Scientific Research Publishing

Applied Mathematics, 2024, 15, 499-507
https://www.scirp.org/journal/am
ISSN Online: 2152-7393
ISSN Print: 2152-7385

Mathematical Modeling of Possibility Markov Chains by Possibility Theory

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How to cite this paper: Uemura, Y., Kazuhisa, T. and Kita, K. (2024) Mathematical Modeling of Possibility Markov Chains by Possibility Theory. *Applied Mathematics*, **15**, 499-507.
https://doi.org/10.4236/am.2024.158031

Received: July 6, 2024
Accepted: August 5, 2024
Published: August 8, 2024

Abstract

Statistical regression models are input-oriented estimation models that account for observation errors. On the other hand, an output-oriented possibility regression model that accounts for system fluctuations is proposed. Furthermore, the possibility Markov chain is proposed, which has a disidentifiable state (posterior) and a nondiscriminable state (prior). In this paper, we first take up the entity efficiency evaluation problem as a case study of the posterior non-discriminable production possibility region and mention Fuzzy DEA with fuzzy constraints. Next, the case study of the ex-ante non-discriminable event setting is discussed. Finally, we introduce the measure of the fuzzy number and the equality relation and attempt to model the possibility Markov chain mathematically. Furthermore, we show that under ergodic conditions, the direct sum state can be decomposed and reintegrated using fuzzy OR logic. We had already constructed the Possibility Markov process based on the indifferent state of this world. In this paper, we try to extend it to the indifferent event in another world. It should be noted that we can obtain the possibility transfer matrix by full use of possibility theory.

Keywords

Possibility Markov Chain, Ergodic Condition, Direct Sum State, Prior Indiscriminate State, Posterior Discriminatory State

1. Introduction

A possibility regression model was proposed by Tanaka *et al.* Furthermore [1], Sakawa *et al.* formulated a multi-objective possibility regression model that introduced the concepts of possibility and necessity [2]. Typical application considerations include the physical test evaluation problem [3] and the entity efficiency evaluation problem [4]. For the entity efficiency evaluation problem, a method (Fuzzy DEA) that can simultaneously evaluate both input and output orientation has been proposed [5]. In this paper, we first introduce the possibility production function and Fuzzy DEA as examples of possibility (logarithmic) linear models. Next, we take the transition from totalitarianism to democracy, the transition from prewar to postwar, as an interpretation of artificial possibility Markov chains [6]-[8]. Finally, a simple possibility state transition diagram is used to explain the non-discriminatory event [9]. Since the posterior non-discriminatory state [10] cannot be represented by a simple possibility state transition diagram, a conceptual diagram of the derivation process for this state is presented [11]. It should be noted that the possibility Markov chain of a direct sum fuzzy event under ergodic conditions can be decomposed and reintegrated by applying fuzzy OR logic. From view viewpoint of the probability theory, our study can be regarded as the Markov Chain upon fuzzy/vague event mapped from the state of nature by the membership function [12] [13]. It should be noted that this possibility Markov Chain has the fuzziness upon fuzzy/vague event. On the other hand, the probability Markov Chain has the fuzziness into the fuzzy buffer.

2. Application Considerations for Entity Efficiency Evaluation Problems

The entity efficiency evaluation problem was once analyzed using Cobb-Douglas type production functions. Subsequently, envelopment analysis methods (e.g. CCR, BCC) were proposed [14]. These are input-oriented analysis methods. Subsequently, the possibility production function, that can perform not only input-oriented but also output-oriented analysis at the same time, was considered. Furthermore, a fusion of the envelopment analysis method and the possibility production function was attempted, and it was shown that it is possible to make new considerations. A fusion model of various analytical models of the envelopment analysis method is also discussed [5].

Focusing on CCR and BCC in the envelopment analysis method, the (posterior) indistinguishable production region can be automatically derived after analyzing each of them separately. First, the posterior indistinguishable production region is specified by fuzzy constraints of a linear function. Next, the fuzzy efficiency is specified by the fuzzy objective of the linear function with the CCR efficiency value and the BCC efficiency value. Finally, by adopting a maximization decision, it becomes an ordinary linear programming problem and the fuzzy optimal efficiency solution can be easily derived [15].

3. Case Study of a Possible Markov Chain

A model in which the transition probability in a normal Markov chain [16] is extended to a fuzzy number, and also in which this fuzzy number transitions state as a fuzzy variable over time, is called a possibility Markov chain [17] [18]. This is also closely related to fuzzy relations and fuzzy graphs [1] [2]. Let us note that possibility Markov chains have indistinguishable states that occur automatically after the fact and indiscriminate states that can be set subjectively by the decision-maker in advance. For example, as an interpretation, in the Pacific War, confusion (ex-post disidentifiable state) occurred after the war ended, and an artificial policy was taken to set up a non-discriminatory state in advance to gradually transition from totalitarianism to democracy, and sequentially transition the (ex-post) disidentifiable state to a crisp state. This is a buffer allocation problem from a systems science perspective. Let us note that this (ex-ante) non-discriminatory event can be subjectively set up as a triangular possibility distribution using the three-point estimation method. In this paper, in addition to This World, we assume the existence of Other World and Another World for convenience in theoretical considerations [19] [20]. These Worlds are assumed to be transitive in a possible Markov chain. The boundaries between these Worlds are obviously blurred, Vague Events (State) on the State of Nature [21]. For example, the transition from This World to Another World can be acute or chronic. In the acute case, This World and Another World are considered to be a direct sum (note: in fuzzy theory, they are considered to be inversions), and the possibility transition matrix is defined by the possibility principal factor rotation matrix [7] [8]. On the other hand, in the chronic case, This World and Another World are not a direct sum, and the possibility transition matrix is specified by the possibility skew factor rotation matrix [9]. Let us note that in the chronic case, a posteriori indistinguishability condition occurs.

4. Mathematical Modeling Efforts for Possibility Markov Chains

In this chapter, regarding fuzzy state transitions in a possible Markov chain, we focus on the indexes (1), (2), (3), and (4) of the fuzzy numbers and (5), (6), (7), and (8) of the equality relation in the possibility theory to construct a possibility state transition model. Here, the fuzzy events are M and N [20],

Their possible distribution is, $\mu_M(U), \mu_N(V)$ are.

$$POS(M \geq N) \triangleq \sup_{U \geq V} \min(\mu_M(U), \mu_N(V)) \tag{1}$$

$$POS(M > N) \triangleq \sup_{U} \inf_{V \geq U} (\mu_M(U), \mu_N(V)) \tag{2}$$

$$NES(M \geq N) \triangleq \inf_{U} \sup_{V \leq U} \max(1 - \mu_M(U), \mu_N(V)) \tag{3}$$

$$NES(M > N) \triangleq 1 - \sup_{U \geq V} \min(\mu_M(U), \mu_N(V)) \tag{4}$$

$$POS(M = N) \triangleq \sup_{u \in R^1} \min(\mu_M(u), \mu_N(u)) \tag{5}$$

$$NES(M \subset N) \triangleq \inf_{u \in R^1} \max(1 - \mu_M(u), \mu_N(u)) \tag{6}$$

$$NES(M \supset N) \triangleq \inf_{u \in R^1} \max(\mu_M(u), 1 - \mu_N(u)) \tag{7}$$

$$NES(M = N) \triangleq \min(NES(M \subset N), NES(N \subset M)) \tag{8}$$

4.1. Possible Markov Chain of Direct Sum States

When fuzzy event M and fuzzy event N are in direct summation, the possibility state transition diagram of the possibility Markov chain is derived as shown in Figure 1.

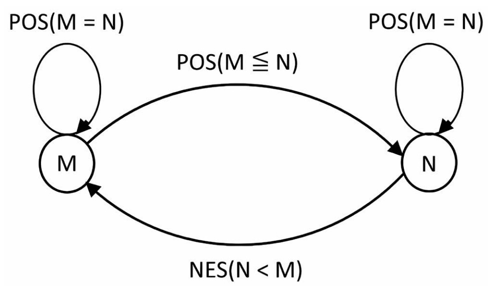


Figure 1. Type-1 fuzzy possibility markov chain.

4.2. Possibility Markov Chain with Prior Nondiscriminatory State Fe

The decision maker subjectively specifies the prior nondiscriminatory event Fe in terms of L-R type fuzzy numbers using the 3-point estimation method by lot drawing. After specifying the prior nondiscriminatory event Fe, fuzzy event M and fuzzy event N are automatically identified after the fact to be in direct summation with Fe (Figure 2). The possibility state transition diagrams of the posterior possibility Markov chain are shown in Figures 3(a) and Figures 3(b). The Type 2 Fuzzy possibility Markov chain is obtained by a two-step transition (Figures 3(c)).

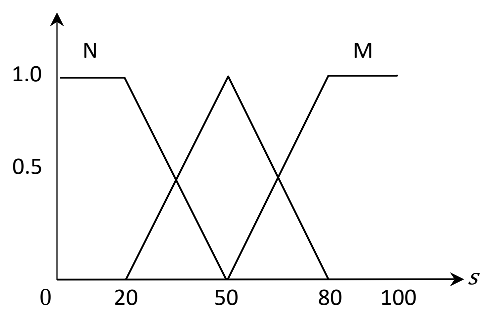
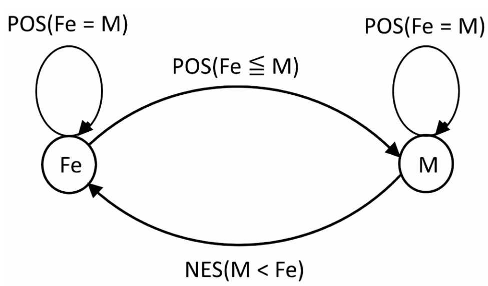
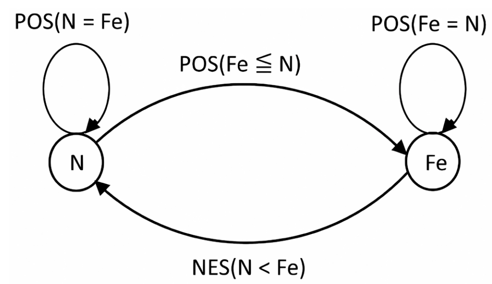


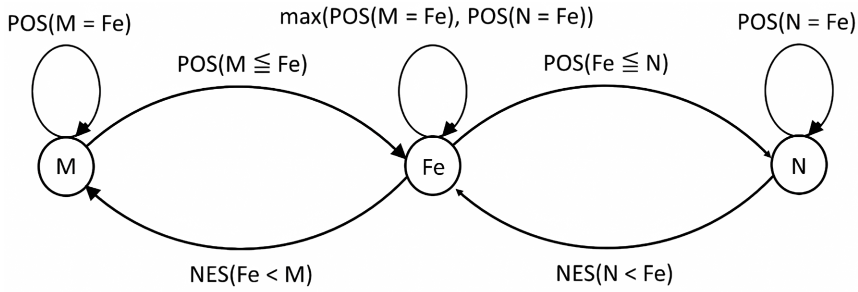
Figure 2. Prior membership function of nom-discriminable events.



(a)



(b)



(c)

Figure 3. (a) Step 1; (b) Step 2; (c) Type 2 fuzzy possibility markov chain.

Here, we propose a possible transition from M to Fe followed by a possible transition from Fe to N in two steps. Of course, a straight transition from M to N without Fe is also possible.

4.3. Possibility Markov Chain with Posterior Indistinguishable State Fe

The posterior disambiguation state Fe after a given fuzzy event M and fuzzy event N is difficult to draw in a simple possibility state transition diagram because it has a conditional disambiguation state. Here, we show a possibility state transition diagram in which fuzzy event M and fuzzy event N encompass the posterior disidentifiable event Fe using the index of equality relation. First, as shown in **Figure 4**, the posterior disidentifiable event Fe is automatically specified to be a direct sum of fuzzy event N and fuzzy event N with each other.

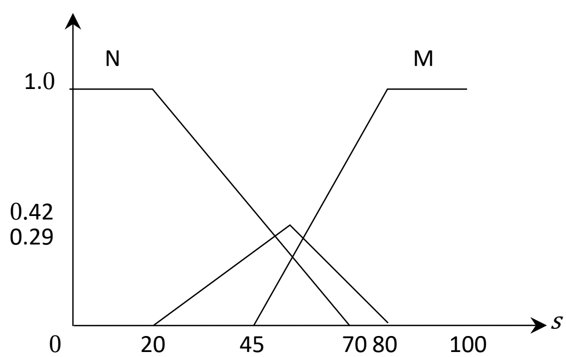


Figure 4. Posterior membership function of disdentifiable events.

The state transitions of a posteriori disambiguated states are decomposed by zones ((s < 20), (20 ≤ s < 45), (45 ≤ s < 70), (70 ≤ s < 80), (80 ≤ s)) on the natural state S and integrated by a weighted sum over each state transition matrix by the amount of possible information per zone is considered as [10]. Thus, using the index of equality relation, the possibility state transition diagrams (**Figures 5(a)-(d)**) are derived for the posterior indistinguishable states where fuzzy events are not direct sum states in the prior.

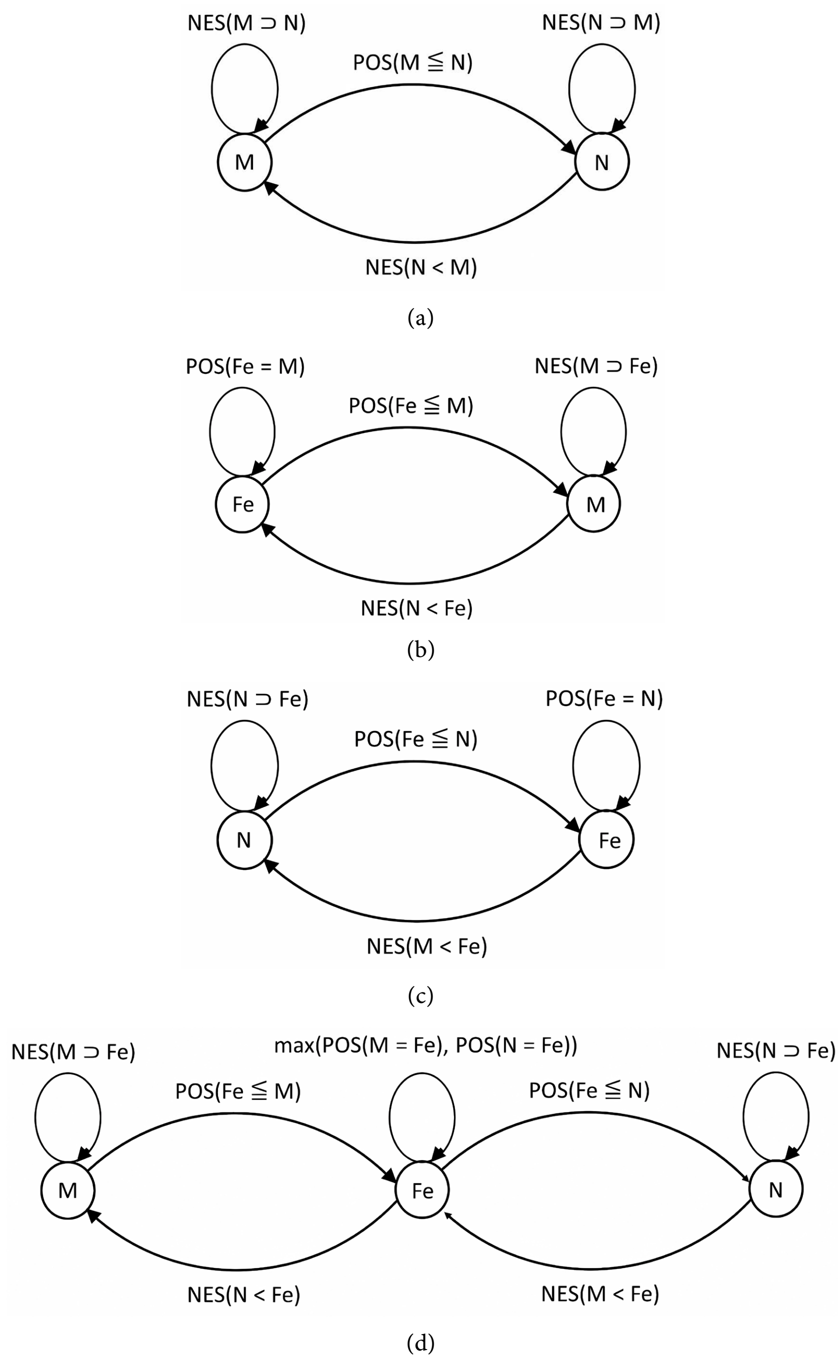


Figure 5. (a) Type 1 vague possibility markov chain; (b) Step 1; (c) Step 2; (d) Type 2 vague possibility markov chain.

4.4. Relationship to Buffer Placement Problem

As shown in the possibility state transition diagram in the previous section, we can see that direct sum state fuzzy events under ergodic conditions can be decomposed and then reintegrated by applying fuzzy OR logic. Let us note that this possibility nondiscriminatory event-setting problem is closely related to the system science buffer allocation problem. In the case of the buffer allocation problem, it has been shown that subsystems may be reintegrated if the rate of variation among the decomposed subsystems is constant [23]. Let us note that since the fuzzy events are in direct summation, the area center of gravity of each decomposed fuzzy system does not change from the fuzzy system before decomposition. In the possibility nondiscriminatory event setting problem, the fuzzy events are in direct summation, while in the buffer allocation problem of systems science, the stochastic events are in exclusion. In the mathematical model, the only difference is that the membership function is orthogonal and the sum of the distribution function in a stochastic event. However, if the membership function is considered as a possibility distribution, it is a natural extension of the likelihood function because it can simultaneously consider possibility and necessity. In addition, the ergodic condition is added in this paper to illustrate the mathematical modeling, although it is a strict condition.

5. Conclusion

First, the biggest problem with Type 2 possibility Markov chains is that there are cases in which a quadratic transformation is performed without going through a first-order transformation. In this case, we are currently examining whether it is appropriate to simply regard the quadratic transformation as a linear transformation, and just use a Type 1 possibility Markov chain to deal with the situation. The next issue to be addressed in the future is the establishment of a pre-setting method for the non-discriminatory state. Here, the three-point estimation method by lottery is the mainstream method, but note that the three-point estimation method is valid and effective when it is set subjectively under compulsion. Regarding the establishment of a new entity efficiency evaluation method, a method that involves ex-ante non-discriminatory producible regions, as well as ex-post non-discriminatory producible regions, is being devised. Our next research plan is to analyze the Bayesian transition from the ex-post to the ex-ante time series using possibility Markov chains. Finally, an example of application of the possibility Markov chain is the state transition from the Lake Wave Case to the Bay Wave Case. The main purpose of introducing a prior non-discriminatory state is to make the transition smooth. Note that the Type 2 Fuzzy possibility Markov chain corresponds to the steady state (calm) case, while the Type 2 Vague possibility Markov chain corresponds to the state transition in rough seas (stormy). It is very difficult problem to set up the membership function from the state of nature to fuzzy/vague event. We had already constructed “vague sets and theory”. For simplicity, we can set up the membership function by the isosceles triangle. However, using triangular fuzzy numbers for simplicity simplifies the computation and avoids computational complexity. In addition, the method proposed in this study may have potential applications in the control of drones and other vehicles in driving in different environmental worlds, such as deep sea, lakes, and land.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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