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A Mathematical Exploration of the Correlation between the Speeds of Light in Adjacent Universes

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Abstract

From the mass-energy equivalence, the energy of a mass moving with velocity V is given by the zeroth component of its four-momentum. In this study, we re-examine this expression and demonstrate that its mathematical formalism admits an additional speed parameter, c_2, determined by the experimental speed of light c_1 and the object's velocity V. Under a set of speculative assumptions—specifically, that energy remains invariant for a mass transitioning between hypothetical "adjacent" universes—the presence of a second mathematical root c_2 suggests the existence of a potential alternative state. In this paper, we explore the speculative possibility that this root describes the physical constants of a coupled adjacent universe. This study explores the mathematical curiosity that the energy of a mass at velocity V is identical whether the invariant speed of the system is c_1 or c_2. Our framework indicates that for low values of V, c_2 is less than c_1, with the relationship reversing beyond a certain critical velocity. If this mathematical duality corresponds to a physical reality, the electromagnetic properties of such an adjacent universe, including vacuum permittivity (ε_{02}) and permeability (μ_{02}), would be constrained by the relation $\varepsilon_{02}\mu_{02} = 1/c_2^2$. This exploration is presented as a theoretical inquiry into the implications of the algebraic structure of relativistic energy-momentum relations.

Keywords

Mass-Energy Equivalence, Vacuum Constants, Adjacent Universes, Mathematical Formalism

1. Introduction

In 1864, Maxwell demonstrated that light is an electromagnetic wave, calculating its speed as $c_1 = 1/\sqrt{\varepsilon_0 \mu_0}$ [1]. According to Maxwell's equations, the speed of light is a vacuum constant, invariant for all observers regardless of their motion [2] [3]. This principle of invariance leads to the Lorentz transformation, where time and space vary between inertial frames to preserve the constancy of c_1. In this four-dimensional framework, physical quantities such as energy and momentum are unified into the four-momentum $P^\mu = mU^\mu$, where the inner product $P^\mu P_\mu = m^2c^2$ remains a Lorentz invariant. While the standard interpretation of special relativity focuses on the physical speed c_1, the algebraic structure of the energy-momentum relation invites further mathematical scrutiny. In this paper, we explore a formal curiosity arising from the rearrangement of the energy component of the four-momentum. By treating the energy relation as a cubic equation, we find that under specific conditions, the system yields three real roots [4]. While the first positive root corresponds to the known invariant speed c_1 in our universe, the existence of a second positive root, c_2, presents an intriguing mathematical possibility. Rather than asserting a new physical reality, this study presents a speculative mathematical exploration of this second root. We investigate the hypothesis that c_2 could characterize the invariant speed of a hypothetical "adjacent" universe, provided that energy remains invariant for a mass transitioning between such domains. Under this framework, we analyze how a variation in the invariant speed would fundamentally alter the physical constants—such as vacuum permittivity (ε_0), permeability (μ_0)—thereby dictating a distinct evolution for the atomic and chemical structures within that domain. This inquiry seeks to determine whether such a second root represents a mere mathematical artifact or a consistent, albeit speculative, extension of relativistic kinematics.

2. Theoretical Framework

2.1. The Five Critical Hypotheses Defining the Physical Nature of the System

To explore the physical significance of c_2, we propose the following set of speculative hypotheses.

1) Existence of Multiple Universes: Multiple universes exist and are positioned in an "adjacent" manner, which are defined as two parallel 3-branes separated by a finite distance in a higher-dimensional bulk [5].

2) Universal Invariance of c_n: Each universe possesses its own distinct invariant speed of light, and the principle of the constancy of light speed holds within each respective universe.

3) Mass Transfer: Defined as the transition of particles across this bulk, where the conservation laws are extended to include both manifolds.

4) Reference Frame: The velocity V is treated relative to the static background of the embedding bulk to avoid ambiguity between the two universes' respective inertial frames.

5) Energy-Based Admissibility for Cross-Universe Transition: A mass moving at velocity V in one universe can transition to an adjacent universe provided that

its total energy remains invariant across the boundary.

Based on these hypotheses, the Modeling Scenario and Physical Setup are defined as follows.

i) Modeling Scenario

We assume a situation where a particle existing in a universe A (3-brane A) transitions to an adjacent universe B (3-brane B) via tunneling or higher-dimensional displacement through the bulk, without any external energy injection. During this transition, the particle moves while maintaining its velocity relative to the stationary background of the bulk.

ii) Boundary Conditions

Energy Continuity: At the boundary (the point of contact between the bulk and the universe), the total energy function of particles must not be discontinuous.

Mass Adaptation: If the speed of light in universe A differs from the speed of light in universe B, the rest mass of a particle is instantly (or during a transition process) redefined from m_A to m_B to satisfy energy conservation.

iii) Constraints to Prevent Model Failure

Causality Constraint: After transitioning to universe B, velocity V must not exceed the speed of light c_B in that universe ($V < c_B$).

Bulk Isotropy: Within the high-dimensional bulk, additional drag forces (resistance) and potential gradients associated with movement from universe A to universe B are considered negligible.

2.2. Derivation of the Other Speed of Light

Based on Hypothesis 5, assuming that an object moving at velocity V in one universe (our universe) can transition to an adjacent universe (Hypothesis 1) while conserving energy, the speed of light in that adjacent universe is derived as follows.

The mass-energy equivalence $E = mc^2$ is also expressed as

$$E = \frac{m_0}{\sqrt{1-\left(\frac{V}{c}\right)^2}} c^2, \tag{1}$$

where m_0 is the rest mass of the object, and V its speed; $V < c$. According to Hypotheses 2, 4, and 5, Equation (1) holds true both in our universe and in the adjacent universe.

As $0 < V/c < 1$, V/c is defined by $\sin\theta \left(0 < \theta < \pi/2\right)$.

From (1),

$$E = \frac{m_0 V^2}{\cos\theta - \cos^3\theta}. \tag{2}$$

Then, $\cos\theta$ is represented as y:

$$c = \frac{V}{\sqrt{1-y^2}} \tag{3}$$

Representing $m_0 V^2$ as P_0, the following third-order equation for y is ob-

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tained from (2):

$$f(y) = y^3 - y + \frac{P_0}{E} = 0. \tag{4}$$

A positive value of E results from any value of θ on the interval $0 < \theta < \pi/2$. In other words, at least one of the three roots of $f(y)$ is real. The minimum value of $g(y) = y^3 - y = y(y+1)(y-1)$; $0 < y < 1$ is $-2/(3\sqrt{3}) \approx -0.384$ at $y = 1/\sqrt{3}$. As the roots of $f(y)$ are intersections of $g(y)$ and $h(y) = -P_0/E$, all three roots are real when $-2/(3\sqrt{3}) < -P_0/E < 0$ (**Figure 1**). Two of them are between 0 and 1 and the remaining root is negative. The two roots between 0 and 1 are denoted as y_1 and y_2. As $0 < y < 1$, the negative root y_3 is neglected. When $-P_0/E = -2/(3\sqrt{3}) \approx -0.384$, $y_1 = y_2 \left(= 1/\sqrt{3} \approx 0.5774\right)$.

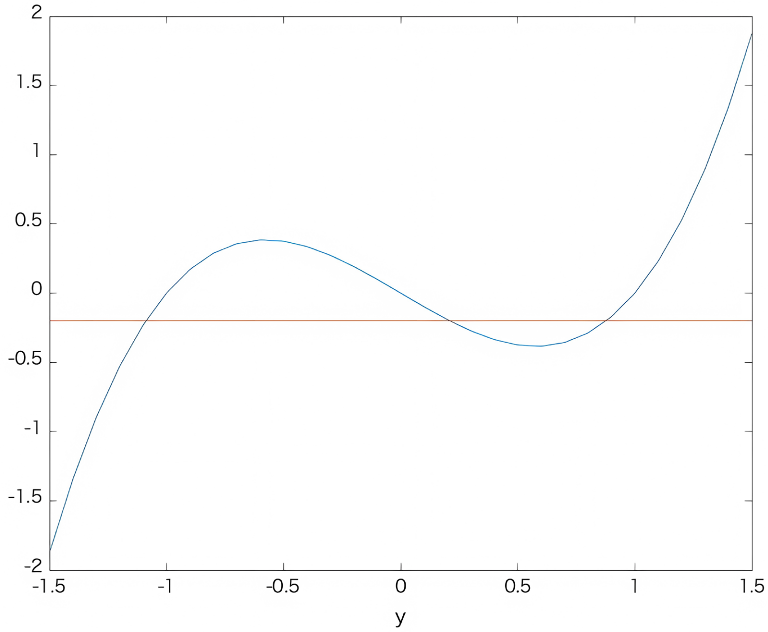

Figure 1. Blue line: $g(y) = y^3 - y = y(y+1)(y-1)$; Red line, $h(y) = -P_0/E$. The minimum value of $g(y)$: $0 < y < 1$ is $-2/(3\sqrt{3}) \approx -0.384$ at $y = 1/\sqrt{3} \approx 0.5774$.

By replacing P_0/E by Q, Equation (4) is transformed to

$$f(y) = y^3 - y + Q = 0. \tag{5}$$

By substituting the experimental speed of light (c_1 = 299,792,458 m/s) demonstrated in the Michelson-Morley into Equation (3),

$$y_1 = \sqrt{1 - (V/c_1)^2}. \tag{6}$$

By substituting y_1 into Equation (5),

$$Q = -y_1^3 + y_1. \tag{7}$$

Thus, it is found that Q can be determined from V and c_1. The solutions of a cubic equation are generally expressed by Cardano's formula; however, since this equation has three real roots when $0 < Q < 2/(3\sqrt{3})$, they can be expressed as follows using Vieta's formulas.

$$Y_k = \frac{2}{\sqrt{3}}\cos\left(\frac{1}{3}\arccos\left(-\frac{3Q}{2}\sqrt{3}\right)+\frac{2\pi k}{3}\right),\ k = 0,1,2. \quad (8)$$

y_1 and y_2 correspond to two of Y_1, Y_2, or Y_3, but the correspondence changes as V increases. In **Figure 2(a)**, the red line represents the change in y_1 as V increases; that is, it represents y_1 as a function of V. The blue polyline represents the behavior of y_2 as V increases. At the point where these two lines intersect, we have $y_1 = y_2$. Before the intersection, $y_2 < y_1$ always holds; after the intersection, $y_1 < y_2$.

From y_2, the corresponding speed of light can be calculated as follows.

$$c_2 = V\Big/\sqrt{1-y_2^2}\ . \quad (9)$$

Figure 2(b) shows c_2 (blue line) changes as V increases. Meanwhile, c_1 (red line) remains constant as the experimental speed of light. These lines intersect at the speed V (= 244,779,516 m/s) where c_2 equals c_1, which corresponds to $y_2 = y_1\left(=1/\sqrt{3} \approx 0.5774\right)$. The corresponding value of V is obtained from the following equation.

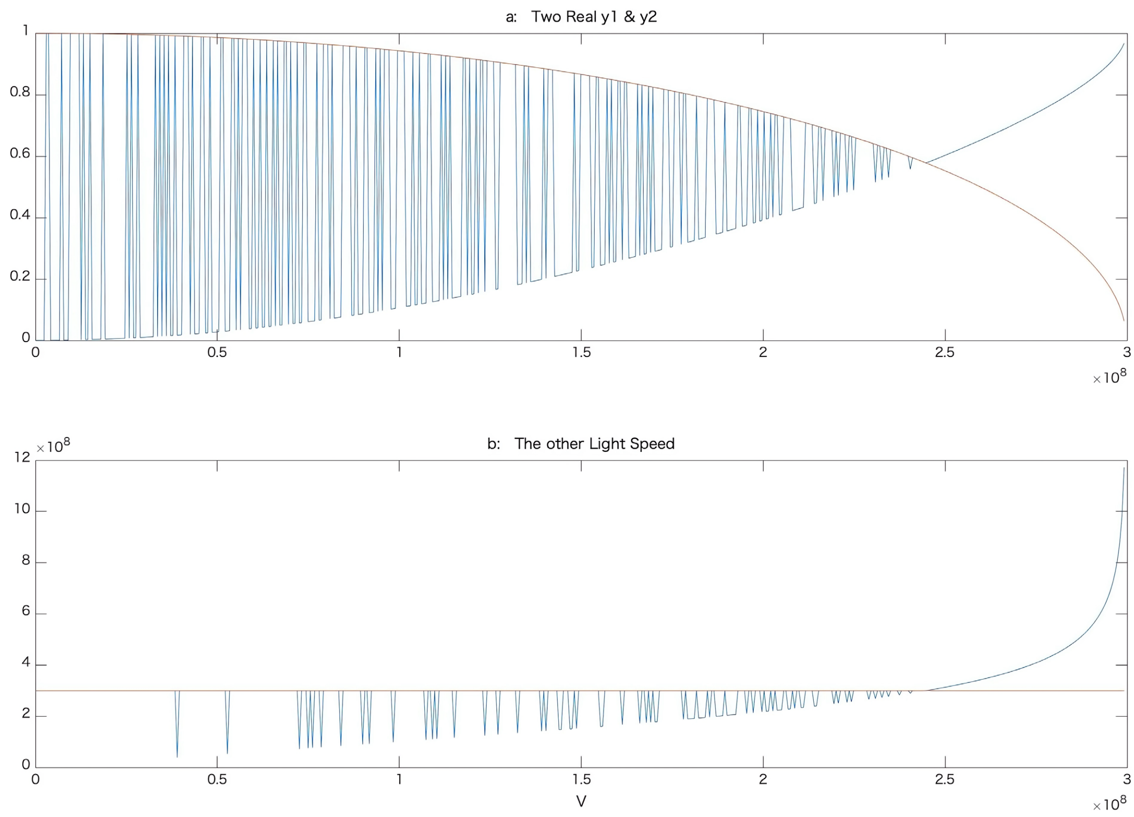

Figure 2. (a) Red line represents the change in y_1. Blue polyline represents the behavior of y_2. (b) Red line shows c_1 (=299,792,458 m/s). Blue line shows c_2 (= the other light speed). V, the speed of a mass (m/s). These lines intersect at the critical speed V_c (=244,779,516 m/s) where c_2 equals c_1, which corresponds to $y_2 = y_1\left(=1/\sqrt{3} \approx 0.5774\right)$. The unit on the horizontal axis is m/s.

$$V_c = c_1\sqrt{1-\left(1/\sqrt{3}\right)^2} = 244,779,516 \ (\text{m/s}) \tag{10}$$

c_2 does not exceed c_1 when V is less than or equal to 244,779,516 m/s; however, after the two lines intersect, c_2 exceeds c_1. We call the speed of V at which they intersect the critical speed (V_c). Below the critical speed, the blue line appears to be discontinuous; however, when magnified, it is clear that the blue line approaches the red line from below so closely that it is not visible. It should be noted that y_2 is also a real root of Equation (7), so since $P_0/E = Q$, E has the same value when y_2 is used. In other words, even in the case of c_2, the energy possessed by the mass is the same as when using the experimental speed of light c_1. This suggests that if the speed of light in an adjacent universe is c_2, it may be possible for mass moving at velocity V to travel from our universe while retaining its energy (Hypothesis 3).

2.3. What Happens to the Results Obtained in Section 2.2 When the Assumed Speed of Light Is Changed?

1) Figure 3 shows the results of examining the relationship between c_1 and c_2 when we assume the speed of light in our universe is, for example, c_1 = 350,792,458 m/s, which is greater than the actual value of 299,792,458 m/s. The critical speed is 286,420,842 m/s.

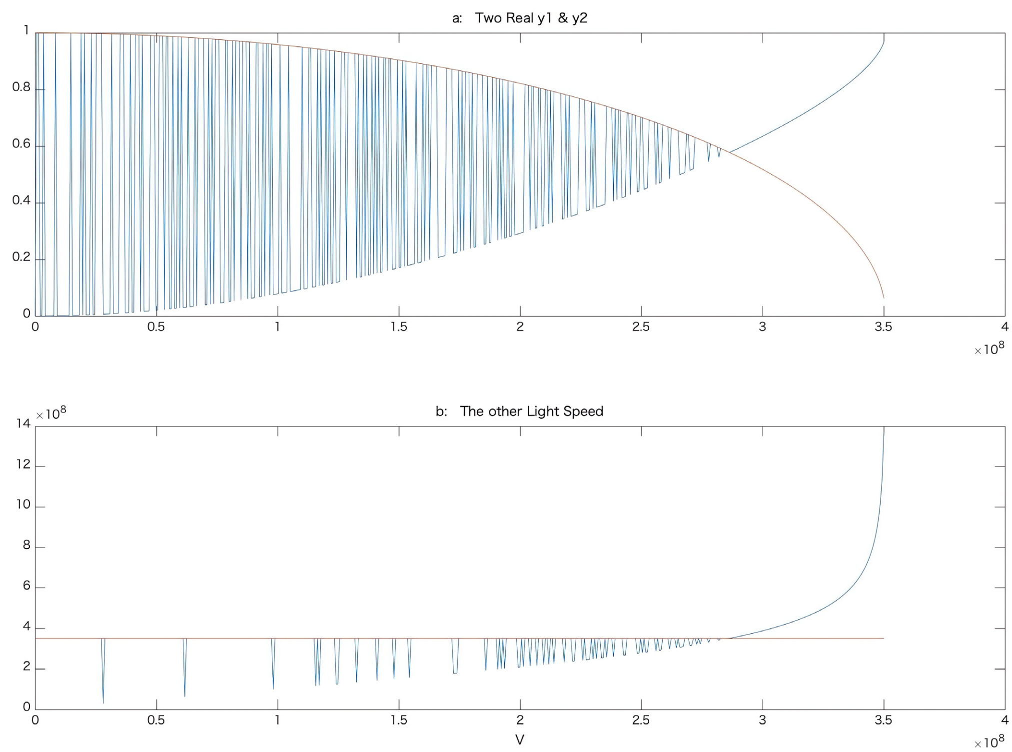

Figure 3. Case of another universe where the assumed speed of light is 350,792,458 m/s. (a) Red line represents the change in y_1. Blue polyline represents the behavior of y_2. (b) Red line shows c_1 (=350,792,458 m/s). Blue line shows c_2 (=the other light speed). V, the speed of a mass (m/s). The critical speed is 286,420,842m/s. The unit on the horizontal axis is m/s.

2) **Figure 4** shows the results of examining the relationship between c_1 and c_2 when we assume the speed of light in our universe is, for example, c_1 = 150,792,458 m/s, which is lesser than the actual value of 299,792,458 m/s. The critical speed is 123,121,526 m/s.

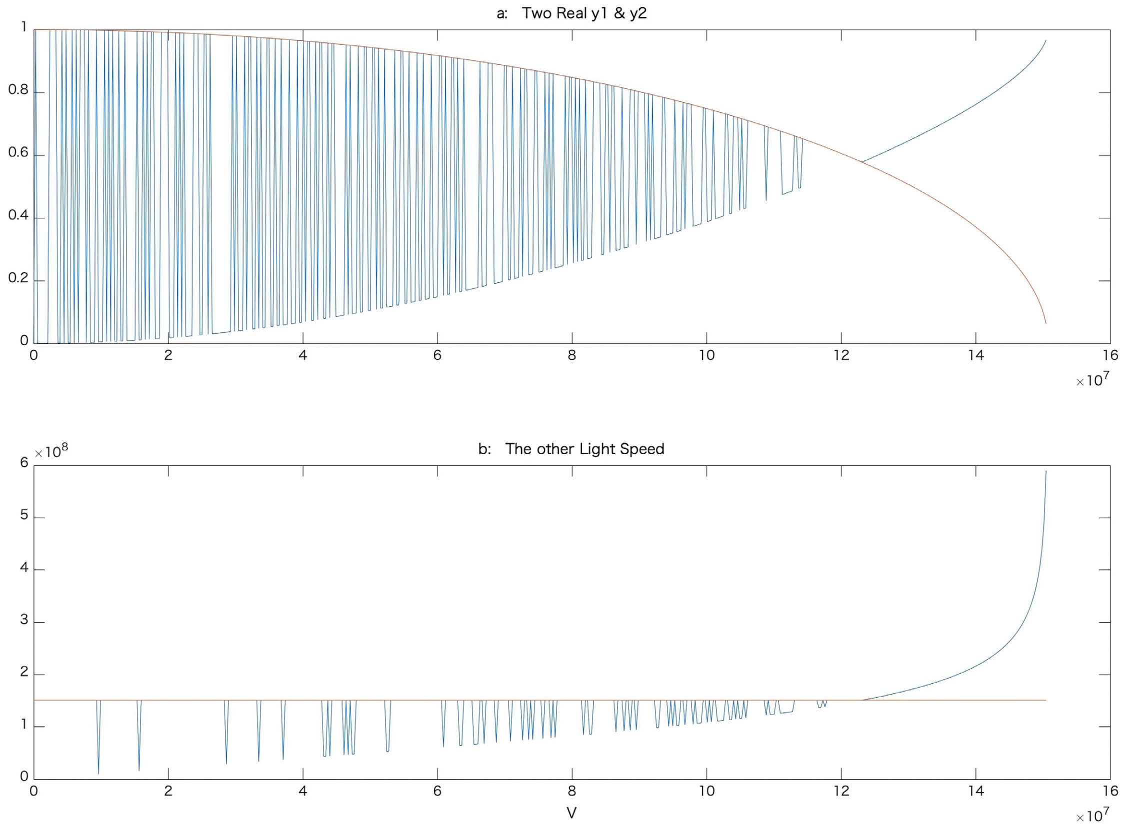

Figure 4. Case of another universe where the assumed speed of light is 150,792,458 m/s. (a) Red line represents the change in y_1. Blue polyline represents the behavior of y_2. (b) Red line shows c_1 (=150,792,458 m/s). Blue line shows c_2 (=the other light speed). V, the speed of a mass (m/s). The critical speed is 123,121,526 m/s. The unit on the horizontal axis is m/s.

Figure 3 and **Figure 4** have the same characteristics as **Figure 2**. Although only two examples are shown here, this feature is the same for other values of the speed of light as well. In other words, if the velocity of the mass is less than the critical speed (V_c), it can have the same energy at the other speed of light (c_2) that is lower than the assumed speed of light (c_1). When the velocity of the mass exceeds the critical speed (V_c), it has the same energy at the other speed of light (c_2) that is greater than the assumed speed of light (c_1). Once the constancy of the speed of light is postulated, the resulting structural properties follow independently of the particular value assigned to the speed of light in the universe. The following can be inferred from the above results. A mass moving at velocity V through an arbitrary universe A, where the speed of light c_A differs from that in our universe, can travel through an adjacent universe with the speed of light $c(c_A, V)$ determined by c_A and V while retaining its energy.

3. Discussion

Einstein established the theory of special relativity based on the principle that the speed of light is constant in all inertial frames. This idea was inspired by the Maxwell's equations, in which the speed of light is expressed in terms of the vacuum permittivity (ε_0) and permeability (μ_0) ($c_0 = 1/\sqrt{\varepsilon_0 \mu_0}$), and is independent of the inertial frame. However, in the International System of Units (SI), these constants are defined so as to be consistent with the experimental speed of light. Therefore, within this system, it is not possible to explain why the experimental speed of light is 299,792,458 m/s. In this study, it has been reconfirmed that, for our universe, the fundamental principle is not the specific numerical value of the speed of light itself, but rather the principle of the invariance of the speed of light. Nevertheless, it remains true that these parameters are the factors that determine the speed of light.

The results presented above are derived from the principle of the constancy of the speed of light. Under those hypotheses, if a mass were capable of moving between universes in contact with each other at a certain velocity, the speeds of light in those universes would be mutually related. The permittivity characterizes the extent to which an electric field can spread in vacuum, while the permeability characterizes how readily a magnetic field can be generated in vacuum. In quantum theory, the vacuum is not regarded as completely empty. Fluctuations of virtual particles occur, and these fluctuations are reflected in the values of the permittivity and permeability. Such fluctuations have been observed under extreme conditions. In neutron stars, the vacuum exhibits birefringence and behaves as if it were a material medium [6]. Moreover, in ultra-strong magnetic fields, electron-positron pair production from the vacuum is predicted to occur [7]. There also exists the fine-structure constant $\alpha\left(= e^2/\left(4\pi\epsilon_0\hbar c\right)\right)$: e, electron charge; $\hbar$, plank constant; c, speed of light; ε_0, vacuum permittivity, which is directly connected to the properties of the vacuum [8]. This is a dimensionless constant determined by four factors. If the speed of light differs, the fundamental constitution of that universe would be entirely distinct, given the constitutive relationship between the speed of light (c), the vacuum permittivity (ε_0) and vacuum permeability (μ_0) ($c = 1/\sqrt{\varepsilon_0 \mu_0}$). Furthermore, a variation in c would necessarily result in a different fine-structure constant (α), thereby fundamentally altering atomic structure, chemical properties, and the overall evolution of the universe. These variations suggest that while the mathematical framework allows for a second root, the resulting physical environment would likely be governed by a different set of spectroscopic and thermodynamic laws. A physico-mathematical approach has revealed the mechanism by which different physical constants are generated across multiple universes [9].

We add a physics-based, concise eligibility criterion for selecting the "second" root used to define c_2 (in addition to the condition that it be a positive real number). Below we describe what this framework predicts when the cubic equation yields no roots in the physical domain.

i) Admissibility Rule: We have clarified the rule for selecting the "second" root c_2. In the case where multiple real roots exist for $f(y) = y^3 - y + Q = 0$, we select the largest positive root. This choice is based on the assumption that the vacuum naturally relaxes into the state that allows for the highest stable signal velocity (minimum energy configuration).

ii) Prediction in Non-Physical Domain: When the parameter Q exceeds the critical threshold ($Q > 2/(3\sqrt{3})$), the equation lacks the required real roots. Our framework interprets this as a physical boundary beyond which a dual-universe system cannot stably exist. This predicts a "forbidden zone" for certain gravitational or electromagnetic coupling strengths, leading to either a vacuum collapse or a phase transition into a unified manifold.

As physical meaning and spacetime structure, c_2 is defined as a distinct invariant speed associated with the spacetime structure of the "adjacent universe". It is not a re-parameterization of the standard kinematics but represents the fundamental causal limit within that specific manifold. The following remarks address whether it is consistent with the special theory of relativity. Within each respective universe, the standard invariant relations among energy (E), momentum (p), and rest mass (m_0) remain valid. Specifically, for the adjacent universe, the relation is given by $E^2 = (pc_2)^2 + (m_0 c_2^2)^2$. We will discuss below whether it is consistent with our universe. The model treats the two universes as distinct manifolds with their own respective invariant speeds (c_1 and c_2). This ensures that the kinematics within our own universe remain strictly governed by the standard value of c_1, preserving the integrity of established special relativity while allowing for speculative interactions across the interface.

If a mass (m_0) moving at a given velocity (V) across these adjacent universes retains the same energy in both, the speed of light in that universe would be c_2. Under a set of those speculative assumptions, it would follow that the speed of light in that universe is not arbitrary but would be determined as c_2. According to the speculative hypotheses, the vacuum permittivity (ε_{02}) and permeability (μ_{02}) in that universe satisfy the relation $\varepsilon_{02}\mu_{02} = 1/c_2^2$. c_2 is smaller than c_1 when V is below the critical velocity, and greater than c_1 when V is exceeds the critical velocity.

The shift in vacuum constants (ε_0 and μ_0) necessarily affects the fine-structure constant $\alpha_2 \left(= e^2/(4\pi\epsilon_{02}\hbar c_2)\right)$, which governs the strength of electromagnetic interactions. For instance, if the proposed relation results in a significantly larger α_2, atomic structures in the adjacent universe might become unstable or exhibit different spectral characteristics. While a comprehensive analysis of stellar nucleosynthesis or atomic stability within Universe 2 is beyond the scope of this study, this internal-consistency check ensures that the mathematical model remains compatible with the fundamental framework of quantum electrodynamics.

4. Conclusion

In this study, we have re-examined the mathematical formalism of the energy-momentum relation and demonstrated that the energy of a mass moving at velocity V admits a dual-root structure for the invariant speed parameter. Our analysis reveals that for any given energy state at velocity V, there exists a second valid speed of light, c_2, distinct from the experimental value c_1. Under the hypothesis of energy invariance across adjacent physical states, this c_2 suggests the existence of a coupled universe with distinct electromagnetic properties, where the vacuum permittivity and permeability are constrained by $\varepsilon_{02}\mu_{02} = 1/c_2^2$. Notably, the relationship between c_1 and c_2 undergoes a reversal at a critical velocity, marking a potential phase transition point in the interaction between these systems. To strengthen the relevance of this theoretical inquiry, future research should focus on three primary avenues. First, a rigorous derivation of the "critical velocity" is required to determine if it falls within experimentally accessible ranges or corresponds to cosmological scales. Second, the mechanism of energy invariance during a hypothetical transition between c_1 and c_2 environments must be explored through the lens of quantum tunneling or multidimensional brane dynamics. Finally, investigating possible observational signatures in high-energy astrophysics—where particles approach these critical velocities—may provide a means to test the validity of this mathematical duality. Such efforts will clarify whether this second root is a mere algebraic curiosity or a window into a broader, multi-layered structure of spacetime.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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