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Natural Transform for Solving Fractional High Order Differential Equations

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Abstract

The natural transform (**nt**), a novel global method for solving fractional high-order differential equations (FDEs), is presented in this work. This technique functions as an all-encompassing integral transform that includes both Sumudu and Laplace transforms. Having inclusive fractional solutions for high-order fractional differential equations is the main goal of this work. The Caputo framework is used to express the fractional derivatives. To demonstrate the effectiveness, simplicity, and ease of use of the suggested approach and its applicability to the solution of differential equations in a variety of scientific domains, three problems are resolved.

Keywords

Natural Transform, Caputo Fractional Derivative, Fractional High Order Differential Equations, Analytical Solutions

1. Introduction

The natural transform (**nt**), first introduced by Khan and Khan [1] as the N-transform, involved an examination of its properties and uses. Subsequently, Belgacem *et al.* [2] [3] established its inverse and examined further essential characteristics of this integral transform, labeling it the natural transform. The **nt** applications in solving symmetrical differential and integral equations, as well as in the distribution and Bohemians spaces, are discussed in [4]-[14]. This work uses the **nt** to provide analytical new examinations of fractional high-order differential equations. It is a universal integral transform of Sumudu (**st**) and Laplace transform (**lt**). For fractional high-order differential equations, the **nt** provides new inclusive fractional solutions. This paper presents a novel application of **nt**. It improved the earlier studies in [15]-[19] by solving high-order fractional differential equations. It helps us comprehend and explain occurrences related to fractional high-order differential equations.

2. Mathematical Preliminaries

In this section, we will go over the basic definitions and theorems of fractional derivatives and the **nt**:

Definition 2.1 [20]

Over the set of functions

$$A=\left\{g(\iota):\exists m,I_1,I_2>0,\left|g(\iota)\right|<m\,\mathrm{e}^{|\iota|/\tau_j},\text{ if }\iota\in(-1)^j\times[0,\infty)\right\}$$

The natural transform of the function $g(\iota)$ is defined by:

$$n\left[g(\iota)\right]=R(\hbar,P)=\int_0^\infty g(P\iota)\mathrm{e}^{-\hbar\iota}\mathrm{d}\iota,\hbar>0,P>0 \tag{1}$$

where $n\left[g(\iota)\right]=R(\hbar,P)$ is the natural transformation of the time function $g(\iota)$ and the variables P and $\hbar$ are the natural transform variables. When $P\equiv1$ in Equation (1) converges to Laplace transform [15]-[17] and $\hbar\equiv1$ in Equation (1) converges to Sumudu transform [18] [19], respectively defined by:

$$\ell\left[g(\iota)\right]=\int_0^\infty g(\iota)\mathrm{e}^{-\hbar\iota}\mathrm{d}\iota, \tag{2}$$

$$\hbar\left[g(\iota)\right]=\int_0^\infty g(\hbar\iota)\mathrm{e}^{-\iota}\mathrm{d}\iota,\hbar\in(-I_1,I_2) \tag{3}$$

Definition 2.2 [20]

Natural-Laplace duality (NSD). If $R(\hbar,P)$ is the natural transform and $G(\hbar)$ is Laplace transform of function $g(\iota)$ in A, then:

$$n\left[g(\iota)\right]=R(\hbar,P)=\frac{1}{\hbar}\int_0^\infty g(\iota)\mathrm{e}^{\frac{-\hbar\iota}{P}}=\frac{1}{\hbar}G\left(\frac{\hbar}{P}\right) \tag{4}$$

Definition 2.3 [20]

Natural-Sumudu duality (NSD). If $R(\hbar,P)$ is the natural transform and $Q(P)$ is Sumudu transform of function $g(\iota)$ in A, then:

$$n\left[g(\iota)\right]=R(\hbar,P)=\frac{1}{P}\int_0^\infty g\left(\frac{\hbar}{P}\iota\right)\mathrm{e}^{-\iota}\,\mathrm{d}\iota=\frac{1}{P}Q\left(\frac{P}{\hbar}\right) \tag{5}$$

Definition 2.4 [20]

If $R(\hbar,P)$ is the natural transform of the function $g(\iota)$, then the natural transform of fractional derivative of order $\triangleright$ is defined as [10]-[12]:

$$n\left[g^{\triangleright}(\iota)\right]=\frac{\hbar^{\triangleright}}{P^{\triangleright}}R(\hbar,P)-\sum_{e=0}^{e-1}\frac{\hbar^{\triangleright-(r+1)}}{P^{\triangleright-r}}g^{r}(0) \tag{6}$$

Definition 2.5 [20]

Convolution theorem of natural transform. If $F(s,u)$, $G(s,u)$ are the natural transform of respective functions $f(t)$, $g(t)$ both defined in set A then,

$$n[y*g]=Py(\hbar,P)G(\hbar,P), \tag{7}$$

where $y*g$ is convolution of two functions y and g.

Definition 2.6 [21]

The fractional derivative of $\rhd(\iota)$ for $\rhd-1<\rhd<\rhd, \rhd\in N, \rhd>0, A, \rhd, \iota\in R$ in the Caputo sense is defined by

$$D_{\iota}^{\rhd} g(\iota)=J^{r-\rhd} D^{\rhd} g(\iota)=\frac{1}{\Gamma(r-\rhd)} \int_{A}^{\iota}(\iota-I)^{r-\rhd-1} g^{(r)}(\iota) \mathrm{d} \iota,\ (\rhd>0) \tag{8}$$

Definition 2.7 [22]

The Mittag-Leffler function with $E_{\rhd}(g)$ is defined via the series representation, valid in the whole complex plane is:

$$E_{\rhd}(g)=\sum_{\iota=0}^{\infty} \frac{g^{\iota}}{\Gamma(\rhd \iota+1)} \tag{9}$$

The natural transform, its inverse, and the Caputo derivative formula are valid for functions $g(\iota)$ that are at least m-times differentiable ($g\in C^{m}$) and of exponential order, within admissible fractional orders $\rhd\in(M-1, M)$. The Caputo derivative requires $M-1<\rhd<M$ and the Laplace-Sumudu transforms rely on $g(\iota)$ being absolutely continuous, enabling the use of standard integer-order initial conditions. We chose the Caputo fractional derivative for our investigation due to these characteristics. The Caputo fractional derivative is useful for problems with non-local features and phenomena with interactions. In this sense, the equation might be compared to a memory. The memory effect is a more finely tuned aspect of the Caputo fractional derivative. It is the best tool for describing recollections. We can therefore conclude that the physical meaning of the fractional Caputo derivative is a memory indicator. The range of physical events shares many characteristics with typical ones. Finally, to replicate real-world problems, the Caputo derivative should be utilized as the fractional operator. This serves as the primary reason for incorporating the Caputo fractional-order derivative operator in various physical applications. Section 2 presents the basic definitions and theorems of fractional derivatives, and the **nt**. Section 3 presents the solutions to three fractional high order differential equations. The results of the study are outlined in Section 3.

3. Applications and Examples

In this section, we use the natural transform to provide general solutions for three fractional boundary value problems. Because the three chosen boundary value problems span a variety of difficulties, from straightforward linear differential equations with straightforward boundary conditions to higher-order equations and systems with or without exact solutions, they serve as representative test cases for the natural transform approach. This makes it possible to confirm how well the natural transform approach handles various differential types and complexities. The resolution of every BVP utilizing the natural transform approach is characterized by the following:

Step 1: Utilizing the natural transform technique on both sides of the BVP and streamlining the outcomes.

Step 2: Utilizing the inverse natural transformation technique on both sides of the boundary value problem.

Step 3: Plugging in $\triangleright \to 4$ into the final outcomes of Step 2 to obtain the classical solution of the BVP and clarifying the effectiveness of the natural transform technique.

Problem 3.1 [23]

Consider a fractional Boundary Value Problem in the form below:

$$D_\iota^{\triangleright}\theta(\iota)=1 \quad 3<\triangleright\le 4, \iota\in(0,1), \quad \theta(0)=\theta'(0)=\theta''(0)=\theta'''(0)=0 \tag{10}$$

At $\triangleright \to 4$ the exact solution is

$$\theta(\iota)=\frac{\iota^4}{4!}. \tag{11}$$

The solution

Applying **nt** on both sides of (10), we have

$$N\left[D_\iota^{\triangleright}\theta(\iota)\right]=[1] \tag{12}$$

On simplifying

$$\frac{\hbar^{\triangleright}}{P^{\triangleright}}R(\hbar,P)-\frac{\hbar^{\triangleright-1}}{P^{\triangleright}}\theta(0)-\frac{\hbar^{\triangleright-2}}{P^{\triangleright-1}}\theta'(0)-\frac{\hbar^{\triangleright-3}}{P^{\triangleright-2}}\theta''(0)-\frac{\hbar^{\triangleright-4}}{P^{\triangleright-3}}\theta'''(0)=\frac{1}{\hbar} \tag{13}$$

i.e.,

$$R(\hbar,P)=\frac{P^{\triangleright}}{\hbar^{\triangleright+1}} \tag{14}$$

Applying the inverse **nt** of (14), we obtain

$$\theta(\iota)=\frac{\iota^{\triangleright}}{\Gamma(\triangleright+1)} \tag{15}$$

In the case $\triangleright \to 4$ from (15), we have

$$\theta(\iota)=\frac{\iota^4}{\Gamma(5)}=\frac{\iota^4}{4!} \tag{16}$$

This is the exact solution of (10) obtained via [23] (See **Figure 1**, **Figure 2**).

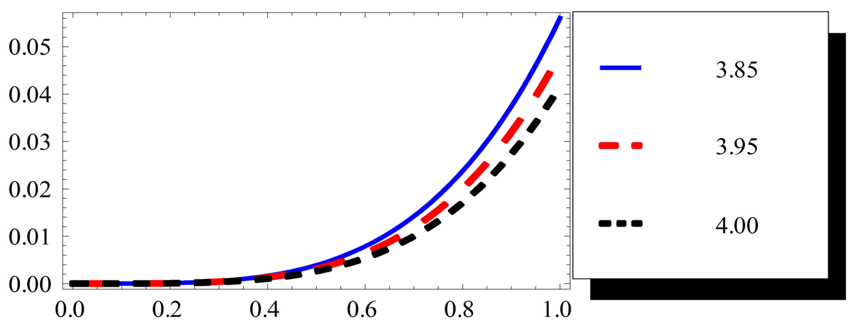

Figure 1. Plot of the solution of (10) at various values of $\triangleright$.

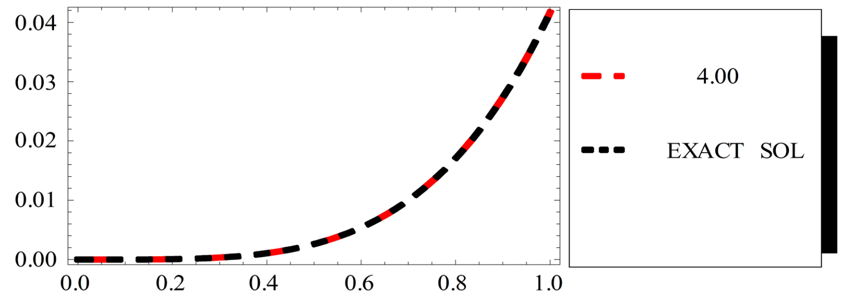

Figure 2. Plot of the solution of (10) at $\triangleright = 4$ in comparison with the exact solution obtained via [23].

Problem 3.2 [23]

Consider the following fractional Boundary Value Problem as follows:

$$\begin{aligned} &D_\iota^{\triangleright}\theta(\iota)-\partial^4\theta(\iota)=0 \quad 3<\triangleright\le 4, \iota\in(0,1), \theta(0)=1, \\ &\theta'(0)=\theta''(0)=\theta'''(0)=0 \end{aligned} \tag{17}$$

At $\triangleright \to 4$ the exact solution is

$$\theta(\iota)=\frac{1}{2}\left[\cos(\partial\iota)+\cosh(\partial\iota)\right] \tag{18}$$

The solution

Applying **nt** on both sides of (17), we have

$$N\left[D_\iota^{\triangleright}\theta(\iota)-\partial^4\theta(\iota)\right]=0 \tag{19}$$

On simplifying

$$\begin{aligned} &\frac{\hbar^{\triangleright}}{P^{\triangleright}}R(\hbar,P)-\frac{\hbar^{\triangleright-1}}{P^{\triangleright}}\theta(0)-\frac{\hbar^{\triangleright-2}}{P^{\triangleright-1}}\theta'(0)-\frac{\hbar^{\triangleright-3}}{P^{\triangleright-2}}\theta''(0) \\ &-\frac{\hbar^{\triangleright-4}}{P^{\triangleright-3}}\theta'''(0)-\partial^4 R(\hbar,P)=0 \end{aligned} \tag{20}$$

i.e.,

$$R(\hbar,P)=\frac{\hbar^{\triangleright-1}}{\hbar^{\triangleright}-P^{\triangleright}\partial^4}, \tag{21}$$

$$\frac{\hbar^{\triangleright-1}}{\hbar^{\triangleright}-P^{\triangleright}\partial^4}=\frac{\hbar^{\triangleright-1}}{\left(\hbar^{\frac{\triangleright}{2}}-\partial^2 P^{\frac{\triangleright}{2}}\right)\left(\hbar^{\frac{\triangleright}{2}}+\partial^2 P^{\frac{\triangleright}{2}}\right)} \tag{22}$$

By partial fraction decomposition, we obtain

$$\frac{\hbar^{\triangleright-1}}{\left(\hbar^{\frac{\triangleright}{2}}-\partial^2 P^{\frac{\triangleright}{2}}\right)\left(\hbar^{\frac{\triangleright}{2}}+\partial^2 P^{\frac{\triangleright}{2}}\right)}=\frac{1}{2}\left[\frac{\hbar^{\frac{\triangleright}{2}-1}}{\hbar^{\frac{\triangleright}{2}}-\partial^2 P^{\frac{\triangleright}{2}}}+\frac{\hbar^{\frac{\triangleright}{2}-1}}{\hbar^{\frac{\triangleright}{2}}+\partial^2 P^{\frac{\triangleright}{2}}}\right] \tag{23}$$

Then,

$$R(\hbar,P)=\frac{1}{2}\left[\frac{\hbar^{\frac{\rhd}{2}-1}}{\hbar^{\frac{\rhd}{2}}-\partial^2P^{\frac{\rhd}{2}}}+\frac{\hbar^{\frac{\rhd}{2}-1}}{\hbar^{\frac{\rhd}{2}}+\partial^2P^{\frac{\rhd}{2}}}\right] \tag{24}$$

Applying the inverse **nt** of (24), we obtain

$$\theta(\iota)=\frac{1}{2}N^{-1}\left[\frac{\hbar^{-1}}{1-\frac{P^{\frac{\rhd}{2}}}{\hbar^{\frac{\rhd}{2}}}\partial^2}+\frac{\hbar^{-1}}{1+\frac{P^{\frac{\rhd}{2}}}{\hbar^{\frac{\rhd}{2}}}\partial^2}\right] \tag{25}$$

Then,

$$\theta(\iota)=\frac{1}{2}E_{\rhd}\left(\partial^2\iota^{\frac{\rhd}{2}}\right)+\frac{1}{2}E_{\rhd}\left(-\partial^2\iota^{\frac{\rhd}{2}}\right), \tag{26}$$

$$\begin{aligned}E_{\frac{\rhd}{2}}\left(-\partial^2\iota^{\frac{\rhd}{2}}\right)&=\sum_{f=0}^{\infty}\frac{\left(-\partial^2\iota^{\frac{\rhd}{2}}\right)^f}{\Gamma\left(\frac{\rhd}{2}f+1\right)},\\ E_{\frac{\rhd}{2}}\left(\partial^2\iota^{\frac{\rhd}{2}}\right)&=\sum_{f=0}^{\infty}\frac{\left(\partial^2\iota^{\frac{\rhd}{2}}\right)^f}{\Gamma\left(\frac{\rhd}{2}f+1\right)}\end{aligned} \tag{27}$$

In the case $\rhd\to 4$ from (27), we have

$$\begin{aligned}E_{2,1}\left(-\partial^2\iota^2\right)&=\sum_{f=0}^{\infty}\frac{\left(-\partial^2\iota^2\right)^f}{\Gamma(2f+1)}=\cos(\partial\iota),\\ E_{2,1}\left(\partial^2\iota^2\right)&=\sum_{f=0}^{\infty}\frac{\left(\partial^2\iota^2\right)^f}{\Gamma(2f+1)}=\cosh(\partial\iota)\end{aligned} \tag{28}$$

i.e.,

$$\theta(\iota)=\frac{1}{2}\left[\cos(\partial\iota)+\cosh(\partial\iota)\right] \tag{29}$$

This is the exact solution of (17) obtained by [23] (See **Figure 3**, **Figure 4**).

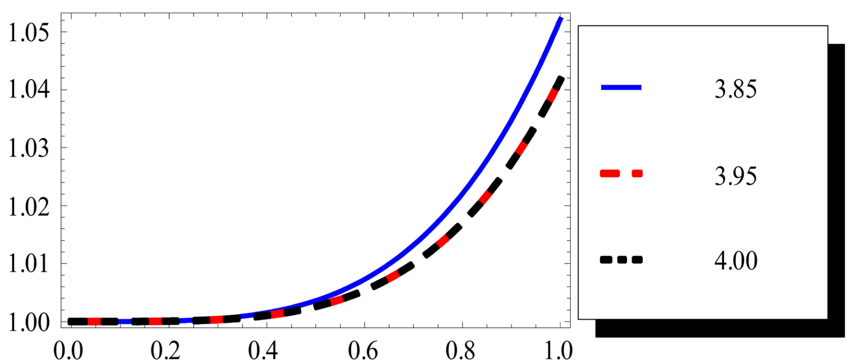

Figure 3. Plot of the solution of (17) at various values of $\rhd$.

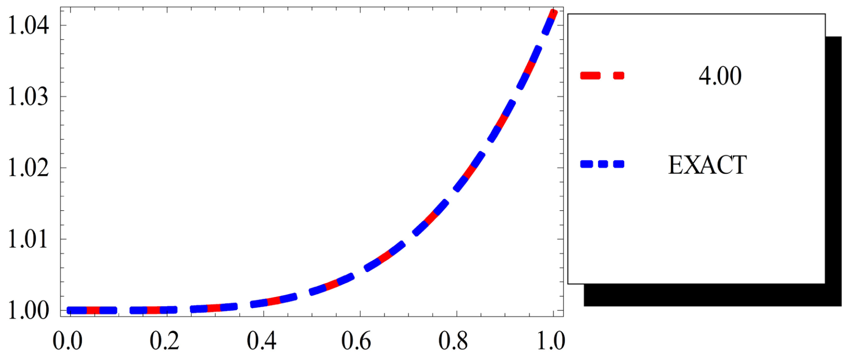

Figure 4. Plot of the solution of (17) at $\rhd = 4$ in comparison with the exact solution obtained via [23].

Problem 3.3 [23]

Consider a fractional Boundary Value Problem in the form below:

$$D_\iota^{\rhd}\theta(\iota) + 2D_\iota^{\frac{\rhd}{2}}\theta(\iota) + \theta(\iota) = \sin\iota \quad 3 < \rhd \le 4, \iota \in (0,1),$$
$$\theta(0) = \theta'(0) = \theta''(0) = \theta'''(0) = 0 \tag{30}$$

At $\rhd \to 4$ the exact solution is

$$\theta(\iota) = \frac{1}{8}\left[3\sin(\iota) - \iota^2\sin(\iota) - 3\iota\cos(\iota)\right]. \tag{31}$$

The solution

Applying **nt** on both sides of (30), we have

$$N\left[D_\iota^{\rhd}\theta(\iota) + 2D_\iota^{\frac{\rhd}{2}}\theta(\iota) + \theta(\iota)\right] = N[\sin\iota] \quad 3 < \rhd \le 4, \iota \in (0,1) \tag{32}$$

On simplifying

$$\frac{\hbar^{\rhd}}{P^{\rhd}}R(\hbar,P) - \frac{\hbar^{\rhd-1}}{P^{\rhd}}\theta(0) - \frac{\hbar^{\rhd-2}}{P^{\rhd-1}}\theta'(0) - \frac{\hbar^{\rhd-3}}{P^{\rhd-2}}\theta''(0) - \frac{\hbar^{\rhd-4}}{P^{\rhd-3}}\theta''''(0)$$
$$+2\left(\frac{\hbar^{\frac{\rhd}{2}}}{P^{\frac{\rhd}{2}}}R(\hbar,P) - \frac{\hbar^{\frac{\rhd}{2}-1}}{P^{\frac{\rhd}{2}-1}}\theta(0) - \frac{\hbar^{\frac{\rhd}{2}-2}}{P^{\frac{\rhd}{2}-2}}\theta'(0)\right) + R(\hbar,P) = \frac{P}{\hbar^2 + P^2} \tag{33}$$

i.e.,

$$\frac{\hbar^{\rhd}}{P^{\rhd}}R(\hbar,P) + 2\frac{\hbar^{\frac{\rhd}{2}}}{P^{\frac{\rhd}{2}}}R(\hbar,P) + R(\hbar,P) = \frac{P}{\hbar^2 + P^2}, \tag{34}$$

$$R(\hbar,P) = \frac{P^{\rhd+1}}{\left(\hbar^2 + P^2\right)\left(\hbar^{\frac{\rhd}{2}} + P^{\frac{\rhd}{2}}\right)^2} \tag{35}$$

Applying the inverse **nt** of (35), we obtain

$$\theta(\iota) = N^{-1}\left[\frac{P}{\hbar^2 + P^2}\frac{P^{\frac{\succ}{2}}}{\hbar^{\frac{\rhd}{2}} + P^{\frac{\rhd}{2}}}\frac{P^{\frac{\rhd}{2}}}{\hbar^{\frac{\rhd}{2}} + P^{\frac{\rhd}{2}}}\right] \tag{36}$$

Using convolution theorem of (36), we have

$$\theta(\iota)=\frac{1}{8}\left[3\sin(\iota)-\iota^{\frac{\triangleright}{2}}\sin\left(\iota^{\frac{\triangleright}{4}}\right)-3\iota^{\frac{\triangleright}{4}}\cos\left(\iota^{\frac{\triangleright}{4}}\right)\right] \tag{37}$$

In the case $\triangleright\to 4$ from (37), we have

$$\theta(\iota)=\frac{1}{8}\left[3\sin(\iota)-\iota^{2}\sin(\iota)-3\iota\cos(\iota)\right] \tag{38}$$

This is the exact solution of (30) obtained by [23] (See **Figure 5**, **Figure 6**).

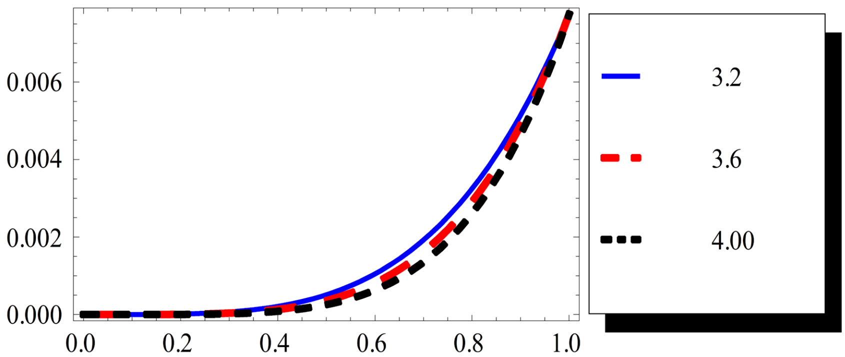

Figure 5. Plot of the solution of (30) at various values of $\triangleright$.

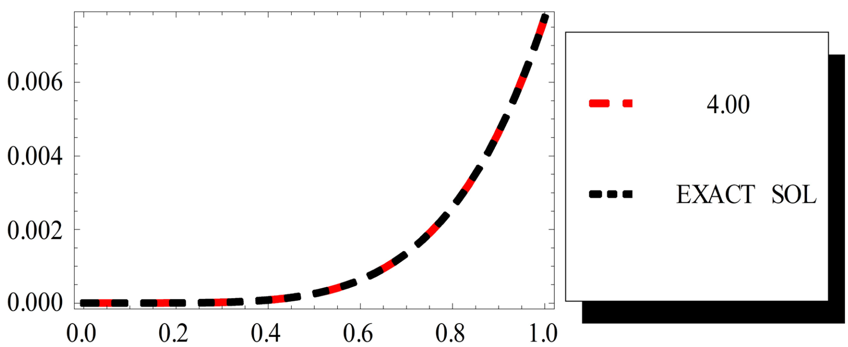

Figure 6. Plot of the solution of (30) at $\triangleright = 4$ in comparison with the exact solution obtained via [23].

4. Conclusion

This study illustrated a novel technique known as the natural transform. It is a general approach of Laplace transform and Sumudu transform. In numerous scientific fields, it offers fresh generic solutions for fractional high-order differential equations. To shed light on the generalization of the algorithm, we compared the outcomes of the current approaches with those of other ways such as the Laplace transform, Sumudu transform and the precise answers. The figures demonstrate the simplicity, symmetric, quality, and generalizability of the proposed algorithm. Finally, but just as importantly, the natural transform is a novel generic method that may be applied to problems in a variety of scientific fields.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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