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Dark Matter Density Profiles of Selected Spiral Galaxies

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Abstract

Understanding the dark matter distribution throughout a galaxy can provide insight into its elusive nature. Numerous density profiles, such as the Navarro, Frenk and White model, have been created in an attempt to study this distribution through analyzing orbital velocities of luminous matter and modeling dark matter distributions to explain these observations. However, we are interested in a simple model to consider the significant fluctuations in rotation curves at larger radii. Therefore, our model is much simpler compared to those previously mentioned. Our model used all the observational data available for four selected galactic rotation curves. These data present a significant variation in the orbital velocity of matter at the same distances. By running real observational data through our model, we show that the density of the dark matter within them shows real complex structure, which is not suggested by other computational models. Our aim of this paper is to model this structure and then speculate as to the cause and implications of these density fluctuations.

Keywords

Dark Matter Density, Galactic Rotation Curves, Spiral Galaxies, NGC 2841, NGC 4527, NGC 4945, NGC 3198

1. Introduction

The presence of dark matter within galaxies is implied by luminous matter orbiting at large galactocentric distances at higher than predicted velocities within those galaxies. By measuring the velocities of this luminous matter, one can calculate the approximate density of dark matter at varying radii in the galaxies. Then, one can compare these approximations to the estimated density at the critical radius, assumed to be the radius at which dark matter becomes more prominent than baryonic matter in the galaxy. The galactic rotation curves exhibit noticeable fluctuations beyond the critical distance, indicative of variations in dark matter density. Our approach involves employing a simple model with minimal free parameters. Finally, these values can be compared to visualize the dark matter density throughout the galaxy and better understand where dark matter tends to exist. These dark matter density profiles have been calculated by different models using measured rotation curves [1]-[3]. For example, Y. P. Jing reinforced the work done by Navarro, Frenk and White which utilized N-body simulations with large amounts of particles in order to model cold dark matter density profiles of galaxies, also known as the NFW density profile [4]. The NFW model, published in 1995, found that most of their observed and simulated galaxies shared similar characteristics in gas and dark matter density profiles, as well as temperature and light emission measurements at the respective radii. However, they chose to neglect radiative cooling on gases within their simulated galaxies. This was the basis for their density model, which has been shown to be a precise model up to several hundred kpc away from the center of the galaxy [5]. The purpose of this paper is not to introduce a new model. In this simulation, we are not assuming the galactic rotation curve is flat at large galactocentric distances. Here, we are using a simple model to show fluctuation structure in dark matter density using four selected galactic rotation curves. From these fluctuations, we can theorize and learn more about these galaxies than one would when assuming a flat rotation curve, as it allows for more accurate calculations.

Rotation curves plot orbital speeds of visible matter versus radial galactocentric distance. The discrepancies shown from between what is expected from most of the luminous matter inhabiting the core area of the galaxy to what is observed produce theories involving dark matter [6]. Without dark matter and with the simplified assumption of the mass concentrated in the center, stars would necessarily revolve around the center at velocities decreasing with greater radii. This is not shown with all galaxies but velocities of many spirals, however, have been observed to decrease with distance [7]. Indeed, the data imply large mass estimates for the galaxy which can be best explained through the assumption of a dark matter halo which surrounds the galaxy [8]. The first publication of such rotation curves was made in 1957 by Henk van de Hulst which were made possible through observing the 21-cm hydrogen radiation from M31, also known as the Andromeda Galaxy [9].

Flat rotation curves at large galactocentric distances are inconsistent with rotation curves generated from observational data. There are two models that are widely used to fit rotation curves. In a forward fitting model, the different parts of a galaxy are described by free parameters and then superimposed to create the rotation curve that fits the galaxy. However, an inverse model takes a look at the rotation curve (tangential velocity at different radii) and derives the mass without any free parameters. Real rotation curve data show that there is significant variation in the velocity of matter within a galaxy at such large distances, especially when one observes past the critical radius where dark matter becomes more prominent than baryonic matter [5]. These variations are present in many galaxies and are larger than the calculated error associated with the data. This indicates that they are not due to instrumental error but are in fact due to physical, structural disruptions within the galaxy.

Density profiles for galaxies generated using simulations show linearly and monotonically decreasing dark matter densities with little to no variations in dark matter density with increasing galactocentric distances [3]. So, we perhaps might expect that the dark matter density profiles for galaxies fitted using observational data would be trending downwards but would exhibit some variations in the density of dark matter throughout the galaxy [10]. However, density profiles which model the galactic rotation curve to be flat at large galactocentric distances do not entirely match the observed data. This discrepancy could originate from multiple different sources, such as a lack of understanding of galaxy formation and dark matter, as well as an incomplete understanding of the theory of gravity [11]. While science is still improving and gaining a better understanding of these topics, we chose to use observational data through our simpler model to be as accurate as possible with our calculations. The observed data show much more variety in the rotation curves, as well as the dark matter density structures. It has also been observed that there is a correlation between the size of a dark matter halo and the size of a galaxy's disk and central bulge, meaning a larger galaxy results in a much larger dark matter halo and vice versa [5].

The following calculation determines simple dark matter density profiles for spiral galaxies using real galactic rotation curve data to gain insight into the nature of dark matter. There are other more complex models that analyze galaxies and their modelled dark matter distributions to a higher precision than our study, such as the work done by Karukes, Salucci and Gentile [12]. Our intention is not to model these distributions to such a high precision, but to acknowledge the changes in density as they relate to the galactic rotation curves. Knowing how mass is distributed and understanding the density of dark matter on a galactic scale can help uncover the nature of dark matter and be a step in understanding it more generally.

Many assumptions have been made about the truth behind dark matter, such as the theory of dark matter being primarily made of primordial black holes which are so small, one could not detect them. Some have also theorized that dark matter and luminous matter interact with a force that we have not yet discovered [13]. While weakly interacting massive particles (WIMPs) were the leading theory for dark matter in the past, they are presently disfavored. There have been many attempts to detect dark matter in the form of WIMPs, as well as neutrinos, but these theories can be supported through forward fitting and inverse models [14]. We then use this knowledge to create the dark matter density profiles from rotation curves that we observe in galaxies. Conversely, there are supporters of the idea that nonbaryonic dark matter does not exist at all due to no true proof of direct detection [15] [16]. We make straight forward assumptions about the galactic system and the dark matter density as a function of radius and orbital velocity, as derived using Newtonian gravity alone. We assume circular orbits in our model. Significant orbital ellipticities would affect our calculations. The results are discussed and theories of the reasoning behind these variations in the velocity of matter at large galactocentric distances are investigated.

Some possible explanations for the varying dark matter density at large distances would include various forms of orbital resonance as seen in many examples of orbital dynamics. Another possibility is the presence of intervening objects. This could include globular clusters, satellites and other dwarf galaxies, among other possibilities. The presence of spiral arms and the phenomenon of spiral density wave theory could be involved as well.

2. Computational Model & Methods

Consider a spiral galaxy. Uniform circular motion and spherical symmetry are assumed for all matter within the galaxy. We assume all significant baryonic matter within the galaxy is enclosed within a spherical Gaussian surface between $0 < r < r_c$, where r_c is the critical radius. We have defined the critical radius to be the radius where dark matter becomes more prominent than baryonic matter, thus making it the leading factor in the effects on velocity and gravitational forces. Dark matter is also present within the critical radius, but it is overshadowed by the effects of baryonic matter.

Assuming circular motion, we find the equation of motion with Kepler's Third Law, relating the radius of an orbit to its period. This gives us

$$\frac{GM(r)}{r^2} = \frac{v^2(r)}{r}, \tag{1}$$

where G is the gravitational constant, $M(r)$ is the mass enclosed within a given radius, and $v(r)$ is the orbital velocity of the piece of matter at a distance r from the galactic center. Rewriting this equation in terms of density, and still assuming spherical symmetry and uniform circular motion,

$$\int_0^r \rho(r') r'^2 \mathrm{d}r' = \frac{rv^2(r)}{4\pi G}. \tag{2}$$

Now, this density is the total density of all matter within the galaxy and is thus a superposition of the baryonic and dark matter densities within the galaxy. Recalling that all baryonic matter within the galaxy is assumed to be enclosed within a spherical Gaussian surface between $0 < r' < r_c$, where r_c is the critical radius,

$$\int_0^{r_c} \rho_{bm}(r') r'^2 \mathrm{d}r' + \int_0^{r_c} \rho_{dm}(r') r'^2 \mathrm{d}r' + \int_{r_c}^{r} \rho_{dm}(r') r'^2 \mathrm{d}r' = \frac{rv^2(r)}{4\pi G}, \tag{3}$$

where ρ_{bm} is the density of baryonic matter and ρ_{dm} is the density of dark matter.

Since all baryonic matter is assumed to be concentrated within a Gaussian surface between $0 < r < r_c$, the amount of baryonic matter is constant. The total amount of dark matter inside this region is also constant since we are only considering radii past the critical radius, r_c. Thus, both of their derivatives are zero.

Taking the derivative of both sides gives us

$$\rho_{dm}(r)r^2 = \frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{rv^2(r)}{4\pi G}\right), \text{ for } r > r_c . \tag{4}$$

Finally, solving for $\rho_{dm}(r)$, yields

$$\rho_{dm}(r) = \frac{1}{4\pi r^2 G}\left[v^2(r) + r\frac{\mathrm{d}\left(v^2(r)\right)}{\mathrm{d}r}\right], \text{ for } r > r_c . \tag{5}$$

We can verify this derivation through comparison with W. J. G. de Blok who used the same final equation for dark matter mass density to create dark matter mass density profiles of low brightness galaxies, focusing mainly on their cores [17]. This calculation for a density profile has also been reinforced by Navarro, Frenk and White, as well as Y. P. Jing and Yoshiaki Sofue [1]-[3]. However, here in this paper we are using this model to show the dark matter density fluctuations at larger radii, respective to the size of the observed galaxies and their dark matter halos. Looking at Equation (5), we can assume that slight change in orbital velocity results in a noticeable change in the dark matter density profile.

A large set of rotation curve data may be found on the website of Yoshiaki Sofue [2]. Their data were used to calculate surface mass density and mass-to-luminosity ratios to understand better galaxy formation and their models, but we only used the rotation curves for objects. When sifting through the data and choosing galaxies to study, it was important to choose galaxies whose rotation curves exhibited significant variations in the orbital velocity of data at large galactocentric distances and that those variations were large enough such that they could not be due to instrumental error. Due to the lack of any published error measurements in the Sofue data, we conducted quadratic fitting over every four points and calculated errors based on this curve, resulting in small error values. The data for the selected galaxies were run through the density profile via a Python code, and four galaxies were selected for further analysis, NGC 2841, NGC 4527, NGC 4945 and NGC 3198.

3. Results and Discussion

Shown below in **Figures 1-4**, are the results for four galaxies as well as the values of their critical radii: NGC 2841, NGC 4527, NGC 4945, and NGC 3198. Recall that the critical radius is the radius at which it is assumed that the galaxy begins to consist primarily of dark matter. Here we have plotted the dark matter density in a logarithmic plot vs. the corresponding radii to portray better the values we calculated.

The fluctuations in rotation curves for these galaxies are larger than the errors in the region farther than the critical distance of $r > r_c$. Due to the nature of our calculations in conjunction with the observed data by Yoshiaki Sofue, the error bars at the smallest radii are largest, but that does not skew our results as we are interested in the larger radii for our density calculations. In this paper, as discussed in our model, the density of the dark matter is calculated for the region of

$r > r_c$, where the fluctuations are overall significantly larger than the error bar. This is a criterion for reliability of the data in our model for calculation of dark matter density. According to the data, we have estimated the critical radii for NGC 2841, NGC 4527, NGC 4945 and NGC 3198 as kpc, 4.25 kpc, 0.3 kpc, 0.42 kpc and 4.15 kpc respectively. The results of density calculation of dark matter for these galaxies show some fluctuations.

Figure 1 shows the results for NGC 2841 with a critical distance of 4.25 kpc. The density profile for this case is calculated to the distance of more than 55 kpc. The density fluctuations in this case are significant up to approximately 35 kpc. NGC 2841 shows in its rotation curve an initial dip, consistent with Keplerian motion around a point or spherically contained source, but then it rises and varies at long distances. The density curve drops quickly, and then oscillates slowly at large distances showing a great deal of structure. This object is a moderately wound barred spiral whose distance is 14.1 Mpc from our own Milky Way Galaxy.

Figure 2 shows the density profile of dark matter for NGC 4527 with a critical radius of 0.3 kpc and the results are shown up to about 12 kpc. In this region oscillations can be seen in the dark matter density. In general, the fluctuations get smaller as distance increases. NGC 4527 shows a variable rotation curve even at larger radii. At a low level, the density curve shows a great deal of structure, but it dies out at larger distances. This object is an intermediate spiral galaxy whose distance is 20.6 Mpc from the Milky Way Galaxy.

Figure 3 shows the results for NGC 4945 with a critical radius of 0.42 kpc. The dark matter profile in this case was calculated up to a radius of more than 18 kpc. NGC 4945 shows a flat rotation curve with mild variations. The density profile shows significant drop followed by significant structure at a low level for some distance. This barred spiral galaxy is bright (V magnitude = 9.3) and close to the Milky Way at a mere 3.6 Mpc distance.

Figure 4 shows the results for NGC 3198 with a critical radius of 4.15 kpc. The dark matter profile in this case was calculated up to a radius of 30 kpc. The density profile shows much smaller fluctuations compared to our other chosen galaxies, but they are still significant. This barred spiral galaxy has an apparent magnitude of 10.3 at a distance of 14.4 Mpc from the Milky Way Galaxy.

To create accurate error bars, we had to account for multiple factors. With all galaxies, the light that one can detect drops off significantly as you approach larger galactocentric distances, which would lead one to believe the error bars would be much larger at those distances. While the lesser amount of light is true, there is still a sufficient amount of light to result in small error bars. Normally, that would be the only consideration and then we would be using S/N statistics alone. However, in our model the galaxy is composed of essentially two parts, the bulge and the disk, which overlap. We are interested in the orbits of the disk (planar stars). The bulge, however, is large, and extends well beyond r_c. The orbits of the stars in the bulge are random in directional orientation and are not generally along the galactic plane. This results in two sources of error.

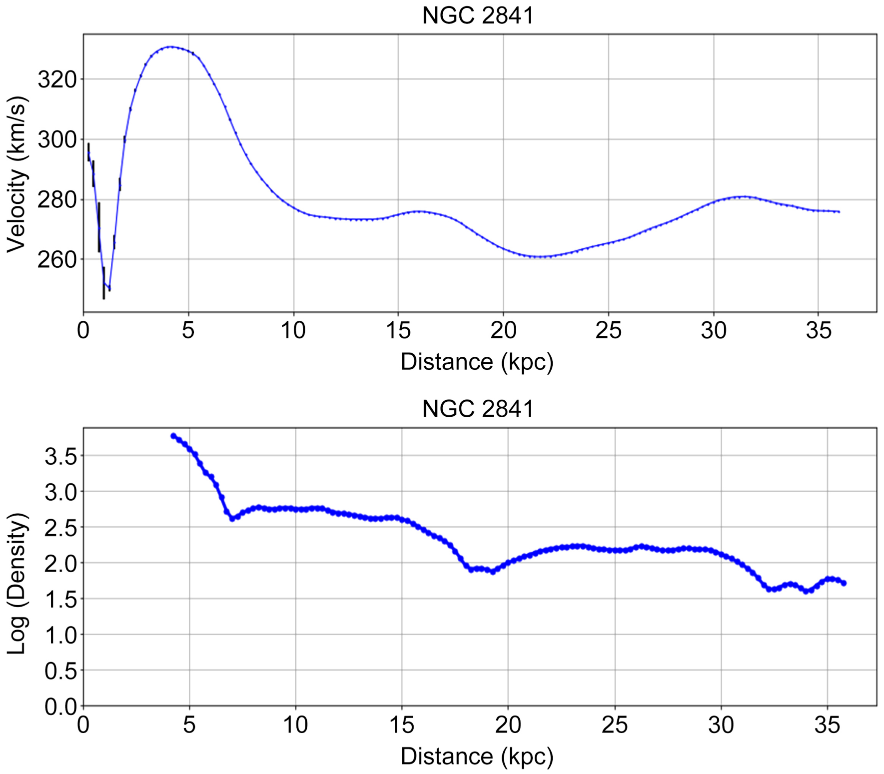

Figure 1. The rotation curve and dark matter density profile for NGC 2841. The critical radius for this case is estimated at 4.25 kpc.

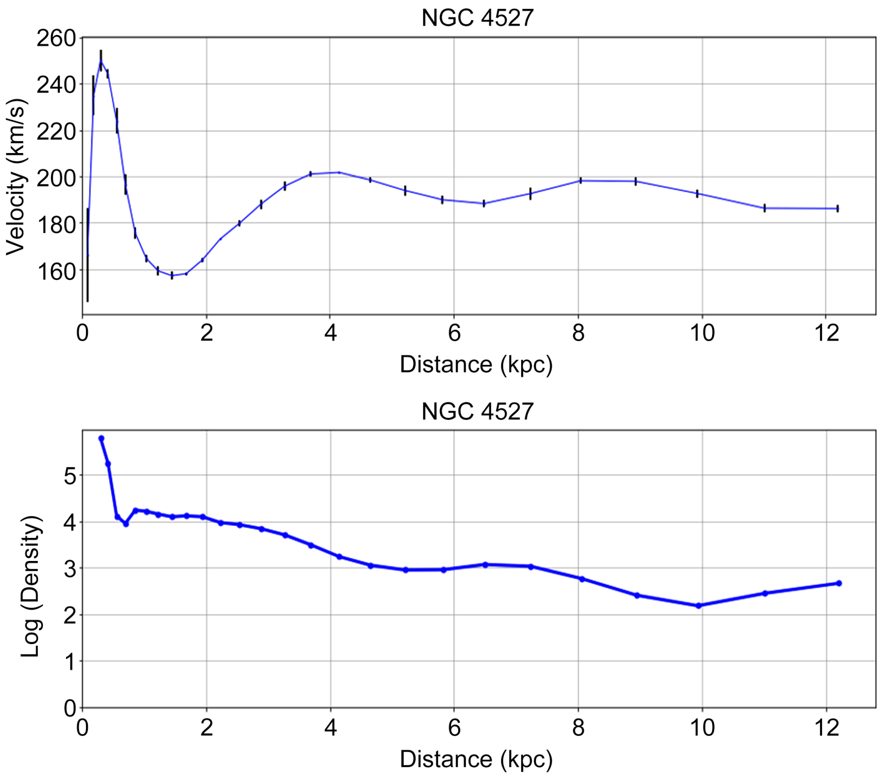

Figure 2. The rotation curve and dark matter density profile for NGC 4527. The critical radius for this case is estimated at 0.3 kpc.

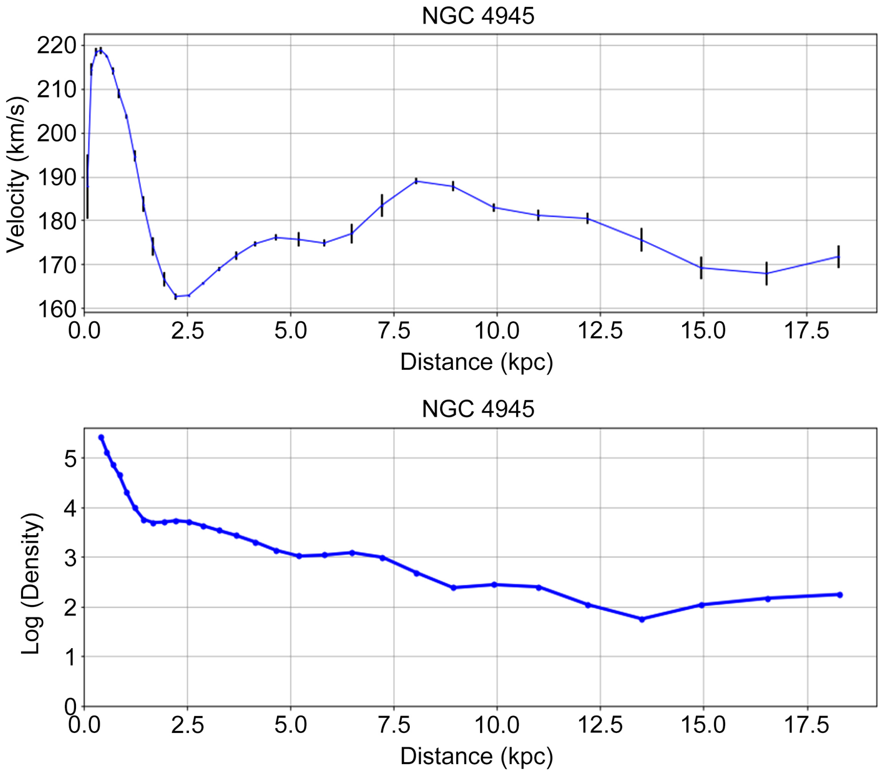

Figure 3. The rotation curve and dark matter density profile for NGC 4945. The critical radius for this case is estimated at 0.42 kpc.

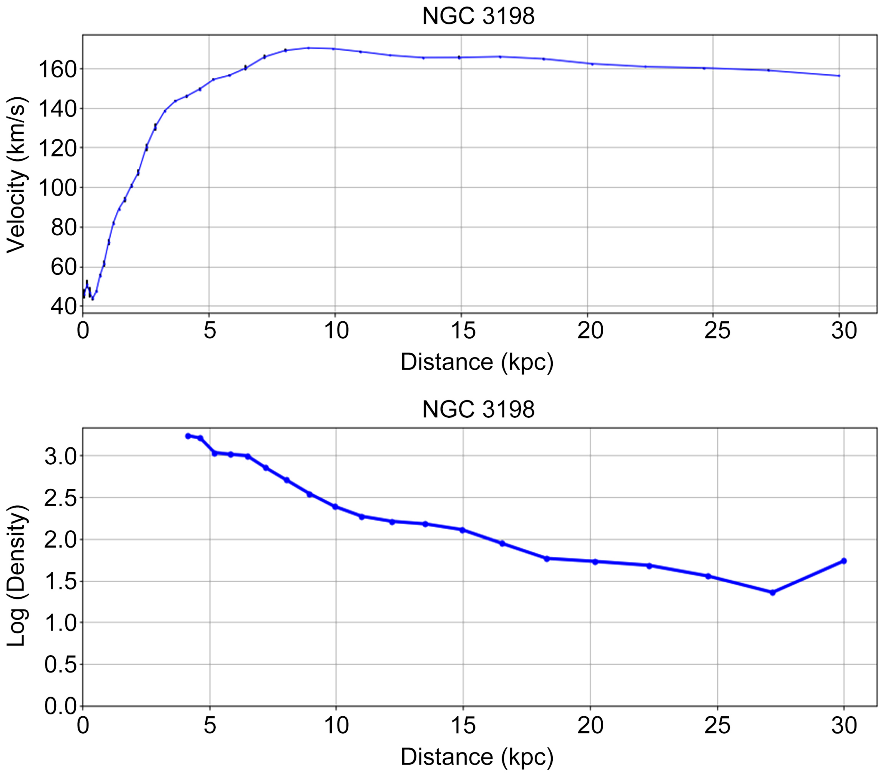

Figure 4. The rotation curve and dark matter density profile for NGC 3198. The critical radius for this case is estimated at 4.15 kpc.

1) Most of the stars are outside the plane, and therefore when slit spectroscopy (aligned perpendicular to the plane of the galaxy) is performed, that light contributes noise to the system, because those recessional velocities are invalid: they are in the wrong place going in the wrong direction.

2) Even if you could somehow consider looking at only those stars in the plane, most of the time the orbits of bulge stars would not be parallel to the orbits of the other stars in the plane, but rather cut at large angles to the defined plane. Therefore, you are introducing further contamination to this observation.

While it is true that the magnitude of the recessional velocity (which is what is directly measured) is large, that is not enough to keep the error small. As we have shown above, we have to contend with contamination, since we do not know on a case by case basis what that component is (for bulge stars found in the plane), and we are forced to include stars outside the plane, which comprise of most of the bulge stars anyway. These two sources of uncertainty are much greater than those presented by S/N statistics alone. The closer to the core of the galaxy, the less of the planar kinematics we are dealing with. The further out, and proportionately the more the disk stars are included and the less the effect of bulge stars. The bulge star population is blending into disk star population. Eventually, we are dealing with disk stars alone. For this reason, the error bars start out large, but as we get further and further out, they shrink.

In all four galaxies, the density of the dark matter is decreasing overall throughout the galaxy, as expected. Not only are there clear disruptions in the distribution of dark matter throughout these four galaxies, but those disruptions align with the variations in the orbital velocity of matter within those galaxies. Indeed, as previously stated, Equation (5) shows that a small change in velocity in the rotational curve implies a change in the density distribution of dark matter. These changes are most visible in the initial few points within the dark matter density plots and rotation curves. One can observe a large initial dip in the rotation curve, which is seen in all four of our chosen galaxies. This is also represented in the large drop of dark matter density in each of our projected density profiles. However, we are more interested in the fluctuations at larger radii. The question then is, what is causing these disruptions in the density of dark matter throughout these galaxies?

Here we may speculate. One possible explanation for these variations in the density of dark matter could be orbital resonance, which is well known to be involved in the structure of Saturn's rings, for example [18]. This occurs when an object, such as an orbiting moon or cloud of orbiting particles, in Saturn's rings orbit at a frequency that exerts a consistent, periodic gravitational pull on other orbiting objects. This phenomenon creates large fluctuations in the formation of Saturn's rings, resulting in varying peaks and dips in luminous matter, thus creating the rings that we observe. The same phenomenon is also known to have a large impact on the evolution of spiral arms and has been modeled in the Milky Way galaxy by T. A. Michtchenko [19]. They used these models to understand better the distribution of stars, as well as the chaos within different regions of galaxies. The dips in observational data seen in galactic rotation curves could be due to voids in luminous matter throughout the galaxy due to orbital resonance.

Another potential explanation could be attributed to perturbing objects (e.g. globular cluster, satellite and interloping galaxies) [20]. Local objects may be gravitationally influencing the dark matter within these galaxies, affecting its orbital velocity at various radii. Dark matter has already been modeled to be distributed between galaxies in galactic clusters, which would support the idea of local objects affecting the dark matter density profiles [21]. This gravitational impact on dark matter by these local objects could be reflected in the galactic rotation curve through the observed variations of the orbital velocity of baryonic matter. Since the effect of perturbating objects would cause more irregular orbital patterns, rather than the periodic orbital patterns caused by orbital resonance, this theory may be more plausible than that of orbital resonance.

It is also possible that the arms of the spiral galaxies could be causing the dips in the data. The space in between the spiral arms of a galaxy contains little luminous matter. Although, according to standard spiral density wave theory there could be a considerable amount of non-luminous matter contained therein but of low baryonic density [22]. When collecting observational data for these galaxies, little data would be collected within the regions in between the galaxy's spiral arms and the Doppler shift information would be slightly contaminated.

Finally, in the derivation of Equation (5) above, the assumption was made that the stellar orbits are circular. For our approach, this is a good assumption. Therefore, we did not include any radial velocity component in our calculations. If you were to include the radial velocity (which cannot be directly observed by Doppler shift methods), the density profile suggested by the rotation curve would imply something a little different.

We have found that a rotation curve which varies at large galactocentric distances may indicate interesting structure in the resultant dark matter density distribution. This, in turn, indicates to us something about galaxy evolution, formation and dynamics which may be a useful investigation to pursue further.

Variations in the rotation curves are not adequately addressed by the modified Newtonian dynamics (MOND), originally proposed by Mordehai Milgrom in 1983 [23]. MOND does indeed have success, however, in predicting rotation curves of low-surface-brightness galaxies [24]. It is possible that these variations in rotation curves could have different explanations depending on the size of the observed galaxies and other factors, such as one of the previously mentioned phenomena or a combination of all of them acting simultaneously.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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