


# Unified Control Theory from PID to ACPID

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**How to cite this paper:** Zeng, Z.Z. and Tang, Y.Q. (2024) Unified Control Theory from PID to ACPID. *Advances in Pure Mathematics*, 14, 523-545.

<https://doi.org/10.4236/apm.2024.147030>

**Received:** June 26, 2024

**Accepted:** July 22, 2024

**Published:** July 25, 2024

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## Abstract

To address the challenge of achieving unified control across diverse nonlinear systems, a comprehensive control theory spanning from PID (Proportional-Integral-Derivative) to ACPID (Auto-Coupling PID) has been proposed. The primary concept is to unify all intricate factors, including internal dynamics and external bounded disturbance, into a single total disturbance. This enables the mapping of various nonlinear systems onto a linear disturbance system. Based on the theory of PID control and the characteristic equation of a critically damping system, Zeng's stabilization rules (ZSR) and an ACPID control force based on a single speed factor have been designed. ACPID control theory is both simple and practical, with significant scientific significance and application value in the field of control engineering.

## Keywords

Insert Nonlinear Systems, PID Control, ACPID Control, Total Disturbance, Unified Control Theory, Zeng's Stabilization Rules (ZSR)

## 1. Introduction

The essence of PID control is to form a control strategy according to the deviation between the actual value and the expected value (control target) of the output of the controlled object. As long as the PID gains are adjusted reasonably and the closed-loop control system is stable, the control target can be achieved. Therefore, the PID control method has been widely used [1]-[3] and has been in the mainstream position in the field of control engineering. So far, PID stabilization methods are mainly divided into two categories: classical stabilization methods for linear systems [1]-[3] and modern optimization methods for nonlinear systems [4]-[13]. Both classical stabilization methods and modern optimization techniques have consistently demonstrated the objective reality of poor gains robustness. Although the PID control law is model-independent, the PID gains

are closely linked to factors such as the models, or operating states, or external disturbances of the controlled object. Therefore, the stabilization value of each PID gain varies significantly depending on the specific controlled object. Even if the PID controller has been set for a particular controlled object, any changes in the operating environment, time-varying parameters, or external disturbances will require re-stabilization of each PID's gain value. Due to the persistent issue of poor gain robustness, PID stabilization has always posed a formidable challenge in the realm of control engineering, significantly constraining the potential control capabilities of PID control theory. At present, the optimization methods of PID parameters mainly include neural network optimization (NNO), fuzzy optimization (FO), particle swarm optimization (PSO), Big Bang-Big Crunch (BBBC) optimization and so on [14]. However, no matter any optimization method cannot make PID control system become an ideal critical damping system.

In order to address the issue of PID tuning, an ACPID control theory based on a single speed factor is proposed [15]. The Chinese scholar Professor Zeng Zhe-zhao argues that the PID control force violates fundamental principles of algebraic operations as it is formed through a weighted summation of dimensionless PID parameters' proportion, integral, and differential errors in [15]. The error, its integral, and differential are distinct physical links; however, Professor Zeng asserts that they are interconnected with the error. Therefore, the three parameters of PID should not be treated as independent entities but rather possess an inherent internal relationship [15]. The problem of dimensionless proportional parameters and mutually independent parameters is addressed by Professor Zeng through the proposal of a PID parameter stabilization rule based on a speed factor, leading to the initial development of Self-Coupling-PID (SCPID) control theory [15]. The SCPID control theory requires stabilizing only one-speed factor in order to achieve tuning for all three parameters of the PID controller. The scientific determination of the stabilization of the speed factor, however, remains uncertain. In order to address the stabilization issue of the speed factor, Professor Zeng proposed a method for stabilizing the speed factor based on the adjustment time of the control system. Through a physical property analysis of the PID control system, he not only provided a scientific explanation for the stabilization rule of PID parameters based on the speed factor but also established internal relationships between the speed factor and integral time constant as well as differential time constant of the PID controller. Furthermore, external relationships between the speed factor and dynamic speed of the control system were also established, and the theoretical framework of the auto-coupling PID (ACPID) control has been enhanced [16]-[18].

The SCPID or ACPID control theory has been extensively applied in various domains of control engineering since its introduction four years ago. These include time-delay system control [19]-[21], strict feedback nonlinear unknown system control [22], wind power system MPPT control [23], nonlinear time-varying system control [24], continuous stirred tank reactor control [25], trajectory tracking control for a three-link robot arm [26], high-order nonlinear

unknown system control [27], unknown nonlinear system control [28], non-affine pure feedback nonlinear system control [29], synchronization control of uncertain chaotic systems [30], accurate final guidance control [31], unmatched nonlinear system control [32], control of underdrive unstable systems with nonlinearity [33], seventh-order chaotic oscillation power system control [34], control of sandwich structure flexible transmission systems [35], hydraulic turbine regulation system control [36], maglev ball system control [37], underdrive bridge crane control [38], permanent magnet synchronous motor speed control [39], system core power control [40], underdrive TORA system control [41], underdrive VTOL (Vertical Take-Off and Landing) vehicle control [42], SCR denitration cascade System Control [43], and flexible spacecraft attitude control [44], among others.

The approach involves defining the internal dynamics and external disturbances of the controlled system as a total disturbance, which can be mapped to a unified form of linear disturbance system for any complex system. A PID control system excited by the total disturbance is then constructed according to the PID control law. To ensure that the PID control system is a critically damped system with desirable dynamic response characteristics, a gain stabilization rule based on a single speed factor (known as Zeng's stabilization rule, ZSR) can be established by utilizing the characteristic equation of the critically damped system to address the issue of stabilizing gains for PID controllers. The ZSR, based on a single speed factor, not only resolves the gain stabilization issue of PID controller but also establishes an internal relationship (or dimension conversion relationship) between each gain. This enables the coordinated control mechanism of proportional control force, integral control force and differential control force to be realized under the unified drive of the speed factor. Moreover, the theoretical defects of dimensionless proportional gain and mutually independent gain are corrected.

The main research contents of this paper are arranged as follows. Section 2 focuses on the unified method of mapping diverse nonlinear systems onto a linear disturbance system. Section 3 delves into the unified control methodology for linear disturbance systems, wherein ZSR and ACPID control theory are established and the robustness of ACPID control system is examined. Section 4 shows the control results of two different nonlinear systems to demonstrate the efficiency and correctness of the ACPID controller proposed. Section V is the conclusions.

## 2. Mapping Method for Diverse Nonlinear Systems

Due to the involvement of numerous nonlinear complex systems, such as strict feedback nonlinear systems [45]-[48], pure feedback nonlinear systems [49], non-affine nonlinear systems [50], and nonlinear time-varying systems [51], etc., control engineering requires a comprehensive approach that considers all known or unknown factors, including internal dynamics, uncertainty, and external disturbances. By treating these factors as a total disturbance, the various linear and

nonlinear systems are transformed into a linear disturbance system, which is more suitable for ACPID control in a unified way.

The term **total disturbance** refers to the comprehensive set of complex factors that comprise both known and unknown internal dynamics of various controlled systems, as well as uncertainties and external bounded disturbances. The introduction of the concept of **total disturbance** serves not only to attenuate the dichotomy between linear and nonlinear systems, but also to integrate control methods for all types of controlled systems through the employment of control theory that is independent from dynamic models. Since ACPID control theory is an advanced control theory that solely relies on the dynamic speed of the controlled system rather than its dynamic model, it is capable of achieving unified control over various linear and nonlinear systems.

### 2.1. Conversion of Diverse Nonlinear Systems into a Linear Disturbance System

#### 1) Mapping of strict feedback non-linear systems

A strict feedback nonlinear system can be described by the differential equations as:

$$\begin{cases} \dot{x}_1 = f_1(x_1) + g_1(x_1)x_2 \\ \dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)(u + d) \\ y = x_1 \end{cases} \quad (1)$$

where,  $u$  and  $y$  represent the control input and output,  $x_1$  and  $x_2$  represent two states,  $f_1(x_1)$ ,  $f_2(x_1, x_2)$ ,  $g_1(x_1)$  and  $g_2(x_1, x_2)$  represent known or unknown nonlinear models respectively, and  $0 < g_1(x_1)g_2(x_1, x_2) \leq b_0$ ,  $d$  represent the external bounded disturbance.

Let  $y_1 = x_1$ , and  $y_2 = f_1(x_1) + g_1(x_1)x_2$ , it follows that  $\dot{y}_1 = y_2$ , and  $\dot{y}_2 = \frac{\partial f_1}{\partial x_1} y_2 + \frac{\partial g_1}{\partial x_1} y_2 x_2 + g_1(x_1)\dot{x}_2$ , and  $= \frac{\partial f_1}{\partial x_1} y_2 + \frac{\partial g_1}{\partial x_1} y_2 x_2 + g_1(x_1)[f_2(x_1, x_2) + g_2(x_1, x_2)(u + d)] = w + b_0 u$ , and  $w = \frac{\partial f_1}{\partial x_1} y_2 + \frac{\partial g_1}{\partial x_1} y_2 x_2 + g_1(x_1)[f_2(x_1, x_2) + g_2(x_1, x_2)(u + d)] - b_0 u$  represents a total disturbance. Therefore, the strict feedback nonlinear system (1) can be transformed into a linear disturbance system as follows:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = w + b_0 u \\ y = y_1 \end{cases} \quad (2)$$

where,  $|w| \leq \varepsilon_0$ ,  $0 < g_1(x_1)g_2(x_1, x_2) \leq b_0$ , and  $b_0$  is the control coefficient.

Let  $y_1$  denote the physical variable of **generalized displacement**,  $y_2$  represents the physical variable of **generalized velocity**, and  $\dot{y}_2$  indicate the physical variable of **generalized acceleration**. According to the principle of dimensional symmetry, both  $w$  and  $b_0 u$  in system (2) are variables of **generalized accelera-**

**tion.** Obviously, the input control force  $b_0u$  of any second-order system should possess the physical dimension of generalized acceleration. Therefore, it is imperative that the control force  $b_0u$  provided by any controller to a second-order system also possesses this same physical dimension; otherwise, it will result in dimensional conflict within the control system.

### 2) Mapping of pure feedback nonlinear systems

A pure feedback nonlinear system can be described by the differential equations as:

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2) \\ \dot{x}_2 = f_2(x_1, x_2) + g(x_1, x_2)(u + d) \\ y = x_1 \end{cases} \quad (3)$$

where,  $u$  and  $y$  represent the control input and output,  $x_1$  and  $x_2$  represent two states,  $f_1(x_1, x_2)$ ,  $f_2(x_1, x_2)$  and  $g(x_1, x_2)$  represent known or unknown nonlinear model respectively,  $0 < \frac{\partial f_1}{\partial x_2} g(x_1, x_2) \leq b_0$ , and  $d$  represents the external bounded disturbance.

Let  $y_1 = x_1$ , and  $y_2 = f_1(x_1, x_2)$ , it follows that  $\dot{y}_1 = y_2$ ,

$$\dot{y}_2 = \frac{\partial f_1}{\partial x_1} y_2 + \frac{\partial f_1}{\partial x_2} \dot{x}_2 = \frac{\partial f_1}{\partial x_1} y_2 + \frac{\partial f_1}{\partial x_2} [f_2(x_1, x_2) + g(x_1, x_2)(u + d)] = w + b_0u.$$

In which,  $w = \frac{\partial f_1}{\partial x_1} y_2 + \frac{\partial f_1}{\partial x_2} [f_2(x_1, x_2) + g(x_1, x_2)(u + d)] - b_0u$  represents a total disturbance, and  $|w| \leq \varepsilon_0$ ,  $0 < \frac{\partial f_1}{\partial x_2} g(x_1, x_2) \leq b_0$ , and  $b_0$  is the control coefficient. Thus, the pure feedback nonlinear system (3) can be transformed into a linear disturbance system, as in (2).

### 3) Mapping of non-affine nonlinear systems

A non-affine nonlinear system can be described by the differential equations as:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = f(y_1, y_2, d, u) \\ y = y_1 \end{cases} \quad (4)$$

where,  $u$  and  $y$  represent the control input and output,  $y_1$  and  $y_2$  represent two states,  $f(y_1, y_2, d, u)$  represents known or unknown non-affine nonlinear models, and  $d$  represents the external bounded disturbance.

A total disturbance can be defined as  $w = f(y_1, y_2, d, u) - b_0u$ , and  $|w| \leq \varepsilon_0$ ,  $b_0 \neq 0$  is the control coefficient. Thus, the non-affine nonlinear system (4) can be transformed into a linear disturbance system, as in (2).

### 4) Mapping of the nonlinear time-varying systems

A nonlinear time-varying system can be described by the differential equations as:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = f(y_1, y_2, \varsigma) + b(d + u) \\ y = y_1 \end{cases} \quad (5)$$

where,  $u$  and  $y$  represent the control input and output,  $y_1$  and  $y_2$  represent two states,  $f(y_1, y_2, \zeta)$  represents the known or unknown nonlinear model,  $b$  represents the time-varying control coefficient,  $\zeta$  represents the time-varying parameter set, and  $d$  represents the external bounded disturbance.

A total disturbance can be defined as  $w = f(y_1, y_2, \zeta) + b(d + u) - b_0 u$ , and  $|w| \leq \varepsilon_0$ ,  $b_0 \leq b_{\max}$ , and  $b_0$  is the control coefficient. Thus, the nonlinear time-varying system (5) can be transformed into a linear disturbance system, as in (2).

The aforementioned analysis demonstrates that various types of nonlinear systems can be uniformly transformed into the same form of linear disturbed system, as in (2), thereby establishing the fundamental prerequisites for designing a unified controller based on system (2).

## 2.2. The Concept of a Unified Control Theory

Up to now, the existing PID control, sliding mode control and ADRC control have failed to achieve a unified control approach for all types of linear and complex nonlinear systems. The main obstacle lies in the fact that these theories involve three or more controller parameters, which vary greatly across different systems. In order to address the problem of unified control theory for various linear and complex nonlinear systems, a theoretical approach to ACPID based on a single speed factor is proposed in this paper.

## 3. The Principle of ACPID Unified Control

Assuming that  $r$  and  $y$  denote the expected and actual outputs of the unified linear disturbance system, as in (2), the tracking error, as well as its integration and differentiation, can be expressed as follows:

$$\begin{cases} e_1 = r - y \\ e_0 = \int_0^t e_1 d\tau \\ e_2 = \dot{e}_1 \end{cases} \quad (6)$$

The conventional PID control force can be mathematically formulated as:

$$b_0 u = k_p (e_1 + e_0/T_i + T_d e_2) = k_p e_1 + k_i e_0 + k_d e_2 \quad (7)$$

where, the parameters  $k_p$ ,  $k_i$  and  $k_d$  correspond to the proportional, integral, and derivative gains of the PID controller respectively,  $k_i = k_p/T_i$  and  $k_d = k_p T_d$ ,  $T_i$  and  $T_d$  are the integral time constant and the differential time constant, respectively, with dimension of second.

### 3.1. A Comprehensive Control Theory from PID to ACPID

1) A comprehensive control theory from PID to ACPID

It can be derived from the linear disturbance system (2) and (6):  $e_2 = \dot{e}_1 = \dot{r} - \dot{y}_2$ , and  $\dot{e}_2 = \ddot{r} - w - b_0 u$ . Based on Equation (7), the PID control system can be established in the following manner:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = \hat{w} - k_p e_1 - k_i e_0 - k_d e_2 \end{cases} \quad (8)$$

where,  $\hat{w} = \ddot{r} - w$  represents the compound total disturbance, and  $|\hat{w}| \leq \varepsilon_1$ .

### 2) Analysis of physical properties for PID control force

Let both  $r$  and  $y$  denote the variables of generalized displacement, then,  $e_1$  also denotes a variable of generalized displacement,  $e_0$  denotes a variable of generalized displacement\*, and  $e_2$  denotes the variable of generalized velocity, therefore, each term within the expression  $(e_1 + e_0/T_i + T_d e_2)$  represents a variable of the generalized displacement. The dimensionless proportional gain  $k_p$  only endows the PID control force  $b_0 u$  with the physical dimension of **generalized displacement**, while the input control force  $b_0 u$  of any second-order system requires the physical dimension of **generalized acceleration**. This creates a dimensional conflict in the PID control system when using a dimensionless  $k_p$ , highlighting an urgent need to correct this theoretical defect. Obviously, by defining the dimension of  $k_p$  as 1/s, the PID control force can be endowed with the same generalized acceleration dimension as that of the input control force of any second-order system, thereby resolving the issue of dimensional conflict in PID control systems.

### 3) Control theory from PID to ACPID

As a controlled error system excited by the  $\hat{w}$ , the PID control system (8) is inherently causal. Let's consider the equation  $E_0(s) = s^{-1} E_1(s)$ , by applying Laplace transform to the system (8), the PID control system can be represented in the complex frequency domain as follows:

$$E_1(s) = \frac{s}{s^3 + k_d s^2 + k_p s + k_i} \hat{W}(s) \quad (9)$$

From Equation (9), the transfer function of the PID control system can be defined as follows:

$$H(s) = \frac{E_1(s)}{\hat{W}(s)} = \frac{s}{s^3 + k_d s^2 + k_p s + k_i} \quad (10)$$

In order to ensure the stability of the PID control system (10), it is necessary to stabilize three gain variables of the PID. However, various online stabilization methods not only entail significant computational costs but also treat  $k_p$ ,  $k_i$  and  $k_d$  as independent parameters for stabilization. This results in proportional control force, integral control force and differential control force being treated independently of each other during the control process, leading to incongruous PID control force.

The **critical damping system** is a stable system with excellent dynamic response, characterized by triple poles located on the real axis of the left half-plane in the complex frequency domain. Therefore, in order to make the PID control system (10) become a critical damping system, by the characteristic equation of the critical damping system:  $(s + z_c)^3 = s^3 + 3z_c s^2 + 3z_c^2 s + z_c^3$ , Zeng's stabilization rules (ZSR) of the PID control system (10) can be defined as [15] [16]:

$$\begin{cases} k_i = z_c^3 \\ k_p = 3z_c^2 \\ k_d = 3z_c \end{cases} \tag{11}$$

where,  $z_c > 0$  represents the speed factor with a dimension of 1/s.

ZSR based on a single speed factor not only scientifically defines the dimensional attributes of each gain of PID, but also establishes the internal relationship among  $k_p$ ,  $k_i$  and  $k_d$  through the  $z_c$ . This theoretically guarantees that PID control system (10) is a **critical damping system** with excellent dynamic response characteristics and stability, thus referred to as an ACPID control system:

$$H(s) = \frac{E_1(s)}{\hat{W}(s)} = \frac{s}{(s + z_c)^3} \tag{12}$$

The stability and model robustness of the ACPID control system (12) is guaranteed when  $z_c > 0$   $z_c$  is solely dependent on the dynamic speed of the controlled system rather than its model. From the ZSR, as in (11), the Proportional-Integral-Derivative control force of ACPID controller can be derived as follows:

$$\begin{cases} b_0 u_p = 3z_c^2 e_1 \\ b_0 u_i = z_c^3 e_0 \\ b_0 u_d = 3z_c e_2 \end{cases} \tag{13}$$

The ACPID control force formed by Equation (13) is:

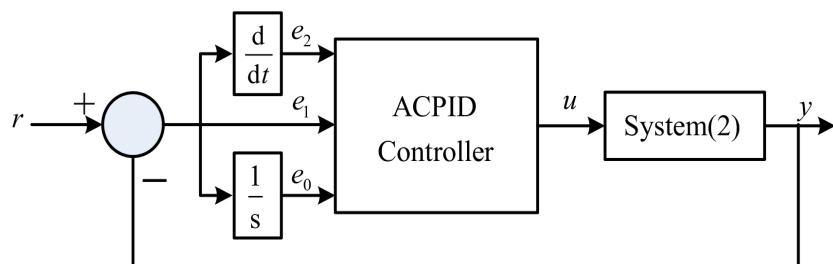
$$b_0 u = b_0 (u_p + u_i + u_d) = z_c^3 e_0 + 3z_c^2 e_1 + 3z_c e_2 \tag{14}$$

where,  $|u| \leq u_m$ , and  $u_m$  represents the maximum amplitude of input that can be applied to the controlled system.

Equation (14) indicates that increasing the value of  $z_c$  can effectively enhance the control force of ACPID controller for a linear disturbance system (2).

Since  $z_c$  is solely dependent on the dynamic speed of the controlled systems, ACPID control theory is a direct approach to controlling the dynamic speed, rather than relying on models of the controlled systems.

To facilitate analysis, **Figure 1** illustrates the block diagram of the ACPID control system.



**Figure 1.** The block diagram of ACPID control system.

### 3.2. Robustness Analysis of ACPID Control System

**Theorem 1.** Assume that the combined total disturbance is bounded:  $|\hat{w}| \leq \varepsilon_1$ , then the steady-state error of ACPID control system is bounded as well:  $|e_1(\infty)| < \varepsilon_1 / (3z_c^2)$ , furthermore, it exhibits commendable resistance to combined total disturbance.

**Proof:** According to ACPID control system (12), the unit impulse response of the system can be derived as follows:

$$h(t) = \dot{h}_1(t) = t(1 - 0.5z_c t) \exp(-z_c t) \varepsilon(t) \quad (15)$$

where,  $h_1(t) = 0.5t^2 \exp(-z_c t) \varepsilon(t)$ , and  $\varepsilon(t)$  represents a unit step function.

From system (12), the time domain model of tracking control error can be written as:

$$e_1(t) = h(t) * \hat{w}(t) = \int_0^t h(\tau) \hat{w}(t - \tau) d\tau \quad (16)$$

When  $|\hat{w}| \leq \varepsilon_1$ ,  $|e_1(t)| \leq \varepsilon_1 \int_0^t |h(\tau)| d\tau$ , then steady-state error can be expressed as:

$$|e_1(\infty)| \leq \varepsilon_1 \int_0^\infty |h(\tau)| d\tau \quad (17)$$

Based on Formula (15):  $h(t) \geq 0$  for  $0 < t \leq 2/z_c$ , and  $h(t) < 0$  for  $2/z_c < t < \infty$ , thus, we have

$$\int_0^\infty |h(\tau)| d\tau = \int_0^{2/z_c} h(\tau) d\tau - \int_{2/z_c}^\infty h(\tau) d\tau \quad (18)$$

Also because that  $\int_0^\infty h(\tau) d\tau = \int_0^{2/z_c} h(\tau) d\tau + \int_{2/z_c}^\infty h(\tau) d\tau = H(0) = 0$ , is based on Formula (18):  $\int_0^\infty |h(\tau)| d\tau = 2 \int_0^{2/z_c} h(\tau) d\tau = 2 [h_1(2/z_c) - h_1(0)] = \frac{2}{e^2 z_c^2} < \frac{1}{3z_c^2}$ ,

where  $e \approx 2.718$  is the Napierian base. Put it into Formula (17), the steady state error is:  $|e_1(\infty)| < \varepsilon_1 / (3z_c^2)$ . *Proof completed.*

From the *proof of theorem 1*, it is known that the steady state error of ACPID control system is inversely proportional to the square value of the speed factor. Hence, increasing the value of the speed factor will significantly improve the control accuracy of steady state and anti-disturbance robustness.

### 3.3. Intrinsic Relationship among $z_c$ , $T_d$ and $T_i$

Given that ACPID comes from PID, hence the  $z_c$  of the ACPID is intrinsically related to  $T_d$  or  $T_i$  of the PID. From  $k_i = k_p / T_i$ ,  $k_d = k_p T_d$ , and Formula (11), we have

$$z_c = 1/T_d = 3/T_i \quad (19)$$

Since  $z_c$  is only related to  $T_d$  or  $T_i$  and unrelated to the model of the controlled system, hence the ACPID control system (12) is a **critical damping system** that has good robust stability and dynamic responsiveness. Its physical significance is the greater the  $z_c$  is, the greater the control force of the ACPID will be, and the faster the dynamic response speed of the ACPID control system will be, and vice versa. Nevertheless, the discussion on the value of the speed factor without con-

sideration of the controlled system is practically meaningless. The consideration of the dynamic speed of the controlled system to stabilize the speed factor is the home to ACPID control theory.

### 3.4. External Relationship between $z_c$ and the Controlled Systems

Let the characteristic time constant of the controlled system (2) as  $\tau_0$ , then the dynamic speed of the system (2) can be described as  $1/\tau_0$ , indicating the smaller  $\tau_0$  means the faster dynamic speed of the system (2), and vice versa. In order for the ACPID controller to have enough control force to harness the controlled system (2), it is required to have the value of speed factor of ACPID greater than the value of dynamic speed of the controlled system (2), *i.e.*:

$$z_c = 1/T_d > 1/\tau_0 \quad (20)$$

Based on the inequality (20), let the minimum speed factor as:  $z_{cm} = \alpha/\tau_0$ , in witch,  $1 < \alpha \leq 10$  represents acceleration factor. Set dynamic process time in the controlled system (2) as  $t_n$  and  $t_r = 10\tau_0$ , then the stabilization model of the minimum speed factor based on  $t_r$  is:

$$z_{cm} = 10\alpha/t_r \quad (21)$$

where,  $1 < \alpha \leq 10$ , and  $t_r$  denotes the dynamic process time (system adjustment time).

The Formula (21) reflects the external relationship between the speed factor of ACPID controller and the dynamic process time of the controlled system (2). Its physical significance is that if the dynamic speed of the controlled system gets faster, it is required to have a greater value of the speed factor in ACPID controller, so that the ACPID controller can have great enough control force on the controlled system. Obviously, the ACPID control theory as a combination of the ACPID controller and the controlled system objectively reflects their physical interaction and physical relationship, hence having definite physical meaning.

Since altering the value of speed factor can alter the control force of ACPID controller, it indicates that ACPID control system is well-suited for regulating various types of dynamic speed objects.

Despite the complexity of the controlled objects, the saturation dynamic speed remains finite, and the  $z_c$  can be stabilized based on the dynamic process time. Hence the ACPID control theory has good universality and accords with the idea of unified control theory.

### 3.5. Adaptive Speed Factor Model

For a controlled system with fast dynamic speed, it is required that such system is controlled by ACPID controller with great value of the speed factor, and vice versa. In addition, based on *Theorem 1*, the greater value of the speed factor, the control accuracy of the steady state of the ACPID control system is higher, so is the anti-disturbance ability. Those indicate the greater the better on the speed factor. However, an overgreat speed factor might easily cause an overshoot phe-

nomenon in the ACPID control systems. To address the conflicts between speedability and overshoot, adaptive speed factor model is proposed as below:

$$z_c = z_{cm} \exp(-\beta |e_2|) \quad (22)$$

where,  $z_{cm} = 10\alpha/t_r$ ,  $1 < \alpha \leq 10$ , and  $\beta = \sqrt{\alpha}$ .

For slow slow-controlled system, the speed factor of the ACPID controller is relatively small, hence there is no need to apply an adaptive speed factor but it can take  $z_c = z_{cm}$  directly.

## 4. Simulation Experiments and Analysis

In order to verify the effectiveness of ACPID control theory, strict-feedback nonlinear systems and non-affine nonlinear systems are respectively taken as controlled objects.

### 4.1. Control Experiments of Strict Feedback Systems

A strict feedback system [47] is:

$$\begin{cases} \dot{x}_1 = f_1(x_1) + g_1(x_1)x_2 \\ \dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)u \\ y = x_1 \end{cases} \quad (23)$$

where,  $u$  and  $y$  represent the input and output,  $x_1$  and  $x_2$  represent two states,  $f_1(x_1) = x_1 e^{-0.5x_1}$ ,  $f_2(x_1, x_2) = x_1 x_2^2$ ,  $g_1(x_1) = 1 + x_1^2$ , and  $g_2(x_1, x_2) = 3 + \cos(x_1 x_2)$ .

For the purpose of facilitating comparison and analysis, we adopt the same expected trajectory and initial state as that in [52], *i.e.*  $r = \sin(t)$ ,  $x_1(0) = 0.5$ ,  $x_2(0) = -0.6$ .

Let  $y_1 = x_1$ ,  $y_2 = f_1(x_1) + g_1(x_1)x_2$ , the system (23) can be transformed into an equivalent linear disturbance system:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = w + b_0 u \\ y = y_1 = x_1 \end{cases} \quad (24)$$

where,  $w = \frac{\partial f_1}{\partial x_1} y_2 + \frac{\partial g_1}{\partial x_1} y_2 x_2 + g_1(f_2 + g_1 u) - b_0 u$ .

Since  $2(1 + x_1^2) \leq g_1 g_2 = (1 + x_1^2)(3 + \cos(x_1 x_2)) \leq 4(1 + x_1^2)$ ,  $0 \leq x_1^2 \leq 1$ , and  $-1 \leq r = \sin(t) \leq 1$ , hence  $2 \leq g_1 g_2 \leq 8$ , so let  $b_0 = 8$ . Set  $t_r = 2$  s,  $\alpha = 4$ , and  $\beta = \sqrt{\alpha} = 2$ , then  $z_{cm} = 10\alpha/t_r = 20$ , and  $z_c = 20 \exp(-2|e_2|)$ .

The following simulation experiments all utilize the ACPID controller with a consistent speed factor, as indicated below:

$$u = (z_c^3 e_0 + 3z_c^2 e_1 + 3z_c e_2) / b_0 \quad (25)$$

where,  $b_0 = 8$ ,  $z_c = 20 \exp(-2|e_2|)$ , and set  $u_m = 4$ .

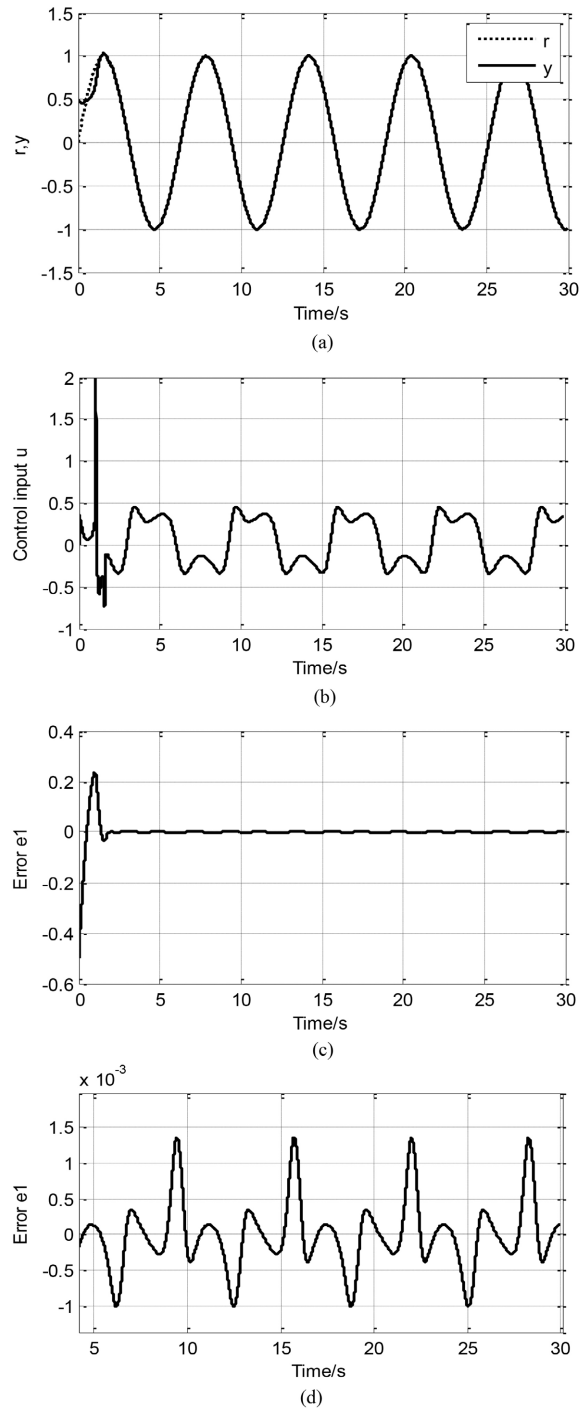
The controller parameters as reported in [47] are outlined below:

$$c_1 = 4, \quad c_2 = 6, \quad \beta_1 = \beta_2 = 15, \quad \gamma_1 = \gamma_2 = 5, \quad \varepsilon_{f_1} = \varepsilon_{f_2} = 0.01, \quad \varepsilon_{g_1} = \varepsilon_{g_2} = 0.05,$$

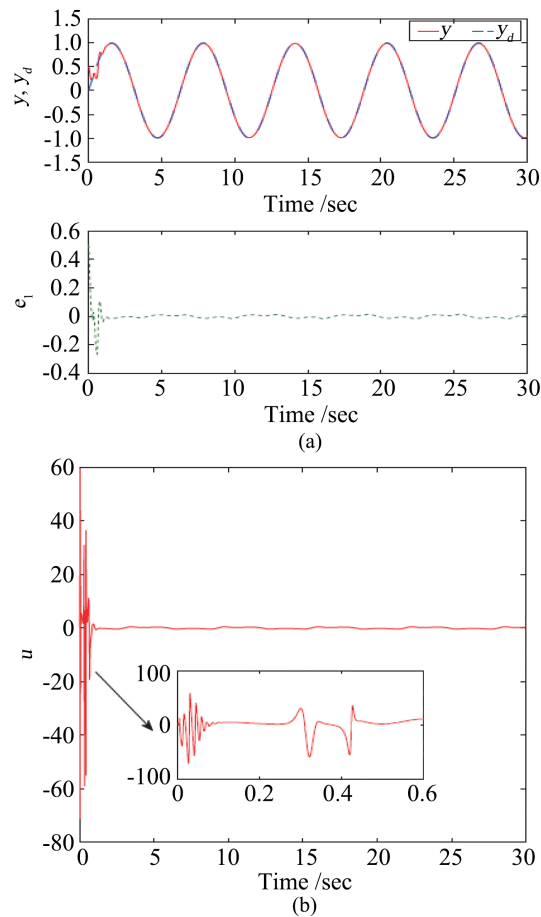
$\tau_1 = 0.005$ ,  $\zeta_0 = 0.02$ , totaling 12 parameters.

**Experiment 4.1** Sinusoidal tracking control

The sinusoidal tracking control results of the strict feedback system (23) utilizing an ACPID controller (25) are depicted in **Figure 2**, while the simulation outcomes from [47] are presented in **Figure 3**.



**Figure 2.** Control results using ACPID. (a) Tracking trajectory, (b) Control input, (c) Tracking error, (d) Steady state error.

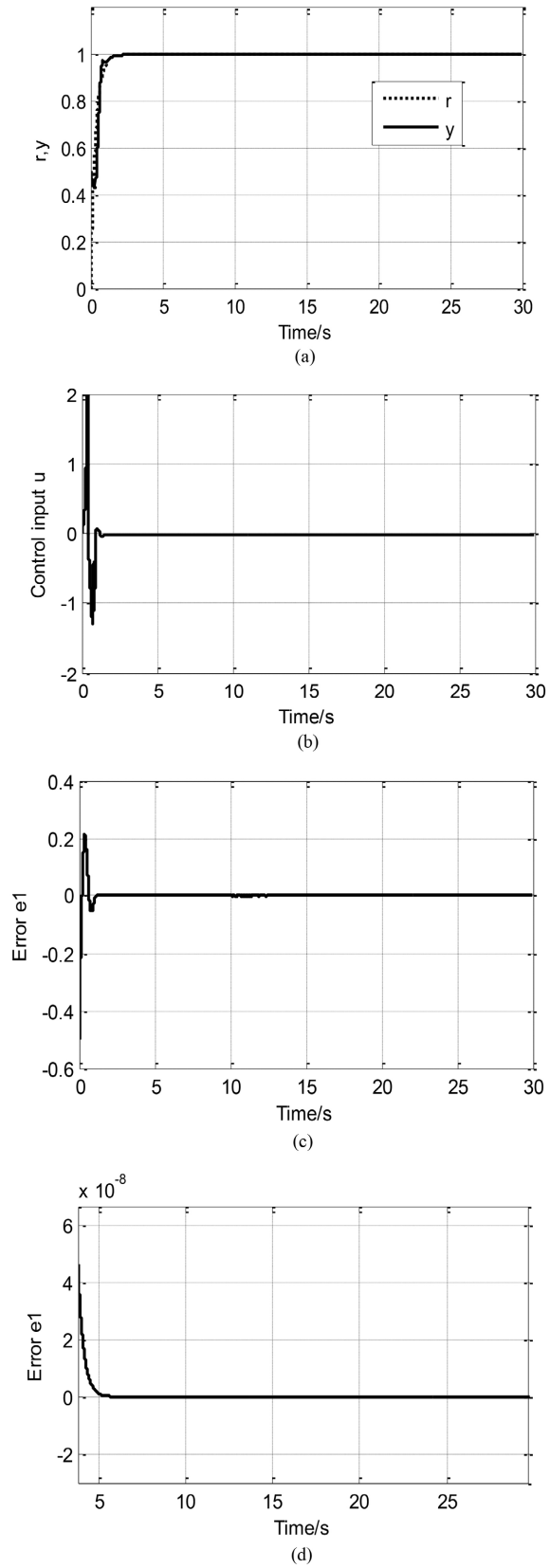


**Figure 3.** Control results in [47]. (a) Tracking trajectory and its error, (b) Control input.

**Figure 2** and **Figure 3** demonstrate that the ACPID control method achieves steady state within 1.5 seconds, with a maximum steady-state error less than  $1.5e-3$ , exhibiting comparable response speed and steady-state accuracy to the control method [47]. However, the maximum output amplitude of ACPID is less than 2, whereas literature [47] reports a maximum amplitude as high as 60. This indicates that the ACPID control method achieves the same level of control effect with only one-thirtieth of the control force required by literature [47], significantly reducing actuator load. In addition, the ACPID controller only requires the stabilization of a single speed factor, resulting in a simple and practical control system structure. However, literature [47] involves stabilizing 12 controller parameters, leading to a complex structure and large calculation requirements.

#### **Experiment 4.2.** Step tracking control simulation

The step tracking control results of the strict feedback system (23) utilizing ACPID controller (25) are depicted in **Figure 4**. As demonstrated by **Figure 4**, the ACPID control strategy can achieve steady state within 2 seconds, with a maximum steady-state error of less than  $4e-8$  and a maximum output amplitude of less than 2 after four seconds. Since there is no existing literature on step tracking experiments in [47], it cannot be compared or analyzed.



**Figure 4.** Control results using ACPID. (a) Tracking trajectory, (b) Control input, (c) Tracking error, (d) Steady state error.

## 4.2. Control Experiments of Non-Affine Nonlinear Systems

Consider a non-affine nonlinear system [15] [16] [52] as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x_1, x_2, u) + d \\ y = x_1 \end{cases} \quad (26)$$

where,  $u$  and  $y$  represent the input and output,  $x_1$  and  $x_2$  represent two states, and  $x_1(0) = 0.5$  and  $x_2(0) = 0.5$  denote the initial states of system,

$f(x_1, x_2, u, d) = x_1^2 + x_2^2 + 0.1(1 + x_2^2)e^u + 0.15u^3 + \sin(0.1u)$  represents the function model of the non-affine nonlinear system,  $d$  denotes an external bounded disturbance as:

$$d = \begin{cases} 1, & 14 \text{ s} \leq t \leq 15 \text{ s} \\ 0, & \text{else} \end{cases} \quad (27)$$

Let  $w = f(x_1, x_2, u) + d - b_0u$ , then, the system (26) can be transformed into an equivalent linear disturbance system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = w + b_0u \\ y = x_1 \end{cases} \quad (28)$$

Let  $b_0 = 2$ , the subsequent simulation experiments all employ the ACPID controller (25) with a consistent speed factor.

### **Experiment 4.3.** Sinusoidal tracking control

The sinusoidal tracking results for the system (26) using ACPID controller (25) are depicted in **Figure 5**. The ACPID control system is capable of achieving a steady state within 2 seconds, with a maximum steady-state error of less than  $0.6e-3$  and strong anti-disturbance ability, as demonstrated in **Figure 5**. Moreover, despite having distinct control coefficients and dynamic models, both system (22) and system (26) achieved satisfactory control performance by utilizing the ACPID controller with identical speed factors. It has been demonstrated that the ACPID controller exhibits strong universality across various dynamic systems with comparable or analogous dynamic speeds.

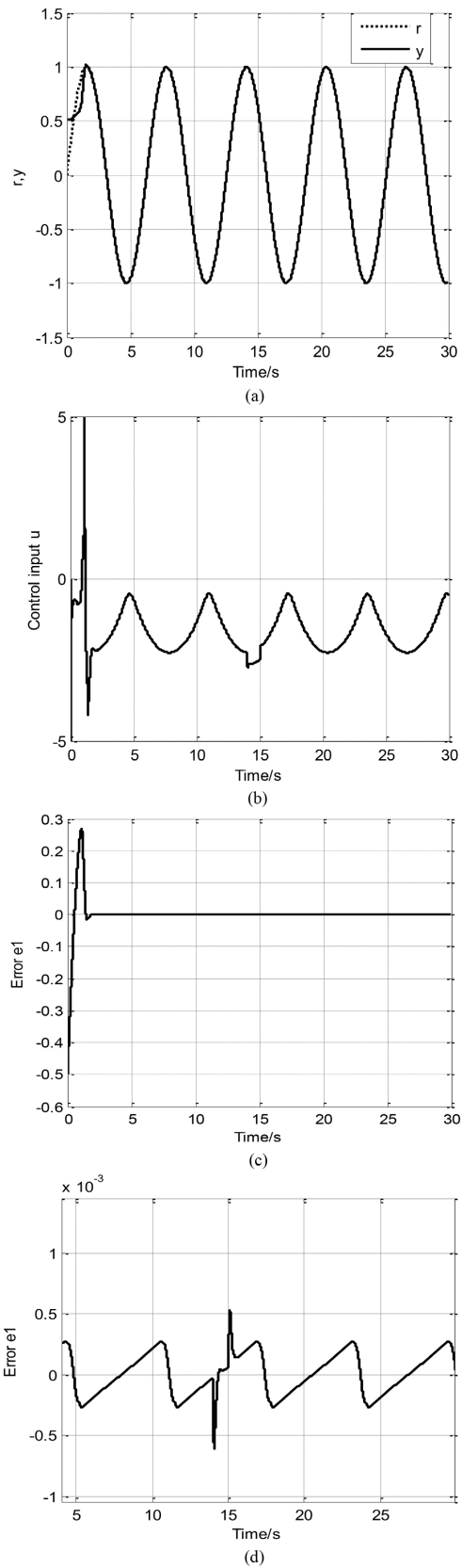
### **Experiment 4.4.** Step tracking control simulation

The step tracking control results of the ACPID controller (25) are depicted in **Figure 6**, and the control results in [15] are depicted in **Figure 7**.

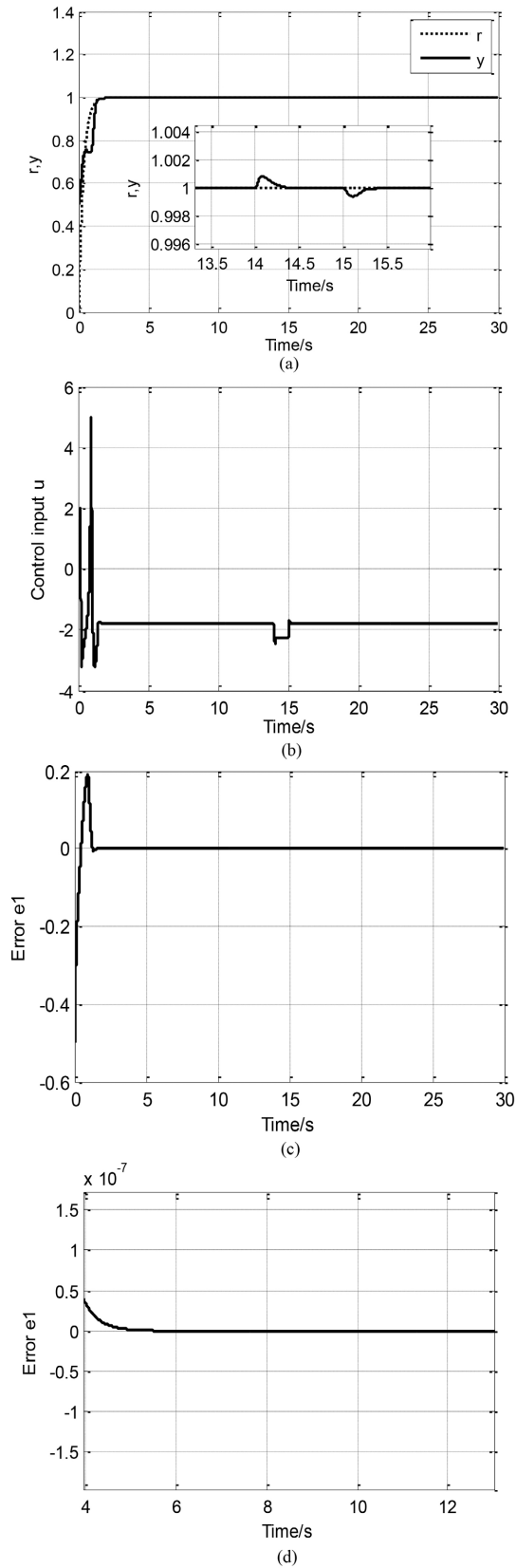
**Note:** SC-PID in [15] is ACPID in this paper, while PID, SMC and ADRC use online optimization methods to stabilize controller parameters.

It can be seen from **Figure 6** that the ACPID control system is capable of achieving steady state within 2 seconds with a maximum steady state error of less than  $0.5e-7$ . Furthermore, even when subjected to external disturbances, the system output can be restored to its original steady state within just 0.4 seconds, thus demonstrating remarkable anti-disturbance capabilities.

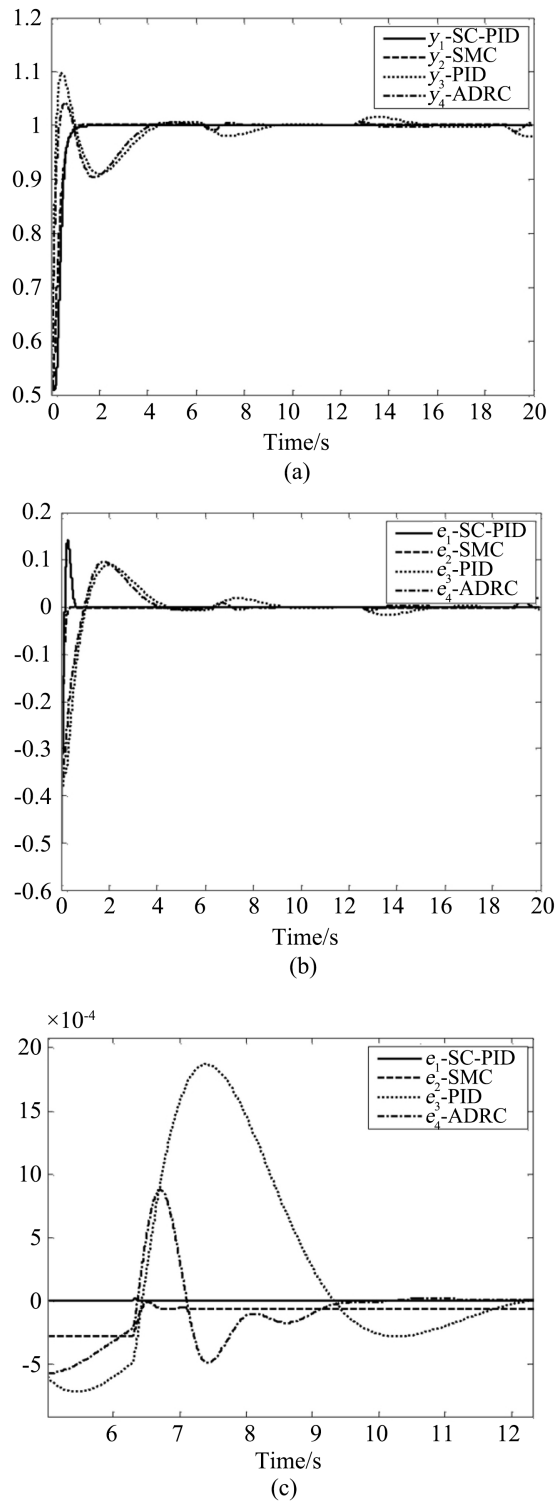
As can be seen from **Figure 7**, both SC-PID and online optimized ADRC in [15] enter the stable tracking state within 2 seconds, and both have strong



**Figure 5.** Control results using ACPID. (a) Tracking trajectory, (b) Control input, (c) Tracking error, (d) Steady state error.



**Figure 6.** Control results using ACPID. (a) Tracking trajectory, (b) Control input, (c) Tracking error, (d) Steady state error.



**Figure 7.** Control results in [15]. (a) Tracking trajectory, (b) Tracking error, (c) Steady state error

anti-disturbance ability, while the online optimized PID and SMC control methods in [15] take 6 seconds to enter the steady state, and both have poor anti-disturbance ability.

The simulation results above demonstrate that the ACPID controller, with a consistent speed factor, can achieve satisfactory control outcomes for two systems with distinct dynamic models. Therefore, the ACPID controller exhibits excellent versatility. Since the ACPID controller possesses a single speed factor for stabilization, it is capable of controlling various linear or nonlinear systems, known or unknown, with comparable dynamic speeds by adjusting the value of speed factor. This grants it unified control capabilities. Furthermore, as the speed factor solely pertains to the controlled object's dynamic speed rather than its model, implementing ACPID control on diverse complex nonlinear systems becomes convenient.

## 5. Conclusions

All the known or unknown complex factors such as the internal dynamics, uncertainty and external disturbance of all kinds of linear and complex nonlinear systems are defined as a total disturbance, so that diverse linear and complex nonlinear disturbance systems can be uniformly mapped to the same form of linear disturbance system, which establishes the fundamental conditions for realizing the unified control theory of ACPID. Its main features and innovations are as follows:

1) The stabilization problem of PID controller is scientifically solved by Zeng's stabilization rules based on a single speed factor. Its innovative point lies in defining the dimensional attribute of proportional gain according to the speed factor, establishing an internal relationship among PID gains, correcting theoretical defects of dimensionless proportional gain and mutually independent gains, and creating a development direction for control theory that follows the principle of dimensional.

2) The ACPID controller, which is based solely on the speed factor, requires stabilization of only one variable. As a result, the control system has a simple structure, low computational requirements and is highly practical.

3) The ACPID control system is not only a critical damping system but also an object-oriented system with dynamic speed rather than a dynamic model. Therefore, the robust stability and anti-disturbance robustness of the ACPID control system are theoretically guaranteed.

4) According to the anti-disturbance robustness analysis of the ACPID control system, it has been scientifically concluded that the upper bound of steady-state error is inversely proportional to the square of speed factor. Therefore, both steady-state accuracy and anti-disturbance ability can be precisely regulated in the ACPID control system.

5) The integration of ACPID control theory and total disturbance definition provides a theoretical basis for the realization of a universal ACPID control method, unifying control theory concepts across various linear and nonlinear complex systems.

The emergence of ACPID control theory enables the critical damping system

to achieve its ideal state and provides a unified method for control theory, which holds significant scientific and practical value in the field of control theory and control engineering.

## Acknowledgements

This work is supported by the Hunan Zhezha Automation Control Technology Co. Ltd.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

## References

- [1] Ziegler, J.G. and Nichols, N.B. (1942) Optimum Settings for Automatic Controllers. *Transaction of ASME*, **64**, 759-765. <https://doi.org/10.1115/1.4019264>
- [2] Ho, W.K., Lim, K.W. and Xu, W. (1998) Optimal Gain and Phase Margin Tuning for PID Controllers. *Automatica*, **34**, 1009-1014. [https://doi.org/10.1016/s0005-1098\(98\)00032-6](https://doi.org/10.1016/s0005-1098(98)00032-6)
- [3] Wang, Q., Zou, B., Lee, T. and Bi, Q. (1997) Auto-Tuning of Multivariable PID Controllers from Decentralized Relay Feedback. *Automatica*, **33**, 319-330. [https://doi.org/10.1016/s0005-1098\(96\)00177-x](https://doi.org/10.1016/s0005-1098(96)00177-x)
- [4] Shen, J., Xin, B., Cui, H. and Gao, W. (2017) Control of Single-Axis Rotation INS by Tracking Differentiator Based Fuzzy PID. *IEEE Transactions on Aerospace and Electronic Systems*, **53**, 2976-2986. <https://doi.org/10.1109/taes.2017.2722558>
- [5] Wu, X., Qin, G., Sun, M., Yu, H. and Xu, Q. (2017) Using Improved Particle Swarm Optimization to Tune PID Controllers in Cooperative Collision Avoidance Systems. *Frontiers of Information Technology & Electronic Engineering*, **18**, 1385-1395. <https://doi.org/10.1631/fitee.1601427>
- [6] Hanifah, R.A., Toha, S.F., Ahmad, S. and Hassan, M.K. (2018) Swarm-Intelligence Tuned Current Reduction for Power-Assisted Steering Control in Electric Vehicles. *IEEE Transactions on Industrial Electronics*, **65**, 7202-7210. <https://doi.org/10.1109/tie.2017.2784344>
- [7] Marin, A., Hernandez R., J.A. and Jimenez, J.A. (2018) Tuning Multivariable Optimal PID Controller for a Continuous Stirred Tank Reactor Using an Evolutionary Algorithm. *IEEE Latin America Transactions*, **16**, 422-427. <https://doi.org/10.1109/tla.2018.8327395>
- [8] Zeng, Z.Z., Xiao, Y.F. and Hao, Y.X. (2012) Nonlinear PID-Like Neuron Network Controller. *Journal of University of Science and Technology Beijing*, **34**, 12-15.
- [9] Li, G.M. and Zeng, Z.Z. (2010) A Nonlinear PID Controller Based on Neural Net-Work Algorithm. *Journal of Central South University*, **41**, 1865-1870.
- [10] Guo, C. (2018) Mold Level Control of Large Size Ingot Based on Fuzzy Self-Tuning PID. *Journal of Mechanical Engineering*, **54**, 212.
- [11] Dong, L.H. and Zhao, P.B. (2013) Grey Predictive PID Control of the Tensioning Device System in Belt Conveyor. *Journal of China Coal Society*, **38**, 342-347.
- [12] Zhao, J.S., Zhang, Y., Cao, X.M., et al. (2016) Multi-PID Controller Parameters Optimization of Fast Forging Hydraulic Press Based on GA. *Journal of Central South*

- University*, **47**, 2650-2660.
- [13] Chen, L.Q., Tan, Y.D., Wu, R., *et al.* (2017) Torque Distribution Control Strategy of Electronically Controlled Four-Wheel Drive Axle Based on Genetic Algorithm. *Journal of Agricultural Machinery*, **48**, 361-367.
- [14] Wang, J. and Kumbasar, T. (2020) Optimal PID Control of Spatial Inverted Pendulum with Big Bang-Big Crunch Optimization. *IEEE/CAA Journal of Automatica Sinica*, **7**, 822-832. <https://doi.org/10.1109/jas.2018.7511267>
- [15] Zeng, Z.Z. and Liu, W.J. (2021) Self-Coupling PID Controllers. *Acta Automatica Sinica*, **47**, 404-422.
- [16] Zeng, Z.Z. and Chen, Z.Y. (2020) On Control Theory of PID and Auto-Coupling PID. *Control Theory & Applications*, **37**, 2654-2662.
- [17] Zeng, Z.Z. (2022) Necessity of Correcting Dimensionless Proportional Gain and Its Improvement Strategy. *Control Engineering of China*, **29**, 1387-1394.
- [18] Zeng, Z.Z. (2023) ACPI Control Theory. *International Journal of Engineering and Science Invention*, **12**, 39-47.
- [19] Zeng, Z.Z. and Liu W.J. (2019) Wisdom PI Control for First-Order Systems with Time-Delay. *Control Theory & Applications*, **36**, 1322-1328.
- [20] Cheng T., Zeng, Z.Z., Du X.F., *et al.* (2019) Predictive Self-Coupling PI Control Method for Large Time-Delay Systems. *Information and Control*, **48**, 672-677.
- [21] Wang W., Zeng, Z.Z. and Zhang Z.H. (2023) Auto-Coupling PID Control Method for Delay Weakening of First Order Systems with Large Time Delays. *Control Engineering of China*.
- [22] Wang, F.Q., Zeng, Z.Z., Cheng, T., *et al.* (2021) Auto-Coupling PID Control Method for Strict Feedback Nonlinear Uncertain Systems. *Control Engineering of China*, **28**, 971-978.
- [23] Su, J. and Zeng, Z.Z. (2021) Maximum Power Point Tracking Based on Auto-Coupling PI Control for a Wind Power Generation System. *Power System Protection and Control*, **49**, 119-127.
- [24] Su, J. and Zeng, Z.Z. (2022) Auto-Coupling PID Control Method for Nonlinear Time Varying Systems. *Control Theory & Applications*, **39**, 299-306.
- [25] Bai, J.Z. and Zeng, Z.Z. (2022) States Adjust of Continuous Stirred Tank Reactor by Auto-Coupling Proportional-Integral-Differential Control. *Control Theory & Applications*, **39**, 1065-1070.
- [26] Chen, L.Q. and Zeng, Z.Z. (2022) Trajectory Tracking Control of Three-Link Manipulator Based on Auto-Coupling PID. *Aerospace Control and Application*, **48**, 70-77.
- [27] Xu, H.Q., Zeng, Z.Z. and Chen, Z.Y. (2022) An ACPI Control Method for High-Order Nonlinear Unknown System. *Electronics Optics & Control*, **29**, 42-46.
- [28] Cheng, Q.Z., Zeng, Z.Z., Cheng, T., *et al.* (2022) Auto-Coupling PD Cooperative Control Method for Unknown Nonlinear Systems. *Control Engineering of China*, **29**, 1437-1442.
- [29] Liu, W.J. and Zeng, Z.Z. (2022) Auto-Coupling PID Control of Non-Affine Pure-Feedback Nonlinear System. *Control Engineering of China*, **29**, 1264-1270.
- [30] Su, J., Zeng, Z.Z. and Xiong, Y.L. (2022) Auto-Coupling PID Cooperative Control Method for Synchronization of Uncertain Chaotic Systems. *Control Engineering of China*, **29**, 1464-1472.

- [31] Ren, S.R. and Zeng, Z.Z. (2022) Auto-Coupling PI Guidance Law with Precision Terminal Guidance. *Fire Control & Command Control*, **47**, 83-86.
- [32] Huang, D., Xu, H.Q., Zhuang, L., *et al.* (2022) ACPID Control Method of Unmatched Nonlinear Systems. *Journal of Electric Power Science and Technology*, **37**, 222-228.
- [33] Xiong, Y.L., Zeng, Z.Z. and Wang, W. (2023) Auto-Coupling PID Control Method for Nonlinear Underactuated Unstable Systems. *Aerospace Control and Application*, **49**, 82-89.
- [34] Zeng, P., Zeng, Z.Z. and Xiong, Y.L. (2023) Auto-Coupling PD Control of Chaotic Oscillation in a Seven-Dimensional Power System. *Power System Protection and Control*, **51**, 13-20.
- [35] Zeng, P. and Zeng, Z.Z. (2023) Auto-Coupling PID Control of Flexible Transmission System with Sandwich Structure. *Aerospace Control and Application*, **49**, 73-79.
- [36] Huang, L.R., Zeng, Z.Z. and Zeng, P. (2023) Auto-Coupling PID Control Method for Hydraulic Turbine Regulation System. *Journal of Hydroelectric Engineering*, **42**, 80-88.
- [37] Zhang, C.Q., Zeng, Z.Z. and Wang, W. (2024) Control Method of Magnetic Levitation Ball System Based on ACPI. *Journal of Railway Science and Engineering*, **21**, 1180-1188.
- [38] Zhang, Z.H., Zeng, Z.Z., Wang, W., *et al.* (2023) Auto-Coupling PID Anti-Swing Control Strategy for Underactuated Bridge Crane. *Mechanical Science and Technology for Aerospace Engineering*.
- [39] Huang, L.R., Zeng, Z.Z. and Zhang, Z.H. (2023) Research on Auto-Coupling PID Control Strategy of Permanent Magnet Synchronous Motor Governing System. *Mechanical Science and Technology for Aerospace Engineering*.
- [40] Lu, Y.X., Zeng, Z.Z., Tang, L.J., *et al.* (2023) Research on a Self-Coupling PID Control Strategy for Reactor Core Power. *Journal of Chinese Society of Power Engineering*, **43**, 1494-1499
- [41] Xiong, Y.L. and Zeng, Z.Z. (2024) Research on a Self-Coupling PID Control Strategy for an Underactuated TORA System. *Control and Decision*, **39**, 853-860.
- [42] Zeng, Z.Z. and Zhang, Z.H. (2023) Auto-Coupling PID Control Method for Underactuated VTOL Aircraft. *Aerospace Control and Application*, **49**, 38-46.
- [43] Zeng, P., Zeng, Z.Z. and Huang, L.R. (2024) Cascaded Self-Coupling PID Control of SCR Denitration System. *Journal of Chinese Society of Power Engineering*, **44**, 84-90.
- [44] Huang, L.R., Zeng, Z.Z. and Zeng, P. (2024) Auto-Coupling PD Attitude Control Method for Flexible Spacecraft. *Aerospace Control and Application*, **50**, 75-83.
- [45] Bechlioulis, C.P. and Rovithakis, G.A. (2009) Adaptive Control with Guaranteed Transient and Steady State Tracking Error Bounds for Strict Feedback Systems. *Automatica*, **45**, 532-538. <https://doi.org/10.1016/j.automatica.2008.08.012>
- [46] Huang, X., Song, Y. and Lai, J. (2019) Neuro-Adaptive Control with Given Performance Specifications for Strict Feedback Systems under Full-State Constraints. *IEEE Transactions on Neural Networks and Learning Systems*, **30**, 25-34. <https://doi.org/10.1109/tnnls.2018.2821668>
- [47] Wu, J.W., Liu, Y.H., Su, C.Y., *et al.* (2021) Adaptive Command Filtered Control of Strict Feedback Systems with Uncertain Control Gains. *Acta Automatica Sinica*, **50**, 1015-1023.

- 
- [48] Tong, S., Min, X. and Li, Y. (2020) Observer-Based Adaptive Fuzzy Tracking Control for Strict-Feedback Nonlinear Systems with Unknown Control Gain Functions. *IEEE Transactions on Cybernetics*, **50**, 3903-3913. <https://doi.org/10.1109/tcyb.2020.2977175>
- [49] Qiu, J., Sun, K., Wang, T. and Gao, H. (2019) Observer-Based Fuzzy Adaptive Event-Triggered Control for Pure-Feedback Nonlinear Systems with Prescribed Performance. *IEEE Transactions on Fuzzy Systems*, **27**, 2152-2162. <https://doi.org/10.1109/tfuzz.2019.2895560>
- [50] Fan, B., Yang, Q., Jagannathan, S. and Sun, Y. (2019) Output-Constrained Control of Nonaffine Multiagent Systems with Partially Unknown Control Directions. *IEEE Transactions on Automatic Control*, **64**, 3936-3942. <https://doi.org/10.1109/tac.2019.2892391>
- [51] Zhang, Y., Wen, C.Y. and Soh, Y.C. (2000) Discrete-Time Robust Backstepping Adaptive Control for Nonlinear Time-Varying Systems. *IEEE Transactions on Automatic Control*, **45**, 1749-1755. <https://doi.org/10.1109/9.880641>
- [52] Zeng, Z.Z., Wu, L.D., Yang, Z.Y., *et al.* (2016) Self-Learning Sliding-Mode Disturbance Rejection Control for Non-Affine Systems. *Control Theory & Applications*, **33**, 980-987.