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Asymptotic Behavior of a Bi-Dimensional Hybrid System

Pedro Gamboa[1], Jaime E. Muñoz[1,2], Octavio Vera[3], Margareth Alves[4]

[1]Departamento de Matemática, Instituto de Matemáticas, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil
[2]Laboratorio Nacional de Computação Científica, Petrópolis, Brazil
[3]Departamento de Matemática, Facultad de Ciencias, Universidad del Bo-Bo, Concepción, Chile
[4]Departamento de Matemática, UFV, Viçosa, Brazil
Email: pgamboa@im.ufrj.br, rivera@im.ufrj.br, rivera@lncc.br, overa@ubiobio.cl, malves@ufv.br

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Abstract

We study the asymptotic behavior of the solutions of a Hybrid System wrapping an elliptic operator.

Keywords

Hybrid System, Compressible, Stabilization, Asymptotic Behavior, Decay Rate, Generator Infinitesimal, Polynomial Decay

1. Introduction

In this paper, we address some issues related to the asymptotic behavior a hybrid system with two types of vibrations of different nature. The model under consideration is inspired in and introduced in [1]. However, there are some important differences between these two models. In [1] the flexible part of the boundary Γ_0 is occupied by a flexible damped beam instead of a flexible. Most of the relevant properties see [2]. In [3] the authors are interested on the existence of periodic solutions of this system. Due to the localization of the damping term in a relatively small part of the boundary and to the effect of the hybrid structure of the system, the existence of periodic solutions holds for a restricted class of non homogeneous terms. Some resonance-type phenomena are also exhibited. Cindea, Sorin and Pazoto [4] consider the motion of a stretched string coupled with a rigid body at one end and we study the existence of periodic solution when a periodic force facts on the body. The main difficulty of the study is related to the weak dissipation that characterizes this hybrid system, which does not ensure a uniform decay rate of the energy. For more examples of hybrid systems see [5] [6]. We refer to [7] for a

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discussion on the model and references therein. In [8] the authors to discern exact controllability properties of two coupled wave equations, one of which holds on the interior of a bounded open domain Ω, and the other on a segment Γ_0 of the boundary $\partial\Omega$. Moreover, the coupling is accomplished through terms on the boundary. Because of the particular physical application involved the attenuation of acoustic waves within a chamber by means of active controllers on the chamber walls control is to be implemented on the boundary only.

We consider the bi-dimensional cavity $\Omega = \Omega_1 \setminus \bar{\Omega}_0$ and that Ω_0 an open class C^2 with limited boundary contained in Ω_1, filled with an elastic, in viscid, compressible fluid, in which the acoustic vibrations are coupled with the mechanical vibration of a string located in the subset $\Gamma_0 = \{(x,0); x \in (0,1)\}$ a part of the boundary of omega of Ω_1, $\Gamma_1 = \partial\Omega_1 \setminus \Gamma_0$ and $\Gamma = \partial\Omega_1 \bigcup \partial\Omega_0$ with $\partial\Omega_1 \bigcap \partial\Omega_0 = \phi$, is boundary of Ω. The subset Γ_1 is assumed to be rigid and we impose zero normal velocity of the fluids on it. The subset Γ_0 is supposed to be flexible and occupied by a flexible string that vibrates under the pressure of the fluid on the plane where Ω lies. The displacement of Γ_0, described by the scalar function $w = w(x,t)$, obeys the one-dimensional dissipative wave equation. As Ω is compressible fluid where the velocity field $\boldsymbol{v}$ is given by the potential $\varphi = \varphi(x,y,t), (\boldsymbol{v} = \nabla\varphi)$. All deformations are supposed to be small enough so that linear theory applies.

The linear motion of this system is described by means of the coupled wave equations

$$\begin{cases} \rho\varphi_{tt} - c\Delta\varphi + \gamma_1\varphi_t = 0 & \text{in } \Omega \times (0,\infty) \\ \dfrac{\partial\varphi}{\partial\nu} = 0 & \text{on } \Gamma_1 \times (0,\infty) \\ \dfrac{\partial\varphi}{\partial\nu} = -w_t & \text{on } \Gamma_0 \times (0,\infty) \\ \varphi = 0 & \text{on } \Gamma_2 \times (0,\infty) \\ w_{tt} - w_{xx} + \gamma_0 w_t + c\varphi_t = 0 & \text{on } \Gamma_0 \times (0,\infty) \\ w_x(0,t) = w_x(1,t) = 0 & \text{for } t > 0 \\ \varphi(0) = \varphi^0, \quad \varphi_t(0) = \varphi^1 & \text{in } \Omega \\ w(0) = w^0, \quad w_t(0) = w^1 & \text{on } \Gamma_0 \end{cases} \tag{1}$$

where ν denote the unit outward normal to Ω.

We define the energy associated with this system. Proceeding formally, multiply the first equation by φ_t and then integrate over Ω.

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega |\varphi_t|^2 \,\mathrm{d}x\mathrm{d}y + \frac{c}{\rho}\int_\Omega \nabla\varphi\cdot\nabla\varphi_t \mathrm{d}x\mathrm{d}y - \frac{c}{\rho}\int_\Gamma \frac{\partial\varphi}{\partial\nu}\cdot\varphi_t \mathrm{d}\sigma + \frac{\gamma_1}{\rho}\int_\Omega |\varphi_t|^2 \,\mathrm{d}x\mathrm{d}y = 0. \tag{2}$$

However, the integral

$$\int_\Gamma \frac{\partial\varphi}{\partial\nu}\cdot\varphi_t \mathrm{d}\sigma = \int_{\Gamma_1} \underbrace{\frac{\partial\varphi}{\partial\nu}}_{=0}\cdot\varphi_t \mathrm{d}\sigma + \int_{\Gamma_0} \underbrace{\frac{\partial\varphi}{\partial\nu}}_{-w_t}\cdot\varphi_t \mathrm{d}\sigma - \int_{\Gamma_2} \frac{\partial\varphi}{\partial\nu}\cdot\underbrace{\varphi_t}_{=0}\sigma,$$

which leads us

$$\int_\Gamma \frac{\partial\varphi}{\partial\nu}\cdot\varphi_t \mathrm{d}\sigma = \int_{\Gamma_0} \frac{\partial\varphi}{\partial\nu}\cdot\varphi_t = -\int_{\Gamma_0} w_t\varphi_t \mathrm{d}x. \tag{3}$$

Replacing (3) into (2) we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{2}\int_\Omega |\varphi_t|^2 \,\mathrm{d}x\mathrm{d}y + \frac{c}{\rho}|\nabla\varphi|^2\right] + \frac{\gamma_1}{\rho}\int_\Omega |\varphi_t|^2 \,\mathrm{d}x\mathrm{d}y - \frac{c}{\rho}\int_{\Gamma_0} w_t\varphi_t \mathrm{d}x = 0. \tag{4}$$

Multiplying by w in the second equation of the system (1) and then integrate over Γ_0

$$\frac{1}{2\rho}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Gamma_0} |w_t|^2 \,\mathrm{d}x - \frac{1}{\rho}\int_{\Gamma_0} w_{xx} w_t \mathrm{d}x + \frac{\gamma_0}{\rho}\int_{\Gamma_0} |w_t|^2 \,\mathrm{d}x + \frac{c}{\rho}\int_{\Gamma_0} \varphi_t w_t \mathrm{d}x = 0. \tag{5}$$

Integrating by parts

$$\int_{\Gamma_0} w_{xx} w_t \mathrm{d}x = \underbrace{w_x}\, w_t \Big|_0^1 - \int_{\Gamma_0} w_x w_{tx} \mathrm{d}x = -\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Gamma_0} \left|w_x\right|^2 \mathrm{d}x.$$

Replacing the above equation over (5) we obtain

$$\frac{1}{2\rho}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Gamma_0} \left|w_t\right|^2 \mathrm{d}x + \frac{1}{2\rho}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Gamma_0} \left|w_x\right|^2 \mathrm{d}x + \frac{\gamma_0}{\rho}\int_{\Gamma_0} \left|w_t\right|^2 \mathrm{d}x + \frac{c}{\rho}\int_{\Gamma_0} \varphi_t w_t \mathrm{d}x = 0, \tag{6}$$

which leads us to assert that, the energy of the system is given by

$$E(t) = E(\varphi, w; t) = \frac{1}{2}\int_{\Omega} \left|\varphi_t\right|^2 + \frac{c}{\rho}\left|\nabla\varphi\right|^2 \mathrm{d}x\mathrm{d}y + \frac{1}{2\rho}\int_{\Gamma_0} w_x^2 + w_t^2 \mathrm{d}x, \tag{7}$$

for each $t \geq 0$.

Remark 1 *The first two terms represents the energy of acoustic wave and the other terms is the energy of bungee wave.*

The system has a natural dissipation. Indeed, to observe this fact multiply the first equation of (1) by φ_t and then the second equation of (1) by w_t, as was done in previous calculations

$$\frac{\mathrm{d}E(t)}{\mathrm{d}t} = -\frac{\gamma_1}{\rho}\int_{\Omega} \left|\varphi_t\right|^2 \mathrm{d}x\mathrm{d}y - \frac{\gamma_0}{\rho}\int_{\Gamma_0} w_t^2 \mathrm{d}x < 0, \tag{8}$$

if $\gamma_1^2 + \gamma_0^2 \neq 0$. Micu, S. in his doctoral thesis [7] shows non-exponential decay of the energy of the hybrid system (1).

2. Mathematical Formulation

Define the face space $\mathcal{H} = H^1(\Omega) \times L^2(\Omega) \times H^1(\Gamma_0) \times L^2(\Gamma_0)$ endowed with the Hilbertian scalar product given by

$$\langle U, V\rangle = \int_{\Omega} \frac{c}{\rho}\nabla\varphi_1 \cdot \nabla\psi_1 + \varphi_2\psi_2 \mathrm{d}x\mathrm{d}y + \frac{1}{\rho}\int_{\Gamma_0} (\varphi_3)_x (\psi_3)_x + \varphi_4\psi_4 \mathrm{d}x, \tag{9}$$

for all $U = (\varphi_1, \varphi_2, \varphi_3, \varphi_4), V = (\psi_1, \psi_2, \psi_3, \psi_4) \in \mathcal{H}$. We can show that the pair $(\mathcal{H}, \langle,\rangle)$ is a Hilbert space.

Since the first and second equation of the system (1), we obtain

$$\varphi_t = \varphi_2 \quad \text{then} \quad \varphi_{2t} = \frac{c}{\rho}\Delta\varphi_1 - \frac{\gamma_1}{\rho}\varphi_2$$

$$w_t = \varphi_4 \quad \text{then} \quad \varphi_{4t} = (\varphi_3)_{xx} - \gamma_0\varphi_4 - c\varphi_2.$$

These equations lead us to define the operator $\mathcal{A}: \mathcal{H} \to \mathcal{H}$ by

$$\mathcal{A} := \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{c}{\rho}\Delta & -\frac{\gamma_1}{\rho} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -c & (\cdot)_{xx} & -\gamma_0 \end{bmatrix},$$

in this sense for all $U = (\varphi_1, \varphi_2, \varphi_3, \varphi_4) \in \mathcal{H}$,

$$\mathcal{A}U = \left(\varphi_2, \frac{c}{\rho}\Delta\varphi_1 - \frac{\gamma_1}{\rho}\varphi_2, \varphi_4, (\varphi_3)_{xx} - \gamma_0\varphi_4 - c\varphi_2\right). \tag{10}$$

Note that $\mathcal{A}U \in \mathcal{H}$ if and only if

$$\varphi_2 \in H^1(\Omega), \quad \frac{c}{\rho}\Delta\varphi_1 - \frac{\gamma_1}{\rho}\varphi_2 \in L^2(\Omega), \quad \frac{c}{\rho}\Delta\varphi_1 - \frac{\gamma_1}{\rho}\varphi_2, \quad \varphi_4 \in H^1(\Gamma_0)$$
$$-c\varphi_2 + (\varphi_3)_{xx} - \gamma_0\varphi_4 \in L^2(\Gamma_0).$$

Now, we consider the problem with Neumann boundary conditions

$$\begin{cases} \frac{c}{\rho}\Delta\varphi_1 = \frac{\gamma_1}{\rho}\varphi_2 + h \in L^2(\Omega), \quad h \in L^2(\Omega) \\ \frac{\partial}{\partial\nu}\varphi_1 = 0 \quad \text{on } \Gamma_1 \\ \frac{\partial}{\partial\nu}\varphi_1 = \varphi_4 \quad \text{on } \Gamma_0 \\ \varphi_1 = 0 \quad \text{on } \Gamma_2, \end{cases} \tag{11}$$

where we can say that $\varphi_1 \in H^2(\Omega)$ see [9]. Similarly, consider the problem

$$\begin{cases} (\varphi_2)_{xx} = c\varphi_2 + \gamma_0\varphi_4 \in L^2(\Gamma_0) \\ (\varphi_3)_x(0) = (\varphi_3)_x(1) = 0. \end{cases} \tag{12}$$

We can say that $\varphi_2 \in H^2(\Gamma_0)$. In this sense we can define the domain of the operator $\mathcal{A}$ which we denote $\mathcal{D}(\mathcal{A})$, as the set of $U = (\varphi_1, \varphi_2, \varphi_3, \varphi_4) \in \mathcal{H}$ such that $\varphi_1 \in H^2(\Omega)$, $\varphi_2 \in H^1(\Omega)$, $\varphi_3 \in H^2(\Gamma_0)$, $\varphi_4 \in H^1(\Gamma_0)$ satisfying

$$\frac{\partial\varphi_1}{\partial\nu} = 0 \quad \text{on } \Gamma_1$$

$$\frac{\partial\varphi_1}{\partial\nu} = \varphi_4 \quad \text{on } \Gamma_0$$

$$\varphi_1 = 0 \quad \text{on } \Gamma_2$$

$$(\varphi_3)_x(0) = (\varphi_3)_x(1) = 0.$$

Remark 2 *By previous observations we can say that the hybrid system* (1) *is equivalent to the Cauchy problem*

$$\begin{cases} U_t(t) = \mathcal{A}U(t), t > 0 \\ U(0) = U_0, \end{cases} \tag{13}$$

where $U_0 = (\varphi^0, \varphi^1, \omega^0, \omega^1) \in \mathcal{D}(\mathcal{A})$ and $U(t) = (\varphi_1, \varphi_2, \varphi_3, \varphi_4) \in \mathcal{D}(\mathcal{A}), t \geq 0$.

3. Solution Existence

We want to show that $\mathcal{A}$ is a dissipative operator and $0 \in \rho(\mathcal{A})$ (The resolvent set of $\mathcal{A}$).

Remark 3 *The operator* $\mathcal{A}$ *is dissipative, ie* $\langle \mathcal{A}U, U \rangle < 0$ *for all* $U = (\varphi_1, \varphi_2, \varphi_3, \varphi_4) \in \mathcal{D}(\mathcal{A})$.

Applying (9), we get

$$\langle \mathcal{A}U, U \rangle = -\frac{\gamma_1}{\rho}\int_\Omega \varphi_2^2 \mathrm{d}x\mathrm{d}y - \frac{\gamma_0}{\rho}\int_{\Gamma_0} \varphi_4^2 \mathrm{d}x < 0.$$

Resolvent Equation:

Given $F \in \mathcal{H}$, *we find* $U \in \mathcal{D}(\mathcal{A})$

$$(\lambda I - \mathcal{A})U = F, \quad \lambda \in \mathbb{C}. \tag{14}$$

In particular, $0 \in \rho(\mathcal{A})$, if and only if, there is

$$U \in \mathcal{D}(\mathcal{A}); \quad -\mathcal{A}U = F,$$

that is,

$$-\varphi_2 = F_1$$

$$-\frac{c}{\rho}\Delta\varphi_1 + \frac{\gamma_1}{\rho}\varphi_2 = F_2$$

$$-\varphi_4 = F_3$$

$$-(\varphi_3)_{xx} + \gamma_0\varphi_4 + c\varphi_2 = F_4,$$

where $F = (F_1, F_2, F_3, F_4)$. By previous observations that there have $U \in \mathcal{D}(\mathcal{A})$. Using the application of Lummer Phillips Theorem [10] [11], we have the following result.

Theorem 1 *The operator* $\mathcal{A}$ *set to* (10) *is the infinitesimal generator of a contraction semigroup* C_0.

Theorem 2 *The* $\mathcal{A}$ *is the infinitesimal generator of a semigroup* C_0 *and verifies* $U_0 \in \mathcal{D}(\mathcal{A}^3)$ *then the solution of* (13) *satisfies*

$$U \in C^1(0,\infty;\mathcal{H}) \cap C^2(0,\infty;\mathcal{D}(\mathcal{A})). \tag{15}$$

4. Asymptotic Behavior

We now show that the energy associated with the system decays exponentially. Multiplying by φ the first equation in (1) and integrating over Ω yields

$$\int_\Omega \varphi_{tt}\varphi \mathrm{d}x\mathrm{d}y - \frac{c}{\rho}\int_\Omega \Delta\varphi\varphi \mathrm{d}x\mathrm{d}y + \frac{\gamma_1}{\rho}\int_\Omega \varphi_t\varphi \mathrm{d}x\mathrm{d}y = 0,$$

equivalently

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega \varphi_t\varphi \mathrm{d}x\mathrm{d}y - \int_\Omega |\varphi_t|^2 \mathrm{d}x\mathrm{d}y + \frac{c}{\rho}\int_\Omega |\nabla\varphi|^2 \mathrm{d}x\mathrm{d}y - \frac{c}{\rho}\int_{\Gamma_0} \frac{\partial\varphi}{\partial\nu}\varphi\sigma + \frac{\gamma_1}{\rho}\int_\Omega \varphi_t\varphi \mathrm{d}x\mathrm{d}y = 0. \tag{16}$$

Observe that

$$-\frac{c}{\rho}\int_{\Gamma_0} \frac{\partial\varphi}{\partial\nu}\varphi\sigma = \frac{c}{\rho}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Gamma_0} \omega\varphi \mathrm{d}x - \frac{c}{\rho}\int_{\Gamma_0} \omega\varphi_t \mathrm{d}x. \tag{17}$$

From the second equation in (1), we obtain

$$-\frac{c}{\rho}\int_{\Gamma_0} \omega\varphi_t \mathrm{d}x = -\frac{1}{\rho}\int_{\Gamma_0} \omega(-\omega_{tt} + \omega_{xx} - \gamma_0\omega)\mathrm{d}x. \tag{18}$$

On the other hand,

$$\frac{1}{\rho}\int_{\Gamma_0} \omega\omega_{tt} \mathrm{d}x = \frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Gamma_0} \omega\omega_t \mathrm{d}x - \frac{1}{\rho}\int_{\Gamma_0} \omega_t^2 \mathrm{d}x. \tag{19}$$

From (17)-(19), we obtain

$$-\frac{c}{\rho}\int_{\Gamma_0} \frac{\partial\varphi}{\partial\nu}\varphi\sigma = \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{\rho}\int_{\Gamma_0} \omega\omega_t \mathrm{d}x + \frac{c}{\rho}\int_{\Gamma_0} \omega\varphi \mathrm{d}x\right] + \frac{1}{\rho}\int_{\Gamma_0} \omega_x^2 \mathrm{d}x - \frac{1}{\rho}\int_{\Gamma_0} \omega_t^2 \mathrm{d}x + \frac{\gamma_0}{\rho}\int_{\Gamma_0} \omega\omega_t \mathrm{d}x. \tag{20}$$

Replacing (20) into (16)

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega \varphi_t\varphi \mathrm{d}x\mathrm{d}y - \int_\Omega |\varphi_t|^2 \mathrm{d}x\mathrm{d}y + \frac{c}{\rho}\int_\Omega |\nabla\varphi|^2 \mathrm{d}x\mathrm{d}y + \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{\rho}\int_{\Gamma_0} \omega\omega_t \mathrm{d}x + \frac{c}{\rho}\int_{\Gamma_0} \omega\varphi \mathrm{d}x\right] \\ &+ \frac{1}{\rho}\int_{\Gamma_0} \omega_x^2 \mathrm{d}x - \frac{1}{\rho}\int_{\Gamma_0} \omega_t^2 \mathrm{d}x + \frac{\gamma_0}{\rho}\int_{\Gamma_0} \omega\omega_t \mathrm{d}x + \frac{\gamma_1}{\rho}\int_\Omega \varphi_t\varphi \mathrm{d}x\mathrm{d}y = 0, \end{aligned} \tag{21}$$

or equivalently

$$\frac{d}{dt}\left[\int_{\Omega}\varphi_t\varphi dxdy+\frac{1}{\rho}\int_{\Gamma_0}\omega\omega_t dx+\frac{c}{\rho}\int_{\Gamma_0}\omega\varphi dx\right]-\int_{\Omega}|\varphi_t|^2 dxdy+\frac{c}{\rho}\int_{\Omega}|\nabla\varphi|^2 dxdy$$
$$+\frac{1}{\rho}\int_{\Gamma_0}\omega_x^2 dx-\frac{1}{\rho}\int_{\Gamma_0}\omega_t^2 dx+\frac{\gamma_0}{\rho}\int_{\Gamma_0}\omega\omega_t dx+\frac{\gamma_1}{\rho}\int_{\Omega}\varphi_t\varphi dxdy=0. \quad (22)$$

Now, since Poincaré inequality we have

$$\left|\frac{\gamma_0}{\rho}\int_{\Gamma_0}\omega\omega_t\right|\le\frac{\gamma_0\lambda_1}{\rho}\left[\int_{\Gamma_0}\omega_x^2\right]^{1/2}\left[\int_{\Gamma_0}\omega_t^2 dx\right]^{1/2}\le\frac{\alpha}{2\rho}\int_{\Gamma_0}\omega_x^2 dx+\frac{\gamma_0\lambda_1}{2\alpha}\int_{\Gamma_0}\omega_t^2, \quad (23)$$

where λ_1 is the Poincaré constant. In a similar way,

$$\left|\frac{\gamma_1}{\rho}\int_{\Omega}\varphi\varphi_t dxdy\right|\le\frac{\theta}{2\rho}\int_{\Omega}|\nabla\varphi|^2 dxdy+\frac{\gamma_1\lambda_1}{2\theta\rho}\int_{\Omega}|\varphi_t|^2 dxdy. \quad (24)$$

From (22), (23) and (24) we have

$$\frac{d}{dt}\left[\int_{\Omega}\varphi_t\varphi dxdy+\frac{1}{\rho}\int_{\Gamma_0}\omega\omega_t dx+\frac{c}{\rho}\int_{\Gamma_0}\omega\varphi dx\right]$$
$$\le\left(1+\frac{\gamma_1\lambda_1}{2c\rho}\right)\int_{\Omega}|\varphi_t|^2 dxdy-\frac{3c}{4\rho}\int_{\Omega}|\nabla\varphi|^2 dxdy+\frac{1}{\rho}\int_{\Gamma_0}\omega_t^2 dx-\frac{1}{2}\int_{\Gamma_0}\omega_x^2 dx. \quad (25)$$

We define the operator

$$\mathcal{L}(t):=nE(t)+\int_{\Omega}\varphi_t\varphi dxdy+\frac{1}{\rho}\int_{\Gamma_0}\omega\omega_t dx+\frac{c}{\rho}\int_{\Gamma_0}\omega\varphi dx,\quad n\in\mathbb{N}. \quad (26)$$

Differentiating (26) and using (8) we obtain

$$\frac{d}{dt}\mathcal{L}(t)=n\frac{d}{dt}E(t)+\frac{d}{dt}\left[\int_{\Omega}\varphi_t\varphi dxdy+\frac{1}{\rho}\int_{\Gamma_0}\omega\omega_t dx+\frac{c}{\rho}\int_{\Gamma_0}\omega\varphi dx\right]$$
$$\le\left[n\frac{\gamma_1}{\rho}-\left(1+\frac{\gamma_1\lambda_1}{2c\rho}\right)\right]\int_{\Omega}|\varphi_t|^2 dxdy-\frac{3c}{4\rho}\int_{\Omega}|\nabla\varphi|^2 dxdy$$
$$-\left[n\frac{\gamma_0}{\rho}-\left(1+\frac{\gamma_0\lambda_1}{2\rho}\right)\right]\int_{\Gamma_0}\omega_t^2 dx-\frac{1}{2}\int_{\Gamma_0}\omega_x^2, \quad (27)$$

Considering *n* large enough, we can obtain a constant *C* such that

$$\frac{d}{dt}\mathcal{L}(t)\le -CE(t). \quad (28)$$

On the other hand, using Poincaré, we can obtain

$$\left|\int_{\Omega}\varphi\varphi_t dxdy\right|\le\frac{c_0\delta\lambda_1}{2}\int_{\Omega}|\nabla\varphi|^2 dxdy+\frac{1}{2\delta}\int_{\Omega}|\varphi_t|^2 dxdy. \quad (29)$$

In a similar way

$$\left|\frac{c}{\rho}\int_{\Gamma_0}\omega\varphi dx\right|\le\frac{c\lambda_1}{\rho}\left(\int_{\Gamma_0}\omega_x^2\right)^{1/2}\left(\int_{\Gamma_0}|\varphi|^2 dx\right)^{1/2}\le\frac{c\lambda_1}{2\rho}\left(\theta\int_{\Gamma_0}\omega_x^2 dx+\frac{1}{\theta}\int_{\Gamma_0}|\varphi|^2 dx\right) \quad (30)$$

Moreover, from trace

$$\int_{\Gamma_0}\varphi^2 dx\le\int_{\Gamma}\varphi^2 dx\le c_3\int_{\Omega}|\nabla\varphi|^2 dxdy. \quad (31)$$

Replacing (31) into (30) we have

$$\left|\frac{c}{\rho}\int_{\Gamma_0}\omega\varphi\right| \le \frac{c\lambda_1\theta}{2\rho}\int_{\Gamma_0}\omega_x^2\mathrm{d}x + \frac{c\lambda_1 c_3}{2\rho\theta}\int_{\Omega}|\nabla\varphi|^2\,\mathrm{d}x\mathrm{d}y. \tag{32}$$

From (23), (29), (32) and (26) we can to claim that there is a constant κ_0 and κ_1 such that

$$\kappa_0 E(t) \le \mathcal{L}(t) \le \kappa_1 E(t), \tag{33}$$

leading to decay exponentially energy

$$E(t) \le c_2 E(0)\mathrm{e}^{-\kappa t}, \quad \forall\, t > 0. \tag{34}$$

where $\kappa = C\kappa_0$. The result follows.

Remark 4 *In the case of* $\gamma_1 = 0$ *can be also said that a power decays exponentially.*

The above results support the conclusion.

Theorem 3 *If* $(\varphi_0, \varphi_1, \omega_0, \omega_1) \in \mathcal{D}(\mathcal{A})$ *and* $\gamma_0 \ne 0$ *then the solution* (φ, ω) *of the hybrid system* (1) *decays exponentially over time.*

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