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# Thin-Shell Wormholes Admitting Conformal Motions in Spacetimes of Embedding Class One

**Peter K. F. Kuhfittig**

Department of Mathematics, Milwaukee School of Engineering, Milwaukee, USA
Email: kuhfitti@msoe.edu

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## Abstract

This paper discusses the feasibility of thin-shell wormholes in spacetimes of embedding class one admitting a one-parameter group of conformal motions. It is shown that the surface energy density $\sigma$ is positive, while the surface pressure $\mathcal{P}$ is negative, resulting in $\sigma + \mathcal{P} < 0$, thereby signaling a violation of the null energy condition, a necessary condition for holding a wormhole open. For a Morris-Thorne wormhole, matter that violates the null energy condition is referred to as "exotic". For the thin-shell wormholes in this paper, however, the violation has a physical explanation since it is a direct consequence of the embedding theory in conjunction with the assumption of conformal symmetry. These properties avoid the need to hypothesize the existence of the highly problematical exotic matter.

## Keywords

Thin-Shell Wormholes, Conformal Symmetry, Embedding Class One, Exotic Matter

## 1. Introduction

Wormholes are handles or tunnels in spacetime connecting different regions of our universe or entirely different universes. While there had been some forerunners, macroscopic traversable wormholes were first discussed in detail by Morris and Thorne in 1988 [1]. The wormhole geometry is described by the following static and spherically symmetric line element

$$\mathrm{d}s^2 = -\mathrm{e}^{\nu(r)}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-\frac{b(r)}{r}} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right), \tag{1}$$

using units in which $c = G = 1$. Here $\nu = \nu(r)$ is usually referred to as the *redshift function*, which must be everywhere finite to prevent the occurrence of an event horizon. The function $b = b(r)$ is called the *shape function* since it determines the spatial shape of the wormhole when viewed, for example, in an embedding diagram. The spherical surface $r = r_0$ is called the *throat* of the wormhole. At the throat, $b = b(r)$ must satisfy the following conditions: $b(r_0) = r_0$, $b(r) < r$ for $r > r_0$, and $b'(r_0) \leq 1$, usually called the *flare-out condition*. This condition can only be satisfied by violating the null energy condition (NEC), which states that for the stress-energy tensor $T_{\alpha\beta}$, we must have

$$T_{\alpha\beta} k^\alpha k^\beta \geq 0 \tag{2}$$

for all null vectors $k^\alpha$. For the outgoing null vector $(1,1,0,0)$, the violation becomes

$$T_{\alpha\beta} k^\alpha k^\beta = \rho + p_r < 0. \tag{3}$$

Here $T^t{}_t = -\rho$ is the energy density, $T^r{}_r = p_r$ is the radial pressure, and $T^\theta{}_\theta = T^\phi{}_\phi = p_t$ is the lateral (transverse) pressure. For a Morris-Thorne wormhole, matter that violates the NEC is called "exotic", a term borrowed from quantum field theory.

The purpose of this paper is to account for the problematical nature of exotic matter by studying the effects of conformal symmetry in conjunction with some well-known classical embedding theorems. More precisely, by conformal symmetry we mean the existence of a conformal Killing vector $\xi$ defined by the action of $\mathcal{L}_\xi$ on the metric tensor:

$$\mathcal{L}_\xi g_{\mu\nu} = \psi(r) g_{\mu\nu}; \tag{4}$$

here $\mathcal{L}_\xi$ is the Lie derivative operator and $\psi(r)$ is the conformal factor. Embedding theorems, which have their origin in classical geometry, depend on Campbell's theorem, which has been used to show that a Riemannian space can be embedded in a higher-dimensional flat space.

## 2. Conformal Killing Vectors

As indicated in the Introduction, we assume in this paper that our static spherically symmetric spacetime admits a one-parameter group of conformal motions, by which we mean motions along which the metric tensor of a spacetime remains invariant up to a scale factor. In other words, there exist conformal Killing vectors such that

$$\mathcal{L}_\xi g_{\mu\nu} = g_{\eta\nu} \xi^\eta{}_{;\mu} + g_{\mu\eta} \xi^\eta{}_{;\nu} = \psi(r) g_{\mu\nu}, \tag{5}$$

where the left-hand side is the Lie derivative of the metric tensor and $\psi(r)$ is the conformal factor [2] [3]. In the usual terminology, the vector $\xi$ generates the conformal symmetry and the metric tensor $g_{\mu\nu}$ is conformally mapped into itself along $\xi$. According to Refs. [4] [5], this type of symmetry has proved to be effective in describing relativistic stellar-type objects. Furthermore, conformal symmetry has led to new solutions, as well as to new geometric and kinematical insights [6]-[9]. Two earlier studies assumed *non-static* conformal symmetry [3]

[10].

To study the effect of conformal symmetry, we wish to make use of Ref. [11], which uses the following form of the line element:

$$\mathrm{d}s^2 = -\mathrm{e}^{\nu(r)}\mathrm{d}t^2 + \mathrm{e}^{\lambda(r)}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right). \tag{6}$$

The Einstein field equations then become

$$\mathrm{e}^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} = 8\pi\rho, \tag{7}$$

$$\mathrm{e}^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right) - \frac{1}{r^2} = 8\pi p_r, \tag{8}$$

and

$$\frac{1}{2}\mathrm{e}^{-\lambda}\left[\frac{1}{2}(\nu')^2 + \nu'' - \frac{1}{2}\lambda'\nu' + \frac{1}{r}(\nu' - \lambda')\right] = 8\pi p_t. \tag{9}$$

Here $\rho$ is the energy density, $p_r$ is the radial pressure, and $p_t$ is the transverse pressure. It is well known that Equation (9) could be obtained from the conservation of the stress-energy tensor, *i.e.*, $T^{\mu\nu}_{\ ;\nu} = 0$. So we need to use only Equations (7) and (8).

As pointed out by Herrera and Ponce de León [4], the subsequent analysis can be simplified somewhat by restricting the vector field in a certain way: we require that $\xi^\alpha U_\alpha = 0$, where $U_\alpha$ is the four-velocity of the perfect fluid distribution, and that fluid flow lines are mapped conformally onto fluid flow lines. According to Ref. [4], the assumption of spherical symmetry then implies that $\xi^0 = \xi^2 = \xi^3 = 0$. Equation (5) now yields the following results:

$$\xi^1\nu' = \psi, \tag{10}$$

$$\xi^1 = \frac{1}{2}\psi r, \tag{11}$$

and

$$\xi^1\lambda' + 2\xi^1_{\ ,1} = \psi. \tag{12}$$

From Equations (10) and (11), we then obtain

$$\mathrm{e}^\nu = Cr^2, \tag{13}$$

where $C$ is an integration constant. Combined with Equation (12), this yields

$$\mathrm{e}^\lambda = \left(\frac{1}{\psi}\right)^2. \tag{14}$$

The arbitrary constant in Equation (13) can be obtained from the junction conditions in the usual way. This is a necessary step since, according to Equation (13), our wormhole spacetime is not asymptotically flat and must therefore be cut off at some $r = a$ and joined to an exterior Schwarzschild spacetime,

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1 - 2M/r} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right). \tag{15}$$

It follows that $\mathrm{e}^{\nu(a)} = Ca^2 = 1 - 2M/a$, so that

$$C = \frac{1-2M/a}{a^2}, \tag{16}$$

where $M$ is the mass of the wormhole as seen by a distant observer. We also have $b(a) = 2M$.

For future reference, let us note that the field Equations (7) and (8) can be rewritten as follows:

$$\frac{1}{r^2}\left(1-\psi^2\right) - \frac{\left(\psi^2\right)'}{r} = 8\pi\rho \tag{17}$$

and

$$\frac{1}{r^2}\left(3\psi^2 - 1\right) = 8\pi p_r. \tag{18}$$

To see why, we get from Equation (14)

$$\mathrm{e}^{-\lambda} = \psi^2 \text{ and } \lambda' = -\frac{2\psi'}{\psi}.$$

Substituting in Equation (7), we get

$$\psi^2\left(-\frac{2\psi'}{\psi r} - \frac{1}{r^2}\right) + \frac{1}{r^2} = -\frac{2\psi\psi'}{r} - \frac{1}{r^2}\psi^2 + \frac{1}{r^2} = \frac{1}{r^2}\left(1-\psi^2\right) - \frac{\left(\psi^2\right)'}{r} = 8\pi\rho.$$

Similarly, combining Equation (8) with Equation (13), yields Equation (18).

## 3. The Role of Embedding

Embedding theorems have a long history in the general theory od relativity. For example, according to Refs. [11] [12], the *vacuum* field equations in five dimensions yield the Einstein field equations *with matter*, called the *induced-matter theory*, to be understood in the following sense: what we perceive as matter is just the impingement of the higher-dimensional space onto ours; this may very well include exotic matter.

According to Campbell's theorem [13], a Riemannian space can be embedded in a higher-dimensional flat space: an $n$-dimensional Riemannian space is said to be of embedding class $m$ if $m+n$ is the lowest dimension $d$ of the flat space in which the given space can be embedded. Given that $d = \frac{1}{2}n(n-1)$, a four-dimensional Riemannian space is of class two since it can be embedded in a six-dimensional flat space, *i.e.*, $d = 6$. Moreover, a line element of class two can be reduced to a line element of class one by a suitable transformation of coordinates [14]-[19]. Such a metric can therefore be embedded in the five-dimensional flat spacetime

$$\mathrm{d}s^2 = -\left(\mathrm{d}z^1\right)^2 + \left(\mathrm{d}z^2\right)^2 + \left(\mathrm{d}z^3\right)^2 + \left(\mathrm{d}z^4\right)^2 + \left(\mathrm{d}z^5\right)^2; \tag{19}$$

the coordinate transformation is given by $z^1 = \sqrt{K}\,\mathrm{e}^{\frac{\nu}{2}}\sinh\frac{t}{\sqrt{K}}$, $z^2 = \sqrt{K}\,\mathrm{e}^{\frac{\nu}{2}}\cosh\frac{t}{\sqrt{K}}$, $z^3 = r\sin\theta\cos\phi$, $z^4 = r\sin\theta\sin\phi$, and $z^5 = r\cos\theta$. The differentials of these components are

$$\mathrm{d}z^1 = \sqrt{K}\,\mathrm{e}^{\frac{\nu}{2}}\frac{\nu'}{2}\sinh\frac{t}{\sqrt{K}}\mathrm{d}r + \mathrm{e}^{\frac{\nu}{2}}\cosh\frac{t}{\sqrt{K}}\mathrm{d}t, \tag{20}$$

$$\mathrm{d}z^2 = \sqrt{K}\,\mathrm{e}^{\frac{\nu}{2}}\frac{\nu'}{2}\cosh\frac{t}{\sqrt{K}}\mathrm{d}r + \mathrm{e}^{\frac{\nu}{2}}\sinh\frac{t}{\sqrt{K}}\mathrm{d}t, \tag{21}$$

$$\mathrm{d}z^3 = \sin\theta\cos\phi\,\mathrm{d}r + r\cos\theta\cos\phi\,\mathrm{d}\theta - r\sin\theta\sin\phi\,\mathrm{d}\phi, \tag{22}$$

$$\mathrm{d}z^4 = \sin\theta\sin\phi\,\mathrm{d}r + r\cos\theta\sin\phi\,\mathrm{d}\theta + r\sin\theta\cos\phi\,\mathrm{d}\phi, \tag{23}$$

and

$$\mathrm{d}z^5 = \cos\theta\,\mathrm{d}r - r\sin\theta\,\mathrm{d}\theta. \tag{24}$$

The substitution yields

$$\mathrm{d}s^2 = -\mathrm{e}^{\nu}\mathrm{d}t^2 + \left(1 + \frac{1}{4}K\mathrm{e}^{\nu}\left(\nu'\right)^2\right)\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right). \tag{25}$$

Metric (25) is therefore equivalent to metric (6) if

$$\mathrm{e}^{\lambda} = 1 + \frac{1}{4}K\mathrm{e}^{\nu}\left(\nu'\right)^2, \tag{26}$$

where $K > 0$ is a free parameter. The result is a metric of embedding class one. Equation (26) can also be obtained from the Karmarkar condition [20]:

$$R_{1414} = \frac{R_{1212}R_{3434} + R_{1224}R_{1334}}{R_{2323}}, \quad R_{2323} \neq 0. \tag{27}$$

It is interesting to note that Equation (26) is a solution of the differential equation

$$\frac{\nu'\lambda'}{1-\mathrm{e}^{\lambda}} = \nu'\lambda' - 2\nu'' - \left(\nu'\right)^2, \tag{28}$$

which is readily solved by separation of variables. So the free parameter $K$ is actually an arbitrary constant of integration.

## 4. Thin-Shell Wormholes

Our first task in this section is to recall from Sec. 1 that for a Morris-Thorne wormhole, the shape function $b = b(r)$ must satisfy the flare-out condition $b'(r_0) \le 1$, a geometric requirement that can only be satisfied by violating the NEC $\rho + p_r < 0$. Our discussion of conformal symmetry has yielded Equations (13) and (14). From Equation (13), $\mathrm{e}^{\nu} = Cr^2$, we obtain

$$\nu' = \frac{2}{r}. \tag{29}$$

Substituting Equations (29) and (14) in Equation (26) from the embedding theory, we obtain

$$\frac{1}{\psi^2} = 1 + \frac{1}{4}K\left(Cr^2\right)\left(\frac{2}{r}\right)^2. \tag{30}$$

The result is

$$\psi^2 = \frac{1}{1+KC}. \tag{31}$$

Returning to Equations (17) and (18), since $\left(\psi^2\right)' = 0$, it follows at once that

$$T_{\alpha\beta}k^{\alpha}k^{\beta} = 8\pi\left(\rho + p_r\right) = 0. \tag{32}$$

Since the NEC is not violated, we do not get a wormhole solution. We will therefore consider instead a thin-shell wormhole by first defining a suitable shape function, making use of Equation (31):

$$b(r) = r\left(1 - \frac{r - r_0}{1 + KC}\right). \tag{33}$$

Observe that we have indeed $b(r_0) = r_0$, while

$$0 < b'(r_0) = 1 - \frac{r_0}{1 + KC} < 1 \tag{34}$$

for $K$ sufficiently large. (Recall that $C$ was obtained from the junction condition, Equation (16)). Conformally symmetric wormholes are also discussed in Ref. [21].

A thin-shell wormhole is constructed by taking two copies of a Schwarzschild spacetime and removing from each the four-dimensional region

$$\Omega = \left\{ r \le a \mid a > 2M \right\}, \tag{35}$$

where $a$ is a constant [22]. By identifying the boundaries, *i.e.*, by letting

$$\partial\Omega = \left\{ r = a \mid a > 2M \right\}, \tag{36}$$

we obtain a manifold that is geodesically complete. In our situation, we take $r = a$ to be the cut-off in Equation (16) since we already know that $b(a) = 2M$; typically, $a \gg r_0$.

To meet this goal, let us consider the surface stresses using the Lanczos equations [23]:

$$\sigma = -\frac{1}{4\pi}\kappa^{\theta}{}_{\theta} \tag{37}$$

and

$$\mathcal{P} = \frac{1}{8\pi}\left(\kappa^{\tau}{}_{\tau} + \kappa^{\theta}{}_{\theta}\right), \tag{38}$$

where $\kappa_{ij} = K^{+}_{ij} - K^{-}_{ij}$ and $K_{ij}$ is the extrinsic curvature. Still following Ref. [23],

$$\kappa^{\theta}{}_{\theta} = \frac{1}{a}\sqrt{1 - \frac{2M}{a}} - \frac{1}{a}\sqrt{1 - \frac{b(a)}{a}}. \tag{39}$$

So by Equation (37),

$$\sigma = -\frac{1}{4\pi a}\left(\sqrt{1 - \frac{2M}{a}} - \sqrt{1 - \frac{b(a)}{a}}\right). \tag{40}$$

Given that the shell is infinitely thin, the radial pressure is zero. If the surface density is denoted by $\sigma$, then the NEC violation $\sigma + p_r < 0$ implies that $\sigma$ is negative, which is completely unphysical. One of the goals in this paper is to show that under the assumption of conformal symmetry in conjunction with the embedding theory, $\sigma$ can be positive. More precisely, if $\mathcal{P}$ denotes the surface pressure, then we must have $\sigma + \mathcal{P} < 0$ to ensure that the NEC is violated on the thin shell itself, even though the NEC is met for the radial outgoing null vector $(1,1,0,0)$, as shown in Inequality (32). Even though $b(a) = 2M$, part of the junction formalism is to assume that the junction surface $r = a$ is an infinitely thin surface having a nonzero density that may be positive or negative. For $\sigma$ to be positive, we must have $\sqrt{1-\frac{2M}{a}} < \sqrt{1-\frac{b(a)}{a}}$, which implies that $b(a) < 2M$. So let us assume for now that $b(a) \approx 2M$ and return to Ref. [23]:

$$K_{\tau}^{\tau+} = \frac{M/a^2}{\sqrt{1-2M/a}} \tag{41}$$

and

$$K_{\tau}^{\tau-} = \frac{1}{2}\nu'(a)\sqrt{1-\frac{b(a)}{a}}. \tag{42}$$

Since $\nu'(a) = 2/r$ by Equation (29), the surface pressure is given by

$$\begin{aligned}\mathcal{P} &= \frac{1}{8\pi}\left[\frac{M/a^2}{\sqrt{1-2M/a}} - \frac{1}{a}\sqrt{1-\frac{b(a)}{a}} + \frac{1}{a}\sqrt{1-\frac{2M}{a}} - \frac{1}{a}\sqrt{1-\frac{b(a)}{a}}\right] \\ &= \frac{1}{8\pi}\frac{1}{\sqrt{1-2M/a}}\left(\frac{M}{a^2} - \frac{1}{a}\sqrt{1-\frac{b(a)}{a}}\sqrt{1-\frac{2M}{a}}\right) \\ &\quad + \frac{1}{8\pi}\left(\frac{1}{a}\sqrt{1-\frac{2M}{a}} - \frac{1}{a}\sqrt{1-\frac{b(a)}{a}}\right).\end{aligned} \tag{43}$$

It now becomes apparent that for $b(a) \lesssim 2M$, the last term on the right-hand side is close to zero. As a result,

$$\begin{aligned}\mathcal{P} &\approx \frac{1}{8\pi}\frac{1}{\sqrt{1-\frac{2M}{a}}}\left[\frac{\frac{1}{2}b(a)}{a^2} - \frac{1}{a}\left(1-\frac{b(a)}{a}\right)\right] \\ &= \frac{1}{8\pi}\frac{1}{\sqrt{1-\frac{2M}{a}}}\cdot\frac{1}{a}\left(-1+\frac{3}{2}\frac{b(a)}{a}\right).\end{aligned} \tag{44}$$

Using our shape function, Equation (33), this leads to

$$\begin{aligned}\mathcal{P} &\approx \frac{1}{8\pi}\frac{1}{\sqrt{1-\frac{2M}{a}}}\cdot\frac{1}{a}\left[-1+\frac{3}{2}\left(1-\frac{a-r_0}{1+KC}\right)\right] \\ &= \frac{1}{8\pi}\frac{1}{\sqrt{1-\frac{2M}{a}}}\cdot\frac{1}{2a}\left(1-\frac{3(a-r_0)}{1+KC}\right).\end{aligned} \tag{45}$$

We know from the flare-out condition, Equation (34), that $1+KC$ is going to be a fixed quantity. Moreover, $a \gg r_0$; so for $a$ sufficiently large, $\mathcal{P}$ is negative and bounded away from zero, while under the assumption that $b \lesssim 2M$, $\sigma$ is close to zero. We therefore get $\sigma+\mathcal{P}<0$, which was to be shown.

The inequality $\sigma+\mathcal{P}<0$ indicates that the NEC has indeed been violated on the thin shell. In a Morris-Thorne wormhole, matter that violates the NEC is referred to as "exotic," a requirement that many researchers consider to be unphysical. In our situation, however, this violation has a physical basis since it is a direct consequence of the embedding in a higher-dimensional spacetime in conjunction with the assumption of conformal symmetry. These properties avoid the need to hypothesize the existence of the highly problematical exotic matter.

## 5. Conclusion

This paper discusses thin-shell wormholes based on the standard cut-and-paste technique. We assume that the wormhole spacetime admits a one-parameter group of conformal motions. We also make use of an embedding theorem that allows a Riemannian space to be embedded in a higher-dimensional flat space. The extra degree of freedom enables us to show that the surface energy density $\sigma$ is positive, while the surface pressure $\mathcal{P}$ is negative, but, in addition, $\sigma+\mathcal{P}<0$. So the null energy condition (NEC) has been violated. For a Morris-Thorne wormhole, matter that violated the NEC is referred to as "exotic", a condition that many researchers consider to be unphysical. In this paper, the violation has a physical explanation since it is a direct consequence of the embedding theory in conjunction with the assumption of conformal symmetry and can therefore be viewed as part of the induced-matter theory.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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