Advances in Pure Mathematics, 2026, 16(1), 69-90
https://www.scirp.org/journal/apm
ISSN Online: 2160-0384
ISSN Print: 2160-0368

Symmetry of the Composite Numbers

Han-Lin Li[1], Shu-Cherng Fang[2], Way Kuo[3], Nianrui Lin[4]

[1]Department of Computer Science, City University of Hong Kong, Hong Kong SAR, China
[2]Department of Industrial and Systems Engineering, North Carolina State University, Raleigh, NC, USA
[3]Hong Kong Institute for Advanced Study and Department of Data Science, City University of Hong Kong, Hong Kong SAR, China
[4]Department of Systems Engineering, City University of Hong Kong, Hong Kong SAR, China
Email: nianrui.lin@my.cityu.edu.hk

How to cite this paper: Li, H.-L., Fang, S.-C., Kuo, W. and Lin, N.R. (2026) Symmetry of the Composite Numbers. *Advances in Pure Mathematics*, **16**, 69-90.
https://doi.org/10.4236/apm.2026.161006

Received: December 25, 2025
Accepted: January 24, 2026
Published: January 27, 2026

Abstract

Traditionally, primes have been viewed as weeds appearing irregularly among the natural numbers, making their locations difficult to predict. We believe composite numbers, the complement of primes, exhibit symmetric patterns in their distribution. This study presents a new perspective: after systematically eliminating integers divisible by small primes such as 2, 3, 5, and 7, the remaining integers can be uniquely represented through a framework of roots and kins. Furthermore, all remaining composite numbers exhibit cyclic and mirror effects, enabling the construction of an extended Cyclic Table of Composites. Using this table, we derive a formula that identifies the locations of primes within any interval. Additional formulas for fast factorization and for locating twin primes, mirror primes, and prime tuples are also obtained. These results suggest that although primes themselves appear structureless, the composites surrounding them obey rich internal symmetries that allow efficient detection of "where primes hide".

Keywords

Primes, Composites, PTP, CTC, eCTC

1. Introduction

Leonhard Euler once remarked, "*Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate*" [1]. Don Zagier similarly noted that prime numbers "*grow like weeds among the natural numbers, seeming to obey no other law than that of chance, and nobody can predict where the next one will sprout*" [2]. In fact, a few patterns have been observed in prime distribution, one of which is that prime numbers demonstrate higher density on the diagonal, horizontal, and vertical lines of the Ulam spiral (equivalently, certain quadratic polynomials generate more primes than others), but with no rigorous proof [3]. Even today, the fundamental question remains: given an interval of integers, where do the primes hide? In other words, we lack an explicit, efficiently computable rule for locating primes inside an arbitrary interval.

Classical sieve methods locate primes by removing all composites, but provide only partial solutions. The ancient Sieve of Eratosthenes [4] and its variants (e.g., Augmented Eratosthenes [4], Segmented Sieve of Eratosthenes [5], and Sieve of Sundaram [6]) exhaustively enumerate composites of certain multiplicative forms. Sieve of Atkin and Bernstein [7] leverages the properties of binary quadratic forms to recognize composites. Although the wheel sieve method [8] provides insight into the cyclic feature of composite distribution, it didn't consider the symmetrical patterns of composites.

The number theory literature [1] [2] [4]-[11] affirms that no existing method provides a direct, effectively computable formula for listing primes solely within a given interval. Gauss's prime number theorem [9] gives a celebrated asymptotic approximation of how many primes lie in an interval, but it cannot tell which values are prime.

Primes play essential roles in data science, color theory [12], reliability engineering [13], and cryptography. Our previous works introduced the Periodic Table of Primes (PTP) [14], periodic prime listings [15], and kernel-based semiprime factorization [16]. In this study, we uncover a striking phenomenon: once small primes are filtered out, the remaining composites form a structure governed by strong horizontal, vertical, and diagonal mirror symmetries. These symmetries allow rapid identification of composites within an interval, and the primes are then simply the remaining entries.

Our main contributions include: 1) Discovery and confirmation of the mirror symmetry in composite numbers, 2) Introduction of the kin-root framework, which shows that although primes appear structureless, composites possess strong governing structure, 3) Development of an extended Cyclic Table of Composites that enables efficient local processing of intervals, and 4) Formulas for prime location, factorization, twin primes, Pythagorean primes, prime tuples, and neighboring primes. This study offers an extension of the method to larger moduli for identifying large primes and factoring large composites.

2. Preliminaries

The Periodic Table of Primes was first derived from the Cyclic Table of Composites (CTC) by Li, Fang, and Kuo [14], which includes composite numbers without factors of 2, 3, 5, and 7 up to 211^2. Though a similar period ($2\times3\times5\times7=210$) appeared in the wheel sieve method [8], CTC is derived through a novel, rigorous method. In this paper, we reorganize the composites and reconstruct the CTC from a different angle, revealing the symmetry properties of the composites.

Our study is based on the following statements, which can be readily verified from [14]-[16].

Statement 1

Let

$$\lambda_7 = 2\times3\times5\times7 = 210 .$$

We can construct a set S_7 containing all integers less than 210 that are not divisible by 2, 3, 5, or 7. This set contains 48 such integers:

$$S_7 = \{11,\ 13,\ 17,\ 23,\ \cdots,\ 103,\ 107,\ \cdots,\ 199,\ 209,\ 211\} .$$

Label these integers as

$$q_1 = 11 ,\ \ q_2 = 13 ,\ \ q_3 = 17 ,\ \ \cdots,\ \ q_{23} = 103 ,\ \ q_{24} = 107 ,\ \ \cdots,\ \ q_{46} = 199 ,\ \ q_{47} = 209 ,\ \ q_{48} = 211 .$$

Statement 2

For any positive integer α that contains no factors of 2, 3, 5, or 7, α can be expressed uniquely as

$$\alpha = r_i + \lambda_7 k = r_i + 210k ,$$

where $r_i \in S_7$ is called the root, k is an integer (the kin), 210 is the period, and $0 \le k \le 210$. Thus, all such integers fall into 48 congruence classes modulo 210.

Statement 3

Every such integer α is either prime or composite. If α is composite, then it can be written as

$$\alpha = r_i + 210k = q_j m ,$$

where $q_j \in S_7$ divides α and m is a positive integer.

Statement 4

To identify all composites and primes within an interval, it suffices to analyze the canonical interval

$$[11 + 210k,\ 211 + 210k],\ \ 0 \le k \le 210$$

Once all composites within this interval are listed, the primes in the same interval follow immediately as the remaining values.

Statement 5

Composites sharing the same root and factor exhibit a cyclic pattern. Let

$$\alpha = r_i + 210k = q_j m ,$$

and consider

$$\alpha' = r_i + 210(k + q_j) .$$

Then, α' has a kin value that is q_j cycles larger than that of α, and satisfies

$$\alpha' = q_j (m + 210) .$$

Thus, composites repeat periodically at intervals proportional to their factor q_j.

Statement 6

Each composite

$$\alpha_0 = r_i + 210k = q_j m$$

has three corresponding "mirror" composites:
Vertical mirror:

$$\alpha_1 = -\alpha_0 + 210q_j .$$

Horizontal mirror:

$$\alpha_2 = -\alpha_0 + 210m .$$

Diagonal mirror:

$$\alpha_3 = \alpha_0 + 210 \times \left(210 - q_j - m\right).$$

These mirror relationships create a symmetrical structure among composites. See Theorem 2 for further details.

Statement 7

Using the cyclic and mirror effects, we construct a 210-based extended Cyclic Table of Composites (eCTC), denoted $eCTC_{210}$, containing tuples of the form

$$\left(k_j,\ t_i\right).$$

This table allows rapid identification of all composites in any interval.

Statement 8

With the eCTC, we derive:

- a formula for locating primes efficiently within an interval,
- a factorization formula based on kin-root relationships, and
- a tuple-prime formula for identifying prime pairs and higher-order prime clusters.

Statement 9

Although Statements 1-8 are based on $\lambda_7 = 210$, the method extends naturally to larger moduli. For example:

$$\lambda_{11} = 2 \times 3 \times 5 \times 7 \times 11 = 2310 ,$$

$$\lambda_{13} = 2 \times 3 \times 5 \times 7 \times 11 \times 13 = 30030 .$$

One may construct:

- a kin-root table $eCTC_{231}0$ to find primes in

$$\left[13 + 2310k,\ 2311 + 2310k\right].$$

- or a kin-root table $eCTC_{30030}$ to find primes in

$$\left[17 + 30030k,\ 30031 + 30030k\right].$$

These generalizations allow the method to scale to increasingly large intervals.

3. Extended Cyclic Table of Composites

In this section, we construct the proposed eCTC and examine its underlying structural properties.

Let

$$\lambda_x = \prod_{p \in \{2, 3, 5, \cdots, x\}} p$$

be the product of all primes from 2 through *x*. For example:

$$\lambda_7 = 2 \times 3 \times 5 \times 7 = 210, \quad \lambda_{11} = 2310, \quad \lambda_{13} = 30030, \quad \lambda_{17} = 510510.$$

3.1. Definition of the Root Set S_x

Let S_x denote the set of positive integers not exceeding $\lambda_x + 1$ that have no prime factors among $2, 3, 5, \cdots, x$. Examples include:

- $S_7 = \{11, 13, 17, 19, 23, 27, \cdots, 211\}$.
- $S_{11} = \{13, 17, 19, 23, 27, \cdots, 2311\}$.
- $S_{13} = \{17, 19, 23, 27, 29, \cdots, 30031\}$.

Let $|S_x|$ denote the number of elements in S_x. For instance, $|S_7| = 48$. In this section, we focus on the case $x = 7$, *i.e.*, $\lambda_7 = 210$.

3.2. Kin-Root Representation

Consider any integer α satisfying

$$11 \le \alpha \le 211^2 \text{ and } \gcd(\alpha, 210) = 1.$$

Then, α can be uniquely expressed as:

$$\alpha = r_i + 210k, \quad i \in \{1, 2, 3, \cdots, |S_7|\}, \quad 0 \le k \le 210. \tag{1}$$

where

- $r_i \in S_7$ is the root,
- 210 is the period,
- k is the kin value of α. In other words, α is the *k*-th generation of r_i.

Thus, (k, r_i) is the kin-root representation of α, serving as the key element in the table proposed later.

The full set S_7 is:

$$S_7 = \{r_1 = 11, r_2 = 13, r_3 = 17, r_4 = 19, r_5 = 23, r_6 = 29, r_7 = 31, \\ r_8 = 37, r_9 = 41, r_{10} = 43, r_{11} = 47, r_{12} = 53, r_{13} = 59, r_{14} = 61, \\ r_{15} = 67, r_{16} = 71, r_{17} = 73, r_{18} = 79, r_{19} = 83, r_{20} = 89, r_{21} = 97, \\ r_{22} = 101, r_{23} = 103, r_{24} = 107, r_{25} = 109, r_{26} = 113, r_{27} = 121, \\ r_{28} = 127, r_{29} = 131, r_{30} = 139, r_{31} = 143, r_{32} = 149, r_{33} = 151, \\ r_{34} = 157, r_{35} = 163, r_{36} = 167, r_{37} = 169, r_{38} = 173, r_{39} = 179, \\ r_{40} = 181, r_{41} = 187, r_{42} = 189, r_{43} = 191, r_{44} = 193, r_{45} = 197, \\ r_{46} = 199, r_{47} = 209, r_{48} = 211\}.$$

3.3. Factor Representation and eCTC Construction

Theorem 1

Let $\alpha = q_j h_{i/j}$, where $q_j, h_{i/j} \in S_7$ and α has no factors of 2, 3, 5, and 7. Then there exist values $k_{i/j} \in \mathbb{N}_0$ (with $0 \le k_{i/j} \le 211$) and $t_{i/j} \in S_7$ such that

$$\alpha = t_{i/j} + 210k_{i/j} = q_j h_{i/j} \tag{2}$$

This theorem enables the construction of $eCTC_{210}$, a 48×48 table whose entries are expressed as:

$$\left(k_{i/j},\ t_{i/j}\right),$$

satisfying Equation (2).

The table has:

- 48 columns, indexed by $j = 1, \cdots, 48$, corresponding to values of q_j.
- 48 rows, indexed by $i = 1, \cdots, 48$, corresponding to values of $h_{i/j}$.

A schematic for $eCTC_{210}$ is shown in Table 1. The full table is presented in Table 2(a) (for $j = 1, \cdots, 23$) and Table 2(b) (for $j = 24, \cdots, 48$).

3.4. Illustrative Entries in the eCTC

From Table 1:

- For $j = 1$ to 23, the corresponding values are $q_1 = 11, \cdots, q_{23} = 107$.
- For $j = 24$ to 46, the values are $q_{24} = 107, \cdots, q_{46} = 199$.
- For $j = 47, 48$, the values are $q_{47} = 209$ and $q_{48} = 211$.

As examples:

- For $(i, j) = (1, 1)$,

$$k_{1/1} = \frac{11 \times 11 - t_{1/1}}{210},$$

given $k_{1/1} = 0$ and $t_{1/1} = 121$.

Thus, the entry is $\left(k_{1/1}, t_{1/1}\right) = (0, 121)$.

- For $(i, j) = (1, 2)$,

$$k_{1/2} = \frac{13 \times 11 - t_{1/2}}{210},$$

yielding $k_{1/2} = 0$ and $t_{1/2} = 143$.

Thus, $\left(k_{1/2},\ t_{1/2}\right) = (0,\ 143)$.

- For $(i, j) = (48, 48)$,

$$k_{48/48} = \frac{211 \times 211 - t_{48/48}}{210},$$

giving $k_{48/48} = 211$ and $t_{48/48} = 211$.

3.5. Mirror and Symmetry Properties

Inspection of $eCTC_{210}$ reveals striking mirror and symmetry phenomena among composites.

The relationships justify dividing $eCTC_{210}$ into areas A, B, C, D, E, each of which can be subdivided into four regions, such as A_1, A_2, A_3, A_4. Table 1(b) specifies the ranges for each region; for example, region A_1 corresponds to $1 \le i \le 22$ and $1 \le j \le 23$.

3.6. Mirror Rules

Remark 1

Given a composite

Table 1. A schematic of $eCTC_{210}$: (a) Divided regions of $eCTC_{210}$; (b) Ranges of *i* and *j* for $eCTC_{210}$ regions.

(a)

j	1	2	3	⋯	23	24	⋯	44	45	46	47	48

i				
1	$\alpha_{a,1}$	$\alpha_{a,3}$	$\alpha_{d,1}$	$\alpha_{d,3}$
2				
3	A_1	A_3	D_1	D_3
⋮				
22				
23				
24				
⋮				
42	A_2	A_4	D_2	D_4
43				
44	$\alpha_{a,2}$	$\alpha_{a,4}$		
45	$\alpha_{b,1}$ B_1	B_3 $\alpha_{b,3}$		
46	$\alpha_{b,2}$ B_2	B_4 $\alpha_{b,4}$	$\alpha_{d,2}$	$\alpha_{d,4}$
47	$\alpha_{c,1}$ C_1	C_3 $\alpha_{c,3}$	E_1	E_3
48	$\alpha_{c,2}$ C_2	C_4 $\alpha_{c,4}$	E_2	E_4

(b)

region	*i* range	*j* range
A_1	$1 \le i \le 22$	$1 \le j \le 23$
A_2	$23 \le i \le 44$	$1 \le j \le 23$
A_3	$1 \le i \le 22$	$24 \le j \le 46$
A_4	$23 \le i \le 44$	$24 \le j \le 46$
B_1	$i = 45$	$1 \le j \le 23$
B_2	$i = 46$	$1 \le j \le 23$
B_3	$i = 45$	$24 \le j \le 46$
B_4	$i = 46$	$24 \le j \le 46$

region	*i* range	*j* range
C_1	$i = 47$	$1 \le j \le 23$
C_2	$i = 48$	$1 \le j \le 23$
C_3	$i = 47$	$24 \le j \le 46$
C_4	$i = 48$	$24 \le j \le 46$
D_1	$1 \le i \le 23$	$j = 47$
D_2	$24 \le i \le 46$	$j = 47$
D_3	$1 \le i \le 23$	$j = 48$
D_4	$24 \le i \le 46$	$j = 48$

region	*i*	*j*
E_1	$i = 47$	$j = 47$
E_2	$i = 48$	$j = 47$
E_3	$i = 47$	$j = 48$
E_4	$i = 48$	$j = 48$

$$\alpha_1 = q_j \times h_{i/j}, \quad q_j,\ h_{i/j} \in S_7,$$

we define

1) Vertical mirror:

$$\alpha_2 = q_j \times \left(210 - h_{i/j}\right).$$

2) Horizontal mirror:

$$\alpha_3 = \left(210 - q_j\right) \times h_{i/j}.$$

3) Diagonal mirror:

$$\alpha_4 = \left(210 - q_j\right) \times \left(210 - h_{i/j}\right).$$

Theorem 2

Let $\left(k_{i/j},\ t_{i/j}\right)$ be an element in region A_1, and

$$\alpha_{a,1} = t_{i/j} + 210k_{i/j} = q_j \times h_{i/j}$$

is a composite.

Then, in **Table 1(a)**, we find that the vertical, horizontal, and diagonal mirrors of $\alpha_{a,1}$:

1) Vertical mirror (region A_2)

$$\left(q_j - 1 - k_{i/j},\ 210 - t_{i/j}\right)$$

corresponds to a composite number

$$\alpha_{a,2} = q_j \left(210 - h_{i/j}\right).$$

2) Horizontal mirror (region A_3)

$$\left(h_{i/j} - k_{i/j} - 1,\ 210 - t_{i/j}\right)$$

corresponds to a composite number

$$\alpha_{a,3} = \left(210 - q_j\right) \times h_{i/j}.$$

3) Diagonal mirror (region A_4)

$$\left(210 - q_j - h_{i/j} + k_{i/j},\ t_{i/j}\right)$$

corresponds to a composite number

$$\alpha_{a,4} = \left(210 - q_j\right)\left(210 - h_{i/j}\right).$$

The vertical, horizontal, and diagonal mirrors of the B, C, D, E regions are derived similarly.

Example 1

Take $q_1 = 11$, and $\left(k_{1/1},\ t_{1/1}\right) = \left(0,\ 121\right)$, so that

$$\alpha_{a,1} = 121.$$

Then,

1) Vertical mirror

$$\left(10,\ 89\right) \Rightarrow 11 \times 89$$

2) Horizontal mirror

$$\left(10,\ 89\right) \Rightarrow 199 \times 11$$

3) Diagonal mirror

$$\left(188,\ 121\right) \Rightarrow 199 \times 199$$

3.7. Consequences of Mirror and Symmetry Effects

The symmetry properties imply that:

- It suffices to generate entries $\left(k_{i/j}, t_{i/j}\right)$ for the regions A_1, B_1, C_1, D_1, E_1.

- All remaining entries in regions A_2, A_3, A_4, B_2, B_3, B_4, C_2, C_3, C_4, D_2, D_3, D_4, E_2, E_3, and E_4 follow directly by symmetry.

Examining **Table 2(a)** and **Table 2(b)**, we conclude:

- Composites without factors 2, 3, 5, and 7 appear in an unexpectedly orderly manner in the eCTC.
- Every such composite has exactly three mirror counterparts.
- The eCTC provides an efficient mechanism for listing all such composites in any interval

$$[11+210k,\ 211+210k],\quad 0 \le k \le 210 .$$

The detailed procedure is given in the next section.

4. Formula of Primes

Based on Statement 5, we attain the following results.

Theorem 3 (Cyclic Effect)

Given an element $(k_{i/j},\ t_{i/j})$ in $eCTC_{210}$ with a corresponding composite

$$\alpha = t_{i/j} + 210k_{i/j} = q_j \times h_{i/j},$$

where $h_{i/j} \in S_7$, let $k^* \in N^+$ with $k^* \le 211$, and suppose

$$k^* = k_{i/j} + q_j m,\ m \in N^+ .$$

Then,

$$\alpha^* = t_{i/j} + 210k^*$$

is also a composite with the same root $t_{i/j}$ and cycle value *m*.

Proof

We have

$$\alpha^* = t_{i/j} + 210(k_{i/j} + q_j m) = q_j h_{i/j} + 210q_j m = q_j (h_{i/j} + 210m) . \quad (3)$$

Hence, α^* is a composite with root value $t_{i/j}$.

Remark 2 (Reduced Kin Value)

For fixed q_j and k^*, define

$$k_j(k^*) = \min\{k^* - q_j m \mid m \in \mathrm{N}_0,\ 0 \le k^* - q_j m \le 211\} .$$

The meaning of $k_j(k^*)$ is: given $q_j \in S_7$ and an integer k^* with $0 \le k^* \le 210$, there exists some *m* such that

$$0 \le k^* - q_j m \le q_j ,$$

and this value is taken as $k_j(k^*)$. If $k^* = 0$, then $m = 0$ and hence $k_j(0) = 0$.

Formula: Listing Composites in $[11+210k^*, 211+210k^*]$

Consider integers of the form

$$\alpha = r_i + 210k^*,\ r_i \in S_7 .$$

Such an α is a composite (without factors 2, 3, 5, 7) if and only if there exists an element $(k_{i/j}, t_{i/j})$ in $eCTC_{210}$ such that

$$k_{i/j} = k_j(k^*),\quad t_{i/j} = r_i .$$

Table 2. (a) A full eCTC$_{210}$ table, for $j = 1, 2, \cdots, 23$; (b) A full eCTC$_{210}$ table, for $j = 24, 25, \cdots, 48$.

(a)

i \ j	$h_{i,j}$ \ q_j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
		11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	101	103
													$(k_{i/j}, t_{i/j})$											
1	11	0 121	0 143	0 187	1 37	1 43	1 109	1 131	1 197	2 31	2 53	2 97	2 163	3 19	3 41	3 107	3 151	3 173	4 29	4 73	4 139	5 17	5 61	5 83
2	13	0 143	0 169	1 11	1 113	1 89	1 167	1 193	2 61	2 113	2 139	2 191	3 59	3 137	3 163	4 31	4 83	4 109	4 187	5 29	5 107	7 179	6 53	6 79
3	17	0 187	1 11	1 79	1 151	1 181	2 73	2 107	3 73	3 67	3 101	3 169	4 61	4 163	4 197	5 89	5 157	5 191	6 83	6 151	7 43	8 163	8 37	8 71
4	19	1 43	1 37	1 113	2 17	2 17	2 131	2 169	4 11	3 149	3 187	4 53	4 167	5 71	5 109	6 13	6 89	6 127	7 31	7 107	8 11	10 131	9 29	9 67
5	23	1 109	1 89	1 181	2 131	2 109	3 37	3 83	5 23	4 103	4 149	5 31	5 169	6 97	6 143	7 71	7 163	10 17	8 137	9 19	9 157	13 83	11 13	11 59
6	29	1 131	1 167	2 73	2 169	3 37	4 59	4 59	5 97	5 139	5 197	6 103	7 67	8 31	8 89	9 53	9 169	10 163	10 191	11 97	12 61	14 67	13 199	14 47
7	31	1 197	1 193	2 107	3 73	3 83	5 23	4 121	6 109	6 11	6 73	6 197	7 173	8 149	10 157	9 187	10 101	12 181	11 139	12 53	13 29	17 19	14 191	15 43
8	37	2 31	2 61	3 67	3 149	4 11	5 139	5 97	7 47	7 47	7 121	8 59	9 71	10 83	11 191	11 169	12 107	14 53	13 193	14 131	15 143	18 197	17 167	18 31
9	41	2 53	2 113	3 101	3 187	4 103	5 197	6 11	7 121	8 83	8 83	9 37	10 73	11 109	12 103	13 17	13 181	14 199	15 89	16 43	17 79	19 181	19 151	20 23
10	43	2 97	2 139	3 169	4 53	4 149	6 103	6 73	8 59	9 37	8 169	9 131	10 179	12 17	13 137	13 151	14 113	16 71	16 37	18 121	18 47	21 149	20 143	21 19
11	47	2 163	2 191	4 61	4 167	5 31	7 67	6 197	9 71	10 73	9 131	10 109	11 181	13 43	15 83	16 191	15 187	18 89	17 143	20 199	19 193	24 101	22 127	23 11
12	53	3 19	3 59	4 163	5 71	5 169	8 31	7 173	10 83	11 109	10 179	11 181	13 79	14 187	17 29	18 173	17 193	20 107	19 197	23 67	22 97	27 53	25 103	28 197
13	59	3 41	3 137	4 197	5 109	6 97	8 89	8 149	10 157	11 191	12 17	13 43	14 187	16 121	17 151	19 97	19 199	21 43	22 41	24 23	25 179	28 37	28 79	29 193
14	61	3 107	3 163	5 89	6 13	6 143	9 53	9 187	11 169	13 17	12 103	13 137	15 83	17 29	19 97	21 79	20 131	23 61	22 199	26 101	28 83	30 199	29 71	32 181
15	67	3 151	4 31	5 157	6 89	7 71	9 169	10 101	12 107	13 181	13 151	15 187	16 191	18 173	20 131	22 137	22 137	24 143	25 43	28 13	30 19	32 167	32 47	34 173
16	71	3 173	4 83	5 191	6 127	7 163	10 17	10 163	12 181	14 53	14 113	16 71	17 193	19 199	21 43	23 61	24 143	25 79	26 149	28 179	30 197	33 151	34 31	35 169
17	73	4 29	4 109	6 83	7 31	8 137	10 191	11 139	13 193	15 89	14 199	17 143	18 89	20 107	22 199	25 43	26 149	27 97	27 97	31 47	33 101	36 103	35 23	38 157
18	79	4 73	4 187	6 151	7 107	9 19	11 97	12 53	14 131	16 43	16 37	18 121	19 197	22 41	24 23	26 101	28 13	28 179	29 151	32 169	35 37	38 71	39 193	40 149
19	83	4 139	5 29	7 43	8 11	9 157	12 61	13 29	15 143	17 79	18 47	19 193	20 199	23 67	25 179	28 83	30 19	30 197	31 47	35 37	37 151	41 23	42 169	43 137
20	89	5 17	5 107	7 179	8 163	10 131	13 83	14 67	17 19	18 197	19 181	21 149	22 97	27 53	28 37	30 199	32 167	33 151	33 101	38 71	41 23	44 169	46 137	47 121
21	97	5 61	6 53	8 37	9 29	11 13	13 199	14 191	17 167	19 151	20 143	22 127	24 101	28 79	29 71	32 47	34 31	35 23	36 103	39 193	42 169	46 137	48 121	49 113
22	101	5 83	6 79	8 71	9 67	11 59	14 47	15 43	18 31	20 23	21 19	23 11	25 103	28 197	29 193	32 181	34 173	35 169	38 157	40 149	43 137	47 121	49 113	50 109
23	103	5 127	6 131	8 139	9 143	11 151	14 163	15 167	18 179	20 187	21 191	23 199	27 107	30 13	31 17	34 29	36 37	37 41	40 53	42 61	45 73	49 89	51 97	52 101
24	107	5 149	6 157	8 173	9 181	11 197	15 11	16 19	19 43	21 59	22 67	24 83	28 109	30 131	31 139	34 163	36 179	37 187	42 107	43 17	46 41	50 73	52 89	53 97
25	109	5 193	7 103	9 31	10 47	12 79	15 127	16 143	19 191	22 13	23 29	25 61	30 113	31 157	32 173	36 11	38 43	39 59	45 109	44 139	47 187	52 41	54 73	55 89
26	113	6 71	7 181	9 167	10 199	13 53	16 149	17 181	21 67	23 131	24 163	27 17	32 11	35 143	35 31	38 127	40 191	42 13	47 163	47 173	51 59	55 187	58 41	59 73
27	121	6 137	8 23	10 59	11 103	13 191	17 113	18 157	22 79	24 167	26 173	28 89	33 13	36 169	36 187	40 109	42 197	44 31	49 59	50 41	53 173	58 139	61 17	62 61
28	127	6 181	8 101	10 127	11 179	14 73	18 19	19 71	23 17	25 121	28 11	29 67	34 121	38 103	38 11	41 167	44 61	45 113	51 113	51 163	55 109	60 107	65 187	64 53
29	131	7 37	8 127	11 19	12 83	15 47	18 193	20 47	24 29	26 157	28 97	30 139	35 17	39 11	39 167	43 149	46 67	47 131	52 61	54 31	58 13	63 59	66 179	67 41
30	137	7 59	8 179	11 53	12 121	15 139	19 41	20 109	24 103	27 29	29 59	31 23	36 19	40 37	40 79	44 73	48 73	48 67	53 167	54 197	58 191	64 43	68 163	68 37
31	139	7 103	9 47	11 121	12 197	16 67	19 157	21 23	25 41	27 193	30 107	33 73	37 127	41 181	41 113	45 131	50 79	49 149	56 11	56 109	60 127	66 11	71 139	70 29
32	143	7 169	9 73	12 13	13 101	16 113	20 121	22 61	26 53	29 19	30 193	33 167	38 23	42 89	43 59	47 113	51 11	51 167	56 169	58 187	63 31	68 173	72 131	73 17
33	149	7 191	9 151	12 47	13 139	17 41	20 179	23 37	26 127	29 101	32 31	35 29	39 131	44 23	43 181	48 37	53 17	52 103	59 13	59 143	66 113	69 157	75 107	74 13
34	151	8 47	10 19	12 149	14 43	17 179	21 143	24 13	27 139	30 137	33 79	36 101	41 29	45 167	45 127	50 19	55 23	54 121	61 67	62 11	69 17	72 109	78 83	79 199
35	157	8 113	10 71	13 41	14 157	18 61	22 107	24 137	28 151	31 173	34 41	37 79	42 31	46 193	47 73	53 59	56 97	56 139	62 173	64 89	70 163	75 61	80 67	81 191
36	163	8 157	10 97	13 109	15 23	18 107	23 13	24 199	29 89	32 127	34 127	37 173	42 137	47 101	48 107	53 193	57 29	58 11	63 121	66 167	71 131	77 29	81 59	82 187
37	167	8 179	10 149	13 143	15 61	18 199	23 71	25 113	29 163	33 163	35 89	38 151	43 139	48 127	49 19	55 41	58 103	58 157	65 17	68 79	73 67	78 13	83 43	84 179
38	169	9 13	11 17	14 103	15 137	19 127	23 187	26 89	30 101	34 199	36 137	40 13	45 37	50 61	50 53	57 23	60 109	60 29	67 71	70 157	75 181	79 191	86 19	87 167
39	173	9 79	11 43	14 137	16 41	19 173	24 151	26 151	31 113	35 71	37 13	40 107	45 143	50 179	52 121	57 157	61 41	62 47	68 19	71 113	76 149	82 143	87 11	88 163
40	179	9 101	11 121	15 29	16 79	20 101	25 173	27 127	31 187	36 107	38 61	41 179	47 41	52 113	54 67	59 139	63 47	62 193	70 73	73 191	79 53	83 127	89 197	91 151
41	181	9 167	11 173	15 97	16 193	20 193	26 79	28 41	32 199	37 61	39 23	42 157	48 43	53 139	55 101	60 197	64 121	66 83	71 179	75 103	80 199	86 79	91 181	93 143
42	187	10 23	11 199	15 131	17 59	21 29	26 137	28 103	33 137	37 143	39 109	43 41	48 149	54 47	56 13	61 121	65 53	67 19	72 127	76 59	81 167	88 47	92 173	94 139
43	191	10 67	12 41	15 199	17 97	21 121	27 43	29 17	34 149	38 97	40 71	44 19	49 151	55 73	57 47	62 179	66 127	68 101	74 23	77 181	83 103	89 31	94 157	96 131
44	193	10 89	12 67	16 23	17 173	21 167	27 101	29 79	35 13	38 179	40 157	44 113	50 47	55 191	57 169	63 103	67 59	69 37	74 181	78 137	84 71	91 193	95 149	97 127
45	197	10 199	12 197	16 193	18 191	22 187	28 181	30 179	36 173	40 169	42 167	46 163	52 157	58 151	60 149	66 143	70 139	72 137	78 131	82 127	88 121	96 113	100 109	102 107
46	199	11 11	13 13	17 17	19 19	23 23	29 29	31 31	37 37	41 41	43 43	47 47	53 53	59 59	61 61	67 67	71 71	73 73	79 79	83 83	89 89	97 97	101 101	103 103
47	209	0 209	6 209	2 209	0 209	7 209	24 209	21 209	2 209	32 209	16 209	14 209	25 209	33 209	51 209	14 209	46 209	7 209	37 209	16 209	63 209	90 209	37 209	25 209
48	211	9 211	5 211	13 211	17 211	14 211	3 211	8 211	33 211	7 211	25 211	31 211	26 211	24 211	8 211	51 211	23 211	64 211	40 211	65 211	24 211	5 211	62 211	76 211

(b)

i \ j	q_j / $h_{i,j}$	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
		107	109	113	121	127	131	137	139	143	149	151	157	163	167	169	173	179	181	187	191	193	197	199	209	211
														$(k_{i/j}, t_{i/j})$												
1	11	5 127	5 149	5 193	6 71	6 137	6 181	7 37	7 59	7 103	7 169	7 191	8 47	8 113	8 157	8 179	9 13	9 79	9 101	9 167	11 173	10 23	10 67	10 89	10 199	11 11
2	13	6 131	6 157	9 31	7 103	7 181	8 23	8 101	8 127	8 179	9 47	9 73	9 151	10 19	10 71	10 97	10 149	11 17	11 43	11 121	15 97	11 199	12 41	12 67	12 197	13 13
3	17	8 139	8 173	10 47	9 167	10 59	10 127	11 19	11 53	11 121	12 13	12 47	12 149	13 41	13 109	13 143	15 137	14 103	14 137	15 29	17 59	15 131	15 199	16 23	16 193	17 17
4	19	9 143	9 181	12 79	10 199	11 103	11 179	12 83	12 121	12 197	13 101	13 139	14 43	14 157	15 23	15 61	18 199	16 41	16 79	16 193	20 193	17 97	17 173	21 167	18 191	19 19
5	23	11 151	11 197	15 127	13 53	13 191	14 73	18 193	15 47	15 139	16 67	16 113	17 41	17 179	18 61	18 107	23 187	19 127	19 173	20 101	26 79	21 29	21 121	27 101	22 187	23 23
6	29	14 163	15 11	16 143	16 149	17 113	18 19	20 47	19 41	19 157	20 121	20 179	21 143	22 107	23 13	23 71	25 113	24 151	26 151	25 173	28 41	26 137	27 43	29 79	28 181	29 29
7	31	15 167	16 19	19 191	17 181	18 157	19 71	24 29	20 109	21 23	26 53	22 61	23 37	24 13	24 137	24 199	30 101	26 89	31 187	27 127	33 137	28 103	29 17	35 13	30 179	31 31
8	37	18 179	19 43	22 13	21 67	22 79	23 17	26 157	24 103	25 41	29 19	26 127	27 139	28 151	29 89	29 163	33 163	31 113	35 71	32 199	37 61	37 143	34 149	38 179	36 173	37 37
9	41	20 187	21 59	23 29	23 131	24 167	25 121	28 11	27 29	27 193	30 107	29 101	30 137	31 173	32 127	34 127	35 89	34 199	37 13	36 107	39 23	39 109	38 97	40 157	40 169	41 41
10	43	21 191	22 67	25 61	24 163	28 89	26 173	30 139	28 97	29 59	33 73	30 193	32 31	33 79	34 41	37 173	38 151	36 137	40 107	38 61	42 157	43 41	40 71	44 113	42 167	43 43
11	47	23 199	24 83	28 109	27 17	32 11	29 67	34 121	31 23	36 19	37 127	33 167	35 29	36 101	37 79	42 137	43 139	40 13	45 143	41 179	48 43	48 149	44 19	50 47	46 163	47 47
12	53	30 13	27 107	31 157	30 113	35 143	33 13	38 103	35 17	40 37	41 181	38 23	39 131	41 29	42 31	47 101	48 127	45 37	50 179	47 41	53 139	54 47	49 151	55 191	52 157	53 53
13	59	31 17	30 131	32 173	35 31	36 187	36 169	39 167	39 11	41 113	43 59	42 89	44 23	45 167	46 193	49 19	50 53	50 61	52 121	52 113	55 101	56 13	55 73	57 169	58 151	59 59
14	61	34 29	31 139	36 11	38 127	40 109	38 11	43 149	40 79	45 131	47 113	43 181	45 127	47 73	48 107	53 193	55 41	57 23	57 157	54 67	60 197	61 121	57 47	63 103	60 149	61 61
15	67	36 37	34 163	38 43	40 191	42 197	41 167	46 67	44 73	48 73	50 79	48 37	50 19	55 23	53 59	57 29	58 103	60 109	61 41	59 139	64 121	65 53	62 179	67 59	66 143	67 67
16	71	37 41	36 179	39 59	42 13	44 31	44 61	47 131	48 67	49 149	51 167	51 11	53 17	56 139	56 97	58 157	60 29	62 47	62 193	63 47	66 83	67 19	66 127	69 37	70 139	71 71
17	73	40 53	37 187	42 107	45 109	47 163	45 113	51 113	52 61	53 167	56 11	52 103	54 121	61 67	58 11	63 121	65 17	67 71	68 19	70 73	71 179	72 127	68 101	74 181	72 137	73 73
18	79	42 61	43 17	44 139	47 173	50 41	49 59	54 31	54 197	56 109	58 187	56 169	59 13	64 89	62 173	66 167	68 79	70 157	71 113	73 191	75 103	76 59	74 23	78 137	78 131	79 79
19	83	45 73	46 41	47 187	51 59	53 173	51 163	58 13	58 191	60 127	63 31	59 143	62 11	69 17	70 163	71 131	73 67	75 181	76 149	79 53	80 199	81 167	77 181	84 71	82 127	83 83
20	89	49 89	50 73	52 41	55 187	58 139	55 109	63 59	64 43	66 11	68 173	69 157	66 113	75 61	77 29	78 13	79 191	82 143	83 127	86 79	88 47	89 31	83 103	91 193	88 121	89 89
21	97	51 97	52 89	54 73	58 41	61 17	60 107	65 187	66 179	68 163	71 139	72 131	72 109	78 83	80 67	81 59	83 43	86 19	87 11	89 197	91 181	92 173	94 157	95 149	96 113	97 97
22	101	52 101	53 97	55 89	59 73	62 61	64 53	67 41	68 37	70 29	73 17	74 13	75 107	79 199	81 191	82 187	84 179	87 167	88 163	91 151	93 143	94 139	96 131	97 127	100 109	101 101
23	103	54 109	55 113	57 121	61 137	64 149	66 157	69 169	70 173	72 181	75 193	76 197	81 103	83 11	85 19	86 23	88 31	91 43	92 47	95 59	97 67	98 71	100 79	101 83	102 107	103 103
24	107	55 113	56 121	58 137	62 169	65 193	70 103	71 23	72 31	74 47	77 71	78 79	84 101	84 127	86 143	87 151	89 167	92 191	93 199	97 13	99 29	100 37	102 53	103 61	106 103	107 107
25	109	57 121	58 137	60 169	65 23	68 71	75 101	73 151	74 167	76 199	80 37	81 53	90 97	87 149	89 181	90 197	93 19	96 67	97 83	100 131	102 163	103 179	113 107	107 17	108 101	109 109
26	113	61 137	62 169	65 23	69 151	73 37	79 47	78 197	80 19	82 83	85 179	91 67	94 199	93 193	96 47	97 79	99 143	103 29	104 61	107 157	110 11	111 43	119 29	114 139	112 97	113 113
27	121	64 149	65 193	68 71	73 37	76 169	81 151	82 179	84 13	86 101	90 23	94 41	97 197	98 121	104 37	102 43	104 131	108 53	109 97	113 19	115 107	116 151	122 187	120 73	120 89	121 121
28	127	66 157	71 23	70 103	75 101	79 47	85 97	85 97	86 149	89 43	92 199	98 107	102 89	101 143	108 199	105 89	107 193	111 139	112 191	116 137	119 31	120 83	128 109	124 29	126 83	127 127
29	131	69 169	72 31	73 151	78 197	82 179	86 149	89 79	90 143	93 61	97 43	99 199	103 193	106 71	110 113	110 53	112 181	116 163	118 17	123 163	124 127	125 191	130 83	129 173	130 79	131 131
30	137	70 173	74 47	74 167	80 19	84 13	89 43	90 143	94 137	94 137	98 131	102 173	106 191	107 187	113 151	111 181	114 107	118 101	119 169	127 71	126 89	127 157	134 31	131 151	136 73	137 137
31	139	72 181	77 71	76 199	82 83	86 101	92 199	93 61	98 131	97 79	101 97	107 29	111 83	115 137	118 103	115 17	117 169	121 187	123 53	132 143	130 13	131 89	139 163	135 107	138 71	139 139
32	143	75 193	78 79	80 37	85 179	90 23	94 41	97 43	99 199	101 97	105 151	108 121	112 187	117 43	120 17	119 191	122 157	128 149	128 89	134 97	135 109	136 197	141 137	141 41	142 67	143 143
33	149	76 197	81 103	81 53	90 97	91 67	97 197	98 107	103 193	102 173	107 29	112 187	117 79	121 181	124 179	121 109	124 83	133 173	130 31	139 169	137 71	138 163	147 59	143 19	148 61	149 149
34	151	83 11	84 127	84 101	93 193	94 199	101 143	102 89	107 187	106 191	111 83	117 43	121 181	126 109	129 131	126 73	129 71	138 197	135 67	145 31	142 167	144 61	152 191	148 163	150 59	151 151
35	157	85 19	86 143	87 149	96 47	98 121	104 37	106 71	110 113	113 151	115 137	120 17	124 179	129 131	132 169	131 37	134 59	142 73	140 103	148 149	148 53	149 169	156 139	154 97	156 53	157 157
36	163	86 23	87 151	89 181	97 79	102 43	105 89	108 199	111 181	115 17	118 103	121 109	126 73	131 37	134 83	134 83	137 121	144 11	143 197	150 103	151 187	153 101	158 113	158 53	162 47	163 163
37	167	88 31	89 167	90 197	99 143	104 131	107 193	110 53	114 107	117 169	119 191	124 83	129 71	134 59	137 121	139 47	139 47	147 97	145 139	154 11	153 149	155 67	162 61	160 31	166 43	167 167
38	169	91 43	92 191	93 19	103 29	108 53	111 139	112 181	118 101	121 187	122 157	128 149	133 173	138 197	142 73	144 11	142 109	152 121	149 23	159 83	157 73	164 107	167 193	163 197	168 41	169 169
39	173	92 47	93 199	96 67	104 61	109 97	112 191	116 163	119 169	123 53	128 89	130 31	135 67	140 103	143 197	145 139	147 97	154 59	154 59	161 37	162 169	166 73	169 167	169 131	172 37	173 173
40	179	95 59	97 13	97 83	107 157	113 19	116 137	118 17	123 163	127 71	132 143	134 97	139 169	145 31	148 149	150 103	149 23	159 83	161 37	166 109	164 131	171 181	175 89	171 109	178 31	179 179
41	181	97 67	99 29	100 131	110 11	115 107	119 31	124 127	126 89	130 13	135 109	137 71	142 167	148 53	151 187	153 149	154 11	162 169	164 131	170 17	170 17	175 113	179 37	177 43	180 29	181 181
42	187	98 71	100 37	102 163	111 43	116 151	120 83	125 191	127 157	131 89	136 197	138 163	144 61	149 169	153 101	155 67	157 73	164 107	166 73	171 181	173 151	177 79	181 11	182 187	186 23	187 187
43	191	100 79	102 53	103 179	113 107	119 29	122 187	128 109	130 83	134 31	139 163	141 137	147 59	152 191	156 139	158 113	162 61	167 193	169 167	175 89	175 113	181 11	184 169	186 143	190 19	191 191
44	193	101 83	103 61	107 17	114 139	120 73	124 29	129 173	131 151	135 107	141 41	143 19	148 163	154 97	158 53	160 31	163 197	169 131	171 109	177 43	179 37	182 187	186 143	188 121	192 17	193 193
45	197	106 103	108 101	112 97	120 89	126 83	130 79	136 73	138 71	142 67	148 61	150 59	156 53	162 47	166 43	168 41	172 37	178 31	180 29	186 23	190 19	192 17	196 13	198 11	196 13	197 197
46	199	107 107	109 109	113 113	121 121	127 127	131 131	137 137	139 139	143 143	149 149	151 151	157 157	163 163	167 167	169 169	173 173	179 179	181 181	187 187	191 191	193 193	197 197	199 199	198 11	199 199
47	209	79 209	67 209	6 209	33 209	100 209	67 209	121 209	46 209	110 209	21 209	63 209	79 209	110 209	100 209	32 209	158 209	51 209	24 209	121 209	180 209	158 209	90 209	180 209	209 209	209 209
48	211	26 211	40 211	105 211	86 211	25 211	62 211	14 211	91 211	31 211	126 211	86 211	76 211	51 211	65 211	135 211	13 211	126 211	155 211	64 211	9 211	33 211	105 211	17 211	207 211	211 211

Proof

We have

$$\alpha = r_i + 210k^* = t_{i/j} + 210k^*.$$

By definition of $k_j\left(k^*\right)$ of Remark 2, we can write

$$k^* = k_{i/j} + q_j m$$

for some $m \in N_0$. Thus,

$$\alpha = t_{i/j} + 210k_{i/j} + 210q_j m.$$

From Theorem 3, we know $t_{i/j} + 210k_{i/j} = q_j r_i'$, for some $r_i' \in S_7$.
Hence,

$$\alpha = q_j\left(r_i' + 210m\right),$$

which shows that α is a composite number.

Example 2: Composites in $[11, 211]$

We first list composites in the base interval [11, 211]. Here,

$$\alpha = r_i + 210k^*,\quad k^* = 0.$$

From eCTC210 (**Table 2(a)** and **Table 2(b)**), when $k_{i/j} = k_j(0) = 0$, the entries with $k_{i/j} = 0$ and values $t_{i/j}$ are:

$$\left(0,\ t_{i/j}\right) \in \left\{(0,\ 121),\ (0,\ 143),\ (0,\ 187),\ (0,\ 143),\ (0,\ 169),\ (0,\ 187)\right\}$$

Thus, the composites in [11, 211] of the form $\alpha = r_i + 210\times 0$ are

$$r_i \in \{121,\ 143,\ 169,\ 187\},$$

as listed in **Table 3(a)**.

Example 3: Composites in $[11+210, 211+210]$

Now consider the interval $[11+210,\ 211+210] = [221,\ 421]$. Here,

$$\alpha = r_i + 210k^*,\quad k^* = 1.$$

Compute

$$k_j(1) = \min\{1 - q_j m\}$$

For all relevant q_j, the minimum is attached at $m = 0$, so $k_j(1) = 1$.

The entries in eCTC210 with $k_{i/j} = 1$ include:

$$\left(1,\ t_{i/j}\right) \in \{(1,\ 43),\ (1,\ 109),\ (1,\ 131),\ (1,\ 197),\ (1,\ 11),\ \cdots,\ (1,\ 193),$$
$$(1,\ 11),\cdots,\ (1,\ 181),\ (1,\ 37),\cdots,\ (1,\ 151)\}$$

Therefore, the composites in [221, 421] (without factors 2, 3, 5, 7) are

$$\alpha = r_i + 210,$$

where r_i belongs to the root set

$$\mathrm{RS}_1 = \{11,\ 37,\ 43,\ 79,\ 89,\ 109,\ 113,\ 131,\ 151,\ 167,\ 181,\ 193,\ 197\},$$

as shown in **Table 3(b)**.

Table 3. Example of a composites listing.

(a)

j	q_j	$k_j(0)$	$h_{i/j}$		
1	11	0	121	143	187
2	13	0	143	169	
3	17	0	187		
4	19	0			
5	23	0			
6	29	0			
7	31	0			
8	37	0			
9	41	0			
10	43	0			
11	47	0			
12	53	0			
13	59	0			
14	61	0			
15	67	0			
16	71	0			
17	7	0			
18	73	0			
19	83	0			
20	89	0			
21	97	0			
22	101	0			
23	103	0			
24	107	0			
25	109	0			
26	113	0			
27	121	0			
28	127	0			

(b)

j	q_j	$k_j(1)$	$h_{i/j}$				
1	11	1	43	109		131	197
2	13	1	11	37	89	167	193
3	17	1	11	79	113	181	
4	19	1	37	113	151		
5	23	1	43	89	189		

Continued

6	29	1	109	167
7	31	1	131	193
8	37	1	197	
9	41	1		
10	43	1		
11	47	1		
12	53	1		
13	59	1		
14	61	1		
15	67	1		
16	71	1		
17	7	1		
18	73	1		
19	83	1		
20	89	1		
21	97	1		
22	101	1		
23	103	1		
24	107	1		
25	109	1		
26	113	1		
27	121	1		
28	127	1		

(c)

j	q_j	$k_j(100)$	$h_{i/j}$			
1	11	1	43	109	131	197
2	13	9	47	73	151	
3	17	15	29	97	131	199
4	19	5	71	109		
5	23	8	137			
6	29	13	83	199		
7	31	7	173			
8	37	26	53	127		
9	41	18	197			
10	43	14	113	199		
11	47	6	103			
12	53	47	41			

Continued

13	59	41	181
14	61	39	167
15	67	33	—
16	71	29	—
17	7	27	97
18	73	21	—
19	83		
20	89		
21	97		
22	101	100	109
23	103		
24	107	100	79
25	109	100	37
26	113	100	131
27	121		
28	127	100	209

Example 4: Composites in $[11+210\times 100, 211+210\times 100]$

Consider the interval

$$[11+210\times 100,\ 211+210\times 100]=[21011,\ 21211]$$

Here,

$$\alpha = r_i + 210\times 100,\quad k^* = 100$$

Compute $k_j(100)$ for various q_j:

$$k_1(100)=\min\{100-11m\}=1,$$

$$k_2(100)=\min\{100-13m\}=9,$$

$$k_3(100)=\min\{100-17m\}=15,$$

$$k_4(100)=\min\{100-19m\}=5,$$

$$k_5(100)=8,\quad k_6(100)=13,\quad k_7(100)=7,\ \cdots,\ k_{28}(100)=209.$$

From eCTC$_{210}$ (Table 2(a) and Table 2(b)) we observe:

- For $k_1(100)=1$, the related $h_{i/j}$ values are 43, 109, 131, 197.
- For $k_2(100)=9$, the related $h_{i/j}$ values are 47, 73, 151, $\cdots$
- For $k_{15}(100)=33$, there are no corresponding $h_{i/j}$ values.
- For $k_{28}(100)=100$, the related $h_{i/j}$ is 209.

Hence, the composites in [21011, 21211] (without factors 2, 3, 5, 7) are

$$\alpha = r_i + 210\times 100,$$

where r_i belongs to the root set

$$\mathrm{RS}_2 = \{29,\ 37,\ 41,\ 43,\ 53,\ 71,\ 73,\ 79,\ 83,\ 97,\ 103,\ 109,\ 113,\ 127,\ 131,\ 151,\ 167,\ 173,\ 181,\ 197,\ 199,\ 209\},$$

as listed in Table 3(c).

Remark 3 (Prime Listing via Composite Removal)

To list all primes in the interval $[11+210k^*,\ 211+210k^*]$, we proceed as follows:

1) First, list all integers in the interval that are coprime to 2, 3, 5, 7, *i.e.*, all $\alpha = r_i + 210k^*$ with $r_i \in S_7$.

2) Then, remove from this set all composites (without factors 2, 3, 5, 7) found by the Formula of listing composites above.

The remaining integers are precisely the primes in that interval.

Formula: Prime Listing in $[11+210k^*, 211+210k^*]$

Integers

$$\alpha = r_i + 210k^*,\quad r_i \in S_7,$$

are prime if and only if they are not among the composites found by formula "Formula of listing composite within $[11+210k^*,\ 211+210k^*]$."

Example 5: Prime Numbers in Several Intervals

1) Primes in $[11,\ 211]$

From Example 2, the composites are 121, 143, 169, 187. Hence, the primes are

$$\alpha = r_i + 210\times 0,\quad r_i \in S_7,\quad r_i \notin \{121,\ 143,\ 169,\ 187\}.$$

2) Primes in $[221, 421]$

From Example 3, the composites correspond to RS_1. Thus, the primes are

$$\alpha = r_i + 210,\quad r_i \in S_7,\quad r_i \notin \mathrm{RS}_1,$$

where RS_1 is given above.

3) Primes in $[21011,\ 21211]$

From Example 4, the composites correspond to RS_2. Thus, the primes are

$$\alpha = r_i + 210\times 100,\quad r_i \in S_7,\quad r_i \notin \mathrm{RS}_2.$$

Formula of Factorization

The table eCTC_{210} can also be used to factor a composite.

Consider a composite α satisfying

$$11 \le \alpha \le 211^2,\quad \gcd(\alpha,\ 210) = 1.$$

Then α has a root $r_i \in S_7$ and period $k^* \le 210$ such that

$$\alpha = r_i + 210k^*.$$

Factorization Formula

The composite α contains a factor $q_j \in S_7$ if and only if there exists an element $(k_{i/j},\ t_{i/j})$ in eCTC_{210} such that

$$k_{i/j} = k^*,\quad t_{i/j} = r_i.$$

Example 6: Factorizing 2431

We have

$$2431 = 121 + 210 \times 11$$

and the corresponding $\left(k^*, r_i\right) = (11, 121)$.

From eCTC210 for $q_2 = 13$, $q_3 = 17$, we have

$$\left(k_{40/2}, t_{40/2}\right) = (11, 121), \left(k_{31/3}, t_{31/3}\right) = (11, 121).$$

Hence, 2431 contains factors 13 and 17. Also, $121 = 11 \times 11$. Therefore,

$$2431 = 13 \times 17 \times 11.$$

Example 7: Factorizing 39203

We write:

$$39203 = 143 + 210 \times 186.$$

Thus, the root is $r_i = 143$ and period $k^* = 186$.

From eCTC210 at $q_j = 197$, we find the entry

$$(186, 143),$$

implying 197 divides 39203.

Therefore,

$$39203 = 197 \times 199.$$

5. Extension and Discussion

In this section, we extend the proposed framework to larger moduli and discuss related formulas, computational complexity, and potential applications.

5.1. Extension from S_7 to S_{11}, S_{13}, …

The eCTC and the prime formulas above are based on the set S_7 and the modulus

$$2 \times 3 \times 5 \times 7 = 210,$$

so that the associated primes are limited to those smaller than 211^2. To study where larger primes hide, we extend the set from S_7 to S_{11}, and further to $S_{13}, S_{17}, \cdots$

Let

$$\lambda_x = 2 \times 3 \times 5 \times \cdots \times x,$$

and let S_x be the set of positive integers up to $\lambda_x + 1$ that are not divisible by any prime in $\{2, 3, 5, \cdots, x\}$. Although the size of S_x increases with x, the mirror and symmetry effects observed in the eCTC for S_7 remain structurally the same for all S_x.

Figure 1 conceptually explains why higher-level sequences S_x share the same features as S_7:

1) Growth of $|S_x|$

Denote $|S_x|$ as the number of elements in S_x. Clearly, $|S_x|$ grows as x increases, reflecting the finer filtration by more primes.

2) Complementary pairs

For each S_x, certain pairs of elements maintain a complementary relationship.

$S_7 = \{11, 13, 17, 19, 23, \cdots, 211\}$ $\quad |S_7| = 48$

$S_{11} = \{13, 17, \quad \cdots, \quad 2{,}311\}$ $\quad |S_{11}| = 48 \times (11 - 1) = 480$

$S_{13} = \{17, 19, \quad \cdots, \quad 30{,}031\}$ $\quad |S_{13}| = 480 \times (13 - 1) = 5760$

$S_{17} = \{17, 23, \quad \cdots, \quad 510{,}511\}$ $\quad |S_{17}| = 5760 \times (17 - 1) = 92{,}160$

$S_{19} = \{23, 29, \quad \cdots, \quad 9{,}699{,}691\}$ $\quad |S_{19}| = 92160 \times (19 - 1) = 1{,}658{,}880$

$S_{23} = \{29, 31, \cdots, \ 223{,}092{,}871\}$ $\quad |S_{23}| = 1658880 \times (23 - 1) = 36{,}495{,}360$

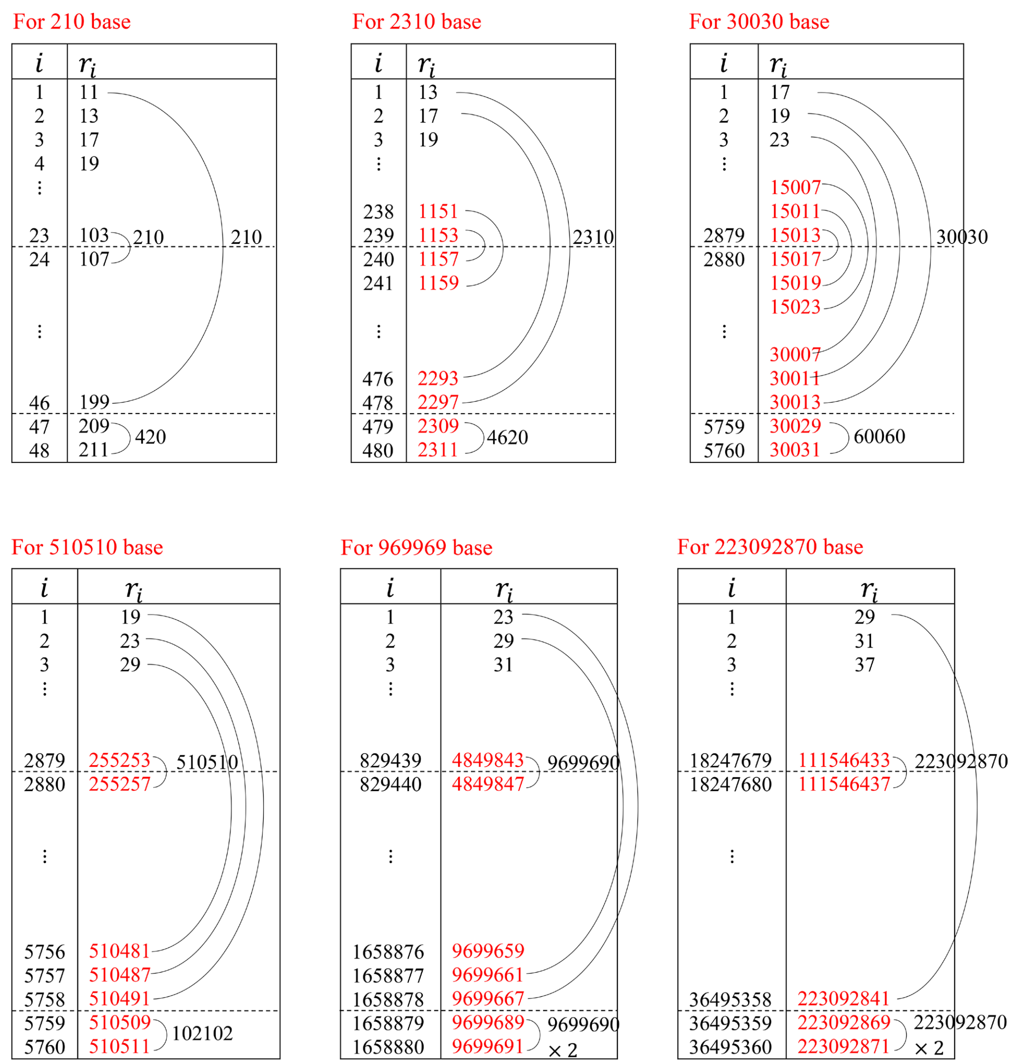

Figure 1. Extension from sequence S_7 to higher sequences.

For example:

- in S_7,

$$11 + 199 = 13 + 197 = \cdots = 103 + 107 = 210 .$$

- in S_{11},

$$13 + 2297 = 17 + 2293 = \cdots = 1153 + 1157 = 2310 .$$

- Even in S_{23},

$$29 + 223092841 = \cdots = 111546433 + 111546437 = 223092870.$$

Thus, complementary pairs always sum to λ_x (or its analogue), preserving a balanced structure.

3) Final rows and doubling property

In each S_x, the last two entries form a special pair whose sum equals twice the product $2 \times 3 \times 5 \times \cdots \times x$.

- In S_7, the last two elements are 209 and 211, with

$$209 + 211 = 2 \times 210.$$

- In S_{11}, the last two are 2309 and 2311, with

$$2309 + 2311 = 2 \times 2310.$$

4) Partition into regions

These structural properties ensure that the eCTC for S_x can always be partitioned into regions A_l, B_l, C_l, D_l, E_l, for $l = 1, 2, 3, 4$, in a manner analogous to eCTC$_{210}$. Each eCTC contains a fixed, structured number of elements:

- In eCTC$_{210}$, there are $(5+6+1) \times 4 = 48$ elements.
- In eCTC$_{2310}$, there are $(59+60+1) \times 4 = 480$ elements.
- In eCTC$_{30030}$, there are $(719+720+1) \times 4 = 5760$ elements.

This extension allows us to locate larger primes and factor larger composites efficiently.

Example 8: Factoring a Semiprime 5,246,767

Consider the semiprime 5,246,767. Since

$$210^2 < 5246767 < 2310^2,$$

we work with eCTC$_{2310}$.

We write

$$5246767 = 757 + 2310 \times 2271.$$

From the eCTC for $\lambda_{11} = 2310$, we identify $r_{100} = 757$. For $j = 100$, we have $q_i = 757$ and $q_j = 2311$.

Moreover,

$$2271 = 757 + 757 \times 2 + 2311 \times 0.$$

Therefore,

$$5246767 = 757 \times (2311 + 2310 \times 2) = 757 \times 6931.$$

5.2. Additional Prime Formulas

We now derive several additional formulas related to prime distribution, twin primes, Pythagorean primes, prime tuples, and neighboring primes.

Let

$$d_i(k) = \begin{cases} 1, & \text{if } \alpha = r_i + 210k \text{ is prime,} \\ 0, & \text{otherwise,} \end{cases}$$

for $i = 1, 2, \cdots, 48$ and $0 \le k \le 210$.

Using the prime-listing formula, we obtain:

1) Formula for Prime Distribution

- the number of primes in the interval $[11+210k,\ 211+210k]$ is

$$\sum_{i=1}^{48} d_i(k).$$

- The total number of primes in a range of blocks, say from $k=k_1$ to $k=k_2$, is

$$\sum_{k=k_1}^{k_2}\sum_{i=1}^{48} d_i(k)$$

2) Formula for the *n*-th prime

Let the largest prime in the interval $[11+210k,\ 211+210k]$ be the n-th prime in the sequence of primes. Then

$$n = 4 + \sum_{j=0}^{k}\sum_{i=1}^{48} d_i(j),$$

where the constant 4 accounts for the primes 2, 3, 5, 7, which are not included in the S_7-based representation (with 2 regarded as the first prime).

3) Formula for Twin Primes

Primes α and α' form a twin prime pair if

$$\alpha = r_i + 210k,\quad \alpha' = r_{i+1} + 210k,\quad r_{i+1} = r_i + 2.$$

In other words, α and α' differ by 2 and are twin primes.

4) Formula for Pythagorean Primes

A prime α is called a Pythagorean prime if $a \equiv 1$ (mod 4), *i.e.*,

$$\alpha = 4l + 1,\quad l \in N^+,$$

and it can be expressed as

$$\alpha = m^2 + n^2,$$

where *m* is even, and *n* is odd.

A prime $\alpha = r_i + 210k$ is Pythagorean if:

a) $r_i = 4l + 1$. The kin value *k* is even,

b) $r_i = 4l + 3$. The kin value *k* is odd,

for some $l \in N^+$.

5) Formula for *l*-Tuple Primes

A sequence of primes p_a, p_b, p_c, $\cdots$, p_h is called an *l*-tuple of primes if

$$p_b - p_a = p_c - p_b = p_d - p_c = \cdots = p_h - p_g = l.$$

In our framework, a sequence of primes $\alpha_1, \alpha_2, \cdots, \alpha_n$ forms an n-tuple of primes with step $210l$ and root r_i (base 210) if:

$$\alpha_1 = r_i + 210k$$

$$\alpha_2 = r_i + 210(k + l)$$

$$\alpha_3 = r_i + 210(k + 2l)$$

$$\vdots$$

$$\alpha_n = r_i + 210(k + (n-1)l)$$

6) Formula for Neighboring Primes

A sequence of primes $\alpha_1, \alpha_2, \cdots, \alpha_n$ are called neighboring primes if there exist r_i and k such that

$$\alpha_1 = r_i + 210k,\quad \alpha_2 = r_{i+1} + 210k,\quad \alpha_3 = r_{i+2} + 210k,\quad \cdots,\quad \alpha_n = r_{i+n-1} + 210k.$$

Example 9: 30-Tuple Primes in [120,000,000, 122,000,000]

We seek primes forming 30-tuples in the interval [120,000,000, 122,000,000].

Since

$$\sqrt{122000000} \approx 11045,$$

we choose S_{13} with modulus

$$2 \times 3 \times 5 \times 7 \times 11 \times 13 = 30030.$$

We consider a 30-tuple of the form:

$$p_a = r_i + 30030k,$$

$$p_b = r_i + 30 + 30030k,$$

$$p_c = r_i + 60 + 30030k,$$

$$p_d = r_i + 90 + 30030k,$$

$$p_e = r_i + 120 + 30030k,$$

$$p_f = r_i + 150 + 30030k.$$

Solving within the target interval yields

$$k = 4035,\quad r_i = 3761,$$

and the corresponding primes in a 30-tuple are:

$$p_a = 121174811,$$

$$p_b = 121174841,$$

$$p_c = 121174871,$$

$$p_d = 121174901,$$

$$p_e = 121174931,$$

$$p_f = 121174961.$$

6. Conclusion

In this study, we derive the kin-root representation for composite numbers without factors 2, 3, 5, and 7. We construct an extended Cyclic Table of Composites using kin-root tuples, and reveal mirror and symmetry properties within the table. We then attain formulas of primes. Primes within a given interval can be located by eliminating all composites related to the table. Additional examples and extensions to larger cases are provided.

Funding

This work is supported in part by CityU 9610556.

Acknowledgements

We are grateful for the in-kind support from NTU and for the constructive comments on the article from the anonymous reviewers.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Dickson, L.E. (2005) History of the Theory of Numbers, Volume II: Diophantine Analysis. Dover Publications.

[2] Zagier, D. (1977) The First 50 Million Prime Numbers. *The Mathematical Intelligencer*, **1**, 7-19. https://doi.org/10.1007/bf03039306

[3] Stein, M.L., Ulam, S.M. and Wells, M.B. (1964) A Visual Display of Some Properties of the Distribution of Primes. *The American Mathematical Monthly*, **71**, 516-520. https://doi.org/10.2307/2312588

[4] Ribenboim, P. (1996) The New Book of Prime Number Records. Springer. https://doi.org/10.1007/978-1-4612-0759-7

[5] Crandall, R. and Pomerance, C. (2005) Prime Numbers: A Computational Perspective. Springer.

[6] Aiyar, V.R. (1934) Sundaram's Sieve for Prime Numbers. *The Mathematics Student*, **2**, 73.

[7] Atkin, A.O.L. and Bernstein, D.J. (2003) Prime Sieves Using Binary Quadratic Forms. *Mathematics of Computation*, **73**, 1023-1030. https://doi.org/10.1090/s0025-5718-03-01501-1

[8] Pritchard, P. (1982) Explaining the Wheel Sieve. *Acta Informatica*, **17**, 477-485. https://doi.org/10.1007/bf00264164

[9] Kumanduri, R. and Romero, C. (1998) Number Theory with Computer Applications. Prentice Hall.

[10] Matiyasevich, Y.V. (1999) Formulas for Prime Numbers. In: Tabachnikov, S., Ed., *Kvant Selecta*: *Algebra and Analysis* (*Vol.* 2), American Mathematical Society, 13-24.

[11] Zagier, D. (1997) Newman's Short Proof of the Prime Number Theorem. *The American Mathematical Monthly*, **104**, 705-708. https://doi.org/10.1080/00029890.1997.11990704

[12] Li, H., Fang, S., Lin, B.M.T. and Kuo, W. (2023) Unifying Colors by Primes. *Light*: *Science & Applications*, **12**, Article No. 32. https://doi.org/10.1038/s41377-023-01073-x

[13] Li, H., Huang, Y., Fang, S. and Kuo, W. (2021) A Prime-Logarithmic Method for Optimal Reliability Design. *IEEE Transactions on Reliability*, **70**, 146-162. https://doi.org/10.1109/tr.2020.3020597

[14] Li, H., Fang, S. and Kuo, W. (2024) The Periodic Table of Primes. *Advances in Pure Mathematics*, **14**, 394-419. https://doi.org/10.4236/apm.2024.145023

[15] Li, H., Fang, S., Kuo, W. and Lin, N. (2025) Listing Prime Numbers Periodically. *Advances in Pure Mathematics*, **15**, 247-268. https://doi.org/10.4236/apm.2025.154012

[16] Li, H., Fang, S., Kuo, W. and Lin, N. (2025) Kernel Factor Pairs for Semiprime Factorization. *Advances in Pure Mathematics*, **15**, 629-642. https://doi.org/10.4236/apm.2025.159032