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An Extended Numerical Method by Stancu Polynomials for Solution of Integro-Differential Equations Arising in Oscillating Magnetic Fields

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Abstract

In this study, the Bernstein collocation method has been expanded to Stancu collocation method for numerical solution of the charged particle motion for certain configurations of oscillating magnetic fields modelled by a class of linear integro-differential equations. As the method has been improved, the Stancu polynomials that are generalization of the Bernstein polynomials have been used. The method has been tested on a physical problem how the method can be applied. Moreover, numerical results of the method have been compared with the numerical results of the other methods to indicate the efficiency of the method.

Keywords

Stancu Polynomials, Collocation Method, Integro-Differential Equations, Linear Equation Systems, Matrix Equations

1. Introduction

Integro-differential equations are eligible equations in many science fields. Because many physical problems can be modelled by these equations in the engineering, mechanics, physics, chemistry, astronomy, biology, economics, potential theory and electrostatics, control theory and financial mathematics [1]-[4]. So the greater part of mathematicians are interested in solving these equtions numerically and analytically.

A class of linear integro-differential equations [2] which describes the charged particle motion for certain configurations of oscillating magnetic fields is very difficult to solve analitically. So the numerical solution of these equations is the most essential in the numerical analysis. Until now, great numbers of studies have been find on numerical methods for solving this physical problem modelled by linear Volterra integro-diffential equations of second order with time-periodic coefficients. These studies screened on the literature are the homotophy perturbation method [2], the Adomian decomposition method [5] [6], the variational iteration method [7], the mixed interpolation collocation method [8], the local polynomial regression method [9], the Legendre multi-wavelets method [10], Chebyshev wavelet technique [11], the thin plate spline collocation method [12], the frequency-domain approach [13], the Galerkin method with Shannon wavelet approximation [14], the local Galerkin integral equation method [15], the generalized fractional order Chebyshev orthogonal functions collocation method [16], the collocation method based on the local multiquadrics [17].

Since polynomials have suitable algebric properties such as countunity, derivability, integrability; the polynomials play very important role to develope the numerical methods for solutions of these equations. Particularly, the Bernstein, Chebyshev, Legendre, Jacobi and Laguerre polynomials are used to numerical methods. In all these polynomials, the Bernstein polynomials are most popular polynomials in terms of effectiveness and efficiency of the numerical methods. From past to present, some numerical methods such as collocation method [18]-[20], spectral collocation and Galerkin methods [21], operational matrix method [22]-[26], Adomian decomposition method [27] and homotopy perturbation method [28] for the solutions of different kinds of integro-differential equations have been produced by using the Bernstein polynomials.

The Stancu polynomials revealed by Dimitrie D. Stancu [29] are a generalization of the Bernstein polynomials. These polynomials are defined on the interval [0,1] as follows:

$$S_n(y;x)=\sum_{i=0}^{n} y\left(\frac{i+\alpha}{n+\beta}\right)\binom{n}{i}x^i(1-x)^{n-i} \tag{1}$$

where $0\le x\le 1$, $0\le\alpha\le\beta$. The Stancu polynomials are Bernstein polynomials for $\alpha=\beta=0$. For this reason, these polynomials have been called the Bernstein-Stancu polynomials by Altomare and Campiti [30]. Here

$$p_{i,n}(x)=\binom{n}{i}x^i(1-x)^{n-i};\ 0\le x\le 1$$

are called the basis polynomials of the Bernstein-type polynomials mentioned in [31] and these polynomials have an important matrix relation that is underlined this study as follows:

Theorem 1.1. There is a relation between the basis polynomials matrix and their derivatives in the form

$$\boldsymbol{P}^{(k)}(x)=\boldsymbol{P}(x)\boldsymbol{N}^k;\ k=0,1,\cdots,m.$$

Here $\boldsymbol{P}(x)=\left[p_{i,n}(x)\right]$, $\boldsymbol{P}^k(x)=\left[p_{i,n}^{(k)}(x)\right]$ are $1\times(n+1)$ matrices,

$\boldsymbol{N} = \left(d_{ij}\right)$ is $(n+1)\times(n+1)$ matrix such that the elements of N are defined by

$$d_{ij} = \begin{cases} n-i; & \text{if } j = i+1 \\ 2i-n; & \text{if } j = i \\ -i; & \text{if } j = i-1 \\ 0; & \text{otherwise} \end{cases}$$

for $i, j = 0,1,\cdots,n$ and $\boldsymbol{N}^0 = I$ is identity matrix [32].

The Stancu polynomials [33] also substantiate the Weierstrass Theorem [34]. This means that the Stancu polynomials converge to a continuous function at the interval [0,1]. Moreover, the Stancu polynomials have algebric properties like positivity, continuity, recursion's relation, differentiability, integrability over the interval [0,1] like the Bernstein polynomials. Furthermore, since the Stancu polynomials depend on the parameters α and β, these polynomials have better approximation than the Bernstein polynomials at the points $x = \frac{i+\alpha}{n+\beta}$ on the interval [0,1]. In the other words, a better approximation of a continuous function y at any x points on the interval [0,1] can be obtained by using the Stancu polynomials depended on the suitable selections of parameters α and β than the Bernstein polynomials [35]. For this reason, the Stancu polynomials approach can be more preferable than the Bernstein polynomials approach because of limited calculation to use the less number of terms.

Taking literature review into account, any Bernstein type polynomials have not been considered to numerical methods for solving the linear differential type equations apart from me. I studied on the collocation method for the numerical solution of the linear differential equation by the Stancu polynomials [36] lately. The study has indicated that the Stancu polynomials approach gives the effective numerical results. Starting from this, the aim of the present study is to probe the collocation method by considering the Stancu polynomials for the numerical solutions of a physical problem modelled by linear Volterra integro-differential equations of the second kind:

$$y''(x) + a(x)y(x) = g(x) + b(x)\int_0^x \cos(\omega_p t)\, y(t)\,\mathrm{d}t \tag{1.2}$$

Under the initial conditions

$$y(0) = \alpha,\; y'(0) = \beta \tag{1.3}$$

where $a(x)$, $b(x)$ and $g(x)$ are given periodic functions of time. These functions may be easily determined in the charged particle dynamics for some field configurations. α and β are real constants and $y(x)$ is an unknown function to be designated. Description of the equation also see [2] [7] in detail.

A brief summary of this paper is as follows: In Section 2, the collocation method with the Stancu polynomials appoach has been presented theoretically. In Section 3, the applicability of the method has been indicated on a physical problem of the charged particle motion for certain configurations of oscillating magnetic fields under the different conditions modelled by linear Volterra integro-differential equations. Moreover, the numerical results have been presented in tabular form to show the approximation rate of the method. Likewise, the numerical results have been compared with the numerical results of the other numerical methods to probe the whether the proposed method converges better than the other methods or not. In Section 4, some inferences have been made about method's advantages and some advises have been given for the future studies.

2. Presentation of the Collocation Method

Theorem 2.1. Let $x_i = \dfrac{i+\alpha}{n+\beta} \in [0,1]$ be collocation points. By means of the Stancu polynomials, linear Volterra integro-differential equation of second kind (1.1) can be modified the following linear matrix equation:

$$\left[\mathbf{P}\mathbf{N}^2 + \mathbf{A}\mathbf{P} - \mathbf{B}\mathbf{V}\right]\mathbf{Y} = \mathbf{G}. \tag{1.4}$$

Here $\mathbf{A} = \text{diag}\left[a(x_i)\right]$, $\mathbf{V} = \left[\mathbf{V}(x_i)\right]$, $\mathbf{P} = \left[\mathbf{P}(x_i)\right]$ are $(n+1)\times(n+1)$ matrices, $\mathbf{Y} = \left[y\left(\dfrac{i+\alpha}{n+\beta}\right)\right]$, $\mathbf{B} = \left[b(x_i)\right]$ and $\mathbf{G} = \left[g(x_i)\right]$ are $(n+1)\times 1$ matrices for $i = 0,1,\cdots,n$.

Proof. Having regard to Theorem (1.1), the Stancu polynomials (1.1) and collocation points $x_i = \dfrac{i+\alpha}{n+\beta}$, unknown function and its first and second derivatives can be written matrix form as follows:

$$y^{(k)}(x_i) \simeq S_n^{(k)}(y;x_i) = \mathbf{P}(x_i)\mathbf{N}^k\mathbf{Y};\ k = 0,1,2. \tag{1.5}$$

Here the collocation points are dependent on selections of the α and β values.

Replacing the matrix equation (1.5) into the main equation (1.2), the following algebraic equation system is attained:

$$\mathbf{P}(x_i)\mathbf{N}^2\mathbf{Y} + a(x_i)\mathbf{P}(x_i)\mathbf{Y} = g(x_i) + b(x_i)\int_0^{x_i} v(x_i,t)\mathbf{P}(t)\mathbf{Y}\mathrm{d}t. \tag{1.6}$$

By considering the following matrix forms into equation (1.6)

$$\mathbf{A} = \begin{bmatrix} a(x_0) & 0 & \cdots & 0 \\ 0 & a(x_1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a(x_n) \end{bmatrix}, \mathbf{B} = \begin{bmatrix} b(x_0) \\ b(x_1) \\ \vdots \\ b(x_n) \end{bmatrix}, \mathbf{G} = \begin{bmatrix} g(x_0) \\ g(x_1) \\ \vdots \\ g(x_n) \end{bmatrix},$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}(x_0) \\ \mathbf{P}(x_1) \\ \vdots \\ \mathbf{P}(x_n) \end{bmatrix} = \begin{bmatrix} p_{0,n}(x_0) & p_{1,n}(x_0) & \cdots & p_{n,n}(x_0) \\ p_{0,n}(x_1) & p_{1,n}(x_1) & \cdots & p_{n,n}(x_1) \\ \vdots & \vdots & \ddots & \vdots \\ p_{0,n}(x_n) & p_{1,n}(x_n) & \cdots & p_{n,n}(x_n) \end{bmatrix} = \left[p_{i,n}(x_i)\right],$$

$$\mathbf{V}=\begin{bmatrix}\mathbf{V}(x_0)\\ \mathbf{V}(x_1)\\ \vdots\\ \mathbf{V}(x_n)\end{bmatrix}=\begin{bmatrix}\int_0^{x_0} v(x_0,t)p_{0,n}(t)\mathrm{d}t & \int_0^{x_0} v(x_0,t)p_{1,n}(t)\mathrm{d}t & \cdots & \int_0^{x_0} v(x_0,t)p_{n,n}(t)\mathrm{d}t\\ \int_0^{x_1} v(x_1,t)p_{0,n}(t)\mathrm{d}t & \int_0^{x_1} v(x_1,t)p_{1,n}(t)\mathrm{d}t & \cdots & \int_0^{x_1} v(x_1,t)p_{n,n}(t)\mathrm{d}t\\ \vdots & \vdots & \ddots & \vdots\\ \int_0^{x_n} v(x_n,t)p_{0,n}(t)\mathrm{d}t & \int_0^{x_n} v(x_n,t)p_{1,n}(t)\mathrm{d}t & \cdots & \int_0^{x_n} v(x_n,t)p_{n,n}(t)\mathrm{d}t\end{bmatrix},$$

The desired matrix equation (1.3) is obtained and this is completed the proof. Moreover, the matrix equation (1.4) and initial conditions (1.3) can be restated the following matrix forms:

$$\mathbf{WY}=\mathbf{G} \text{ or } [\mathbf{W};\mathbf{G}];\ \ \mathbf{W}=\mathbf{PN}^2+\mathbf{AP}-\mathbf{BV}, \tag{1.7}$$

$$\mathbf{P}(0)\mathbf{Y}=\alpha \text{ and } \mathbf{P}(0)\mathbf{NY}=\beta. \tag{1.8}$$

The matrix equation (1.7) remarks a linear algebraic system including unknown coefficients $y_0,y_1,\cdots,y_n$. In order to solve the matrix equation system (1.7) under the matrix form of the initial conditions (1.8), we can use the technique of adding or technique of displacement. As the additive technique is used , the elements of the row matrices (1.8) are added to the end of the matrix (1.7). Then an augmented matrix $[\tilde{\mathbf{W}};\tilde{\mathbf{G}}]$ is attained. The dimension of this matrix is $(n+3)\times(n+1)$. On the other hand, as the displacement technique is used, whatever rows of the augmented matrix (1.7) are displaced with the rows of the matrix (1.8). Then a square matrix $[\mathbf{W}^*;\mathbf{G}^*]$ is attained. In order that the unknown coefficients $y_i; i=0,1,\cdots,n$ of the system can be determinated uniquely, the condition $rank(\tilde{\mathbf{W}})=rank[\tilde{\mathbf{W}};\tilde{\mathbf{G}}]=n+1$ should be provided.

3. Applications of a Physical Problem

In this section, a physical problem of the charged particle motion for certain configurations of oscillating magnetic fields given for four states have been considered in order to show that the Stancu collocation method can be applied to the each problem. The numerical results of every problems have been given on different collocation points in according to values of α and β. In addition, the numerical results were calculated using the MATLAB 7.1 program. Moreover, comparisons of the numerical results with the other methods have been presented as the tables to see how much the Stancu collocation method is useful and effective.

Definition 3.1. Let $y(x)$ is an exact solution and $S_n(y;x)$ is a Stancu approximate solution of the equation (1.2). The absolute and maximum errors can be expressed on the collocation points by the following relations:

$$e=\left|y(x_i)-S_n(y;x_i)\right|,\ \ E_{\max}=\frac{1}{n}\sum_{i=1}^{n}\left|y(x_i)-S_n(y;x_i)\right|.$$

Example 3.1. Consider the equation (1.2) with

$$\alpha = 1,\ \beta = 0,\ \omega_p = 2,\ a(x) = \cos x,\ b(x) = \sin\left(\frac{x}{2}\right),$$

$$g(x) = \cos x - x\sin x + (\cos x)(x\sin x + \cos x) - \sin\left(\frac{x}{2}\right)\left(\frac{2}{9}\sin(3x) - \frac{x}{6}\cos(3x) + \frac{x}{2}\cos(x)\right)$$

where $y(x) = x\sin(x) + \cos(x)$ is an exact solution of the equation.

Table 1. Comparison of the $E_{\max}$ errors for Example 3.1.

	Stancu Collocation Method					Homotopy Perturbation Method [2]	Galerkin Method [14]
n	$\alpha = 0.25$ $\beta = 0.25$	$\alpha = 0$ $\beta = 0$	$\alpha = 10^{-4}$ $\beta = 0.75$	$\alpha = 0$ $\beta = 1$	$\alpha = 0$ $\beta = 2$		
3	5.3e−002	3.8e−002	2.3e−002	1.9e−002	9.8e−003	1.3e−005	2.6e−008
4	5.1e−002	1.2e−003	4.3e−004	3.2e−004	1.2e−004	1.2e−007	3.4e−011
5	4.1e−002	1.7e−004	6.5e−005	6.2e−005	2.5e−005	7.7e−010	-
6	3.4e−002	4.6e−006	9.9e−006	1.4e−006	5.0e−007	3.4e−012	-
7	3.0e−002	4.0e−007	1.1e−005	1.5e−007	5.9e−008	-	1.6e−012
8	2.6e−002	9.0e−009	9.3e−006	2.9e−009	1.0e−009	-	5.2e−015
10	2.1e−002	1.3e−011	7.7e−006	4.2e−012	1.5e−012	-	-
14	1.5e−002	6.9e−015	5.7e−006	2.7e−015	2.6e−015	-	-

In Table 1, the numerical results of the proposed method have been compared with the numerical results of the Homotopy perturbation method [2] and the Galerkin method [14]. The numerical results of the proposed method have been given for different values of α and β. Privately, the proposed method has been modified the Bernstein collocation method for $\alpha = \beta = 0$. Moreover, the numerical results of maximum errors have also been calculated with additive technique. Table 1 indicates that the numerical values of the proposed method converge zero more and more values of n. As to Table 1, the best approximation has been attined for $\alpha = 0$ and $\beta = 2$ and $n = 14$. Let remark that the best approximation belongs to Stancu collocation method. It also means that the numerical results calculated on the points $x_i = \frac{i}{n+2}$; $n = 0,1,\cdots,n$ are better than the numerical results calculated on the points $x_i = \frac{i}{n}$; $n = 0,1,\cdots,n$.

Example 3.2. Secondly, consider the Equation (1.2) with

$$\alpha = 2,\ \beta = -5,\ \omega_p = 3,\ a(x) = 1,\ b(x) = \sin x + \cos x,$$

$$g(x) = -x^3 + x^2 - 11x + 4 - (\sin x + \cos x)\left(-\frac{x^3}{3}\sin 3x - \frac{x^2}{3}\cos 3x - \frac{13}{27}\cos 3x - \frac{13}{9}x\sin 3x + \frac{x^2}{3}\sin 3x + \frac{16}{27}\sin 3x + \frac{2}{9}x\cos 3x + \frac{13}{27}\right)$$

where $y(x) = -x^3 + x^2 - 5x + 2$ is an exact solution of the equation.

Table 2. Comparison of the E_{max} errors for Example 3.2.

Stancu Collocation Method				Bernstein Collocation Method [20]	Homotopy Perturbation Method [2]
n	$\alpha=0.0001$ $\beta=0.0001$	$\alpha=0$ $\beta=1$	$\alpha=0$ $\beta=2$	$\alpha=0$ $\beta=0$	
3	1.6e−004	3.1e−016	1.9e−016	1.1e−016	3.2e−005
4	1.2e−004	2.2e−016	3.1e−016	0	3.9e−007
5	1.0e−004	4.4e−016	4.4e−016	4.4e−016	3.1e−009
6	8.3e−005	8.9e−016	0	6.7e−016	1.8e−011

Table 3. Comparison of the e errors for Example 3.1.

Stancu Collocation Method					Chebyshev Wavelet Method [11]	LPR Method [9]	GFCFs Collocation Method [16]
$n=5$	$\alpha=0.0001$ $\beta=0.0001$	$\alpha=0$ $\beta=0$	$\alpha=0$ $\beta=1$	$\alpha=0$ $\beta=2$	$n=6$	$n=30$	$n=5$
0	1.0e−004	0	0	0	3.8e−014	3.3e−005	0
0.1	9.5e−005	2.5e−016	3.4e−016	0	4.9e−015	1.0e−004	0
0.2	9.0e−005	3.2e−017	3.0e−016	0	3.2e−014	-	0
0.3	8.4e−005	6.3e−016	2.1e−017	0	6.0e−015	1.6e−004	0
0.4	7.8e−005	8.7e−016	3.8e−017	0	2.9e−014	-	0
0.5	7.1e−005	8.3e−016	1.1e−016	0	4.8e−015	1.9e−005	0
0.6	6.3e−005	7.8e−016	1.1e−016	0	3.1e−014	-	0
0.7	5.6e−005	4.4e−016	2.2e−016	0	5.5e−015	4.3e−005	0
0.8	4.8e−005	8.9e−016	4.4e−016	0	5.8e−015	-	0
0.9	4.0e−005	0	0	0	6.1e−015	3.6e−005	0

In Table 2 and Table 3, the maximum and absolute errors of the proposed method have been presented for the different values of $\alpha=\beta$ and $\alpha<\beta$. Moreover, the numerical results of the errors have been worked out additive technique. In Table 2, the numerical results of the maximum errors have been compared with the numerical results of the Bernstein collocation method [20] and the Homotopy perturbation method [2]. The numerical results of the extended method are better than the others for $\alpha=0$, $\beta=2$ and increasing values of n. This means that numerical results calculated on the points $x_i=\frac{i}{n+2}$; $n=0,1,\cdots,n$ are more effective than the numerical results calculated on the points $x_i=\frac{i}{n}$; $n=0,1,\cdots,n$. In Table 3, the numerical results of the absolute errors have been compared with the Chebyshev wavelet collocation method [11], Local polynomial regression method [9] the generalized fractional order of the Chebyshev orthogonal functions collocation method [16] on the tiered points. As to Table 3, the extended method has the most effective results in the others for $\alpha = 0$, $\beta = 2$ and $n = 5$. Considering both of the Tables, the numerical values of the proposed method get better for $\alpha < \beta$. Besides, when the exact solution of the equation is polynomial function, the best numerical results of the extended method are attained.

Example 3.3. Thereafter, consider the Equation (1.2) with

$$\alpha = 0,\ \beta = 0,\ \omega_p = 2,\ a(x) = 1 + \cos x,\ b(x) = \sin\left(\frac{x}{2}\right),$$

$$g(x) = 2 + \cosh x + (1 + \cos x)\left(x^2 + \cosh x - 1\right)$$
$$-\frac{\sin\left(\frac{x}{2}\right)e^{-t}}{10}\left(10x^2 e^x \cos x \sin x + 2e^{2x}\cos^2 x - 2\cos^2 x + 10xe^x \cos^2 x\right.$$
$$\left. + 4e^{2x}\cos x \sin x + 4\cos x \sin x - 15e^x \cos x \sin x - e^{2x} - 5xe^x + 1\right)$$

where $y(x) = x^2 + \cosh x - 1$ is an exact solution of the equation.

Table 4. Comparison of the $E_{\max}$ errors for Example 3.1.

	Stancu Collocation Method				Spline Collocation Method [12]
n	$\alpha = 0.005$ $\beta = 0.005$	$\alpha = 0$ $\beta = 0$	$\alpha = 0$ $\beta = 1$	$\alpha = 0$ $\beta = 2$	
5	2.1e−003	3.9e−005	1.4e−005	5.5e−006	-
10	1.1e−003	1.1e−012	3.5e−013	1.3e−013	9.5e−005
15	7.2e−004	7.1e−015	3.3e−015	1.1e−015	-
20	5.4e−004	3.4e−014	2.7e−014	6.7e−015	1.7e−005

In Table 4, the numerical results of the extended method have been compared with the numerical results of the Spline collocation method [12]. The numerical values of the maximum errors converge to zero rapidly for $\alpha < \beta$ and increasing values of n. Moreover, numerical results of the proposed method are more better than the numerical results of the other method.

Example 3.4. Finally, consider the Equation (1.2) with

$$\alpha = \sqrt{2},\ \beta = \frac{\sqrt{2}}{4},\ \omega_p = 2,\ a(x) = \mathrm{In}\left(\pi + \frac{3}{1 + \cos^2(2t)}\right),$$

$$b(x) = \cos(x)\sin^2\left(\frac{x}{2}\right)$$

where $y(x) = \sqrt{2 + \sin x}$ is an exact solution of the equation.

Table 5. Comparison of the $E_{\max}$ errors for Example 3.1.

n	Stancu Collocation Method				Local Multiquadrics Collocation Method [17]
	$\alpha = 0.0025$ $\beta = 0.0025$	$\alpha = 0$ $\beta = 0$	$\alpha = 0$ $\beta = 1$	$\alpha = 0$ $\beta = 2$	
5	1.8e−004	3.8e−006	1.2e−006	5.2e−007	4.8e−003
9	9.8e−005	1.6e−006	6.9e−010	3.1e−010	2.5e−003
17	5.2e−005	2.2e−015	1.9e−014	4.3e−015	4.3e−004

In Table 5, the numerical results of the extended method are compared with those of the local multiquadrics collocation method [17]. Considering the numerical results of the maximum errors the extended method is more effective then the other method for different values of n. Moreover, the numerical results of the proposed method converge to zero rapidly for $\alpha < \beta$ and increasing values of n. Furthermore, the numerical values of the extended method are the best for $\alpha = 0$ and $\beta = 2$ in the values of the Table 5.

4. Conclusions and Discussions

In this study, the Berntein collocation method has been extended to Stancu collocation method in terms of the Stancu polynomials that are generalization of the Bernstein polynomials. The theory of the method has been placed on the linear integro-differential equations that describe the charged particle motion for certain configurations of oscillating magnetic fields. Then the extended method has been applied to four numerical examples of the Equation (1.2) under the initial conditions (1.3). The collocation points of the method have been selected depending on the values of α and β for the numerical results. Moreover, in order to compute the numerical results, the additive technique has been used. Then, the obtained numerical results of the method have been presented tabular. In tables, the numerical absolute and maximum errors of the proposed method have been compared with the numerical absolute and maximum errors of the other methods.

In all the study, we can deduce a lot of significant positive inferences: The theory of the extended method is easy comprehensible and applicable to a physical problem of the charged particle motion for certain configurations of oscillating magnetic fields modelled by linear integro-differential equations. The collocation points of the method are more general than the collocation points $x_i = \frac{i}{n}$, because of the $x_i = \frac{i+\alpha}{n+\beta}$; $0 \le \alpha \le \beta$. That is to say, the extended method works with by far collocation points instead of the collocation points $x_i = \frac{i}{n}$. Thereby, the method has been provide an opportunity for the comparisons of the numerical results with the Bernstein collocation method. The numerical values of the method converge to zero rapidly for $\alpha < \beta$ and increasing values of n. In other words, the Stancu collocation method has better approximation with relevant selection of the parameters α and β at any collocation points x on the interval [0,1] than the Bernstein collocation method. This means that the Stancu polynomials approach supplies to limit calculation to less number of terms. Moreover, the method has the best numerical results for $\alpha = 0$, $\beta = 2$. Besides, the method can be used easily for finding the numerical solution of linear Volterra integro-differential equations, when point 1 is not included by the collocation points depended on the values of α and β. Finally, the numerical results computed with the additive technique are better and more consistent than the numerical results computed with the displacement technique for increasing values of n.

Taking all the above inferences into account, the Stancu collocation method in the general form of the Berntein collocation method can be applied for numerical solution of any linear differential type equations by modelling physical and engineering problems for the future studies. Moreover, this study leads to new studies on collocation methods introduced in terms of the Bernstein-type polynomials such as q-Bernstein polynomials [37], Stancu-Cholodowsky polynomials [38]. Likewise, new numerical methods can be tested on the physical problems modelled by any linear integro-differential equations. Furthermore, the stability of the Stancu collocation method can be studied for different problem configurations.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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