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Numerical Modeling of the Dynamic Problem of Building Foundation Interaction

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Abstract

This article considers the direct solution of the dynamic problem of building-foundation interaction. A mathematical model of the interaction between a building and a deformable foundation is presented. A numerical method of successive approximations of velocities and accelerations, varying over time, is proposed. A numerical algorithm and a computer program have been developed, and studies of free and forced oscillations of the dynamic model of the building have been carried out. Numerical experiments were conducted, and results were obtained using a frame building as an example. The results are given for a simplified model considering linear and angular displacements of the foundation slab.

Keywords

Seismic Response, Soft Soil, Interaction Effect, Rocking, Homogeneous Foundation, Shear Wave, Dynamic Model, Damping Matrix

1. Introduction

During earthquakes, seismic vibrations from the source are transmitted to the building through the foundation. As is known, for a building constructed on a rock foundation, during an earthquake, the force created in the form of an overturning moment and shearing force will not cause deformation of the foundation. In this case, the seismic response depends only on the properties of the building structure. However, for soft soils used as a foundation, soil deformation under seismic loading affects the response of the building and creates a building-soil interaction (BSI) effect. The BSI effect has been studied in the works [1]-[5], etc.

The BSI effect produces two types of interaction: kinematic—deviation of the foundation's motion from the motion in the free field; inertial—the foundation sets the superstructure in motion and inertial forces develop in the structure. The inertia-induced shear force and overturning moment developed in the foundation soil cause additional deformations in the soil. The foundation, excited in this way, becomes a source that propagates waves through the soil into infinity. This leads to a particular consideration of damping forces.

2. Equations of Motion

The interaction between the foundation soil and the structure is a determining factor influencing the dynamic responses of the system under seismic loading. Let us consider a multi-story frame building on an elastic homogeneous foundation subjected to the action of a shear wave (**Figure 1(a)**), which propagates upward from below at a speed

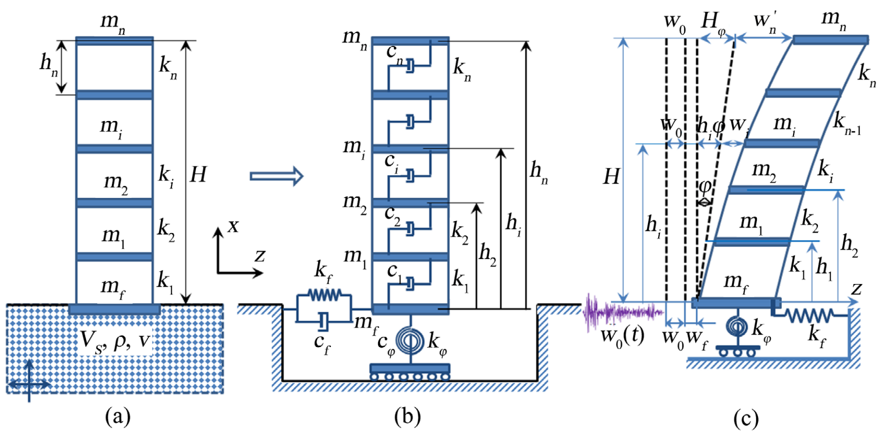


Figure 1. Dynamic model of a building considering foundation compliance.

$$V_s = \sqrt{G/\rho}, \quad G = E/2(1+\nu),$$

where G, E, ν, ρ —are the shear modules, Young's modulus, Poisson's ratio, and soil density. The dynamic model of the building with the corresponding stiffness coefficients and viscous damping elements is shown in **Figure 1(b)**. Under seismic impact, it is assumed that the deformability of the foundation causes the building to move as a whole object with a rotation angle $\varphi(t)$ and a linear horizontal displacement $w_f(t)$ (**Figure 1(c)**). Consequently, the dynamic model presented in **Figure 1(b)** has a certain number of degrees of freedom.

It is assumed that, under the action of a transverse wave, the dynamic model shown in **Figure 1(b)**, as a single entity, performs a translation and rotational movement by amounts $w_f(t)$ and, and then deforms (**Figure 1(c)**). It is also assumed that the mass of the foundation has two degrees of freedom, while the remaining masses $m_1, \cdots, m_n$ have one degree of freedom each.

Based on the primary system of the displacement method, the equations of motion for the dynamic model of the building can be formulated. By sequentially analyzing the dynamic equilibrium of each mass in the primary system, the corresponding equations are obtained. According to Alembert's principle; inertia forces are applied to the masses and treated as external forces. Considering these forces, the equilibrium conditions of the dynamic model in its deformed state are expressed using the equations of the displacement method [6].

$$\boldsymbol{RZ} + \boldsymbol{R_P} = \boldsymbol{0}, \quad \boldsymbol{Z} \to \boldsymbol{W}, \quad \boldsymbol{R_P} = \boldsymbol{M\ddot{W}}^*,$$

$$\boldsymbol{M\ddot{W}}^* + \boldsymbol{RW} = \boldsymbol{0} \tag{1}$$

As follows from the deformed state of the model (**Figure 1(c)**), the displacement of each mass consists of four components:

$$w_i^* = w_0 + w_f + h_i\varphi + w_i$$
$$i = 1, 2, \cdots, n$$

where w_0 is the prescribed displacement due to the earthquake, w_f —is the horizontal displacement of the foundation, $h_i\varphi$ —is the linear displacement due to the rotation of the foundation by an angle φ, w_i —is the relative displacement of the mass due to the deformation of the structure. The elements of the inertia force vector in (1) are represented as follows:

$$m_i\ddot{w}_i^* = m_i\left(\ddot{w}_0 + \ddot{w}_f + h_i\ddot{\varphi} + \ddot{w}_i\right) \tag{2}$$
$$i = 1, 2, \cdots, n$$

The matrix Equation (1), taking into account (2), is presented in expanded form.

$$\begin{gathered} m_1\ddot{w}_1 + m_1\ddot{w}_f + m_1h_1\ddot{\varphi} - k_1w_f + \left(k_1 + k_2\right)w_1 - k_2w_2 = -m_1\ddot{w}_0\left(t\right), \\ m_i\ddot{w}_i + m_i\ddot{w}_f + m_ih_i\ddot{\varphi} - k_iw_{i-1} + \left(k_i + k_{i+1}\right)w_i - k_{i+1}w_{i+1} = -m_i\ddot{w}_0\left(t\right), \\ i = 2, 3, \cdots, n-1, \\ m_n\ddot{w}_n + m_n\ddot{w}_f + m_nh_n\ddot{\varphi} - k_nw_{n-1} + k_nw_n = -m_n\ddot{w}_0\left(t\right). \end{gathered} \tag{3}$$

The other two equations are obtained from the equilibrium of the foundation slab, which has two degrees of freedom. The first of these two equations, corresponding to the horizontal motion of the mass of the foundation slab, is written as

$$m_1\ddot{w}_1 + \cdots + m_n\ddot{w}_n + m_{sf}\ddot{w}_f + \sum\nolimits_{i=1}^{n} m_ih_i\ddot{\varphi} + k_fw_f = -m_{sf}\ddot{w}_0\left(t\right) \tag{4}$$

$$m_{sf} = \sum\nolimits_{i=1}^{n} m_i + m_f$$

where k_f —the soil foundation stiffness coefficient in shear is denoted. The indices s and f correspond to the structural and foundation parts of the building, respectively. The second equation, which describes the rotation (rocking) of the building due to soil compliance, is expressed as

$$m_1h_1\ddot{w}_1 + \cdots + m_nh_n\ddot{w}_n + \sum\nolimits_{i=1}^{n} m_ih_i\ddot{w}_f + I_{sf}\ddot{\varphi} + k_\varphi\varphi = -\ddot{w}_0\left(t\right)\sum\nolimits_{i=1}^{n} m_ih_i \tag{5}$$

$$I_{sf} = I_f + \sum\nolimits_{i=1}^{n} I_i + \sum\nolimits_{i=1}^{n} m_ih_i^2$$

where I_f, I_i —the moments of inertia of the foundation mass and the mass of the

i -th floor of the building are taken with respect to an axis y, perpendicular to the plane of the drawing k_φ —is the rotational stiffness coefficient of the soil.

The moment of inertia of a rectangular plate with plan dimensions—along the z -axis and b —along the y-axis, thickness δ, and mass m, is determined by the formula [7].

$$I_i = I_y = m\left(a^2 + \delta^2\right)/12$$

where $m = \rho ab\delta$ —mass (ts²/m), $\rho = \gamma/g$ —material density (t/m³), γ —unit weight (t/m³).

Equations (3)-(5), which describe the motion of the building taking into account the soil compliance, can be written in matrix form

$$\boldsymbol{M}\ddot{\boldsymbol{W}} + \boldsymbol{K}\boldsymbol{W} = \boldsymbol{M}\boldsymbol{r}\ddot{w}_0(t) \tag{6}$$

where the square mass matrix of order $n+2$ can be represented in block form.

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_s & \boldsymbol{M}_{sf} \\ \boldsymbol{M}_{sf}^{\mathrm{T}} & \boldsymbol{M}_f \end{bmatrix} \quad \boldsymbol{M}_f = \begin{bmatrix} m_f + \sum_{i=1}^{n} m_i & \sum_{i=1}^{n} m_i h_i \\ \sum_{i=1}^{n} m_i h_i & I_f + \sum_{i=1}^{n} I_i + \sum_{i=1}^{n} m_i h_i^2 \end{bmatrix}$$

$\boldsymbol{M}_s$ —the diagonal mass matrix of order n corresponding to the part of the building above the foundation.

$$\boldsymbol{M}_s = diag\left(m_1 \quad m_2 \quad \cdots \quad m_n\right)$$

$\boldsymbol{M}_{sf}$ —a rectangular matrix of size $n \times 2$ $\boldsymbol{M}_f$ —a square matrix of second order. The square mass matrix of order $n+2$ can be represented as follows:

$$\boldsymbol{M} = \begin{bmatrix} m_1 & & & 0 & m_1 & m_1 h_1 \\ & m_2 & & & m_2 & m_2 h_2 \\ & & \ddots & & \vdots & \vdots \\ 0 & & & m_n & m_n & m_n h_n \\ m_1 & m_2 & \cdots & m_n & \Sigma_1 & \Sigma_2 \\ m_1 h_1 & m_2 h_2 & \cdots & m_n h_n & \Sigma_2 & \Sigma_3 \end{bmatrix}$$

where $\Sigma_1 = m_f + \sum_{i=1}^{n} m_i \quad \Sigma_2 = \sum_{i=1}^{n} m_i h_i \quad \Sigma_3 = I_f + \sum_{i=1}^{n} I_i + \sum_{i=1}^{n} m_i h_i^2$.

The stiffness matrix of the studied object has the form

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{K}_s & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{K}_f \end{bmatrix}$$

where $\boldsymbol{K}_s$ —is the banded stiffness coefficient matrix of order n.

$$\boldsymbol{K}_s = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & \cdots & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & -k_{n-1} & k_{n-1} + k_n & -k_n & 0 \\ 0 & 0 & \cdots & 0 & -k_n & k_n \end{bmatrix}$$

$\boldsymbol{K}_f$ —the stiffness coefficient matrix of the soil foundation.

$$\boldsymbol{K}_f = \begin{bmatrix} k_f & 0 \\ 0 & k_\varphi \end{bmatrix}$$

The acceleration and displacement vectors are represented as:

$$\ddot{\boldsymbol{W}} = \left\{ \ddot{w}_1 \ \ \ddot{w}_2 \ \cdots \ \ddot{w}_n \ \ \ddot{w}_f \ \ \ddot{\varphi} \right\}^{\mathrm{T}} \quad \boldsymbol{W} = \left\{ w_1 \ \ w_2 \ \cdots \ w_n \ \ w_f \ \ \varphi \right\}^{\mathrm{T}}$$

The mass column vector on the right-hand side of (6), as follows from (3)-(5), consists of the following elements:

$$\boldsymbol{m} = \boldsymbol{Mr} = \left\{ m_1, m_2, \cdots, m_n, \left(m_f + \sum_{i=1}^{n} m_i \right), \sum_{i=1}^{n} m_i h_i \right\}^{\mathrm{T}}$$

$\boldsymbol{r} = \{0, \cdots, 0, 1, 0\}^{\mathrm{T}}$ —influence vector.

The system of Equations (6), accounting for damping, is presented in its expanded form

$$\begin{bmatrix} \boldsymbol{M}_s & \boldsymbol{M}_{sf} \\ \boldsymbol{M}_{sf}^{\mathrm{T}} & \boldsymbol{M}_f \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{W}}_s \\ \ddot{\boldsymbol{W}}_f \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C}_s & 0 \\ 0 & \boldsymbol{C}_f \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{W}}_s \\ \dot{\boldsymbol{W}}_f \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_s & \mathbf{0} \\ \mathbf{0} & \boldsymbol{K}_f \end{bmatrix} \begin{Bmatrix} \boldsymbol{W}_s \\ \boldsymbol{W}_f \end{Bmatrix} = -\begin{Bmatrix} \boldsymbol{m}_s \\ \boldsymbol{m}_f \end{Bmatrix} \ddot{w}_0 \tag{7}$$

$$\boldsymbol{m}_s = \left\{ m_1 \quad m_2 \quad \cdots \quad m_n \right\}^{\mathrm{T}}, \quad \boldsymbol{m}_f = \left\{ \left(m_f + \sum_{i=1}^{n} m_i \right) \quad \sum_{i=1}^{n} m_i h_i \right\}^{\mathrm{T}}$$

where the damping coefficient matrix (damping matrix) can be represented according to Rayleigh [6] [8].

$$\boldsymbol{C}_s = a\boldsymbol{M}_s + b\boldsymbol{K}_s$$

a, b —arbitrary proportionality coefficients. The damping matrix corresponding to the oscillatory process in the soil foundation (**Figure 1(b)**) has the form

$$\boldsymbol{C}_f = \begin{bmatrix} c_f & 0 \\ 0 & c_\varphi \end{bmatrix}$$

The stiffness and damping coefficients of the soil in shear and during building rotation in the vertical plane for a rectangular foundation in plan, obtained from the solution of the problem of vibrations of a footing on an elastic foundation, are determined by the formulas [1] [2] [9]-[12], etc.

$$k_f = \frac{8GR_f}{2-\nu} \quad k_\varphi = \frac{8GR_\varphi^3}{3(1-\nu)} \quad B_\varphi = \frac{3(1-\nu)I_\varphi}{8\rho R_\varphi}$$

$$R_f = \sqrt{ab/\pi} \quad R_\varphi = \sqrt{\sqrt{a^3 b/3\pi}}$$

$$I_\varphi = m\left(a^2 + \delta^2\right)/12 = I_y \quad m = \rho ab\delta$$

$$c_f = \frac{4,6R_f^2\sqrt{\rho G}}{2-\nu} \quad c_\varphi = \frac{0.8R_\varphi^4\sqrt{\rho G}}{(1-\nu)\left(1+B_\varphi\right)}$$

where a, b, δ —width, length, and thickness of the foundation slab, respectively; G —is the shear modulus of the soil; ν —nu ν is the Poisson's ratio; R_f, R_φ —are the equivalent radii of the foundation and $\rho = \gamma/g$ —is the density of the soil. It should be noted that the methods for determining the soil stiffness and damping coefficients are also discussed in [13]-[18].

3. Numerical Modeling

The system of differential Equations (7) is solved using the numerical method of successive approximations [19], according to which the accelerations and velocity at time t_j are represented in general form as follows

$$\ddot{w}_{ij} = \frac{\alpha_1}{\tau^2}\left(w_{ij} - w_{i,j-1}\right) - \frac{\alpha_2}{\tau}\dot{w}_{i,j-1} - \alpha_3\ddot{w}_{i,j-1}$$

$$\dot{w}_{ij} = \frac{\beta_1}{\tau}\left(w_{ij} - w_{i,j-1}\right) - \beta_2\dot{w}_{i,j-1} - \tau\beta_3\ddot{w}_{i,j-1} \tag{8}$$

$$i = 1, 2, \cdots, n; \quad j = 1, 2, \cdots, N \tag{9}$$

where α_k, β_k—are numerical coefficients and τ—is the integration step. Angular accelerations and angular velocities are approximated in the same way:

$$\ddot{\varphi}_j = \frac{\alpha_1}{\tau^2}\left(\varphi_j - \varphi_{j-1}\right) - \frac{\alpha_2}{\tau}\dot{\varphi}_{j-1} - \alpha_3\ddot{\varphi}_{j-1} \tag{10}$$

$$\dot{\varphi}_j = \frac{\beta_1}{\tau}\left(\varphi_j - \varphi_{j-1}\right) - \beta_2\dot{\varphi}_{j-1} - \tau\beta_3\ddot{\varphi}_{j-1} \tag{11}$$

By substituting Equations (8)-(11) into Equation (3), written at time t_j, we obtain a system of n equations with $n+2$ unknown linear and angular displacements.

$$\left(m_1\frac{\alpha_1}{\tau^2} + c_1\frac{\beta_1}{\tau} + k_1 + k_2\right)w_{1j} - k_2 w_{2j} + \left(m_1\frac{\alpha_1}{\tau^2} - k_1\right)w_{fj} + m_1 h_1\frac{\alpha_1}{\tau^2}\varphi_j = B_{1j},$$

$$-k_2 w_{1j} + \left(m_2\frac{\alpha_1}{\tau^2} + c_2\frac{\beta_1}{\tau} + k_2 + k_3\right)w_{2j} - k_3 w_{2j} + m_2\frac{\alpha_1}{\tau^2}w_{fj} + m_2 h_2\frac{\alpha_1}{\tau^2}\varphi_j = B_{2j},$$

$$-k_n w_{n-1j} + \left(m_n\frac{\alpha_1}{\tau^2} + c_n\frac{\beta_1}{\tau} + k_n\right)w_{nj} + m_n\frac{\alpha_1}{\tau^2}w_{fj} + m_n h_n\frac{\alpha_1}{\tau^2}\varphi_j = B_{nj}, \tag{12}$$

$$j = 1, 2, 3, \cdots, N$$

where the free terms are represented in the form

$$\begin{aligned} B_{ij} = &-m_i\ddot{w}_{0j} + m_i\left(\frac{\alpha_1}{\tau^2}w_{i,j-1} + \frac{\alpha_2}{\tau}\dot{w}_{i,j-1} + \alpha_3\ddot{w}_{i,j-1}\right) \\ &+ m_i\left(\frac{\alpha_1}{\tau^2}w_{f,j-1} + \frac{\alpha_2}{\tau}\dot{w}_{f,j-1} + \alpha_3\ddot{w}_{f,j-1}\right) \\ &+ m_i h_i\left(\frac{\alpha_1}{\tau^2}\varphi_{j-1} + \frac{\alpha_2}{\tau}\dot{\varphi}_{j-1} + \alpha_3\ddot{\varphi}_{j-1}\right) \\ &+ c_i\left(\frac{\beta_1}{\tau}w_{i,j-1} + \beta_2\dot{w}_{i,j-1} + \tau\beta_3\ddot{w}_{i,j-1}\right), \end{aligned} \tag{13}$$

$$i = 1, 2, 3, \cdots, n; \quad j = 1, 2, 3, \cdots, N.$$

By substituting Equations (8)-(11) into the differential Equations (4) and (5), also written at time t_j, we obtain a system of two algebraic equations with $n+2$ unknowns.

$$\frac{\alpha_1}{\tau^2}\sum_{i=1}^{n} m_i w_{ij} + \left[\frac{\alpha_1}{\tau^2}\left(m_f + \sum_{i=1}^{n} m_i\right) + c_f\frac{\beta_1}{\tau} + k_f\right]w_{f,j} + \frac{\alpha_1}{\tau^2}\left(\sum_{i=1}^{n} m_i h_i\right)\varphi_j = B_{n+1j} \tag{14}$$

$$\frac{\alpha_1}{\tau^2}\sum_{i=1}^{n} m_i h_i w_{ij} + \frac{\alpha_1}{\tau^2} w_{fj} \sum_{i=1}^{n} m_i h_i + \left(\frac{\alpha_1}{\tau^2} I_{sf} + c_\varphi \frac{\beta_1}{\tau} + k_\varphi\right)\varphi_j = B_{n+2j} \tag{15}$$

$$I_{sf} = I_f + \sum_{i=1}^{n} I_i + \sum_{i=1}^{n} m_i h_i^2$$

where the right-hand side of these equations is represented as follows:

$$\begin{aligned} B_{n+1,j} = &-\left(m_f + \sum_{i=1}^{n} m_i\right)\ddot{w}_{0j} + \sum_{i=1}^{n} m_i \left(\frac{\alpha_1}{\tau^2} w_{i,j-1} + \frac{\alpha_2}{\tau}\dot{w}_{i,j-1} + \alpha_3 \ddot{w}_{i,j-1}\right) \\ &+\left(m_f + \sum_{i=1}^{n} m_i\right)\left(\frac{\alpha_1}{\tau^2} w_{fj-1} + \frac{\alpha_2}{\tau}\dot{w}_{fj-1} + \alpha_3 \ddot{w}_{fj-1}\right) \\ &+\sum_{i=1}^{n} m_i h_i \left(\frac{\alpha_1}{\tau^2}\varphi_{j-1} + \frac{\alpha_2}{\tau}\dot{\varphi}_{j-1} + \alpha_3\ddot{\varphi}_{j-1}\right) \\ &+ c_f\left(\frac{\beta_1}{\tau} w_{fj-1} + \beta_2 \dot{w}_{fj-1} + \tau\beta_3 \ddot{w}_{fj-1}\right), \end{aligned} \tag{16}$$

$$\begin{aligned} B_{n+2,j} = &-\ddot{w}_{0j}\sum_{i=1}^{n} m_i h_i + \sum_{i=1}^{n} m_i h_i \left(\frac{\alpha_1}{\tau^2} w_{i,j-1} + \frac{\alpha_2}{\tau}\dot{w}_{i,j-1} + \alpha_3\ddot{w}_{i,j-1}\right) \\ &+\sum_{i=1}^{n} m_i h_i \left(\frac{\alpha_1}{\tau^2} w_{fj-1} + \frac{\alpha_2}{\tau}\dot{w}_{fj-1} + \alpha_3\ddot{w}_{fj-1}\right) \\ &+ I_{sf}\left(\frac{\alpha_1}{\tau^2}\varphi_{j-1} + \frac{\alpha_2}{\tau}\dot{\varphi}_{j-1} + \alpha_3\ddot{\varphi}_{j-1}\right) \\ &+ c_\varphi\left(\frac{\beta_1}{\tau}\varphi_{j-1} + \beta_2\dot{\varphi}_{j-1} + \tau\beta_3\ddot{\varphi}_{j-1}\right) \end{aligned} \tag{17}$$

The systems of algebraic Equations (12)-(17) form a solvable system of equations of order $n+2$ with $(w_1, w_2, \cdots, w_n, w_f, \varphi)$ unknowns. This system of equations, corresponding to the time t_j, is written in matrix form as follows:

$$\boldsymbol{A}\boldsymbol{W}_j = \boldsymbol{B}_j \tag{18}$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & 0 & 0 & a_{1f} & a_{1\varphi} \\ a_{21} & a_{22} & a_{23} & & 0 & a_{2f} & a_{2\varphi} \\ 0 & a_{32} & \cdots & \cdots & 0 & \vdots & \vdots \\ 0 & & \cdots & \cdots & a_{n-1,n} & a_{n-1f} & a_{n-1\varphi} \\ 0 & 0 & & a_{n,n-1} & a_{nn} & a_{nf} & a_{n\varphi} \\ a_{n+1,1} & a_{n+1,2} & \cdots & \cdots & a_{n+1,n} & a_{n+1,f} & a_{n+1,\varphi} \\ a_{n+2,1} & a_{n+2,2} & \cdots & \cdots & a_{n+2,n} & a_{n+2,f} & a_{n+2,\varphi} \end{bmatrix} \begin{Bmatrix} w_{1j} \\ w_{2j} \\ \vdots \\ \vdots \\ w_{nj} \\ w_{fj} \\ \varphi_j \end{Bmatrix} = \begin{Bmatrix} B_{1j} \\ B_{1j} \\ \vdots \\ \vdots \\ B_{nj} \\ B_{fj} \\ B_{\varphi j} \end{Bmatrix} \tag{19}$$

where the elements of matrix $\boldsymbol{A}$ have the following form:

$$a_{11} = m_1\frac{\alpha_1}{\tau^2} + c_1\frac{\beta_1}{\tau} + k_1 + k_2, \quad a_{12} = -k_2,$$

$$a_{1n+1} = a_{1f} = m_1\frac{\alpha_1}{\tau^2}, \quad a_{1n+2} = a_{1\varphi} = m_1 h_1\frac{\alpha_1}{\tau^2},$$

$$a_{21} = -k_2, \quad a_{22} = m_2\frac{\alpha_1}{\tau^2} + c_2\frac{\beta_1}{\tau} + k_2 + k_3, \quad a_{23} = -k_3,$$

$$a_{2n+1} = a_{2f} = m_2\frac{\alpha_1}{\tau^2}, \quad a_{2n+2} = a_{2\varphi} = \frac{\alpha_1}{\tau^2} m_2 h_2,$$

$$a_{nn+1} = a_{nf} = m_n\frac{\alpha_1}{\tau^2}, \quad a_{nn+2} = a_{n\varphi} = m_n h_n\frac{\alpha_1}{\tau^2},$$

$$a_{nn-1} = -k_{n-1}, \quad a_{nn} = m_n \frac{\alpha_1}{\tau^2} + c_n \frac{\beta_1}{\tau} + k_n, \tag{20}$$

$$a_{n+1,1} = m_1 \frac{\alpha_1}{\tau^2}, \quad a_{n+1,2} = m_2 \frac{\alpha_1}{\tau^2}, \quad \cdots, \quad a_{n+1,n} = m_n \frac{\alpha_1}{\tau^2},$$

$$a_{n+1,n+1} = a_{n+1f} = \Sigma_1 \frac{\alpha_1}{\tau^2} + c_f \frac{\beta_1}{\tau} + k_f, \quad a_{n+1,n+2} = a_{n+1\varphi} = \Sigma_2 \frac{\alpha_1}{\tau^2},$$

$$a_{n+2,1} = m_1 h_1 \frac{\alpha_1}{\tau^2}, \quad a_{n+2,2} = m_2 h_2 \frac{\alpha_1}{\tau^2}, \quad \cdots, \quad a_{n+2,n} = m_n h_n \frac{\alpha_1}{\tau^2},$$

$$a_{n+2,n+1} = a_{n+2f} = \Sigma_2 \frac{\alpha_1}{\tau^2}, \quad a_{n+2,n+2} = a_{n+2\varphi} = \Sigma_3 \frac{\alpha_1}{\tau^2} + c_f \frac{\beta_1}{\tau} + k_\varphi,$$

$$\Sigma_1 = m_f + \sum_{i=1}^{n} m_i \quad \Sigma_2 = \sum_{i=1}^{n} m_i h_i \quad \Sigma_3 = I_f + \sum_{i=1}^{n} I_i + \sum_{i=1}^{n} m_i h_i^2$$

Thus, based on the conditions of convergence and stability of the solutions, by selecting the integration step τ, and specifying the soil stiffness and damping coefficients, the matrix $\boldsymbol{A}$ of the system of Equations (18) is formed. The vector of free terms, which consists of the elements from (13), (16), and (17), is formed depending on the external load and the values of displacements, velocities, and accelerations corresponding to the previous time step. Consequently, the system of Equations (18) is solved at each time step to determine the displacement vector, after which the velocity and acceleration vectors are calculated. At the next stage, internal forces are determined, including the overturning moment and the shear force in the support part of the structure. It should be noted that the matrix $\boldsymbol{A}$ is block-structured, and the Gaussian elimination method is applied to solve the system of algebraic Equations (18) [20].

4. Numerical Modeling Results

Based on the presented mathematical model, a computer program BCO-3-El Centro was developed in the Fortran language, and results of free and forced vibrations of the studied object under various loads were obtained. The numerical simulation results were obtained with an integration step $\tau = \Delta t / NT$, where $\Delta t = 0.02$, $NT \geq 4$—the sampling interval of the El Centro accelerogram.

Example 1. Study of free vibrations of a building considering the soil foundation compliance. A 10-story framed building is considered, with plan dimensions of 36 × 18 m a column grid of 6 × 6 m and a floor height of $h = 3$. The column cross-section is 0.5 × 0.5 the beam cross-section is 0.3 × 0.45 m, and the slab thickness is 0.2 m. The building rests on a foundation slab 0.5 m thick [21].

The results of free vibrations were obtained from the action of a uniformly distributed initial velocity of $v_0 = 1$ m/s). To analyze the convergence and stability of the solutions, free vibrations of the building without considering soil compliance were first studied for various time step values. Numerical experiments showed that when varying the time step within the range from $\Delta t/20$ to $\Delta t/4$, the results practically coincide. Figure 2 shows the free vibration graphs obtained at $\tau = \Delta t/20 = 0.001$ seconds). The fundamental period of free vibrations is found to be 0.84 seconds.

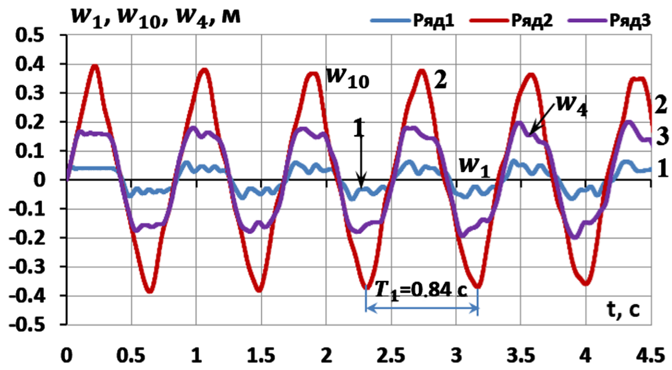


Figure 2. Free vibrations of the building model with a rigid base.

Similar results were obtained for the model considering soil flexibility. **Figure 3** shows the vibration graphs of masses m_f m_1 and m_{10}, obtained without considering damping, using the following soil stiffness coefficients [22].

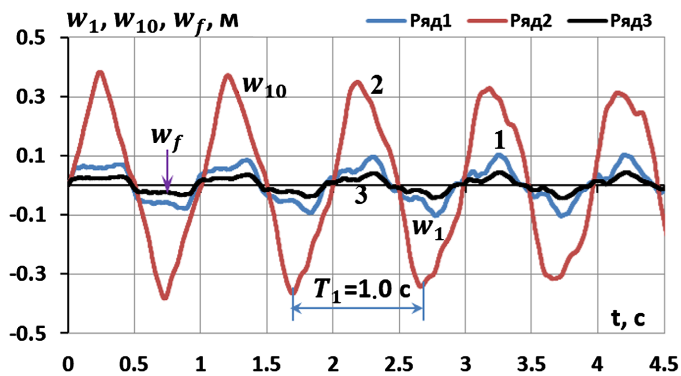


Figure 3. Free vibrations of the building model considering soil flexibility.

$$k_f = 1.91\times10^5 \ \ (\text{t/m}),\ \ k_\varphi = 7.53\times10^7 \ \ (\text{t/m}).$$

As follows from the obtained results, the fundamental period of free vibrations of the building with soil flexibility is 1.0 s, which is 19% greater than that with a rigid base.

Example 2. Forced vibrations of the dynamic model under the action of a model accelerogram.

$$\ddot{w}_0(t) = A\sin\theta t$$

$$A = 4.0 \ \ \text{m/s}^2,\ \ \theta = 2\pi/T_0$$

Acting in the foundation part of the building. A building model with the initial data provided in Example 1 is considered. Numerical modeling results were obtained for various values of the harmonic excitation frequency. For the purpose of comparison and to verify the reliability of the results, a dynamic model of the building was first considered, in which the stiffness coefficients of the foundation soil tend to infinity. The vibration graphs of the m_1 and m_{10}, obtained with damping at $T_0 = 1.0\ \text{s}$, are shown in **Figure 4**. It can be seen that the dynamic process corresponds to the beating effect, which indicates the closeness of the vibration periods of the external excitation and the natural vibrations of the building ($T_1 = 0.84$ c , Figure 2). The beating period is equal to:

$$T_b = \frac{2\pi}{|\omega_0 - \omega_1|} = \frac{T_0 T_1}{|T_1 - T_0|} = \frac{1 \cdot 0.84}{|0.84 - 1.0|} = 5.25$$

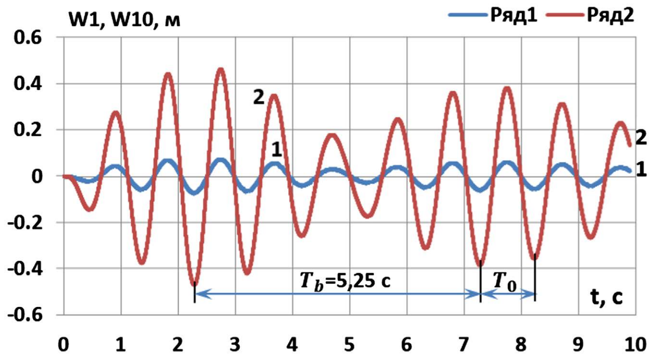


Figure 4. Beating phenomenon arising from the superposition of two oscillations with close frequencies.

Next, the results were obtained taking into account the foundation compliance under harmonic excitation at $T_0 = 1.0$ s . Figure 5 shows the results obtained for $k_f = 1.91 \times 10^5$ t/m $k_\varphi = 7.53 \times 10^7$ t/m without damping (Curve 1), and with damping taken into account at $c_f = 2.19 \times 10^4$ ts/m $c_\varphi = 2.26 \times 10^6$ ts/m (Curve 2). As expected, the amplitude of the oscillations increases indefinitely, which corresponds to a resonance mode. This confirms the reliability of the results related to the study of free vibrations of the dynamic building model with consideration of foundation compliance.

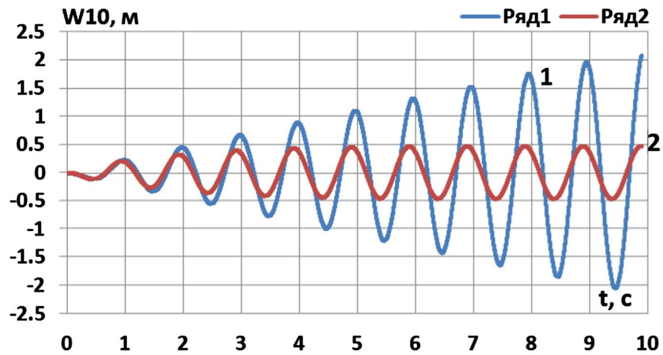
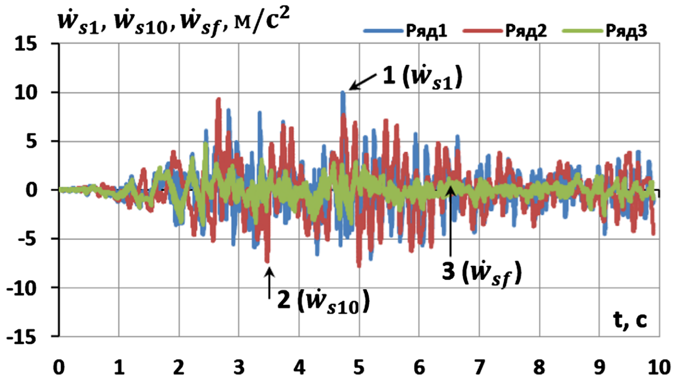
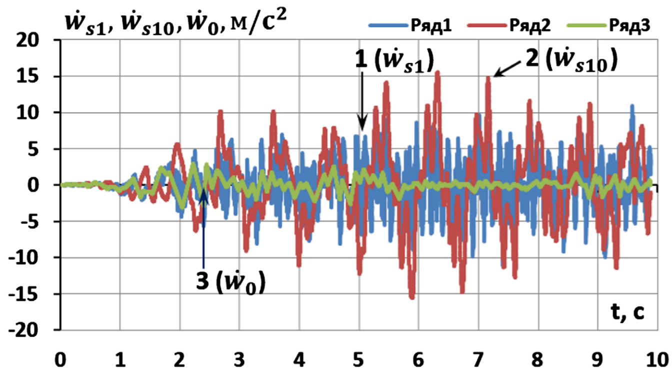


Figure 5. Resonance mode of the oscillatory process in the dynamic model considering the compliance of the foundation soil.

Example 3. Numerical solution of the dynamic problem for calculating the building response considering foundation compliance under seismic loading in the form of a given earthquake accelerogram. The building model with the data from Example 1. is considered. Figure 6(a) shows the graphs of the total acceleration response obtained under the action of the El Centro accelerogram for the following values of stiffness and damping coefficients in the foundation soil.



(a)



(b)

Figure 6. Acceleration response graphs obtained from the given El Centro earthquake accelerogram: (a)—with compliant (flexible) foundation, (b)—with rigid foundation.

$$k_f = 1.91\times10^5 \ \text{t/m}, \quad k_\varphi = 7.53\times10^7 \ \text{tm}$$

$$c_f = 2.19\times10^4 \ \text{tf/m}, \quad c_\varphi = 2.26\times10^6 \ \text{t·m·s}$$

The damping matrix is assumed to be proportional to the mass matrix with a proportionality coefficient $b = 2\xi\omega_1$, where the damping parameter $\xi = 0.02$ $\omega_1 = 2\pi/T_1$, $T_1 = 1.0$ corresponds to the fundamental period of the free vibrations of the building model. For comparison, Figure 6(b) presents the graphs of total acceleration response for the case of a rigid foundation.

It can be seen that the maximum acceleration in the case of a rigid foundation is approximately 1.5 times greater than that with a compliant foundation. It should be noted that in the model with a rigid foundation, the highest total acceleration is experienced by mass m_{10}, whereas in the model with a compliant foundation, it is mass m_1. At the same time, the maximum acceleration of the foundation slab mass is 4.65 m/s^2, which is 48.5% higher than the peak acceleration of the given El Centro accelerogram.

5. Conclusion

Based on the obtained results, it can be concluded that the developed numerical modeling algorithm and computer program enable the study of buildings taking into account the compliance of the foundation soil. The comparison shows that considering the soil compliance in the horizontal direction and during the building's rotation in the vertical plane leads to a significant reduction in total accelerations. The proposed calculation method can be used at the preliminary design stage of buildings and structures under seismic impacts.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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