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Pareto Optimization as the Basic for Selecting Robotic Mechanic Assembly Technologies

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Abstract

The task of selecting robotic mechanic assembly technologies (RMAT) is considered as a multi-criteria optimization task, which in this formulation is solved on the set of previously obtained solutions regarding the selection of RMAT. The purpose of the paper is to increase the efficiency of technological preparation of robotic mechanical assembly production of machine and instrument engineering due to a new approach to the selection of RMAT using Pareto optimization and the peculiarities of the selection task formulation. The novelty consists in the further development of a science-based approach to solving multi-criteria selection task, based on the first proposed formalisms of the specified process, which reflect the peculiarities of the selection task formulation, its meaningful essence and the content of the Pareto optimization method. The practical value of the research lies in the proposed engineering-acceptable approach to solving applied multi-criteria selection tasks on the example of RMAT selection, which is invariant to the statement of the selection task, the dimension of the task, and its meaningful essence. The methods of discrete optimization, fuzzy multi-criteria selection of alternatives, and the Pareto optimization method were used for the research. The main results of this work consist of the development of formalisms and the demonstration of the efficiency of the proposed approach for the applied task of RMAT selection. The peculiarity of the developed approach is the combination of Pareto optimization, performed on a discrete set of local criteria. Directions for further research are presented.

Keywords

Multicriteria Optimization, Efficiency, Fuzzy Multicriteria Selection of Alternatives

1. Introduction

A characteristic feature of many modern industries, including mechanical assembly industries of machine and instrument engineering, is the use of such valuable technological equipment as industrial robots (IR). A characteristic feature of IR manufacture is the relative stabilization of their annual production and introduction in the post-war period in various industries. According to the International Federation of Robotics (IFR), today this figure makes 5% with an obvious predicted trend of its growth to 7% by 2026 [1].

This indicates the necessity and expediency of conducting research on certain content and setting selection tasks in the field of industrial robotics with the general goal of increasing the efficiency (in the accepted sense) of robotic mechanical assembly production (RMAP).

As the practice of designing any technical systems and technologies implemented in them shows, the number of obtained solutions usually represents a certain finite set of them [2]. Such situations often arise during the design/synthesis of robotic mechanical assembly technologies (RMAT) in various branches of modern automated production, including robotic mechanical assembly production (RMAP) [3] [4]. At the same time, it is one of the mandatory substantive components of the technological training of robotic mechanical assembly production (TPp RMAP). The effectiveness of the latter in its various manifestations and the efficiency of the operation of similar technological structures depend on how scientifically justified decisions are made at the TPg RMAP. Therefore, in the context of the above-mentioned, the scientific justification of the selection process when solving the task of choosing the optimal RMAT from the set of predefined known solutions is undoubtedly important.

The presence of more than one technological solution, for example, in relation to the number of generated RMATs, creates the problem of ordering the obtained decisions, that is, determining the optimal suitable decision. In this case, it is the choice of the optimal RMAT. In terms of content, the above-mentioned is related to the problems of multi-criteria optimization in the part of the tasks for fuzzy multi-criteria alternatives selection (FMCAS) [5].

The general features of solving FMCAS tasks are as follows: the tasks refer to multi-criteria optimization tasks; the correct final solution of any of these problems is unknown a priori and a posteriori; the content of the choice is the optimization process, which in the context of this RMAT selection task content is precisely the process of arranging the input selection elements.

Along with the above-mentioned general features of FMCSA tasks, the feature of the selection task considered in this paper is the following: the input alternatives are the finite input set of known solutions for the selection of the RMAT, previously obtained according to the developed original methods; each element in their DSLC, *i.e.* RMAT manifestations, has a different physical nature, content and different measurement scales; ordered sets of local criteria obtained by different methods contain different sequences of local criteria in previously obtained solutions.

The content of the RMAT selection process in the formulation considered here is reduced to the selection of the best solution from the previously obtained, *i.e.* known solutions. They are obtained automatically using the original software product FMCSA (Fuzzy Multi-criteria Selection of Alternatives). It has been tested and confirmed its performance in the automated solution of FMCSA problems in the part of multi-criteria RMAT selection for cases based on the respective comparisons with the worst alternative and with the worst criterion (*WMS*) [6] and with the best (quasi-best) alternative and with the most important criterion (*QBMS*) [7]. Other input decision data, which are input alternatives, are obtained for the conditions of the corresponding comparisons based on such mean parameters as: arithmetic mean (*MMS*:*A*), median (*MMS*:*M*), root mean square (*MMS*:*S*) and geometric mean (*MMS*:*G*).

All the obtained results of the alternatives preliminary selection, *i.e.* the final set of RMAT, were obtained based on the same results of a rigorous expert survey. At the same time, evaluations by experts were made without repetition, the set of experts $E = \left(Ei \mid i = \overline{1,n}\right)$ on the discrete set of local criteria $S = \left(Sj \mid j = \overline{1,m}\right)$ with the number of experts $n = 10$ and number of local selection criteria, which are elements of DSLC, $n = 12$. Local criteria, which are manifestations of RMAT, in this case are numbered and appropriately marked local criteria: $S1 = Gm$ (Geometric); $S2 = Kn$ (Kinematic); $S3 = Dn$ (Dynamic); $S4 = Ct$ (Control); $S5 = En$ (Energy); $S6 = Tr$ (Trajectory); $S7 = \tau(Q)$ (Time (Productivity)); $S8 = Rl$ (Reliability); $S9 = Ec$ (Economy); $S10 = Ac$ (Accuracy); $S11 = Fc$ (Force); $S12 = Fopt$ (the component which is determined by other types of optimization criteria (e.g. technical and economic)). The experts assessed the importance of each criterion based on the following: the least important criterion from the DSLC was assessed at 1 point, and the most important at 12 points. In the process of calculations, methodologically determined fuzzy assessments of the importance of each of the local selection criteria (elements of the DSLC) were found, the sorting (ordering) of which from the largest to the smallest determined the final decision on the set of DSLC.

2. Literature Review

In general, multicriteria optimization tasks are solved by various methods, techniques, and approaches. Below, we analyze some information sources that could be used to some extent in solving the applied problem of selecting a RMAT in the formulation described above.

The Analytic Hierarchy Process (AHP) [8] [9] [10] [11], which is also known as the Saaty's pairwise comparison method, is widely known. The method consists of decomposing the problem into simpler components and prioritizing the evaluated components step by step using pairwise comparisons. The first stage identifies the most important elements of the problem. The second is the best way to verify observations, test and evaluate the elements. The third step is to develop a way to apply the solution and evaluate its quality. The entire process is subject to verification and redefinition until it is certain that the process has captured all the important characteristics needed to represent and solve the problem. The process can be carried out over a sequence of hierarchies. In this case, the results obtained in one hierarchy are used as input to the next.

Saaty calls the simplest hierarchy as the dominant hierarchy and identifies three levels in it: the top level is the goal (or goals), the middle level is the criteria, and the bottom level is the list of alternatives. The process can be carried out over a sequence of hierarchies. In this case, the results obtained in one of them are used as input data for the next one.

A significant disadvantage of the AHP is its rather high labor intensity both in terms of human involvement in determining the elements of matrices with a certain level of subjectivity, systematic understanding of the problem in the formation of hierarchies of different levels, and matrix calculations. Each survey (judgments) matrix at different levels has the property of inverse symmetry, *i.e.*, it contains only integers on the nine-point Saaty scale, or their inverse values, which are used in further matrix processing. In matrices, the lower-level elements (alternatives, options) are compared in pairs with respect to the criteria, and the criteria are compared with respect to the goal. The process of selecting the best alternative from a set of pre-selected solutions is characterized by significant labor intensity, if such a task were solved.

It is obvious that the hierarchical, multi-level, and in some cases cyclical nature of calculations when using the ANP method is characterized by a large amount of subjectivism. Therefore, the final result cannot guarantee obtaining the optimal solution in the accepted sense, although this does not contradict the essence of the FMCSA in terms of the a priori and a posteriori uncertainty of the correct solution availability.

Multiple-criteria decision-making (MCDM) has been widely used to solve problems similar in content to the problem of this work, namely: PROMETHEE II (Preference Ranking Organization Method for Enrichment Evaluation II), TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) and GRA (Gray Relational Analysis).

The PROMETHEE II method [12] is widely used for multi-disciplinary tasks aimed at implementing FMCSA. Its functioning is characterized by the use of independent and conflicting criteria and the selection criteria weights known in advance. The method is based on the results of alternative pair comparisons, taking into account their deviations, which (alternatives) show compliance with each criterion. The PROMETHEE II method provides a complete ranking of the final alternatives set and thus turns the unordered set of input alternatives into an ordered one (from worst to best). The PROMETHEE II method allows obtaining a full rating both on a set of local selection criteria and on a set of pre-selected results obtained by different methods.

The TOPSIS method [13] is based on the following principle (concept): the chosen alternative should have the shortest geometric distance from the positive ideal solution and the longest geometric distance from the negative solution. Here, a positive ideal solution is a hypothetical solution for which all values of the criteria correspond to the maximum values, and a negative ideal solution is a hypothetical solution for which all values of the criteria correspond to the minimum values in the DSLC. Therefore, the TOPSIS methodology provides a solution that is not only the closest to the hypothetical best, but also the furthest from the hypothetical worst. Thus, TOPSIS is a way of assigning ranks based on the weight and influence of input factors on the final decision. The end product of using this method is the ordering of input criteria.

GRA [14] is an MCDM method that uses a specific notion of information. It defines situations in which there is no information as black, and situations with perfect information as white. But in real problems of choice, the situation with such information occurs extremely rarely. Situations between these opposites contain scattered knowledge (partial information) and are therefore described as gray, hazy or unclear (it accounts for the name of the method). This method is used to resolve relationships between multiple characteristics or parameters by optimizing gray relational classes. GRA uses the generation of gray relations and calculates the coefficients of gray relations to solve uncertain systematic problems in the status of only partially known information. The grey relation coefficient expresses the relationship between desired and actual outcomes, and the gray relational class is simultaneously calculated and used to select and rank alternatives.

In general, the PROMETHEE II, TOPSIS, and GRA methods have the ordering of the initially unordered selection criteria as a final result. In addition, they allow the use of the obtained results for further generalizing studies. The selection of FMCSA by PROMETHEE II, TOPSIS and GRA methods was automated by the authors in the FMCSA software.

One of the methods for solving such tasks is Pareto optimization [15] [16]. Other terms that are often used in this context are the following: Pareto-optimum, Pareto principle, Pareto-efficient distribution, Pareto's highest economic point, Pareto efficiency, Pareto optimality, etc. In general, this term(s) describes the state of a system in which the value of each criterion describing the state of a system cannot be improved without degrading the values of other elements of that system.

Thus, the Pareto principle in the context of the task being solved makes it possible to reduce the number of possible alternatives, that is, RMATs affecting the final solution of such problems, and to exclude deliberately non-competitive RMATs from consideration. And the final choice is made on the basis of additional information, taking into account the preference of the making decision (MD).

In general, Pareto principle applies to a wide class of multi-criteria selection tasks, in which the relations of advantages the DM, as well as the set of possible solutions, are unclear.

In a simplified form, it can be expressed as follows: in those tasks of multicriteria selection, which conform to certain axioms, the fuzzy selection should be made only inside the Pareto set. It requires the formation of the Pareto set itself. The following statements are valid for the Pareto set, on which the actual FMCS of RMAT is performed [17] [18] [19]:

- the Pareto set includes alternatives formed by the input set of the RMAT preliminary selection on the DSLC set, which are no worse than others by at least one criterion;
- the Pareto set consists of non-comparable decision alternatives, or only one dominant decision alternative;
- The Pareto set is called the set of non-improvable solutions.

The specified features of Pareto optimization largely determine the possibility of using this particular optimization to solve the given selection task.

Thus, the set of obtained solutions with ordered fuzzy estimates for each local criterion from the discrete set serves the basis for making a Pareto set from a certain number of non-dominating alternatives, that is, one or more of the known input solutions-alternatives, where one should search Pareto-optimal solution.

It should be noted that Pareto optimization is used in solving a number of applied problems. The use of Pareto-optimal choice in forming a portfolio of IT project orders is considered. As a result of the Pareto set construction, ordered versions of this process scenarios were obtained according to two criteria, namely profitability and prospects, in which it is impossible to determine the benefit. An obvious advantage of the work is obtaining scenarios options in the order of alternatives importance decreasing with the dimensions of this problem $n \times m = 10 \times 2$, where n is the number of alternatives and m is the number of selection criteria.

The use of Pareto optimization when choosing software development technologies is also known [20]. The problem has dimensions of $n \times m = 6 \times 8$. At the same time, a five-point scale of each criterion's importance assessments and a non-strict expert survey were used. The result of solving this task is also the construction of the final Pareto set in the number of two alternatives, which is much fewer than the initial number of alternatives. It is noted that a possible way of obtaining a single solution, that is, the presence of one element in the Pareto set, can either repeat the procedure for identifying the relative importance of criteria or involve an additional criterion and conduct a similar analysis.

In works [21] [22] [23] [24] the selection process is performed according to local selection criteria, and not on a set of already obtained solutions according to the same criteria, which is one of the distinctive features of the problem considered in this work.

Thus, the authors aim to investigate the possibility of selecting Pareto-optimal RMATs from a set of pre-selected solutions obtained by the above developed methods, the content of which is based on various computational procedures for fuzzy multi-criteria selection of alternatives. The proposed scientifically based approach to selecting Pareto-optimal RMATs on a set of known alternative solutions is an obvious component of TPg RMAP. Its content and use are aimed at increasing the validity of the decisions made in this process and minimizing the time for RMAT selection. It obviously increases the efficiency of TPg RMAP and the value and importance of the applied aspect of Pareto optimization.

3. Research Methodology

Taking into account the above-mentioned (see point 1), the input alternatives for this RMAT selection task are the following solutions-alternatives, on the set of which the Pareto optimal RMAT selection is actually performed, the designations of which are clear from the above presented:

$$
\begin{aligned}
QBMS &= \left\langle \frac{Kn=S2}{0.19354},\frac{En=S5}{0.19015},\frac{Tr=S6}{0.18514},\frac{Gm=S1}{0.18074},\frac{Rl=S8}{0.18029},\frac{Ct=S4}{0.17900},\right.\\
&\qquad \left.\frac{\tau(Q)=S7}{0.17603},\frac{Ac=S10}{0.17118},\frac{Fc=S11}{0.16525},\frac{Fopt=S12}{0.16415},\frac{Ec=S9}{0.15259},\frac{Dn=S3}{0.15139}\right\rangle;\\
MMS:A &= \left\langle \frac{Ct=S4}{0.21844},\frac{Fopt=S12}{0.21839},\frac{Rl=S8}{0.21638},\frac{En=S5}{0.21630},\frac{Gm=S1}{0.21613},\frac{Tr=S6}{0.21562},\right.\\
&\qquad \left.\frac{Kn=S2}{0.20353},\frac{Fc=S11}{0.20353},\frac{Ec=S9}{0.20347},\frac{Ec=S10}{0.20255},\frac{\tau(Q)=S7}{0.20152},\frac{Dn=S3}{0.18836}\right\rangle;\\
MMS:M &= \left\langle \frac{En=S5}{0.22376},\frac{Fopt=S12}{0.22370},\frac{Ct=S4}{0.22342},\frac{Gm=S1}{0.21961},\frac{Tr=S6}{0.21910},\frac{Rl=S8}{0.21910},\right.\\
&\qquad \left.\frac{Kn=S2}{0.21599},\frac{Ec=S9}{0.20866},\frac{Fc=S11}{0.20838},\frac{Ac=S10}{0.20417},\frac{\tau(Q)=S7}{0.19567},\frac{Dn=S3}{0.19307}\right\rangle;\\
MMS:S &= \left\langle \frac{Fopt=S12}{0.21737},\frac{Ct=S4}{0.21689},\frac{Rl=S8}{0.21638},\frac{En=S5}{0.21589},\frac{Gm=S1}{0.21576},\frac{Tr=S6}{0.21567},\right.\\
&\qquad \left.\frac{Kn=S2}{0.21530},\frac{Ec=S9}{0.20248},\frac{Fc=S11}{0.20202},\frac{Ac=S10}{0.20152},\frac{\tau(Q)=S7}{0.20090},\frac{Dn=S3}{0.18689}\right\rangle;\\
MMS:G &= \left\langle \frac{Ct=S4}{0.22011},\frac{Fopt=S12}{0.21937},\frac{Rl=S8}{0.21853},\frac{En=S5}{0.21702},\frac{Gm=S1}{0.21684},\frac{Tr=S6}{0.21659},\right.\\
&\qquad \left.\frac{Kn=S2}{0.21594},\frac{Fc=S11}{0.20515},\frac{Ec=S9}{0.20202},\frac{Ac=S10}{0.20362},\frac{\tau(Q)=S7}{0.20214},\frac{Dn=S3}{0.18994}\right\rangle;\\
WMS &= \left\langle \frac{Kn=S2}{0.12432},\frac{En=S5}{0.10399},\frac{Gm=S1}{0.09038},\frac{Dn=S3}{0.07654},\frac{\tau(Q)=S7}{0.07085},\frac{Ct=S4}{0.06701},\right.\\
&\qquad \left.\frac{Ac=S10}{0.06697},\frac{Rl=S8}{0.06450},\frac{Tr=S6}{0.05102},\frac{Ec=S9}{0.03908},\frac{Fopt=S12}{0.03616},\frac{Fc=S1}{0.03433}\right\rangle.
\end{aligned}
\tag{1}
$$

Each of the input alternatives in expression (1) is ordered by the fuzzy scores (this is the denominator of the conditional fraction in (1)) of each of the local selection criteria (this is the numerator of the conditional fraction in (1)).

Thus, the Pareto-optimal RMAT selection task is performed on the set represented by expression (1). Each of its elements, *i.e.*, each decision-alternative, in turn, is the result of ordering the fuzzy estimates of each element from the DSLC, obtained according to the methods indicated above. In other words, it is necessary to choose the best solution from previously obtained alternative solutions from expression (1). In such a setting, the specified problem was not solved before. According to its content and essence of the selection process, it can be attributed to the FMCSA tasks [3].

Essentially, the task of choosing Pareto-optimal RMAT, taking into account the above-mentioned peculiarity of its formulation, means choosing a Pareto-optimal RMAT from the set (1) based on the appropriate procedures for analyzing fuzzy estimates ${}^{k}a_{S\langle j\rangle}$ of each S_j-th parameter, which is performed on the entire set of initial solutions-alternative.

Here, k is an index indicating the type of the corresponding input solution for the k-th alternative, $k=(QBMS, MMS:A, MMS:M, MMS:S, MMS:G, WMS)$.

Given the above mentioned, the RMAT selection process using Pareto optimization at a certain level of abstraction can be represented by the following formalized expression:

$$\varphi:\left({}^{k}S_{\langle j\rangle}\to {}^{k}S_{(j)}\right)\to\left({}^{p}S_{(j)}\to {}^{p}S_{\langle j\rangle}\right), \tag{2}$$

where φ is a set of decision-making functions (hereinafter the same term or simply functions is used), $\varphi=\left(\varphi_i \mid i=\overline{1,n_\varphi}\right)$, which is a methodologically determined ordered set of elementary φ_i-th decision-making functions with a total number of n_φ; ${}^{k}S_{\langle j\rangle}$ is the set of input solutions-alternatives according to expression (1); p is the upper left (further on, the lower left) index as an indicator of the set of input solutions-alternatives entry into the formed Pareto set, which is also denoted as p, but in uppercase: p = {*QBMS*, *MMS*:*A*, *MMS*:*M*, *MMS*:*S*, *MMS*:*G*, *WMS*}; brackets {…} mean that at least one element can be selected from the specified set of elements (but not necessarily all at the same time); → is a logical sequence symbol (implication); $\langle\ldots\rangle$ is an ordered set (tuple); $\left|{}^{p}a_j\right|\le\left|{}^{k}a_j\right|$ is the number of Pareto set fuzzy estimates $\left|{}^{p}a_j\right|$ (the power of this set) does not exceed the number of evaluations of the input alternatives set of ${}^{k}a_j$ [8] This actually means that the Pareto set is a subset of the input set of analyzed solutions.

Thus, the selection process in this task is performed by a methodically determined sequence of executing φ_i decision-making functions. A brief summary of each φ_i-th function is given below.

According to the function φ_1, the input data-alternatives are ordered according to the sequence of local criteria specified in item 1 of local criteria from their DSLC, *i.e.* *S*1, *S*2, ..., *S*12. The elements of this set are not the ordered (right index (j)) fuzzy estimates of each S_j-th local criterion from their DSLC, but are ordered (right index $\langle j\rangle$) by the numbers of the local criteria mentioned above. A formal representation of this is the following formalized expression:

$$\varphi_1:\left({}^{k}a_{S\langle j\rangle}\to {}^{k}a_{S(j)}\right)\mid \forall S_j,\ j=\overline{1,m} \tag{3}$$

The implementation of the function φ_2 compares fuzzy estimates ${}^{l}a_{S(j)}$ and ${}^{r}a_{S(j)}$ of the same name according to each local criterion, where the symbols l and r here and further denote certain types of received input solutions-alternatives from their k types when they are compared in pairs for each of the input alternatives according to each S_j-th local criterion by determining the ratio R between them. The ratio R takes the values >, <, = for all local criteria from their DSLC in the compared corresponding estimates of each local criterion in pairs-alternatives from the set (1). This is necessary for the further determination of either the non-dominant or the dominant alternative, which in turn is necessary for the further use of Pareto optimization as such (see below):

$$\varphi_2 : \left({}^{l}a_{S(j)} R\, {}^{r}a_{S(j)} \mid R = \{>,<,=\}; \forall (l,r) \in k; l \neq r; \forall S_j, j = \overline{1,m} \right). \quad (4)$$

Here and further, the upper left indices *l* and *r* denote any pair of compared alternatives from the set *k*, *i.e.* input solutions-alternatives according to expression (1).

When performing the function φ_3, the analysis of the performed pairwise comparisons is carried out and dominant and non-dominant alternatives are determined. Dominant alternatives are removed from further consideration, and non-dominant ones remain for further consideration. A mandatory condition for assigning any of the analyzed pairs of input alternatives to dominant alternatives is a strict comparison of fuzzy estimates for all local criteria, for example, (${}^{l}a_{S(j)} > {}^{r}a_{S(j)}$), is the condition $|pR| = m$, that is, the ratio of full dominance must be preserved for all *m* local criteria in pairs of input alternatives. In this case, the analyzed pair is not included in the Pareto set *p*. In the opposite case, that is, when $|\neg pR| \neq m$, the pair of analyzed alternatives belongs to the Pareto set *p*:

$$\begin{aligned} \varphi_3 : \forall \Bigg(& \left(\left(\left| {}^{l}a_{S(j)} > {}^{r}a_{S(j)} \right| j = \overline{1,m} \right) \rightarrow \left(\left({}^{l}S_{S(j)} \ni {}^{l}a_{S(j)} \right) \vee \left({}^{r}S_{S(j)} \ni {}^{r}a_{S(j)} \right) \neg \in p \right) \right) \\ & \vee \left(\left(\left| {}^{l}a_{S(j)} > {}^{r}a_{S(j)} \right| < m \right) \rightarrow \left(\left({}^{l}_{p}S_{S(j)} \ni \left({}^{l}a_{S(j)} = {}^{l}_{p}a_{S(j)} \right) \right) \vee \left({}^{r}_{p}S_{S(j)} \ni \left({}^{r}_{p}a_{S(j)} = {}^{l}_{p}a_{S(j)} \right) \right) \in p \right) \right) \Bigg) \end{aligned} \quad (5)$$

Here, the symbols $\in$ and $\neg\in$ mean, respectively, belonging and non-belonging of the pair of *l*-th and *r*-th input alternatives from the set *k* to the Pareto set *p*; $|\ldots|$ is the module of a number.

Implementation of the function φ_4 involves the analysis of fuzzy estimates for each of the local criteria of the alternatives from the formed Pareto set *p*. At the same time, the difference of fuzzy estimates $\Delta {}^{l,r}_{p}a_{S(j)} = \left({}^{l}_{p}a_{S(j)} - {}^{r}_{p}a_{S(j)} \right)$ for each ${}_{p}S_j$-th local criterion in each alternative of the formed Pareto set (here and then the lower left index *p*) is calculated and the most important criterion ${}^{l}_{p}S_j max$ is determined, for which $\left| \Delta {}^{l,r}_{p}a_{S(j)} \right| max$, as well as the least important criterion ${}^{l}_{p}S_j min$, for which $\left| \Delta {}^{l,r}_{p}a_{S(j)} \right| min$:

$$\begin{aligned} & \varphi_4 : \left(\forall \left(\left({}^{l}_{p}S_j \subset p \right) \ni {}^{l}_{p}a_{S(j)} \right) \rightarrow \left(\Delta {}^{l,r}_{p}a_{S(j)} \mid \forall j = \overline{1,m} \right) \right. \\ & \left. \rightarrow \left(\left| \Delta {}^{l,r}_{p}a_{S(j)} \right| max \rightarrow {}^{l,r}_{p}S_j max \right); \left(\left| \Delta {}^{l,r}_{p}a_{S(j)} \right| min \rightarrow {}^{l,r}_{p}S_j min \right) \right) \end{aligned} \quad (6)$$

The function φ_5 realizes the preferences of the DM regarding the most important ${}^{l,r}_{p}S_j max$ and the least important ${}^{l,r}_{p}S_j min$ criteria, which are components of the Pareto set. This is done based on Pareto principles by decreasing the fuzzy estimate $\left| {}^{l,r}_{p}a_{S(j)} \right| max$ of the most important local criterion ${}^{l,r}_{p}S_j max$ and increasing the fuzzy estimate $\left| {}^{l,r}_{p}a_{S(j)} \right| min$ for the least important criterion ${}^{l,r}_{p}S_j min$.

The correction of fuzzy estimates of the most important ${}^{l,r}_{p}S_j max$ and the least important ${}^{l,r}_{p}S_j min$ local criterion from the alternatives of the Pareto set is performed from the following equation regarding their potential equality:

$$\left|{}^{l,r}_{p}a_{S(j)}\right| * min = \left|{}^{l,r}_{p}a_{S(j)}\right| * max = 0.5 . \tag{7}$$

Hereinafter, elements with the symbol * denote new values of fuzzy estimates after their Pareto correction; $\left|{}^{l,r}_{p}a_{S(j)}\right| * min$ and $\left|{}^{l,r}_{p}a_{S(j)}\right| * max$ are the values of fuzzy estimates of the corresponding criteria after correction.

Then the coefficient of the criteria relative importance $\theta_{Sjmin,Sjmax}$ equals:

$$\theta_{Sjmin,Sjmax} = \frac{\left|{}^{l,r}_{p}a_{S(j)}\right| * min}{\left|{}^{l,r}_{p}a_{S(j)}\right| * min + \left|{}^{l,r}_{p}a_{S(j)}\right| * max} = \frac{0.5}{0.5+0.5} = \frac{0.5}{1} = 0.5 . \tag{8}$$

This makes it possible to determine new values of the criteria ${}^{l,r}_{p}S_j * min$ and ${}^{l,r}_{p}S_j * max$ with fuzzy estimates, respectively, $\left|{}^{l,r}_{p}a_{S(j)}\right| * min$ and $\left|{}^{l,r}_{p}a_{S(j)}\right| * max$ for each of the alternatives of the above Pareto set. For the *l**-th and *r**-th type of the received input decision-alternatives of their k types (symbol $\subset$) it looks as follows:

for

$$(l* \subset k): {}^{l}_{p}a_{S(j)} * min = \theta_{Sjmin,Sjmax} \cdot {}^{l}_{p}a_{S(j)}min + \left(1-\theta_{Sjmin,Sjmax}\right) \cdot {}^{l}_{p}a_{S(j)}max;$$

$${}^{l}_{p}a_{S(j)} * max = \theta_{Sjmin,Sjmax} \cdot {}^{l}_{p}a_{S(j)}max + \left(1-\theta_{Sjmin,Sjmax}\right) \cdot {}^{l}_{p}a_{S(j)}min;$$

for

$$(r* \subset k): {}^{r}_{p}a_{S(j)} * max = \theta_{Sjmin,Sjmax} \cdot {}^{r}_{p}a_{S(j)}max + \left(1-\theta_{Sjmin,Sjmax}\right) \cdot {}^{r}_{p}a_{S(j)}min;$$

$${}^{r}_{p}a_{S(j)} * min = \theta_{Sjmin,Sjmax} \cdot {}^{r}_{p}a_{S(j)}min + \left(1-\theta_{Sjmin,Sjmax}\right) \cdot {}^{r}_{p}a_{S(j)}max; \tag{9}$$

The function φ_6 analyzes the new comparisons by the fuzzy estimates of the corresponding **l*-th and **r*-th input decision alternatives by analogy with the function φ_4 according to expression (6) and taking into account the adjustments made according to expression (9). At the same time, the difference of fuzzy estimates $\Delta^{*l,*r}_{\;p}a_{S(j)} = \left|{}^{*l}_{p}a_{S(j)} - {}^{*r}_{p}a_{S(j)}\right|$ for each ${}_{p}S_j$-th local criterion in each alternative of the formed Pareto set p is calculated. The results of the comparisons are analyzed and serve the base for determining either non-belonging to the Pareto set ($\subset \neg p$) and, accordingly, the absence of Pareto solutions ($\neg F^p$), or (∇ is the symbol of the logical operation of exclusive or) belonging to the Pareto set ($\subset p$) and the presence of Pareto solutions (F^p).

Formally, this is defined by the following expression:

$$\begin{aligned}
&\varphi_6 : \left(\forall\left(\left({}^{*l}_{p}S_j \subset p\right) \ni {}^{*l}_{p}a_{S(j)}\right) \to \left(\Delta^{*l,*r}_{\;p}a_{S(j)} \mid \forall j = \overline{1,m}; *l \neq *r; (*l,*r) \in k\right)\right. \\
&\to \left(\left({}^{*l}_{p}a_{S(j)} \in {}^{*l}_{p}S_{\langle j\rangle}\right) R \left({}^{*r}_{p}a_{S(j)} \in {}^{*r}_{p}S_{\langle j\rangle}\right) \mid R = \{>,<,=\}\right) \\
&\to \left(\left(\left(|R| < m\right) \to \left\{{}^{*l}_{p}S_{\langle j\rangle}, {}^{*r}_{p}S_{\langle j\rangle}\right\} \subset p\right) \to F^p\right) \\
&\nabla\left(\left(\left(|R| = m\right) \to \left\{{}^{*l}_{p}S_{\langle j\rangle}, {}^{*r}_{p}S_{\langle j\rangle}\right\} \subset \neg p\right) \to \neg F^p\right)
\end{aligned} \tag{10}$$

Summarizing the above mentioned, we state that an obvious feature of the methodology, which takes into account the formulation and solution of a Pareto-optimal RMAT selection, lies in the selection process performed on the finite set of solutions to the problem a priori as a problem of fuzzy multicriteria selection of alternatives using fuzzy multicriteria optimization, set theory, fuzzy and discrete mathematics. These solutions were previously obtained using the aforementioned original *WMS*, *QBMS*, *MMS*:*A*, *MMS*:*M*, *MMS*:*S*, and *MMS*:*G* methods. In this case, the process of selecting Pareto-optimal alternatives is performed on the set $n = 6$ (the number of results of solving NBVA problems by the above methods) and $m = 12$ (the number of local selection criteria), which indicates the dimension of this problem $n \times m = 6 \times 12$.

This formulation of the problem is fundamentally different from the possible generally accepted approach to solving it based on the results of a strict expert ranking by questionnaire. In this case, the process of selecting Pareto-optimal RMATs would be performed on a set of $n = 10$ experts and $m = 12$ local selection criteria and would be reduced to the implementation of certain computational procedures for each local selection criterion.

The paper formalisms are represented by expressions (2)-(10) and are developed for the first time. At a certain level of abstraction, which is typical for most mathematical models expressed by such formalisms, they have a number of important features. Firstly, their orderly presentation and implementation obviously covers the content essence of the optimization process when selecting Pareto-optimal alternatives (in this case, RMAT), and, secondly, formalisms (2)-(10) are mathematical models of the formal content of the methodology for selecting Pareto-optimal alternatives It is demonstrated by the selection of Pareto-optimal RMAT. These formalisms can be considered as a necessary component of theoretical and practical research in the development of new methods and techniques, their varied combinations, for example, in the study of small-dimensional and/or large-dimensional selection problems, etc. The formalisms developed on this basis allow for their algorithmization, software implementation, and practical use in the automated solution of Pareto-optimal problems. It indicates the fundamental development of this type of optimization in terms of, first of all, its practical implementation and, in general, indicates an increase in the efficiency of using Pareto-optimization in solving a number of theoretical and practical problems.

The formulation and solution of problems using Pareto-optimization in this paper are fundamentally different from the possible generally accepted approach to solving it (the problem) based on the results of a strict expert ranking by questionnaire. In this case, the process of selecting Pareto-optimal RMATs would be performed on a set of $n = 10$ experts and $m = 12$ local selection criteria and would be reduced to the implementation of certain computational procedures for each local selection criterion.

In general, the above mentioned can be interpreted as a manifestation of a two-stage approach to solving the problem of Pareto-optimal RMAT selection. At the first stage, a set of solutions to the fuzzy multicriteria RMAT selection problem is generated according to the local criteria for RMAT manifestation. This problem has dimension $n \times m = 10 \times 12$, where n is the number of experts, m is the number of local selection criteria, *i.e.*, RMAT manifestations, and is not considered in this paper. However, its results (see expression (1)) are the input data for solving the problem of choosing RMATs that are optimal according to Pareto, which is the content of solving the problems presented in this paper. In general, this problem (*i.e.*, the problem of the second stage) has the dimension $n \times m = 6 \times 12$, where n is the number of the above-mentioned solutions by the *WMS*, *QBMS*, *MMS*:*A*, *MMS*:*M*, *MMS*:*S*, *MMS*:*G*), m is the number of local selection criteria, *i.e.*, RMAT manifestations. The latter is solved on the basis of the principal provisions of Pareto optimization.

In fact, the above-mentioned approach can already be called a two-stage approach. The paper presents it only in terms of highlighting the content of the second stage tasks. The approach can be used to solve the problems of selecting any Pareto-optimal (if possible according to the task statement) research objects of any origin (production, non-production), any content and dimensions.

4. Results and Discussion

According to the methodology described above, the following outcomes were obtained, which are illustrated below by the corresponding results after performing each φ_i-th decision-making function in the methodological-ly determined sequence of their implementation. The results of the decision-making function φ_1 are presented in **Table 1**.

Table 1. Ordered by the numbers of local criteria of input data-alternatives as a result of the decision-making function φ_1.

	*S*1 - *Gm*	*S*2 - *Kn*	*S*3 - *Dn*	*S*4 - *Ct*	*S*5 - *En*	*S*6 - *Tr*
${}^{QBMS}a_{S(j)}$	0.18074	0.19354	0.15139	0.17900	0.19015	0.18514
${}^{MMS:A}a_{S(j)}$	0.21630	0.21562	0.18836	0.21844	0.21638	0.21613
${}^{MMS:M}a_{S(j)}$	0.21961	0.21599	0.19307	0.22342	0.22376	0.21910
${}^{MMS:S}a_{S(j)}$	0.21576	0.21530	0.18689	0.21689	0.21589	0.21567
${}^{MMS:G}a_{S(j)}$	0.21684	0.21594	0.18994	0.22011	0.21702	0.21659
${}^{WMS}a_{S(j)}$	0.09038	0.12432	0.07643	0.06701	0.10399	0.05102
	*S*7 - $\tau(Q)$	*S*8 - *Rl*	*S*9 - *Ec*	*S*10 - *Ac*	*S*11 - *Fc*	*S*12 - *Fopt*
${}^{QBMS}a_{S(j)}$	0.17603	0.18029	0.15259	0.17118	0.16525	0.16415
${}^{MMS:A}a_{S(j)}$	0.20152	0.21744	0.20347	0.20255	0.20353	0.218i39
${}^{MMS:M}a_{S(j)}$	0.19567	0.21910	0.20866	0.20417	0.20838	0.22370
${}^{MMS:S}a_{S(j)}$	0.20090	0.21638	0.20248	0.20152	0.20202	0.21737
${}^{MMS:G}a_{S(j)}$	0.20214	0.21853	0.20444	0.20362	0.20515	0.21937
${}^{WMS}a_{S(j)}$	0.07085	0.06450	0.03908	0.06697	0.03433	0.03616

Here, the rows denote the fuzzy estimates ${}^{k}a_{S(j)}$ of each local criterion from their discrete set for each input decision-alternative technique (upper left indices). The columns denote the set of local selection criteria, which are manifestations of RMAT, and, in fact, the above-mentioned fuzzy estimates.

Table 2. Examples of comparing fuzzy estimates of alternatives pairs as a result of the decision-making function φ_2 implementation.

1	${}^{QBMS}a_{S(j)} < {}^{MMS:A}a_{S(j)}$	${}^{QBMS}a_{S(j)} < {}^{MMS:A}a_{S(j)}$	${}^{QBMS}a_{S(j)} < {}^{MMS:A}a_{S(j)}$	${}^{QBMS}a_{S(j)} < {}^{MMS:A}a_{S(j)}$
2	$S1$ - Gm	$S2$ - Kn	$S3$ - Dn	$S4$ - Ct
3	${}^{QBMS}a_{S(j)} < {}^{MMS:A}a_{S(j)}$	${}^{QBMS}a_{S(j)} < {}^{MMS:A}a_{S(j)}$	${}^{QBMS}a_{S(j)} < {}^{MMS:A}a_{S(j)}$	${}^{QBMS}a_{S(j)} < {}^{MMS:A}a_{S(j)}$
4	$S5$ - En	$S6$ - Tr	$S7$ - $\tau(Q)$	$S8$ - Rl
5	${}^{QBMS}a_{S(j)} < {}^{MMS:A}a_{S(j)}$	${}^{QBMS}a_{S(j)} < {}^{MMS:A}a_{S(j)}$	${}^{QBMS}a_{S(j)} < {}^{MMS:A}a_{S(j)}$	${}^{QBMS}a_{S(j)} < {}^{MMS:A}a_{S(j)}$
6	$S9$ - Ec	$S10$ - Ac	$S11$ - Fc	$S12$ - $Fopt$
…	…	…	…	…
7	${}^{MMS:M}a_{S(j)} > {}^{MMS:G}a_{S(j)}$	${}^{MMS:M}a_{S(j)} > {}^{MMS:G}a_{S(j)}$	${}^{MMS:M}a_{S(j)} > {}^{MMS:G}a_{S(j)}$	${}^{MMS:M}a_{S(j)} > {}^{MMS:G}a_{S(j)}$
8	$S1$ - Gm	$S2$ - Kn	$S3$ - Dn	$S4$ - Ct
9	${}^{MMS:M}a_{S(j)} > {}^{MMS:G}a_{S(j)}$	${}^{MMS:M}a_{S(j)} > {}^{MMS:G}a_{S(j)}$	${}^{MMS:M}a_{S(j)} < {}^{MMS:G}a_{S(j)}$	${}^{MMS:M}a_{S(j)} > {}^{MMS:G}a_{S(j)}$
10	$S5$ - En	$S6$ - Tr	$S7$ - $\tau(Q)$	$S8$ - Rl
11	${}^{MMS:M}a_{S(j)} > {}^{MMS:G}a_{S(j)}$	${}^{MMS:M}a_{S(j)} > {}^{MMS:G}a_{S(j)}$	${}^{MMS:M}a_{S(j)} > {}^{MMS:G}a_{S(j)}$	${}^{MMS:M}a_{S(j)} > {}^{MMS:G}a_{S(j)}$
12	$S9$ - Ec	$S10$ - Ac	$S11$ - Fc	$S12$ - $Fopt$

Table 2 shows as examples only two pairs of fuzzy estimates comparisons for each local criterion (RMAT manifestations). Here, the pairs of alternatives are presented in odd rows, and the ">" sign indicates the advantage (for all $m = 12$ local criteria) of the alternative located to the left of this sign. The even rows of Table 2 indicate the corresponding local criteria from their discrete set, whose fuzzy estimates are used for comparison. In the first case (see rows 1 - 6 in Table 2), the results of solutions for the fuzzy evaluations of the input pairs of alternatives ${}^{QBMS}a_{S(j)}$ and ${}^{MMS:A}a_{S(j)}$, are presented, and in the second case (see rows 7 - 12 in Table 2), the results of comparisons by fuzzy estimates of the other pair ${}^{MMS:A}a_{S(j)}$ and ${}^{MMS:G}a_{S(j)}$, are presented. Hereinafter, the upper left indices next to the values of fuzzy estimates $a_{S(j)}$ for each S_j-th criterion indicate the methods by which the corresponding fuzzy estimates were obtained.

For the first pair of comparisons in Table 2 with fuzzy estimates ${}^{QBMS}a_{S(j)}$ and ${}^{MMS:A}a_{S(j)}$, the alternative with the estimates of ${}^{MMS:A}a_{S(j)}$, *i.e. MMS:A* has an advantage in comparison estimates by absolutely all local criteria. Therefore, this comparison pair is classified as the dominant alternative and is removed for further calculations.

Similar comparisons were made for other pairs of input alternatives. The generalized results of the comparisons are presented in Table 3. Here, in the column of dominant alternatives, the symbol "+" indicates that certain alternatives belong to the set of dominant alternatives, and, accordingly, the symbol "–" in the column of non-dominant alternatives indicates that the input alternatives belong to the set of non-dominant alternatives.

The data presented in **Table 3** should be interpreted as follows: 14 out of 15 comparison pairs of input al-ternatives have an R-ratio between them either ">" or "<", there are no R-ratios with the symbol "=". Each of them is obtained on the entire set of comparison pairs of local criteria. These pairs of comparisons are included in the set of dominant alternatives and are not used in further calculations. The exception is line 11, marked in green, where the results of comparing the input sets of alternatives *MMS*:*M* and *MMS*:*G* with their respective fuzzy estimates ${}^{MMS:M}a_{S(j)}$ and ${}^{MMS:G}a_{S(j)}$ are indicated. These are the so-called non-comparable sets, which, according to the Pareto principle, are actually included in the Pareto set *p*.

The above-mentioned, taking into account expression (5), is the content of the decision-making function φ_3.

Table 3. The result of forming a Pareto set *p* by performing functions φ_3 and φ_6 (highlighted in blue).

№ Comparison	Compared alternatives (fuzzy estimates of comparisons see **Table 1**)	The results of of comparisons by the ratio R:	Alternatives	Alternatives	Pareto p sets
		> = <	dominant (excluded)	non-dominant	
1	${}^{QBMS}a_{S(j)}$ vs ${}^{MMS:A}a_{S(j)}$	0 0 12	+	–	–
2	${}^{QBMS}a_{S(j)}$ vs ${}^{MMS:M}a_{S(j)}$	0 0 12	+	–	–
3	${}^{QBMS}a_{S(j)}$ vs ${}^{MMS:S}a_{S(j)}$	0 0 12	+	–	–
4	${}^{QBMS}a_{S(j)}$ vs ${}^{MMS:G}a_{S(j)}$	0 0 12	+	–	–
5	${}^{QBMS}a_{S(j)}$ vs ${}^{WMS}a_{S(j)}$	12 0 0	+	–	–
6	${}^{MMS:A}a_{S(j)}$ vs ${}^{MMS:M}a_{S(j)}$	0 0 12	+	–	–
7	${}^{MMS:A}a_{S(j)}$ vs ${}^{MMS:S}a_{S(j)}$	12 0 0	+	–	–
8	${}^{MMS:A}a_{S(j)}$ vs ${}^{MMS:G}a_{S(j)}$	0 0 12	+	–	–
9	${}^{MMS:A}a_{S(j)}$ vs ${}^{WMS}a_{S(j)}$	12 0 0	+	–	–
10	${}^{MMS:M}a_{S(j)}$ vs ${}^{MMS:S}a_{S(j)}$	12 0 0	+	–	–
11	${}^{MMS:M}a_{S(j)}$ vs ${}^{MMS:G}a_{S(j)}$	11 0 1	–	+	*MMS*:*M*; *MMS*:*G*
12	${}^{MMS:M}a_{S(j)}$ vs ${}^{WMS}a_{S(j)}$	12 0 0	+	–	–
13	${}^{MMS:S}a_{S(j)}$ vs ${}^{MMS:G}a_{S(j)}$	0 0 12	+	–	–
14	${}^{MMS:S}a_{S(j)}$ vs ${}^{WMS}a_{S(j)}$	12 0 0	+	–	–
15	${}^{MMS:G}a_{S(j)}$ vs ${}^{WMS}a_{S(j)}$	12 0 0	+	–	–
*16	${}^{MMS:M}a_{S(j)}$ vs ${}^{MMS:G}a_{S(j)}$	12 0 0	+	–	–

The implementation of the function φ_4 ensures the calculation of fuzzy estimates difference for each local criterion with their DSLC. Taking into account expression (6), the most important criterion is determined, which is *S*12 - *Fopt*, for which the difference between fuzzy estimates when compared with the least

important criterion $S7$ - $\tau(Q)$ is as follows
$\Delta_{p}^{l,r}a_{S12} = {}^{MMS:M}a_{S12} - {}^{MMS:G}a_{S12} = 0.22370 - 0.21937 = 0.01855$ (in **Table 4**, highlighted in green). For the least important criterion, *i.e.*, $S7$ - $\tau(Q)$, the difference between the fuzzy estimates when compared to the most important criterion equals: $\Delta_{p}^{l,r}a_{S7} = {}^{MMS:M}a_{S7} - {}^{MMS:G}a_{S7} = 0.19567 - 0.20214 = -0.00674$ (highlighted in yellow in **Table 4**).

Table 4. The results of determining the fuzzy estimates differences (comparisons) of each local criterion for each solutions set of the formed Pareto set *p* by the function φ_4.

	$S1$ - Gm	$S2$ - Kn	$S3$ - Dn	$S4$ - Ct	$S5$ - En	$S6$ - Tr
${}^{MMS:M}a_{S(j)}$	0.21961	0.21599	0.19307	0.22342	0.22376	0.21910
${}^{MMS:G}a_{S(j)}$	0.21684	0.21594	0.18994	0.22011	0.21702	0.21659
$\Delta^{k}a_{S(j)}$	0.00277	0.00005	0.00313	0.00331	0.00674	0.00251
	$S7$ - $\tau(Q)$	$S8$ - Rl	$S9$ - Ec	$S10$ - Ac	$S11$ - Fc	$S12$ - $Fopt$
${}^{MMS:M}a_{S(j)}$	0.19567	0.21910	0.20866	0.20417	0.20838	0.22370
${}^{MMS:G}a_{S(j)}$	0.20214	0.21853	0.20444	0.20362	0.20515	0.21937
$\Delta^{k}a_{S(j)}$	−0.00674	0.00057	0.00422	0.00055	0.00323	0.01855

For the final selection of the Pareto-optimal solution in the fuzzy multicriteria selection of RMAT, it is necessary to have information about the DM's preferences for each of the local criteria set $S1$, ..., $S12$. Functionally, this is realized by the function φ_5.

According to Pareto and taking into account expressions (7)-(9), to increase the importance of the criterion $S7$ - $\tau(Q)$, its fuzzy estimate ${}_{p}^{l,r}a_{S7} = -0.00674$ increases and at the same time the fuzzy estimate ${}_{p}^{l,r}a_{S12} = 0.01855$ of the criterion $S12$ - $Fopt$ (see **Table 4**) reduces. In this case, the following equation must be observed, which reflects the preferences of the DM:

$$\left|{}_{p}^{l,r}a_{S7}\right| * min = \left|{}_{p}^{l,r}a_{S12}\right| * max = 0.5.$$

Then the coefficient of local criteria relative importance $*S7$ and $*S12$ is as follows:

$$\theta_{S7,?S12} = \frac{\left|{}_{p}^{l,r}a_{S7}\right| * min}{\left|{}_{p}^{l,r}a_{S7}\right| * min + \left|{}_{p}^{l,r}a_{S12}\right| * max} = \frac{0.5}{0.5+0.5} = \frac{0.5}{1} = 0.5.$$

This determines the new fuzzy estimates values of the criteria $*S7$ and $*S12$ for each of the above-mentioned Pareto set alternatives, *i.e.* $*MMS{:}M$ and $*MMS{:}G$, which are as follows:

$$*MMS:M: {}_{p}^{l*}a_{S7} * min = 0.5 \times 0.19567 + (1-0.5) \times 0.2237 = 0.209715;$$

$${}_{p}^{l*}a_{S7} * max = 0.5 \times 0.2237 + (1-0.5) \times 0.19567 = 0.209715;$$

$$*MMS:G: {}_{p}^{r*}a_{S12} * min = 0.5 \times 0.20214 + (1-0.5) \times 0.20515 = 0.20364;$$

$${}_{p}^{r*}a_{S12} * max = 0.5 \times 0.20515 + (1-0.5) \times 0.20214 = 0.20364.$$

By performing φ_6, we compare the new (after correction) fuzzy estimates of the criteria $^*S7$, $^*S12$ and others, taking into account expression (9). These estimates differ from the data in **Table 4** only in the estimates for $^*S7$, $^*S12$, which is determined by their adjustment by function φ_5.

The comparison results are listed in **Table 5** and highlighted in yellow (for *S7) and green (for *S12), respectively.

Table 5. The results of determining the differences (comparisons) by the function φ_6 after adjusting the fuzzy estimates of each local criterion for each solution set of the formed Pareto set $^*p$.

	$S1$ - Gm	$S2$ - Kn	$S3$ - Dn	$S4$ - Ct	$S5$ - En	$S6$ - Tr
$^{MMS:M}a_{S(j)}$	0.21961	0.21599	0.19307	0.22342	0.22376	0.21910
$^{MMS:G}a_{S(j)}$	0.21684	0.21594	0.18994	0.22011	0.21702	0.21659
$\Delta^k a_{S(j)}$	0.00277	0.00005	0.00313	0.00331	0.00674	0.00251

	$^*S7$ - $\tau(Q)$	$S8$ - Rl	$S9$ - Ec	$S10$ - Ac	$S11$ - Fc	$^*S12$ - $Fopt$
$^{MMS:M}a_{S(j)}$	0.209715	0.21910	0.20866	0.20417	0.20838	0.22370
$^{MMS:G}a_{S(j)}$	0.20214	0.21853	0.20444	0.20362	0.20515	0.20374
$\Delta^k a_{S(j)}$	0.007575	0.00057	0.00422	0.00055	0.00323	0.01996

The result of the adjusted fuzzy estimates comparisons for the Pareto-set p alternatives is presented in **Table 3**, row *16 on the blue background. The data in this row indicate that the $^*MMS{:}M$ and $^*MMS{:}G$ comparison pair is dominated by the $^*MMS{:}M$ alternative, *i.e.*, this pair is not incomparable. This means that with such initial data, there is no unambiguous decision on the selection of the Pareto-optimal RMAT.

To find a single solution, *i.e.*, to obtain a one-element Pareto set p, one can repeat the procedure for identifying the relative importance of the criteria or involve an additional criterion for analysis. This is the subject of separate studies and is not considered here.

5. Conclusions

A new approach to the fuzzy multicriteria selection of RMAT as a component of the TPp of RMAP is presented. Its peculiarity is the selection process performed on the set of previously obtained results of tasks solved by FMCSA methods, which reproduce the final set of selection results, which are the input alternatives for the Pareto selection of RMAT. This approach has the following obvious advantages. First: the number of analyzed alternatives is reduced. The input number in this task is 6 (see expression (1)), the output number is 2 alternatives (see **Table 3**). Secondly, the obtained Pareto solution allows to increase the total number of solutions to the FMCS of RMAT problems in the unchanged formulation (the input solutions-alternatives were 6, the total number of solutions, taking into account the solution obtained here, is 7) and provides the possibility of further generalization and correct research of solving the FMCS of RMAT tasks, for example, according to the rating of each of the 7 solutions, the tasks of the FMCS of RMAT, the degree of their overall and pairwise consistency, taking into account the peculiarities of the formulation and the content and essence of such tasks, their correlation, etc. The above-mentioned stuff in an in-depth form can be considered as one of the directions for further research.

In addition, using Pareto-optimization, it is possible to order the input alternatives using such relations between the local criteria of each alternative as strict preference, equivalence, and incomparability, with the use of transitivity checks. The obtained solutions can be investigated using the methods PROMETHEE II, TOPSIS and GRA or their combination, provided that the correctness of such studies is observed, etc.

In general, the novelty of the presented material is determined by both scientific and applied components. The scientific component is determined by the newly generated formalisms for the process of determining Pareto-optimal alternatives, which are generally characterized by process-event phenomena. The developed formalisms can be interpreted as formally fixed new knowledge characterized by the revealed substantive consistency and completeness. In contrast, previous expressions of similar content are characterized by fragmentary formalisms that were not systematic and were largely descriptive. The applied component of the paper novelty indicates further possibility of automated solutions to such problems, which significantly reduces the duration of their solution due to the possibility of developing appropriate algorithmic software and its implementation.

Thus, the development of new methods and approaches to solving FMCSA tasks, including RMAT, analysis and study of the results obtained in this way is an obvious and main direction of the authors' further research in the context of the topic considered in this paper.

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Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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