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The Adomian Decomposition Method for Solving Volterra-Fredholm Integral Equation Using Maple

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Abstract

In this paper, Adomian decomposition method (ADM) is used to solve the Volterra-Fredholm integral equation. A number of examples have been presented to explain the numerical results, which is the comparison between the exact solution and the numerical solution, and it is found through the tables and the amount of error between the exact solution and the numerical solution, it is very small and almost non-existent and is also illustrated through the graph how the exact solution of completely applies to the numerical solution This proves the accuracy of the method, which is the Adomian decomposition method (ADM) for solving the Volterra Fredholm integral equation using Maple 18. And that this method is characterized by ease, speed and great accuracy in obtaining numerical results.

Keywords

Volterra-Fredholm Integral Equation, Adomian Decomposition Method, Maple18

1. Introduction

The current research intends to the Adomian decomposition method for solving Volterra-Fredholm integral equation using Maple18.

Integral equations are the basic sciences in our real life, and they explain physical, chemical, engineering, and medical phenomena, and more than that, they contribute greatly to reaching analytical and numerical solutions to these phenomena in various areas of our lives [1] [2]. There are several studies of Adomian decomposition metod, convergence and accuracy of Adomian's decomposition method for the solution of Lorenz equationsis studied in [3]. Solving Riccati differential equation using Adomian's decomposition method is given in [4]. An adaptation of Adomian decomposition for numeric-analytic integration of strongly nonlinear and chaotic oscillatorsis studied in [5]. The extended Adomian decomposition method for fourth order boundary value problems is given in [6]. The use of the Adomian decomposition method for solving multi-point boundary value problems is mentioned in [7]. [8] developed a new algorithm for evaluating Adomian polynomials. [9] studied an efficient algorithm for the multivariable Adomian polynomials. [10] found convenient analytic recurrence algorithms for the Adomian polynomials. A review of the Adomian decomposition method and its applications to fractional differential equations is given in [11]. [12] covers a bibliography of the theory and applications of the Adomian decomposition method. We find that solutions of nonlinear integral equations are more difficult to solve than linear integral equations and there are many analytical and numerical methods for solving linear and nonlinear integral equations mentioned in the references [13] [14] [15] [16]. We discuss the numerical solution of the integral Volterra equation of the second type using an implicit trapezoidal [17] [18]. The Adomian decomposition method of the Fredholm integral equation of the second kind using MATLAB and Maple is demonstrated in [19]. The Adomian decomposition method was applied to solve the Fredholm integral equation of the second kind [20]. Also, Modified analysis method for solving the Volterra integral equation of the second kind using Maple is discussed in [21].

In this article we have applied the Adomian decomposition method used by using the Maple algorithm by applying this algorithm to different examples, including finding the approximate solution and then comparing it to the exact solution and finding out the amount of error between the approximate solution and the exact solution.

The main objective of this work is to use the Adomian decomposition method in solving the Volterra-Fredholm integral equation of the second kind using Maple18.

The paper is arranged as follows: In Section 2, the Adomian decomposition method; in Section 3, numerical examples are also considered to show the ability of the proposed method, and the conclusion is drawn in Section 4.

2. The Adomian Decomposition Method

To clarify the basic idea of this method, we consider the following general non-linear differential equation:

$$u(x) = f(x) + \lambda_1 \int_a^x K_1(x,t)u(t)\,\mathrm{d}t + \lambda_2 \int_a^b K_2(x,t)u(t)\,\mathrm{d}t. \tag{1}$$

where L is assumed invertible and L^{-1} is an inverse operator.

The standard Adomian method defines the solution $u(x)$ by the series

$$u(x) = \sum_{n=0}^{\infty} u_n(x). \tag{2}$$

The modified decomposition method

$$u_0(x) = f(x) \tag{3}$$

$$u_1(x) = f(x) + L^{-1}\left(\lambda_1 \int_a^x K_1(x,t) u_0(t)\,dt\right) + L^{-1}\left(\lambda_2 \int_a^b K_2(x,t) u_0(t)\,dt\right), \tag{4}$$

$$u_{n+1}(x) = L^{-1}\left(\lambda_1 \int_a^x K_1(x,t) u_n(t)\,dt\right) + L^{-1}\left(\lambda_2 \int_a^b K_2(x,t) u_n(t)\,dt\right). \tag{5}$$

The use of a modified decomposition method not only reduces the calculations but avoids the use of the human polynomial arrangement of boundaries in such cases.

3. Numerical Examples

In this section, we solve some examples, and we can compare the numerical results with the exact solution.

Example 1. Consider the Volterra Fredholm integral equation

$$u(x) = x - \frac{1}{3}x^3 + \int_0^x t u(t)\,dt + \int_{-1}^1 t^2 u(t)\,dt, \tag{6}$$

the exact Solution $u(x) = x$.

Applying the Adomian decomposition method using Maple18 we find (Table 1 & Figure 1).

Example 2. Consider the Volterra Fredholm integral equation

$$u(x) = \sin(x) - \cos(x) - \int_0^x u(t)\,dt + \int_0^{\frac{\pi}{2}} u(t)\,dt \tag{7}$$

the exact Solution $u(x) = \sin(x)$.

Applying the Adomian Decomposition Method using Maple18 we find (Table 2 & Figure 2).

Example 3. Consider the Volterra Fredholm integral equation

$$u(x) = 3x + 4x^2 - x^3 - x^4 - 2 + \int_0^x t u(t)\,dt + \int_{-1}^1 t u(t)\,dt. \tag{8}$$

the exact Solution $u(x) = 3x + 4x^2$.

Table 1. Approximation solution and exact solution of Volterra Fredholm integral equations for example 1.

x	$u = x$	$\text{Exact1} = x$	$\text{Error} = \lvert \text{Exact1} - u \rvert$
0.10000	0.1000000	0.1000000	0.0000000
0.20000	0.2000000	0.2000000	0.0000000
0.30000	0.3000000	0.3000000	0.0000000
0.40000	0.4000000	0.4000000	0.0000000
0.50000	0.5000000	0.5000000	0.0000000
0.60000	0.6000000	0.6000000	0.0000000
0.70000	0.7000000	0.7000000	0.0000000
0.80000	0.8000000	0.8000000	0.0000000
0.90000	0.9000000	0.9000000	0.0000000
1.00000	1.0000000	1.0000000	0.0000000

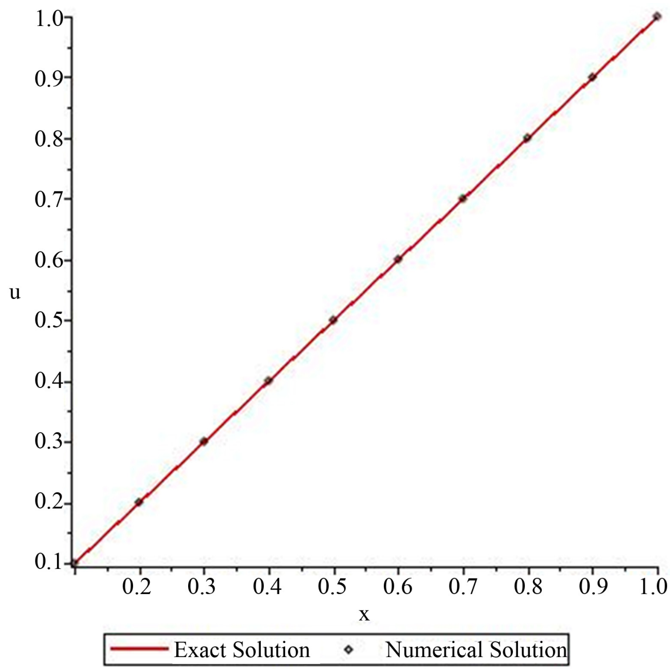

Figure 1. Plot of the solutions of Volterra Fredholm integral equation for example 1.

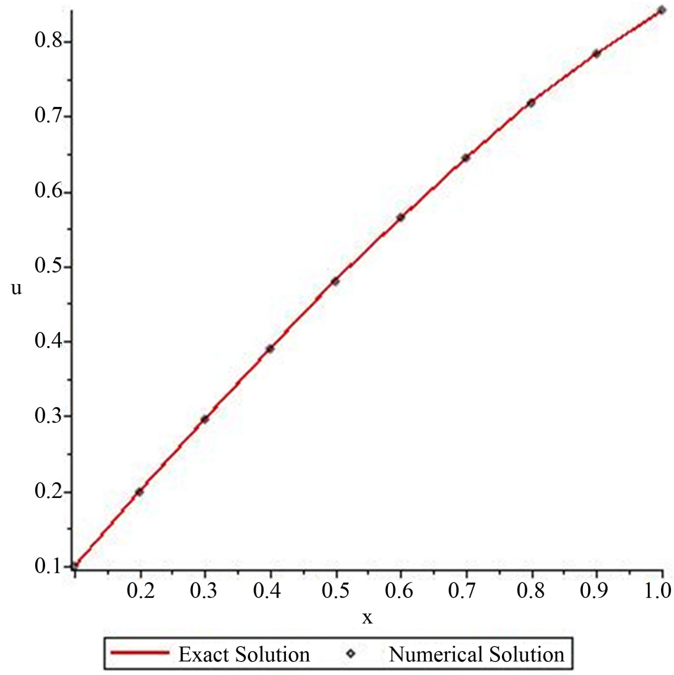

Figure 2. Plot of the solutions of Volterra Fredholm integral equation for example 2.

Table 2. Approximation solution and exact solution of Volterra Fredholm integral equations for example 2.

x	u	$\text{Exact2} = \sin(x)$	$\text{Error} = \lvert \text{Exact2} - u \rvert$
0.10000	0.0997464	0.0998334	0.0000870
0.20000	0.1986230	0.1986693	0.0000463

Continued

0.30000	0.2954967	0.2955202	0.0000235
0.40000	0.3894071	0.3894183	0.0000113
0.50000	0.4794205	0.4794255	0.0000050
0.60000	0.5646404	0.5646425	0.0000021
0.70000	0.6442169	0.6442177	0.0000008
0.80000	0.7173558	0.7173561	0.0000003
0.90000	0.7833268	0.7833269	0.0000001
1.00000	0.8414710	0.8414710	0.0000000

Table 3. Approximation solution and exact solution of Volterra Fredholm integral equations for example 3.

x	u	$\text{Exact3} = 3x + 4x^2$	$\text{Error} = \lvert \text{Exact3} - u \rvert$
0.10000	0.3400000	0.3399998	0.0000002
0.20000	0.7600000	0.7599998	0.0000002
0.30000	1.2600000	1.2599996	0.0000004
0.40000	1.8400000	1.8399994	0.0000006
0.50000	2.5000000	2.4999988	0.0000012
0.60000	3.2400000	3.2399976	0.0000024
0.70000	4.0600000	4.0599948	0.0000052
0.80000	4.9600000	4.9599883	0.0000117
0.90000	5.9400000	5.9399731	0.0000269
1.00000	7.0000000	6.9999375	0.0000625

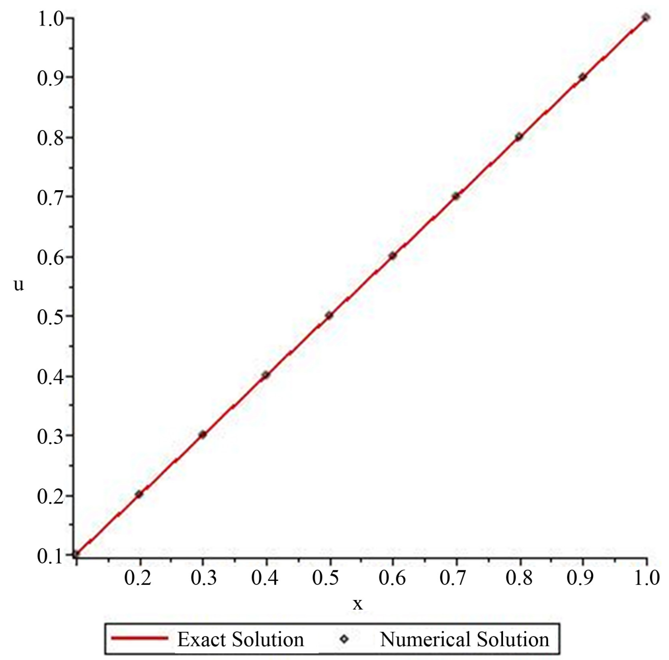

Figure 3. Plot of the solutions of Volterra Fredholm integral equation for example 3.

Applying the Adomian Decomposition Method using Maple18 we find (Table 3 & Figure 3).

Example 4. Consider the Volterra Fredholm integral equation

$$u(x) = -2 - 2x + 2e^x + \int_0^x (x-t)u(t)\,dt + \int_0^1 xu(t)\,dt \tag{9}$$

the exact Solution $u(x) = xe^x$.

Applying the Adomian Decomposition Method using Maple we find (Table 4 & Figure 4).

Example 5. Consider the Volterra Fredholm integral equation

$$u(x) = x^3 - \frac{9}{20}x^5 - \frac{1}{4}x + \frac{1}{5} + \int_0^x (x+t)u(t)\,dt + \int_0^1 (x-t)u(t)\,dt \tag{10}$$

the exact Solution $u(x) = x^3$.

Table 4. Approximation solution and exact solution of Volterra Fredholm integral equations for example 4.

x	u	$\text{Exact4} = xe^x$	$\text{Error} = \lvert \text{Exact4} - u \rvert$
0.10000	0.1105171	0.1093187	0.0011984
0.20000	0.2442806	0.2418630	0.0024176
0.30000	0.4049576	0.4012789	0.0036788
0.40000	0.5967299	0.5917260	0.0050039
0.50000	0.8243606	0.8179448	0.0064158
0.60000	1.0932713	1.0853320	0.0079393
0.70000	1.4096269	1.4000262	0.0096006
0.80000	1.7804327	1.7690040	0.0114287
0.90000	2.2136428	2.2001875	0.0134553
1.00000	2.7182818	2.7025662	0.0157157

Table 5. Approximation solution and exact solution of Volterra Fredholm integral equations for example 5.

x	u	$\text{Exact5} = x^3$	$\text{Error} = \lvert \text{Exact5} - u \rvert$
0.10000	0.0010000	0.0007782	0.0002218
0.20000	0.0080000	0.0083499	0.0003499
0.30000	0.0270000	0.0280883	0.0010883
0.40000	0.0640000	0.0659947	0.0019947
0.50000	0.1250000	0.1280596	0.0030596
0.60000	0.2160000	0.2202426	0.0042426
0.70000	0.3430000	0.3484413	0.0054413
0.80000	0.5120000	0.5184507	0.0064507
0.90000	0.7290000	0.7359143	0.0069143
1.00000	1.0000000	1.0062804	0.0062804

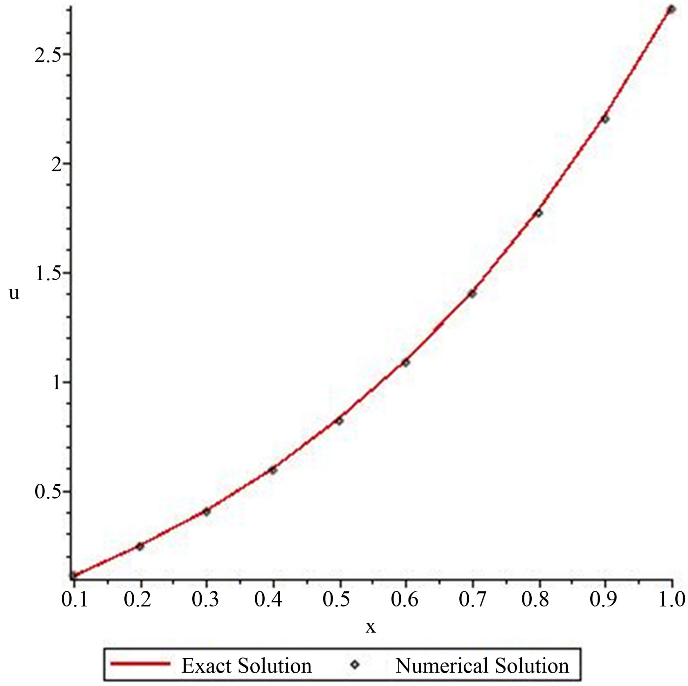

Figure 4. Plot of the solutions of Volterra Fredholm integral equation for example 4.

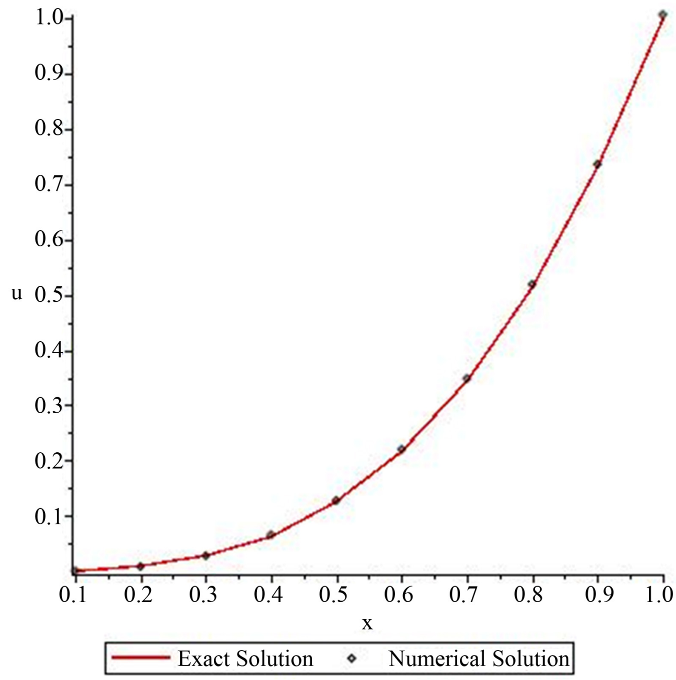

Figure 5. Plot of the solutions of Volterra Fredholm integral equation for example 5.

Applying the Adomian Decomposition Method using Maple18 we find (Table 5 & Figure 5).

4. Conclusion

In this paper, the Adomian decomposition method was applied to solve the integral Volterra Fredholm equation using program Maple18. The results are obtained in the tables and drawn in the figures. **Tables 1-5** show the correct solution and the numerical solution. **Tables 1-5** represent the exact and numerical results of the examples in this article. **Figures 1-5** readily show the comparison of exact solution and ap-proximate solution. Comparing the numerical results, we find that the numerical solution is largely applied to the exact solution, which proves the efficiency of the method used and the ability to obtain the numerical solution corresponding to the exact solution easily and conveniently with a program Maple 18. Moreover, the high accuracy of the results is obtained.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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