselves completely lifeless, such as atoms and molecules. Such a process is not reversible, however, even in principle: living organisms tell us nothing about the particles in the fundamental theory. In fact, these properties are not even relevant in the resulting *effective model*, thereby illustrating the often surprising or unexpected outcomes that characterize emergent phenomena. Moreover, noncommutative geometry in the form discussed in the previous section is an example of a fundamental property, thereby calling attention to the fact that quantum mechanics generally incorporates many such fundamental phenomena. More precisely, it is argued in Ref. [29] that in this context, emergence is based on entanglement. For our purposes we simply acknowledge that two entangled particles are connected by a type of wormhole called an Einstein-Rosen bridge [2]. This can now be seen as a fundamental property, while the emergent property is a macroscopic wormhole. Of course, such wormholes have never been observed, but the very possibility of emergence seems to greatly increase the probability that such wormholes actually exist.

This observation is made more concrete in Ref. [30]. Using Equation (33), the shape function becomes

$$\begin{aligned} b(r) &= r_0 + \int_{r_0}^{r} 8\pi (r')^2 \rho(r') \mathrm{d}r' \\ &= \frac{4m}{\pi}\left[\tan^{-1}\frac{r}{\sqrt{\beta}} - \sqrt{\beta}\frac{r}{r^2+\beta} - \tan^{-1}\frac{r_0}{\sqrt{\beta}} + \sqrt{\beta}\frac{r_0}{r_0^2+\beta}\right] + r_0 \\ &= \frac{4m}{\pi}\frac{1}{r}\left[r\tan^{-1}\frac{r}{\sqrt{\beta}} - \sqrt{\beta}\frac{r^2}{r^2+\beta} - r\tan^{-1}\frac{r_0}{\sqrt{\beta}} + \sqrt{\beta}\frac{r_0 r}{r_0^2+\beta}\right] + r_0. \end{aligned} \tag{44}$$

The reason is that we can now define a new shape function $B = b/\sqrt{\beta}$:

$$\begin{aligned} &\frac{1}{\sqrt{\beta}} b(r) = B\left(\frac{r}{\sqrt{\beta}}\right) \\ &= \frac{4m}{\pi}\frac{1}{r}\left[\frac{r}{\sqrt{\beta}}\tan^{-1}\frac{r}{\sqrt{\beta}} - \frac{\left(\frac{r}{\sqrt{\beta}}\right)^2}{\left(\frac{r}{\sqrt{\beta}}\right)^2+1} - \frac{r}{\sqrt{\beta}}\tan^{-1}\frac{r_0}{\sqrt{\beta}} + \frac{r}{\sqrt{\beta}}\frac{\frac{r_0}{\sqrt{\beta}}}{\left(\frac{r_0}{\sqrt{\beta}}\right)^2+1}\right] + \frac{r_0}{\sqrt{\beta}}. \end{aligned} \tag{45}$$

It follows that

$$B\left(\frac{r_0}{\sqrt{\beta}}\right) = \frac{r_0}{\sqrt{\beta}}, \tag{46}$$

the analogue of $b(r_0) = r_0$. So the throat size can be macroscopic, confirming the claim that we are indeed dealing with an emergent property.

11. Conclusions

Geometrically speaking, wormholes may be compared to black holes, but they are subject to severe restrictions from quantum field theory. This raises some questions regarding the number and severity of these constraints, as well as conditions needed to allow a wormhole to be used for interstellar travel. It is proposed in this paper that controlling the radial tidal forces, usually associated with black holes, can essentially eliminate the other obstacles. To keep the analysis tractable, we assume zero tidal forces (Sec. 4), an assumption that immediately yields a stable solution (Sec. 5). The same assumption enables us to invoke $f(R)$ modified gravity (Sec. 6), thereby eliminating the need for exotic matter at the thoat of a Morris-Thorne wormhole. The reason is that $f(R)$ modified gravity can be kept arbitrarily close to Einstein gravity (Sec. 6).

It is shown in Sec. 7 that the zero-tidal-force assumption would make the wormhole solution incompatible with quantum field theory in classical general relativity but would be acceptable in $f(R)$ modified gravity.

It is noted in Sec. 1 that we may hypothesize a basic wormhole structure (discussed in Sec. 2), which necessarily includes a shape function. Moreover, it is pointed out in Sec. 3 that any valid shape function can be approximated by a member of the family $b_\eta(r) = r_0 (r/r_0)^{1-\eta}$, $0 < \eta < 1$, and in Sec. 8 that a Morris-Thorne wormhole can only exist on very large scales, *i.e.*, with a large $r = r_0$, which is consistent with the discussion following Equation (32).

Finally, Sec. 9 discusses the feasibility of this approach by introducing several additional conditions to ensure that the radial tidal forces can indeed be kept sufficiently low, while Sec. 10 discusses wormholes as emergent phenomena.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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