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Developing a New Reformulation of Single Level Capacitated Lot Sizing Problem (SLCLSP) with Set up, Shortage and Inventory Costs

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Abstract

Formulation of SLCLSP given by Pochet and Wolsey [1] had set up, variables, inventory and shortage cost. We give a new reformulation where SLCLSP is reduced to set up and inventory variables. We find that this reformulation has less number of real variables than the reformulation of Pochet and Wolsey [1]. It is argued that this leads to computations advantages, and this is supported by the empirical investigation that we carried out.

Keywords

Reformulation of SLCLSP

1. Introduction and Literature Review

Capacitated lot sizing problem (CLSP) is well studied in literature, see Verma [2], and Verma and Sharma [3] [4] [5] for a summary of recent works on CLSP. For literature on reformulation of CLSP, see Pochet and Wolsey [1] and Miller and Nemhauser *et al.* [6] for a detailed exposition on reformulations of CLSP. In this paper we give a new approach which leads to a better reformulation of CLSP.

2. Formulation by Pochet and Wolsey [1]

Indices Used

t: Set of the Time period from 1, ⋯, n, for which we are taking decisions;

Definition of Constant

f_t : Fixed setup cost in time period "t";

p_t : Per unit production cost in time period "t";

d_t : Demand in time period "t", here demand is independent;

h_t : Per unit inventory carrying cost in time period "t";

sh_t : Per unit shortage cost in time period "t";

c_t : Production capacity in the time period "t";

Definition of Variables

x_t : Number of product produced in time period "t";

y_t : Binary variable takes value "1" if machine setup to produce in time period "t", "0", otherwise;

I_t : In stock inventory at the end of time period "t";

s_t : Backlog in the end of period "t";

Mathematical Model

$$\text{Minimize } Z = \sum_{t=1}^{n} f_t * y_t + \sum_{t=1}^{n} p_t * x_t + \sum_{t=0}^{n-1} h_t * I_t + \sum_{t=1}^{n} sh_t * s_t \tag{1}$$

Production balance constraints

$$x_t + \left(s_t - s_{t-1}\right) = d_t + \left(I_t - I_{t-1}\right),\ 1 \le t \le n \tag{2}$$

Capacity constraints

$$x_t \le c_t * y_t,\ 1 \le t \le n \tag{3}$$

$$I_o = s_n = 0 \tag{4}$$

Pochet and Wolsey [1] gave the following constraint that lead to reformulation:

$$x_t = y_t * c_t \tag{5}$$

Non-negativity constraints

$$x_t, r_t, s_t \ge 0 \tag{6}$$

SLCLSP as given by Pochet and Wolsey [1] is Model A1: Min (1); s.t. (2), (3), (4) and (6). By using (5) in place of (3) lead to reformulation (called Model A2: min (1); s.t. (2), (4), (5) and (6). Model A1 has less number of variables as variable "x" is eliminated.

We add a new constraint given below (see, [7]) that can be used in place of (2):

$$I_0 + \sum_{t=1}^{t_1} x_t + s_{t_1} = \sum_{t=1}^{t_1} D_t + I_{t_1},\ \forall t_1 = 1, \cdots, T \tag{7}$$

Using (5) we get the following (called Model A3): Min Z1 (or (8)); s.t. (4), (5), (6), (7).

$$\text{Min } Z1 = \sum_{t=1}^{T} f_t * y_t + \sum_{t=1}^{T} p_t * c_t * y_t + \sum_{t=1}^{T} h_t * I_t + \sum_{t=1}^{T} sh_t * s_t \tag{8}$$

We use (7) to eliminate s_t from the problem A2 to get: Min Z2 (or (9)); s.t. (4), (5), (6).

$$\text{Min } Z2 = \sum_{t=1}^{T} f_t * y_t + \sum_{t=1}^{T} p_t * c_t * y_t + \sum_{t=1}^{T} h_t * I_t + \sum_{t=1}^{T} sh_t * \left(\sum_{t=1}^{t_1} D_t + I_{t_1} - I_0 - \sum_{t=1}^{t_1} c_t * y_t \right) \tag{9}$$

It can be seen that model A3 has least number of variables; it is followed by A2 that has less number of variables compared to model A1 which is well known reformulation (Pochet and Wolsey [1]. We solve model A1, model A2, and

Table 1. Problem for 50 time period (Z value and No. of node processed in brand and bound procedure).

Serial Number	Z Value			No. of Nodes		
	A1	**A2**	**A3**	**A1**	**A2**	**A3**
1	3,743,474,690	3,772,080,223	3,729,968,134	5993	5928	5337
2	4,087,263,963	4,082,641,822	4,051,323,308	36,118	16,984	6789
3	4,278,583,249	4,242,536,589	4,147,905,553	88,941	31,926	28,253
4	2,657,721,758	2,651,489,861	2,651,489,861	18,356	20,115	14,209
5	3,769,493,580	4,638,033,887	4,523,870,372	77	112	157
6	4,424,505,091	4,301,967,396	4,290,588,881	524	203	147
7	4,283,068,415	3,681,840,081	3,668,626,558	570	2817	2406
8	3,442,589,568	3,432,273,130	3,378,960,995	287,834	9244	3783
9	4,592,366,594	3,958,939,510	3,924,766,601	187	24,835	10,575
10	3,668,355,694	3,727,174,504	3,698,909,501	1650	86,561	45,619
11	3,088,312,983	288,601,626	287,732,844	267,534	3636	1956
12	3,444,265,960	2,385,462,383	2,348,656,449	13,810	13,246	9216
13	2,890,919,916	319,566,707	315,170,702	749,374	4256	2616
14	3,777,498,273	401,799,199	399,302,094	297,212	4359	3843
15	3,954,626,394	3,698,592,414	3,698,592,414	15,096	11,135	4418
16	3,462,169,188	4,117,003,500	4,091,197,410	67,681	1616	868
17	3,083,733,256	3,867,520,394	3,777,265,922	533,601	2747	2300
18	3,702,311,188	3,263,031,295	3,263,031,295	132,638	4458	3399
19	3,413,172,042	3,784,324,541	3,784,324,541	2,630,215	459	339
20	297,731,294	3,977,685,998	3,958,558,821	2895	21,236	11,028
21	2,376,689,404	4,185,624,481	4,185,624,481	11,985	4230	3722
22	330,678,453	4,708,536,900	4,698,411,882	2756	1842	828
23	3,001,054,672	3,698,592,414	3,698,592,414	272,260	11,135	4418
24	2,634,799,375	3,413,786,217	3,392,672,356	181,070	14,498	10,402
25	399,617,912	3,948,613,833	3,924,169,034	9609	18,015	10,176
26	4,056,860,648	2,978,614,594	2,955,820,597	109	14,124	6961
27	3,719,004,873	3,593,077,095	3,537,841,781	9606	22,312	14,489
28	4,157,622,531	371,084,977	367,217,332	1544	8673	5716
29	3,931,770,463	479,425,663	479,425,663	2752	3819	1494
30	3,260,332,825	449,667,615	447,808,961	4267	8612	6392
31	3,757,225,879	394,281,224	392,939,379	1607	110,009	8117
32	2,793,729,505	429,249,899	424,300,345	358,736	5447	3218
33	3,518,672,484	447,012,193	442,518,534	58,503	16,848	15,289
34	3,066,219,455	394,784,161	390,195,788	5,139,325	23,809	14,939
35	3,973,415,489	401,806,697	398,576,087	27,498	5845	2910
36	3,814,691,442	377,810,560	373,051,019	52,490	3189	1664
37	3,654,522,375	469,436,504	469,436,504	252,824	3197	2734
38	5,076,651,047	471,817,933	467,846,010	73	8961	3575
39	4,196,445,388	344,956,193	342,401,612	6910	53,543	5990
40	4,686,709,286	449,733,822	447,872,965	1740	8730	6140
41	3,255,605,457	381,239,619	381,239,619	37,394	20,546	17,064
42	3,087,665,098	455,636,028	455,004,336	589,854	39,006	9762
43	3,052,704,246	594,321,585	586,259,295	1,639,057	5433	3384
44	4,434,168,032	656,218,451	650,254,876	69,217	911	548
45	3,719,004,873	364,670,837	364,273,762	9606	84,467	33,349
46	3,456,909,380	351,107,592	351,107,592	14,722	44,222	24,581
47	4,216,993,709	364,892,115	364,892,115	15,970	22,806	17,007
48	3,014,612,688	349,406,839	345,927,711	20,113	24,790	11,000
49	3,721,096,794	348,757,814	343,882,576	18,994	25,567	7667
50	3,036,834,473	501,568,451	500,864,792	2,556,386	13,651	6358

Table 2. Problem for 50 time period (Iteration and execution time in GAMS).

Serial Number	Iteration			Execution Time		
	A1	**A2**	**A3**	**A1**	**A2**	**A3**
1	12,300	12,540	10,970	0.56	0.75	0.41
2	75,969	37,249	15,133	2.47	2.36	0.59
3	184,762	76,943	50,826	5.63	4.83	2.64
4	41,814	47,591	32,570	1.17	2.86	1.17
5	355	252	55	0.02	0.03	0.02
6	1514	386	327	0.05	0.03	0
7	1352	6142	5326	0.05	0.34	0.22
8	539,012	22,539	8742	16.72	1.52	0.3
9	547	51,978	21,909	0.03	4.19	1
10	3907	187,053	94,645	0.14	12.97	4
11	125,232	5987	3434	7.84	0.27	0.16
12	32,737	28,395	19,127	1.05	2.16	0.84
13	1,570,386	7223	4367	47.58	0.56	0.22
14	589,601	8036	6697	19.05	0.63	0.33
15	30,613	22,114	8365	1.05	2.09	0.42
16	142,522	3434	1768	5.05	0.13	0.11
17	1,022,302	5041	3828	31.89	0.25	0.16
18	264,804	9126	7044	7.22	0.67	0.31
19	5,246,146	930	698	166.7	0.06	0.05
20	5029	46,352	44,016	0.19	3.94	0.27
21	25,722	9730	8103	0.84	0.66	0.33
22	5095	3582	1809	0.2	0.14	0.08
23	562,119	22,114	8365	18.55	2.17	0.41
24	351,278	35,387	24,552	10.92	2.8	1
25	16,284	35,489	20,288	0.64	1.13	0.94
26	464	28,650	14,000	0.05	2.45	0.61
27	19,147	46,962	30,319	0.59	3.7	1.34
28	3669	14,176	9803	0.16	1.17	0.5
29	5488	6116	2779	0.2	0.45	0.18
30	8893	14,884	10,183	0.3	1.19	0.55
31	3494	19,233	14,394	0.11	1.58	0.75
32	695,827	9619	5756	21.56	0.59	0.27
33	111,470	29,821	20,012	3.8	2.5	1.19
34	10,698,820	42,148	26,635	288.53	3.5	1.33
35	51,882	9280	4632	2.47	0.72	0.22
36	106,707	5786	2987	3.78	0.41	0.14
37	517,323	5243	4841	17.61	0.31	0.23
38	245	14,249	5877	0.02	1.09	0.3
39	15,291	91,463	11,775	0.5	7.31	1.3
40	3564	19,895	10,013	0.11	1.19	0.53
41	76,716	17,707	9292	2.56	1.39	1.72
42	1,079,584	64,819	10,684	37.44	5.36	0.44
43	3,117,900	19,700	6075	96.86	5.81	0.33
44	148,392	1750	814	4.86	0.08	0.02
45	19,147	42,188	14,982	0.63	3.56	1.04
46	31,767	69,784	25,564	1.08	6.5	1.63
47	29,865	24,259	17,658	1.42	2.83	1.5
48	43,544	46,436	18,173	1.5	3.98	1
49	38,066	46,991	13,588	1.38	4.02	0.73
50	4,885,648	23,807	11,239	164.81	2.08	0.59

Table 3. Problem for 60 time period (Z value and No. of nodes).

SL	Z Value			Node		
	A1	A2	A3	A1	A2	A3
1	4,690,405,488	4,686,843,376	4,640,146,230	8071	8099	2814
2	4,942,175,901	4,686,879,703	4,646,739,641	1209	1763	664
3	4,026,748,035	4,012,399,867	3,945,646,725	1188	2171	836
4	3,834,684,997	3,865,674,646	3,848,141,195	177,892	240,386	90,528
5	3,388,653,181	3,426,469,921	3,362,668,371	12,725	12,504	8060
6	3,704,244,277	3,704,048,262	3,704,048,262	3596	2365	745
7	3,455,003,458	3,409,044,379	3,364,708,897	8501	10,335	4753
8	4,556,745,207	4,690,205,153	4,690,205,153	892	1415	863
9	4,694,304,922	4,714,569,074	4,702,537,977	93,969	83,377	37,951
10	2,879,898,344	2,907,597,665	286,315,535	775,659	1,684,532	464,630
11	3,107,852,372	3,082,251,489	3,012,079,197	5970	3582	1013
12	4,353,593,305	4,342,423,470	4,314,841,901	211,554	162,445	133,640
13	4,409,925,978	4,415,335,303	4,395,705,093	44,293	44,184	14,059
14	5,929,108,104	5,923,080,031	5,466,104,213	223	89	39
15	4,111,385,645	4,070,582,829	4,055,880,497	51,680	21,521	16,643
16	4,687,235,436	4,652,160,822	4,607,862,044	959,798	477,212	138,813
17	4,235,416,065	4,272,842,064	4,261,023,388	730	1339	540
18	4,260,571,964	4,245,828,562	4,232,562,302	967,816	452,814	162,929
19	2,611,374,340	2,577,029,890	2,553,793,759	3674	3217	1160
20	2,336,144,807	2,309,640,453	2,300,813,943	669,140	472,088	365,487
21	4,323,409,419	4,311,774,093	4,252,408,853	43,377	16,706	13,706
22	3,719,068,960	3,597,389,360	3,563,661,098	469,631	442,077	256,355
23	4,174,000,078	4,096,103,762	4,096,103,762	152,880	85,825	83,523
24	4,780,392,415	4,771,984,379	4,755,238,510	899,284	1,053,503	301,860
25	1,937,842,216	1,947,423,174	1,942,272,356	2,704,373	3,009,166	1,645,326
26	2,270,644,648	2,254,999,232	2,240,211,283	37,687	45,325	29,653
27	2,071,257,407	2,078,391,746	2,064,339,811	84,174	63,538	48,651
28	1,604,837,467	1,612,424,216	1,612,424,216	4173	8877	2389
29	1,569,499,494	1,579,594,148	1,574,368,041	11,978	14,582	7395
30	1,559,462,959	1,580,155,697	1,560,772,554	43,444	78,898	40,765
31	1,733,621,817	1,724,665,321	1,708,094,975	61,913	81,098	74,472
32	1,833,422,008	1,801,731,617	1,801,731,617	558,557	289,834	155,345
33	2,137,715,113	2,074,988,385	2,062,720,687	9663	8844	6838
34	1,949,723,121	1,926,337,591	1,922,911,150	9727	6792	4687
35	1,698,093,660	1,698,093,660	1,755,809,065	230,811	113,438	81,644
36	2,086,663,406	2,083,248,601	2,081,397,235	9441	11,028	7179
37	1,554,206,641	1,554,966,625	1,554,966,625	1187	840	650
38	2,714,570,892	2,618,703,255	2,612,412,931	15,899	8857	4640
39	2,247,971,204	2,248,320,525	2,246,131,484	230,842	95,857	12,216
40	2,277,730,008	2,278,562,498	2,278,562,498	246,853	285,599	70,236
41	1,999,443,356	1,999,443,356	1,995,585,067	175,568	181,155	119,248
42	531,507,860	524,940,891	523,394,518	597,836	314,640	189,345
43	674,870,478	652,360,168	652,360,168	26,560	25,222	13,212
44	489,632,687	490,027,989	487,887,285	130,152	92,299	40,231
45	485,113,122	483,412,665	483,412,665	14,334	47,008	20,675
46	685,963,429	699,786,979	692,172,784	4325	10,338	2918
47	608,117,858	612,169,049	611,923,818	159,238	323,704	196,769
48	557,099,497	580,350,856	580,199,304	37,038	17,506	13,731
49	668,245,439	636,311,587	630,006,170	7876	35,492	13,932
50	495,189,891	493,394,606	490,192,272	200,790	216,293	81,276

Table 4. Problem for 60 time period (Iteration and execution time).

SL	Iteration			Execution Time		
	A1	A2	A3	A1	A2	A3
1	18,491	18,327	5383	0.08	1.78	0.03
2	2811	3521	503	0.09	0.22	0.06
3	2635	4214	1772	0.14	0.34	0.13
4	266,016	579,151	199,516	10.81	52.73	10.44
5	26,638	26,954	9158	1.3	3.14	1.08
6	7396	4512	1756	0.38	0.39	0.09
7	17,629	22,604	10,103	0.77	2.31	0.59
8	2159	3027	2141	0.09	0.14	0.09
9	186,806	180,994	116,351	8.72	17.72	9.63
10	1,831,965	4,229,982	1,051,470	67.55	358.89	51.98
11	16,579	9521	2217	0.69	0.72	0.36
12	518,132	355,143	18,534	20.88	35.84	9.98
13	98,905	98,741	31,936	4.5	10.23	2.33
14	478	209	45	0.03	0.03	0.01
15	114,348	54,113	16,377	4.52	5.03	1.3
16	2,018,060	1,045,471	980,875	84.09	108.06	51.81
17	1665	2884	814	0.08	0.22	0.06
18	2,068,000	1,109,211	826,199	76.03	89.89	47.66
19	9331	7243	6519	0.33	0.56	0.27
20	1,267,578	874,740	401,226	56.83	95.13	47.09
21	96,856	40,772	34,302	3.61	3.53	1.7
22	959,958	941,229	500,384	39.63	89.59	26.91
23	363,886	180,704	113,897	14.25	19.59	11.5
24	1,817,125	2,494,929	988,712	71.77	222.83	148.48
25	4,944,234	5,647,452	1,173,054	208.44	585.91	371.05
26	71,511	94,381	55,060	3.73	9.44	3.5
27	164,375	126,100	93,642	7.06	13.13	5.27
28	8028	16,470	11,447	0.33	1.77	0.64
29	40,356	29,049	14,050	2.3	3.45	0.72
30	81,681	157,192	78,127	5.64	15.98	4.28
31	103,150	143,584	103,015	5.05	16.73	7.42
32	986,774	504,544	86,905	48.05	60.48	23.96
33	18,011	17,331	13,654	0.92	1.56	0.75
34	18,202	11,830	5699	0.94	1.38	0.9
35	387,885	197,989	136,863	22.59	22.91	6.84
36	17,644	19,910	12,533	0.81	2.75	0.78
37	2492	1654	1150	0.16	0.13	0.6
38	31,362	17,861	11,481	1.94	1.83	0.98
39	443,174	189,783	112,152	17.97	19.27	2.32
40	459,116	573,012	129,497	19.56	59.77	7.77
41	337,127	335,078	234,501	18.97	38.5	9.78
42	1,106,229	600,595	369,974	34.02	51.92	17.89
43	50,817	148,172	41,267	2.05	5.98	0.34
44	257,576	170,949	79,037	8.06	14.53	1.67
45	25,553	85,116	37,443	0.98	7.55	1.95
46	8509	18,684	4911	0.33	1.73	0.55
47	292,322	595,259	166,155	9.63	52	16.3
48	76,788	43,400	15,727	2.86	2.73	0.39
49	14,868	65,503	25,183	0.48	5.7	1.64
50	363,746	401,966	91,760	12.02	35.02	1.27

Table 5. Problem for 100 time period (Z value and No. of nodes).

SL	Z Value			Node		
	A1	A2	A3	A1	A2	A3
1	784,3029,491	7,877,537,710	7,769,189,120	72,643	88,209	48,317
2	6,914,636,403	6,923,965,921	6,834,887,462	95,815	40,820	32,312
3	8,421,575,167	8,408,950,995	8,268,564,952	62,608	50,555	17,993
4	8,804,608,364	8,843,852,648	8,752,092,110	812,984	590,304	118,227
5	7,608,091,814	7,655,271,854	7,597,769,696	405,728	983,090	298,997
6	9,086,521,855	8,965,639,966	8,876,600,315	970,899	519,518	113,691
7	9,198,926,496	9,198,548,698	9,189,804,759	241,318	200,548	114,138
8	9,811,211,753	9,791,454,768	9,591,181,293	942,632	2,478,344	592,710
9	10,000,502,947	10,019,792,845	1.00E+10	8,242,681	3,383,989	1,919,028
10	10,133,441,392	10,082,021,214	1.00E+10	18,186,819	5,379,121	21,683,216
11	6,149,103,272	6,132,801,814	6,049,216,186	1,145,606	915,648	486,000
12	2,809,539,439	2,828,300,995	2,801,445,751	15,180,655	2,847,293	14,633,913
13	1,553,786,668	1,549,451,275	1,504,845,350	17,319,620	3,037,272	14,576,855
14	5,853,627,774	5,835,360,213	5,697,780,856	15,391,471	3,282,344	13,915,507
15	897,949,271	899,432,520	873,604,181	37,667,783	15,896,324	7,162,707
16	846,213,980	857,374,315	835,617,832	15,950,921	3,682,615	17,050,248
17	894,239,781	899,179,505	896,806,826	7,802,904	6,391,928	30,241,852
18	867,083,832	859,306,485	841,409,442	221,347	450,214	48,686
19	812,385,714	815,714,061	794,644,508	15,294,760	6,762,016	7,378,909
20	863,630,108	870,412,370	846,485,778	58,188,682	4,488,389	30,856,058
21	1,062,373,133	1,048,364,550	1,014,939,150	49,434,679	5,316,731	9,887,054
22	1,070,030,635	1,083,753,761	1,058,863,922	13,821	14,496	3664
23	902,889,803	881,643,335	858,227,194	5,117,916	7,741,987	2,278,118
24	950,964,795	942,789,081	919,860,364	10,174,194	20,126,148	6,223,031
25	839,796,537	837,822,000	828,270,785	1,541,016	264,307	139,755
26	1,000,442,148	946,808,231	917,732,145	2,263,845	1,473,302	49,900
27	835,942,848	846,163,976	841,478,230	1,068,173	276,559	448,216
28	1,099,119,265	1,101,190,286	1,080,124,395	2848	2026	1025
29	880,989,931	880,989,931	880,989,931	27,414,101	36,616,372	3,376,684
30	1,369,426,395	1,311,852,696	1,384,064,938	14,369,322	19,430,396	2,772,482
31	2,253,129,303	2,238,728,229	2,238,261,389	56,697	55,886	34,763
32	2,795,000,648	2,703,823,834	2,852,338,520	13,439,638	14,401,641	2,430,554
33	2,141,187,892	2,130,400,457	2,084,855,815	14,083,082	2,947,477	14,793,955
34	2,858,371,963	2,812,423,030	2,883,562,568	16,911,118	16,488,084	3,367,739
35	6,505,955,854	6,505,552,267	6,443,019,138	215,897	389,712	159,723
36	7,120,089,635	7,029,560,900	7,192,693,324	17,437,449	20,640,443	3,174,595
37	7,417,990,481	7,412,033,641	7,372,388,498	2,358,941	5,309,850	658,972
38	7,998,954,487	7,351,949,793	7,248,240,359	4256	1899	1307
39	7,959,199,539	7,972,784,306	7,875,269,348	188,336	395,097	61,776
40	6,193,201,606	6,158,185,283	6,232,881,651	13,478,527	15,212,209	3,394,554
41	1,098,953,281	1,076,540,340	1,069,021,798	990,700	933,922	261,497
42	1,046,033,021	1,036,261,751	1,002,018,404	36,350,711	5,400,390	3,309,850
43	1,051,488,430	996,574,063	978,324,659	12,803	11,799	4684
44	901,471,618	903,580,808	867,415,059	2,834,680	4,150,237	1,514,086
45	1,033,200,071	1,004,850,326	978,567,738	42,561	27,185	23,514
46	733,978,831	721,026,906	738,059,281	12,658,974	38,298,907	4,922,622
47	851,455,157	847,833,266	843,145,487	1,620,112	2,471,798	1,443,618
48	1,229,444,379	1,256,108,184	1,256,108,184	957,777	335,103	312,973
49	607,804,230	585,665,620	608,147,092	15,061,323	14,665,814	3,176,671
50	1,000,377,286	963,151,682	944,911,401	1,256,616	1,694,760	454,551

Table 6. Problem for 100 time period (Iteration and execution time).

SL	Iteration			Execution Time		
	A1	A2	A3	A1	A2	A3
1	160,720	199,563	103,925	8.73	36.08	8.7
2	231,028	98,985	76,959	10.31	15.64	6.19
3	158,166	119,211	44,489	6.98	18.22	3.66
4	2,062,294	1,470,813	296,590	82.94	194.94	17.78
5	855,027	2,056,419	662,418	47.52	314.67	41.16
6	2,159,006	1,158,509	272,845	98.66	176.61	17.48
7	567,313	481,066	254,392	24.35	62.61	17.39
8	2,512,756	5,818,447	1,534,131	95.86	830.53	89.38
9	17,926,270	7,856,951	4,412,712	820.36	1140.75	308.97
10	37,428,780	11,239,981	16,162,834	1682.45	2007.62	3698.95
11	2,486,358	2,031,471	1,060,316	115.41	326.84	72.61
12	24,766,560	4,786,546	24,921,192	1319.08	1018.2	2060.33
13	29,807,413	4,897,202	23,570,556	1503.27	996.47	2169.47
14	25,294,333	5,447,783	22,625,452	1379.17	1122.73	1913.77
15	68,893,251	47,586,392	13,069,843	2997.34	2016.38	1053.75
16	27,775,595	6,219,174	29,345,929	1381.63	1055.73	2656.64
17	14,508,444	11,000,994	51,710,689	670.02	2280.02	4457.28
18	421,758	866,848	91,507	18.95	150.53	7.55
19	26,873,039	10,939,481	12,431,540	1103.06	2085.75	1039.75
20	1.01E+08	7,392,928	52,800,281	4409.89	1401.33	3964.23
21	91,277,538	9,305,823	18,316,119	3950.31	2003.77	1437.06
22	27,324	28,340	7099	1.86	5	0.55
23	9,433,353	15,460,266	4,281,016	380.36	2487.64	298.58
24	19,095,269	9.22E+09	11,346,296	905.88	2314.71	806.61
25	3,014,801	463,192	158,344	115.19	84.16	88.02
26	3,814,705	2,564,479	80,820	162.22	421.94	7.47
27	2,111,080	513,290	874,164	135.86	82.13	38.34
28	5582	3730	2004	0.31	0.57	0.14
29	47,646,947	66,350,119	5,976,211	1981.63	4314	1023
30	22,725,853	35,418,693	4,497,866	1307.55	3030.52	975.7
31	100,564	102,912	67,370	6.87	18.73	5.86
32	20,943,805	23,364,145	3,913,268	1135.72	2025.3	850.14
33	22,549,359	4,963,552	26,068,655	1213.59	1010.94	2089.88
34	28,414,236	26,294,210	5,651,969	1544.09	2289.92	1228.81
35	6,589,712	5,475,236	1,256,874	321.56	215.8	88.94
36	38,103,576	48,452,323	6,561,811	1687.86	3168.47	1087.75
37	6,589,423	9,890,077	1,589,427	642.8	864.25	235.6
38	10,360	4870	3049	0.48	0.48	0.23
39	461,338	914,409	145,340	20.14	134.34	9.64
40	23,824,141	28,930,061	5,950,545	1203.98	2200.64	1188.61
41	1,649,860	1,611,377	452,427	86.3	268.2	35.11
42	70,663,429	9,555,353	5,890,077	3486.75	1756.73	482.3
43	24,210	22,903	8850	1.64	4.78	0.78
44	4,905,127	4,905,127	2,647,763	308.86	1402.73	206.67
45	73,987	46,008	40,641	4.5	10.72	4.13
46	36,984,512	68,027,223	8,258,810	2597.63	5217.41	1608.94
47	3,142,292	4,829,845	2,672,200	252.42	834.39	191.75
48	1,853,086	643,340	596,360	90	113.84	45.25
49	25,325,701	25,495,568	5,273,071	1100.64	1880.19	1038.63
50	2,405,128	3,388,555	819,376	107.83	576.91	60.11

model A3 by using student version of GAMS available at IIT Kanpur; and find that our reformulations (model A3 and A2) have superior computational advantages than model has A1.

3. Preparing Test Problems and Results

We created problem instances with set up, inventory carrying, shortage and production cost are normally distributed with mean and variance given below:

Fixed cost: mean 100000 and variance 10000
Shortage cost: mean 5000 and variance 500
Inventory carrying cost: mean 600 and variance 60
Variable Production cost: mean1500 and variance 100

Demand and capacity were chosen from uniform distribution in the range of 10,000 - 15,000. In the case of infeasible solution, the capacity values are increased or demand values are decreased keeping other costs same. We created 50 problem instances each for periods 50, 60 and 100. Models A1, A2 and A3 were coded in GAMS and were solved in GAMS; and they were run in branch and bound mode. The GAMS solver returns a satisfactory solution obtainable in reasonable time. It is to be noted that these problems are NP-HARD and will take few billion centuries to come to optimal solution. Detailed data are given in appendix see Tables 1-6; and consolidated results of "t" test are given in Tables 7-9 below. Models A1, A2 and A3 are compared on the criteria of execution time,

Table 7. 50 Time period: t values.

	A1 A3	A2 A3	A1 A2
Iteration	3.078**	5.179***	−1.406
Execution Time	2.742**	5.78***	4.81***
No. of Nodes Processed	2.736**	3.833***	−0.207

Table 8. 60 Time period: t values.

	A1 A3	A2 A3	A1 A2
Iteration	3.217**	2.546*	0.152
Execution Time	0.57	3.29**	2.572*
No. of Nodes Processed	3.693***	2.938**	−0.385

Table 9. 100 time period: t values.

	A1 A3	A2 A3	A1 A2
Iteration	4.147***	4.409***	−1.942
Execution Time	2.735**	4.596***	2.776**
No. of Nodes Produced	3.691***	4.582***	−0.686

*Significant at 0.05 level; **significant at 0.01 level; ***significant at 0.001 level; In Table 7: 2.742**means that model A3 takes less time than model A1 and is significant at 0.01 level.

number of iterations and number of nodes evaluated in the search tree). On an average, A3 is superior to A1 and A2 and A2 is superior to A1 on most criteria; and large positive "t" values in the Tables 7-9 give adequate support in favor of A3.

4. Discussion and Conclusion

Thus it can be seen that model A3 has superior results in general (except for the case of execution time in 60 period problems) (here A3 is better than A1, but not statistically significant). This shows that the new formulation given by us is superior to models available in literature. This is the useful contribution we make. The three reformulations presented in this paper use Equation (5) and this leads to $\sum_{t=1}^{T} x_t \geq \sum_{t=1}^{T} D_t$ To get $\sum_{t=1}^{T} x_t = \sum_{t=1}^{T} D_t$, we need to develop good heuristics, and show that the duality gap is as less as possible. We have already started work on this, and will come back with results as soon as possible.

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