

# An Inventory Model for Deteriorating Items Having Price, Stock and Advertisement Dependent Demand under Credit Period

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## Abstract

Managing inventory for products that deteriorate over time is a significant challenge for retailers. This model assumes that items follow a two-parameter Weibull distribution to represent the decay rate. In practice, demand is rarely constant and is influenced by factors such as price, stock availability, advertisement efforts, and seasonal trends. By taking inspiration from these real-life situations, we developed an inventory model for deteriorating items with price, stock, and advertisement-dependent demand. The model deals with a two-level partial credit period. The primary goal of this inventory model is to evaluate the optimal selling price and optimal replenishment cycle length in order to minimize the retailer's total cost per unit time. Numerical examples are illustrated to demonstrate the model, and sensitivity analysis highlights the impact of various parameters on the optimal solutions.

## Keywords

Inventory Model, Price, Stock and Advertisement-Dependent Demand, Deterioration, Weibull Distribution of Two Parameters, Trade Credit

## 1. Introduction

In inventory management, managing inventory for deteriorating items such as fresh salads, organic juices, natural cosmetics, nutritional supplements, and frozen meals has become increasingly important due to their limited shelf life and dynamic market demand. These products often follow a two-parameter Weibull-type deterioration process, where their quality or quantity declines at an accelerating rate over time. To effectively manage such items, businesses frequently adopt a two-echelon inventory model involving a supplier and a retailer. Financial

strategies, particularly two-level trade credit policies, add another layer of complexity and opportunity where the supplier offers credit to the retailer, who may in turn extend partial credit to customers requiring an upfront payment and allowing a delay for the remaining amount, thereby encouraging purchases while managing risk. In today's competitive market, selling price is a critical factor that influences customer purchasing decisions. Setting the right selling price is vital for a product's success, as it directly affects both profitability and demand. For instance, strategic pricing of organic juice can lead to significant shifts in customer demand. Typically, a higher price tends to decrease demand, while a lower price can have the contrary effect. Finding a balance is key to ensure both customer satisfaction and business profitability. Advertising plays a vital role in building brand awareness and connecting with customers. It helps customers comprehend how product and service can satisfy their needs and desires. For instance, increased advertising of a nutritional supplement can lead to significant shifts in customer demand. A well-crafted advertisement has the potential to influence consumer purchasing decisions, ultimately leading to increased sales and revenue.

## 2. Literature Review

Pankaj Narang *et al.* [1] developed a model for a deteriorating item under price, stock and advertisement dependent demand. De *et al.* [2] created an inventory model under inflation and partial blacklogging for deteriorating goods with stock and price-dependent demand. Nita Shah *et al.* [3] established an inventory model under credit financing with reliability and inflation for deteriorating items when demand depends on the stock displayed. Sunil Tiwari *et al.* [4] developed a model under two-level trade credit for imperfect quality and deteriorating items. Y.-F. Huang [5] designed a model under trade credit financing by considering retailer's ordering policies. Abubakar Musa *et al.* [6] established inventory policies under delay in Payments, for delayed deteriorating items. Nita Shah and Vaghela [7] generated an EPQ model under two-level trade credit financing for deteriorating items where demand is selling price dependent. Mamta Kumari and De [8] created an EOQ model for deteriorating items to analyze retailers' optimal strategy under trade credit and return policy with nonlinear demand. Monalisha Tripathy *et al.* [9] obtained an EOQ inventory model under progressive credit for a non-instantaneous deteriorating item with constant demand. Ummeferva *et al.* [10] established an inventory model under greening degree dependent demand and reliability under a two-level trade credit policy. Bhaskar *et al.* [11] presented an inventory model for perishable items when the demand rate is selling price and time-dependent under price discount and delay in payment. Aggarwal and Jaggy [12] introduced ordering policies for deterioration items. Mukharjee and Mahata [13] formed an inventory model for optimal replenishment and credit policy under two-level trade credit policies when demand depends on time and credit period. Alaa Fouad Mommenna *et al.* [14] designed a two-storage model where holding cost is time-varying and quantity discounts are considered with a trade credit pol-

icy. Singh *et al.* [15] generated an EOQ model for deterioration items where the deterioration rate is a function of preservation technology under trade credit policy and preservation technology with stock-dependent demand. Mahata [16] invented the EPQ model for deterioration items, where demand and replenishment rate both are constant under a partial trade credit policy. Jaggi *et al.* [17] built an optimal replenishment credit policy under a two-level credit period. Chuan Zhang *et al.* [18] designed retailer's optimal credit policy at risk of customer default under partial trade credit where demand is a positive exponential function of customer's credit period. Santosh Gite [19] developed EOQ Model for deteriorating items with a quadratic time-dependent demand rate under permissible delay in payment.

Extensive research has been conducted on deteriorating items and trade credit under various demand scenarios. However, no prior study has addressed deterioration as a Weibull distribution within a two-level trade credit framework where demand depends on selling price, stock and advertisement.

The objective of this study is to develop a mathematical model that captures these interdependencies for deteriorating items under price and advertisement-dependent demand within a two-level trade credit framework. The model seeks to determine the optimal selling price, replenishment cycle length, and total cost, helping businesses enhance profitability, reduce waste due to spoilage, and maintain service efficiency so that the retailer can better manage their financial expenses. A numerical example supported by sensitivity analysis is included to illustrate the model's practical application and assess how changes in key parameters affect outcomes.

### 3. Notations and Assumptions

#### 3.1. Notations

- $A$ : Frequency of advertisement
- $a$ : Initial demand parameter
- $b$ : price elasticity index
- $c$ : parameter related to stock level
- $\gamma$ : The shape parameter of advertisement
- $\alpha$ : Deterioration parameter  $0 < \alpha < 1$
- $\beta$ : Deterioration parameter  $\beta > 0$
- $S$ : The initial inventory level at  $t = 0$
- $M$ : Retailer's credit period offered by the supplier in years
- $N$ : Customer's credit period offered by the retailer in the years
- $C_o$ : Ordering cost
- $C_d$ : Cost of deterioration per unit item
- $C_h$ : Holding cost per unit item per unit time
- $C_A$ : Advertisement cost
- $C_p$ : Purchase cost per unit item
- $I_p$ : Interest paid by the retailer to the supplier

$I_c$ : Interest earned by the retailer

$t_1$ : Time at which deterioration starts

$T$ : Total time of inventory cycle

$I(t)$ : The stock level at any time  $t$  where  $0 \leq t \leq T$

$TC$ : Total cost per unit time

### 3.2. Assumptions

The demand rate  $D(A, p, I(t))$  is impacted by the frequency of advertisement ( $A$ ), selling price ( $p$ ) and inventory level ( $I(t)$ ), i.e.,

$D(A, p, I(t)) = A^\gamma [a - bp + cI(t)]$  where  $a, b, c, \gamma$  and  $A$  are integers.

1) Deterioration is taken as a Weibull distribution.

2) Shortages are not permitted.

3) The infinite planning horizon is considered.

4) Two-level trade credit policies are implemented in this model according to this policy, the supplier offers a trade credit period to the retailer and retailer's offers a partial trade credit period to the customer.

### 4. Mathematical Formulation

The inventory level  $I(t)$  decreases due to demand during the period  $[0, t_1]$  and in the period  $[t_1, T]$ , the stock level gradually decreases due to deterioration and customer's demand.

The differential equations of the inventory level are expressed as follows:

$$\frac{dI(t)}{dt} = -A^\gamma [a - bp + cI(t)], \quad 0 < t < t_1 \quad (1)$$

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1}I(t) = -A^\gamma [a - bp + cI(t)], \quad t_1 < t < T \quad (2)$$

Under conditions  $I(t) = S$  at  $t = 0$ ;  $I(t) = 0$  at  $t = T$

Solve Equations (1) and (2) to obtain the following solution of  $I(t)$

$$I(t) = \frac{a - bp}{c} (e^{-A^\gamma ct} - 1) + S e^{-A^\gamma ct}, \quad 0 < t < t_1 \quad (3)$$

$$I(t) = -A^\gamma (a - bp) \left[ (t - T) + \frac{\alpha}{\beta + 1} (t^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t^2 - T^2) \right] e^{-(\alpha t^\beta + A^\gamma ct)}, \quad t_1 < t < T \quad (4)$$

Now considering the continuity of  $I(t)$  at  $t = t_1$  we get the initial stock as

$$S = -A^\gamma (a - bp) \left[ (t_1 - T) + \frac{\alpha}{\beta + 1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] e^{-\alpha t_1^\beta} - \frac{a - bp}{c} (1 - e^{-A^\gamma ct_1}) \quad (5)$$

The total order quantity for the retailer is obtained as  $Q = S$ ; therefore, the value of  $Q$  is

$$Q = -A^\gamma (a - bp) \left[ (t_1 - T) + \frac{\alpha}{\beta + 1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] e^{-\alpha t_1^\beta} - \frac{a - bp}{c} (1 - e^{-A^\gamma ct_1}) \quad (6)$$

The different costs associated with the model are evaluated as follows:

Ordering cost (*OC*):

$$C_o \tag{7}$$

Advertisement Cost:

$$C_A A \tag{8}$$

Purchasing Cost (*PC*):

$$C_p Q = C_p \left[ -A^\gamma (a - bp) \left[ (t_1 - T) + \frac{\alpha}{\beta + 1} (t_1^{\beta + 1} - T^{\beta + 1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] e^{-\alpha t_1^\beta} - \frac{a - bp}{c} (1 - e^{A^\gamma c t_1}) \right] \tag{9}$$

Sales revenue (*SR*):

$$= p \left[ \left[ (a - bp) t_1 + \frac{a - bp}{A^\gamma c} (1 - e^{-A^\gamma c t_1}) - t_1 + \frac{a - bp}{A^\gamma c} (1 - e^{A^\gamma c t_1}) (1 - e^{-A^\gamma c t_1}) \right] - A^\gamma (a - bp) \left[ (t_1 - T) + \frac{\alpha}{\beta + 1} (t_1^{\beta + 1} - T^{\beta + 1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] e^{-\alpha M^\beta} M - \frac{1}{2} A^\gamma c M^2 \right] + \left[ (a - bp) \frac{T^2 - t_1^2}{2} - c A^\gamma (a - bp) \frac{T^3 - t_1^3}{3} - c A^\gamma (a - bp) \frac{\alpha}{\beta + 1} \frac{T^{\beta + 3} - t_1^{\beta + 3}}{\beta + 3} \right] \tag{10}$$

Deterioration Cost (*DC*)

$$= Cd \left[ A^\gamma (a - bp) e^{-(\alpha T^\beta + A^\gamma c T)} \alpha \beta \left( 1 + \frac{\alpha}{\beta + 1} (\beta + 1) T^\beta + A^\gamma c T \right) \frac{(T - t_1)^2}{2} \right] \tag{11}$$

Holding cost:

$$HC = C_h \left[ \left[ -A^\gamma (a - bp) \left[ (t_1 - T) + \frac{\alpha}{\beta + 1} (t_1^{\beta + 1} - T^{\beta + 1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] \right] + \left[ -A^\gamma (a - bp) \left[ \frac{T^2 - t_1^2}{2} + \frac{\alpha}{\beta + 1} \left( \frac{T^{\beta + 2}}{\beta + 2} - \frac{t_1^{\beta + 2}}{\beta + 2} - T^{\beta + 1} (T - t_1) \right) \right] + \frac{A^\gamma c}{2} \left( \frac{T^3}{3} - \frac{t_1^3}{3} - T^2 (T - t_1) \right) \right] \right] \tag{12}$$

### Trade Credit

**Scenario 1:**  $M < N$

In this scenario, the credit period supplied by the supplier to a retailer is less than the period supplied by the retailer to the customer.

**Case I:**  $0 < M < N < t_1 < T$

In this case, the interest paid and the interest earned are calculated as follows

$$IP = C_p I_p \left[ \left[ \frac{a - bp}{c} (e^{A^\gamma c t} - 1) (t_1 - M) - \frac{(a - bp) A^\gamma e^{A^\gamma c t_1}}{c} \left( \frac{t_1^2 - T^2}{2} \right) - A^\gamma (a - bp) \left[ (t_1 - T) + \frac{\alpha}{\beta + 1} (t_1^{\beta + 1} - T^{\beta + 1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] (t_1 - M) \right] \right]$$

$$\begin{aligned}
 &+ A^{2\gamma}c(a-bp)\left(\frac{t_1^2-T^2}{2}\right)\left[\left(t_1-T\right)+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-T^{\beta+1}\right)+\frac{A^\gamma c}{2}\left(t_1^2-T^2\right)\right] \\
 &+ \left[-A^\gamma(a-bp)\left[\frac{T^2-t_1^2}{2}+\frac{\alpha}{\beta+1}\left(\frac{T^{\beta+2}}{\beta+2}-\frac{t_1^{\beta+2}}{\beta+2}-T^{\beta+1}(T-t_1)\right)\right.\right. \\
 &\left.\left.+\frac{A^\gamma c}{2}\left(\frac{T^3}{3}-\frac{t_1^3}{3}-T^2(T-t_1)\right)\right]\right] \tag{13}
 \end{aligned}$$

$$\begin{aligned}
 IE &= pI_e \left[ (a-bp)M + \frac{a-bp}{A^\gamma c} (1 - e^{-A^\gamma cM}) - M + \frac{a-bp}{A^\gamma c} (1 - e^{-A^\gamma ct_1^2}) (1 - e^{-A^\gamma ct_1^2}) \right] \\
 &- A^\gamma (a-bp) \left[ (M-T) + \frac{\alpha}{\beta+1} (M^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (M^2 - T^2) \right] e^{-\alpha M^\beta} M \tag{14} \\
 &- \frac{1}{2} A^\gamma cM^2
 \end{aligned}$$

The total cost is obtained as

$$TC_1 = \frac{1}{T} [[SR + IE] - OC - PC - DC - HC - AC - IP]$$

$$\begin{aligned}
 TC_1 &= \frac{1}{T} \left[ p \left[ \left[ (a-bp)t_1 + \frac{a-bp}{A^\gamma c} (1 - e^{-A^\gamma ct_1}) - t_1 + \frac{a-bp}{A^\gamma c} (1 - e^{-A^\gamma ct_1}) (1 - e^{-A^\gamma ct_1}) \right] \right. \right. \\
 &- A^\gamma (a-bp) \left[ (t_1-T) + \frac{\alpha}{\beta+1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] e^{-\alpha M^\beta} M - \frac{1}{2} A^\gamma cM^2 \left. \right] \\
 &+ \left[ (a-bp) \frac{T^2-t_1^2}{2} - cA^\gamma (a-bp) \frac{T^3-t_1^3}{3} - cA^\gamma (a-bp) \frac{\alpha}{\beta+1} \frac{T^{\beta+3}-t_1^{\beta+3}}{\beta+3} \right] \\
 &+ pI_e \left[ (a-bp)M + \frac{a-bp}{A^\gamma c} (1 - e^{-A^\gamma cM}) - M + \frac{a-bp}{A^\gamma c} (1 - e^{-A^\gamma ct_1^2}) (1 - e^{-A^\gamma ct_1^2}) \right] \\
 &- A^\gamma (a-bp) \left[ (M-T) + \frac{\alpha}{\beta+1} (M^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (M^2 - T^2) \right] e^{-\alpha M^\beta} M - \frac{1}{2} A^\gamma cM^2 - C_o \\
 &- C_p \left[ -A^\gamma (a-bp) \left[ (t_1-T) + \frac{\alpha}{\beta+1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] e^{-\alpha t_1^\beta} - \frac{a-bp}{c} (1 - e^{-A^\gamma ct_1}) \right] \\
 &- C_d \left[ A^\gamma (a-bp) e^{-(\alpha T^\beta + A^\gamma cT)} \alpha \beta \left( 1 + \frac{\alpha}{\beta+1} (\beta+1) T^\beta + A^\gamma cT \right) \frac{(T-t_1)^2}{2} \right] \\
 &- C_h \left[ -A^\gamma (a-bp) \left[ (t_1-T) + \frac{\alpha}{\beta+1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] \right] \\
 &+ \left[ -A^\gamma (a-bp) \left[ \frac{T^2-t_1^2}{2} + \frac{\alpha}{\beta+1} \left( \frac{T^{\beta+2}}{\beta+2} - \frac{t_1^{\beta+2}}{\beta+2} - T^{\beta+1}(T-t_1) \right) \right. \right. \\
 &\left. \left. + \frac{A^\gamma c}{2} \left( \frac{T^3}{3} - \frac{t_1^3}{3} - T^2(T-t_1) \right) \right] \right] - C_A A \\
 &- C_p I_p \left[ \frac{a-bp}{c} (e^{A^\gamma ct} - 1) (t_1 - M) - \frac{(a-bp) A^\gamma e^{-A^\gamma ct_1}}{c} \left( \frac{t_1^2 - T^2}{2} \right) \right]
 \end{aligned}$$

$$\begin{aligned}
 & -A^\gamma(a-bp)\left[\left(t_1-T\right)+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-T^{\beta+1}\right)+\frac{A^\gamma c}{2}\left(t_1^2-T^2\right)\right]\left(t_1-M\right) \\
 & +A^{2\gamma}c(a-bp)\left(\frac{t_1^2-T^2}{2}\right)\left[\left(t_1-T\right)+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-T^{\beta+1}\right)+\frac{A^\gamma c}{2}\left(t_1^2-T^2\right)\right] \\
 & +\left[-A^\gamma(a-bp)\left[\frac{T^2-t_1^2}{2}+\frac{\alpha}{\beta+1}\left(\frac{T^{\beta+2}}{\beta+2}-\frac{t_1^{\beta+2}}{\beta+2}-T^{\beta+1}\left(T-t_1\right)\right)\right]\right. \\
 & \left.+\frac{A^\gamma c}{2}\left(\frac{T^3}{3}-\frac{t_1^3}{3}-T^2\left(T-t_1\right)\right)\right]\right] \tag{15}
 \end{aligned}$$

**Case II:**  $0 < t_1 < M < N < T$

In this case, the interest paid and the interest earned are calculated as follows

$$\begin{aligned}
 IP = C_p I_p \left[ -A^\gamma(a-bp)\left[\frac{T^2-M^2}{2}+\frac{\alpha}{\beta+1}\left(\frac{T^{\beta+2}}{\beta+2}-\frac{M^{\beta+2}}{\beta+2}-T^{\beta+1}(T-M)\right)\right] \right. \\
 \left. +\frac{A^\gamma c}{2}\left(\frac{T^3}{3}-\frac{M^3}{3}-T^2(T-M)\right)\right] \tag{16}
 \end{aligned}$$

$$\begin{aligned}
 IE = pI_e \left[ \left[ (a-bp)t_1 + \frac{a-bp}{A^\gamma c}\left(1-e^{-A^\gamma ct_1}\right) - t_1 + \frac{a-bp}{A^\gamma c}\left(1-e^{A^\gamma ct_1^2}\right)\left(1-e^{-A^\gamma ct_1^2}\right) \right] \right. \\
 -A^\gamma(a-bp)\left[\left(t_1-T\right)+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-T^{\beta+1}\right)+\frac{A^\gamma c}{2}\left(t_1^2-T^2\right)\right]e^{-\alpha M^\beta}t_1-\frac{1}{2}A^\gamma ct_1^2 \tag{17} \\
 \left. +\left[(a-bp)\frac{M^2-t_1^2}{2}-cA^\gamma(a-bp)\frac{M^3-t_1^3}{3}-cA^\gamma(a-bp)\frac{\alpha}{\beta+1}\frac{M^{\beta+3}-t_1^{\beta+3}}{\beta+3}\right] \right]
 \end{aligned}$$

The total cost is obtained as

$$\begin{aligned}
 TC_2 & = \frac{1}{T}[[SR + IE] - OC - PC - DC - HC - AC - IP] \\
 TC_2 & = \frac{1}{T} \left[ p \left[ \left[ (a-bp)t_1 + \frac{a-bp}{A^\gamma c}\left(1-e^{-A^\gamma ct_1}\right) - t_1 + \frac{a-bp}{A^\gamma c}\left(1-e^{A^\gamma ct_1^2}\right)\left(1-e^{-A^\gamma ct_1^2}\right) \right] \right. \right. \\
 & -A^\gamma(a-bp)\left[\left(t_1-T\right)+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-T^{\beta+1}\right)+\frac{A^\gamma c}{2}\left(t_1^2-T^2\right)\right]e^{-\alpha M^\beta}M-\frac{1}{2}A^\gamma cM^2 \tag{17} \\
 & \left. +\left[(a-bp)\frac{T^2-t_1^2}{2}-cA^\gamma(a-bp)\frac{T^3-t_1^3}{3}-cA^\gamma(a-bp)\frac{\alpha}{\beta+1}\frac{T^{\beta+3}-t_1^{\beta+3}}{\beta+3}\right] \right] \\
 & + pI_e \left[ \left[ (a-bp)t_1 + \frac{a-bp}{A^\gamma c}\left(1-e^{-A^\gamma ct_1}\right) - t_1 + \frac{a-bp}{A^\gamma c}\left(1-e^{A^\gamma ct_1^2}\right)\left(1-e^{-A^\gamma ct_1^2}\right) \right] \right. \\
 & -A^\gamma(a-bp)\left[\left(t_1-T\right)+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-T^{\beta+1}\right)+\frac{A^\gamma c}{2}\left(t_1^2-T^2\right)\right]e^{-\alpha M^\beta}t_1-\frac{1}{2}A^\gamma ct_1^2 \tag{17} \\
 & \left. +\left[(a-bp)\frac{M^2-t_1^2}{2}-cA^\gamma(a-bp)\frac{M^3-t_1^3}{3}-cA^\gamma(a-bp)\frac{\alpha}{\beta+1}\frac{M^{\beta+3}-t_1^{\beta+3}}{\beta+3}\right] \right] - C_o \\
 & -C_p \left[ -A^\gamma(a-bp)\left[\left(t_1-T\right)+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-T^{\beta+1}\right)+\frac{A^\gamma c}{2}\left(t_1^2-T^2\right)\right]e^{-\alpha t_1^\beta}-\frac{a-bp}{c}\left(1-e^{A^\gamma ct_1}\right) \right]
 \end{aligned}$$

$$\begin{aligned}
 & -C_d \left[ A^\gamma (a-bp) e^{-(\alpha T^\beta + A^\gamma cT)} \alpha \beta \left( 1 + \frac{\alpha}{\beta+1} (\beta+1) T^\beta + A^\gamma cT \right) \frac{(T-t_1)^2}{2} \right] \\
 & -C_h \left[ \left[ -A^\gamma (a-bp) \left[ (t_1-T) + \frac{\alpha}{\beta+1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] \right] \right. \\
 & + \left[ -A^\gamma (a-bp) \left[ \frac{T^2 - t_1^2}{2} + \frac{\alpha}{\beta+1} \left( \frac{T^{\beta+2}}{\beta+2} - \frac{t_1^{\beta+2}}{\beta+2} - T^{\beta+1} (T-t_1) \right) \right. \right. \\
 & \left. \left. + \frac{A^\gamma c}{2} \left( \frac{T^3}{3} - \frac{t_1^3}{3} - T^2 (T-t_1) \right) \right] \right] - C_A A \\
 & -C_p I_p \left[ -A^\gamma (a-bp) \left[ \frac{T^2 - M^2}{2} + \frac{\alpha}{\beta+1} \left( \frac{T^{\beta+2}}{\beta+2} - \frac{M^{\beta+2}}{\beta+2} - T^{\beta+1} (T-M) \right) \right. \right. \\
 & \left. \left. + \frac{A^\gamma c}{2} \left( \frac{T^3}{3} - \frac{M^3}{3} - T^2 (T-M) \right) \right] \right] \tag{18}
 \end{aligned}$$

**Scenario 2:**  $M > N$

In this scenario, the credit period supplied by the supplier to the retailer is greater than the period supplied by the retailer to the customer.

**Case III:**  $0 < N < M < t_1 < T$

In this case, the interest paid and the interest earned are calculated as follows

$$\begin{aligned}
 IP = C_p I_p & \left[ \left[ \frac{a-bp}{c} (e^{A^\gamma ct} - 1) (t_1 - M) - \frac{(a-bp) A^\gamma e^{A^\gamma ct_1}}{c} \left( \frac{t_1^2 - T^2}{2} \right) \right. \right. \\
 & - A^\gamma (a-bp) \left[ (t_1 - T) + \frac{\alpha}{\beta+1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] (t_1 - M) \\
 & \left. \left. + A^{2\gamma} c (a-bp) \left( \frac{t_1^2 - T^2}{2} \right) \left[ (t_1 - T) + \frac{\alpha}{\beta+1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] \right] \right. \\
 & + \left[ -A^\gamma (a-bp) \left[ \frac{T^2 - t_1^2}{2} + \frac{\alpha}{\beta+1} \left( \frac{T^{\beta+2}}{\beta+2} - \frac{t_1^{\beta+2}}{\beta+2} - T^{\beta+1} (T-t_1) \right) \right. \right. \\
 & \left. \left. + \frac{A^\gamma c}{2} \left( \frac{T^3}{3} - \frac{t_1^3}{3} - T^2 (T-t_1) \right) \right] \right] \tag{19}
 \end{aligned}$$

$$\begin{aligned}
 IE = pI_e & \left[ \left[ \left[ (a-bp)N + \frac{a-bp}{A^\gamma c} (1 - e^{-A^\gamma cN}) \right] - N + \frac{a-bp}{A^\gamma c} (1 - e^{-A^\gamma ct_1^2}) (1 - e^{-A^\gamma ct_1^2}) \right] \right. \\
 & - A^\gamma (a-bp) \left[ (N-T) + \frac{\alpha}{\beta+1} (N^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (N^2 - T^2) \right] e^{-\alpha N^\beta} N - \frac{1}{2} A^\gamma c N^2 \left. \right] \\
 & + \left[ (a-bp)(M-N) + (a-bp)(-A^\gamma) \frac{M^2 - N^2}{2} + (a-bp) A^\gamma t_1 (M-N) \right. \\
 & \left. + (a-bp) A^\gamma ct_1 \frac{M^2 - N^2}{2} - A^\gamma (a-bp) \right] \left[ (t_1 - T) + \frac{\alpha}{\beta+1} (\beta+1) t_1^\beta (t_1 - T) \right] (M-N) \\
 & - A^\gamma (a-bp) \left[ (t_1 - T) + \frac{\alpha}{\beta+1} (\beta+1) t_1^\beta (t_1 - T) \right] A^\gamma c \frac{M^2 - N^2}{2} \tag{20}
 \end{aligned}$$

The total cost is obtained as

$$TC_3 = \frac{1}{T} [[SR + IE] - OC - PC - DC - HC - AC - IP]$$

$$\begin{aligned}
 TC_3 = & \frac{1}{T} \left[ p \left[ \left[ (a-bp)t_1 + \frac{a-bp}{A^\gamma c} (1 - e^{-A^\gamma ct_1}) - t_1 + \frac{a-bp}{A^\gamma c} (1 - e^{A^\gamma ct_1}) (1 - e^{-A^\gamma ct_1}) \right] \right. \right. \\
 & - A^\gamma (a-bp) \left[ (t_1 - T) + \frac{\alpha}{\beta+1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] e^{-\alpha M^\beta} M - \frac{1}{2} A^\gamma c M^2 \left. \right] \\
 & + \left[ (a-bp) \frac{T^2 - t_1^2}{2} - c A^\gamma (a-bp) \frac{T^3 - t_1^3}{3} - c A^\gamma (a-bp) \frac{\alpha}{\beta+1} \frac{T^{\beta+3} - t_1^{\beta+3}}{\beta+3} \right] \\
 & + p I_e \left[ \left[ (a-bp) N + \frac{a-bp}{A^\gamma c} (1 - e^{-A^\gamma cN}) - N + \frac{a-bp}{A^\gamma c} (1 - e^{A^\gamma ct_1^2}) (1 - e^{-A^\gamma ct_1^2}) \right] \right. \\
 & - A^\gamma (a-bp) \left[ (N - T) + \frac{\alpha}{\beta+1} (N^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (N^2 - T^2) \right] e^{-\alpha N^\beta} N - \frac{1}{2} A^\gamma c N^2 \left. \right] \\
 & + \left[ (a-bp)(M - N) + (a-bp)(-A^\gamma) \frac{M^2 - N^2}{2} + (a-bp) A^\gamma t_1 (M - N) \right. \\
 & + (a-bp) A^\gamma ct_1 \frac{M^2 - N^2}{2} - A^\gamma (a-bp) \left. \right] \left[ (t_1 - T) + \frac{\alpha}{\beta+1} (\beta+1) t_1^\beta (t_1 - T) \right] (M - N) \\
 & - A^\gamma (a-bp) \left[ (t_1 - T) + \frac{\alpha}{\beta+1} (\beta+1) t_1^\beta (t_1 - T) \right] A^\gamma c \frac{M^2 - N^2}{2} \left. \right] - C_o \\
 & - C_p \left[ -A^\gamma (a-bp) \left[ (t_1 - T) + \frac{\alpha}{\beta+1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] e^{-\alpha t_1^\beta} - \frac{a-bp}{c} (1 - e^{A^\gamma ct_1}) \right] \\
 & - C_d \left[ A^\gamma (a-bp) e^{-(\alpha T^\beta + A^\gamma cT)} \alpha \beta \left( 1 + \frac{\alpha}{\beta+1} (\beta+1) T^\beta + A^\gamma cT \right) \frac{(T - t_1)^2}{2} \right] \tag{21} \\
 & - C_h \left[ -A^\gamma (a-bp) \left[ (t_1 - T) + \frac{\alpha}{\beta+1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] \right] \\
 & + \left[ -A^\gamma (a-bp) \left[ \frac{T^2 - t_1^2}{2} + \frac{\alpha}{\beta+1} \left( \frac{T^{\beta+2}}{\beta+2} - \frac{t_1^{\beta+2}}{\beta+2} - T^{\beta+1} (T - t_1) \right) \right. \right. \\
 & \left. \left. + \frac{A^\gamma c}{2} \left( \frac{T^3}{3} - \frac{t_1^3}{3} - T^2 (T - t_1) \right) \right] \right] - C_A A \\
 & - C_p I_p \left[ \left[ \frac{a-bp}{c} (e^{A^\gamma ct} - 1) (t_1 - M) - \frac{(a-bp) A^\gamma e^{A^\gamma ct_1}}{c} \left( \frac{t_1^2 - T^2}{2} \right) \right. \right. \\
 & - A^\gamma (a-bp) \left[ (t_1 - T) + \frac{\alpha}{\beta+1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] (t_1 - M) \\
 & + A^{2\gamma} c (a-bp) \left( \frac{t_1^2 - T^2}{2} \right) \left[ (t_1 - T) + \frac{\alpha}{\beta+1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] \left. \right] \\
 & + \left[ -A^\gamma (a-bp) \left[ \frac{T^2 - t_1^2}{2} + \frac{\alpha}{\beta+1} \left( \frac{T^{\beta+2}}{\beta+2} - \frac{t_1^{\beta+2}}{\beta+2} - T^{\beta+1} (T - t_1) \right) \right. \right. \\
 & \left. \left. + \frac{A^\gamma c}{2} \left( \frac{T^3}{3} - \frac{t_1^3}{3} - T^2 (T - t_1) \right) \right] \right] \left. \right]
 \end{aligned}$$

**Case IV:**  $0 < t_1 < N < M < T$

In this case, the interest paid and the interest earned are calculated as follows

$$IP = C_p I_p \left[ -A^\gamma (a - bp) \left[ \frac{T^2 - M^2}{2} + \frac{\alpha}{\beta + 1} \left( \frac{T^{\beta+2}}{\beta + 2} - \frac{M^{\beta+2}}{\beta + 2} - T^{\beta+1} (T - M) \right) + \frac{A^\gamma c}{2} \left( \frac{T^3}{3} - \frac{M^3}{3} - T^2 (T - M) \right) \right] \right] \tag{22}$$

$$IE = p I_e \left[ \left[ (a - bp)t_1 + \frac{a - bp}{A^\gamma c} (1 - e^{-A^\gamma ct_1}) - t_1 + \frac{a - bp}{A^\gamma c} (1 - e^{-A^\gamma ct_1^2}) (1 - e^{-A^\gamma ct_1^2}) \right] - A^\gamma (a - bp) \left[ (t_1 - T) + \frac{\alpha}{\beta + 1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] e^{-\alpha M^\beta} t_1 - \frac{1}{2} A^\gamma ct_1^2 \right] + \left[ (a - bp) \frac{N^2 - t_1^2}{2} - cA^\gamma (a - bp) \frac{N^3 - t_1^3}{3} - cA^\gamma (a - bp) \frac{\alpha}{\beta + 1} \frac{N^{\beta+3} - t_1^{\beta+3}}{\beta + 3} \right] + \left[ (a - bp) \frac{M^2 - N^2}{2} - cA^\gamma (a - bp) \frac{M^3 - N^3}{3} - cA^\gamma (a - bp) \frac{\alpha}{\beta + 1} \frac{M^{\beta+3} - N^{\beta+3}}{\beta + 3} \right] \right] \tag{23}$$

The total cost is obtained as

$$TC_4 = \frac{1}{T} \left[ [SR + IE] - OC - PC - DC - HC - AC - IP \right]$$

$$TC_4 = \frac{1}{T} \left[ p \left[ \left[ (a - bp)t_1 + \frac{a - bp}{A^\gamma c} (1 - e^{-A^\gamma ct_1}) - t_1 + \frac{a - bp}{A^\gamma c} (1 - e^{-A^\gamma ct_1}) (1 - e^{-A^\gamma ct_1}) \right] - A^\gamma (a - bp) \left[ (t_1 - T) + \frac{\alpha}{\beta + 1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] e^{-\alpha M^\beta} M - \frac{1}{2} A^\gamma cM^2 \right] + \left[ (a - bp) \frac{T^2 - t_1^2}{2} - cA^\gamma (a - bp) \frac{T^3 - t_1^3}{3} - cA^\gamma (a - bp) \frac{\alpha}{\beta + 1} \frac{T^{\beta+3} - t_1^{\beta+3}}{\beta + 3} \right] \right] + p I_e \left[ \left[ (a - bp)t_1 + \frac{a - bp}{A^\gamma c} (1 - e^{-A^\gamma ct_1}) - t_1 + \frac{a - bp}{A^\gamma c} (1 - e^{-A^\gamma ct_1^2}) (1 - e^{-A^\gamma ct_1^2}) \right] - A^\gamma (a - bp) \left[ (t_1 - T) + \frac{\alpha}{\beta + 1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] e^{-\alpha M^\beta} t_1 - \frac{1}{2} A^\gamma ct_1^2 \right] + \left[ (a - bp) \frac{N^2 - t_1^2}{2} - cA^\gamma (a - bp) \frac{N^3 - t_1^3}{3} - cA^\gamma (a - bp) \frac{\alpha}{\beta + 1} \frac{N^{\beta+3} - t_1^{\beta+3}}{\beta + 3} \right] + \left[ (a - bp) \frac{M^2 - N^2}{2} - cA^\gamma (a - bp) \frac{M^3 - N^3}{3} - cA^\gamma (a - bp) \frac{\alpha}{\beta + 1} \frac{M^{\beta+3} - N^{\beta+3}}{\beta + 3} \right] \right] - C_o - C_p \left[ -A^\gamma (a - bp) \left[ (t_1 - T) + \frac{\alpha}{\beta + 1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] e^{-\alpha t_1^\beta} - \frac{a - bp}{c} (1 - e^{-A^\gamma ct_1}) \right] - C_d \left[ A^\gamma (a - bp) e^{-(\alpha T^\beta + A^\gamma cT)} \alpha \beta \left( 1 + \frac{\alpha}{\beta + 1} (\beta + 1) T^\beta + A^\gamma cT \right) \frac{(T - t_1)^2}{2} \right] - C_h \left[ -A^\gamma (a - bp) \left[ (t_1 - T) + \frac{\alpha}{\beta + 1} (t_1^{\beta+1} - T^{\beta+1}) + \frac{A^\gamma c}{2} (t_1^2 - T^2) \right] \right] \right]$$

$$\begin{aligned}
 & + \left[ -A^\gamma (a - bp) \left[ \frac{T^2 - t_1^2}{2} + \frac{\alpha}{\beta + 1} \left( \frac{T^{\beta+2}}{\beta + 2} - \frac{t_1^{\beta+2}}{\beta + 2} - T^{\beta+1} (T - t_1) \right) \right. \right. \\
 & \left. \left. + \frac{A^\gamma c}{2} \left( \frac{T^3}{3} - \frac{t_1^3}{3} - T^2 (T - t_1) \right) \right] \right] - C_A A \\
 & - C_p I_p \left[ -A^\gamma (a - bp) \left[ \frac{T^2 - M^2}{2} + \frac{\alpha}{\beta + 1} \left( \frac{T^{\beta+2}}{\beta + 2} - \frac{M^{\beta+2}}{\beta + 2} - T^{\beta+1} (T - M) \right) \right. \right. \\
 & \left. \left. + \frac{A^\gamma c}{2} \left( \frac{T^3}{3} - \frac{M^3}{3} - T^2 (T - M) \right) \right] \right] \tag{24}
 \end{aligned}$$

The retailer’s total cost in each case is the function of two variables,  $T$  and  $p$ . To minimize total cost with respect to selling price  $p$  and cycle Time  $T$ , the necessary conditions are

$$\frac{\partial TC_i}{\partial T} = 0 \quad \text{and} \quad \frac{\partial TC_i}{\partial p} = 0 \quad \text{where } i = 1, 2, 3, 4$$

The critical point  $(T^*, p^*)$  can be found by solving  $\frac{\partial TC_i}{\partial T} = 0$  and  $\frac{\partial TC_i}{\partial p} = 0$  simultaneously for each case and for minimising cost, the following condition should be satisfied for the critical point.

$$\frac{\partial^2 TC_i}{\partial T^2} \cdot \frac{\partial^2 TC_i}{\partial p^2} - \frac{\partial^2 TC_i}{\partial T \partial p} \cdot \frac{\partial^2 TC_i}{\partial p \partial T} < 0 \quad \text{and} \quad \frac{\partial^2 TC_i}{\partial T^2} > 0, \frac{\partial^2 TC_i}{\partial p^2} > 0$$

Using this approach, we calculate the total cost in each case for the numerical example provided in the following section

### 5. Numerical Illustration

We consider the following parameters

**Case I:**

$A = 5, a = 20, b = 0.5, c = 1, t_1 = 0.3, \alpha = 0.1, \beta = 0.3, \gamma = 0.3, M = 0.041, N = 0.08200, I_p = 0.15, I_e = 0.10; C_o = 50, C_h = 0.5, C_p = 10, C_a = 3, C_d = 1$ . Then the optimal solution is as follows

$$P^* = 99.3572, T = 3.1301, TC = 13526.00$$

**Case II:**

$A = 5, a = 20, b = 0.5, c = 1, t_1 = 0.3, \alpha = 0.1, \beta = 0.3, \gamma = 0.3, M = 0.3013, N = 0.3150, I_p = 0.15, I_e = 0.10; C_o = 50, C_h = 0.5, C_p = 10, C_a = 3, C_d = 1$ . Then the optimal solution is as follows

$$P^* = 124.7984, T = 3.3643, TC = 24029.2613.$$

**Case III:**

$A = 5, a = 20, b = 0.5, c = 1, t_1 = 0.3, \alpha = 0.1, \beta = 0.3, \gamma = 0.3, M = 0.082, N = 0.041, I_p = 0.15, I_e = 0.10; C_o = 50, C_h = 0.5, C_p = 10, C_a = 3, C_d = 1$ . Then the optimal solution is as follows

$$P^* = 95.7250, T = 3.0627, TC = 11571.5496.$$

**Case IV:**

$A = 5, a = 20, b = 0.5, c = 1, t_1 = 0.3, \alpha = 0.1, \beta = 0.3, \gamma = 0.3, M = 0.3150, N =$

0.3013,  $I_p = 0.15$ ,  $I_e = 0.10$ ;  $C_o = 50$ ,  $C_h = 0.5$ ,  $C_p = 10$ ,  $C_a = 3$ ,  $C_d = 1$ . Then the optimal solution is as follows

$$P^* = 136.3396, T = 3.5759, TC = 34159.16940.$$

## 6. Sensitivity Analysis

To study the effect of the changes in the numerical values of different parameters on the optimal solution of the current inventory model, we have performed the sensitivity analysis by changing the numerical values of each parameter while keeping the initial values of the remaining parameters the same, which is given in **Table 1**.

**Table 1.** A sensitivity analysis is performed for case I, changing the inventory parameters.

Parameter	change	$P^*$	$T^*$	$TC$
$a$	18	61.6195	2.5559	2373.781777
	19	78.1417	2.8212	5796.249698
	20	99.3572	3.1301	13425.289882
	21	127.0256	3.4945	30670.765140
	22	163.8926	3.9366	71355.780808
$b$	0.46	82.6318	2.6400	4484.140550
	0.48	90.1509	2.8668	7770.253752
	0.50	99.3572	3.1301	13425.289882
	0.52	110.7293	3.4381	23421.131722
	0.54	124.9709	3.8038	41766.364551
$c$	0.98	89.7755	2.9548	8766.470768
	0.99	94.3635	3.0394	10830.789984
	1.00	99.3572	3.1301	13425.289882
	1.01	104.8128	3.2277	16711.432050
	1.02	110.7966	3.3330	20907.307624
$\alpha$	0.06	78.1837	2.7567	4875.384276
	0.08	87.6601	2.9258	8011.489815
	0.10	99.3572	3.1301	13425.289882
	0.12	114.1661	3.3804	23267.810164
	0.14	133.5106	3.6951	42449.602196
$\beta$	0.1	118.4523	3.4811	26567.408276
	0.2	107.1571	3.2763	18077.875969
	0.3	99.3572	3.1301	13425.289882
	0.4	93.7967	3.0225	10640.427401
	0.5	89.7690	2.9421	8874.390582

## Continued

	0.28	76.7511	2.6743	4323.214688
	0.29	86.7879	2.8819	7548.352292
$\gamma$	0.30	99.3572	3.1301	13425.289882
	0.31	115.4566	3.4317	24673.650031
	0.32	136.6726	3.8074	47666.071993
	0.26	83.6721	2.7699	6476.398664
	0.28	91.3595	2.9495	9461.859281
	0.30	99.3572	3.1301	13425.289882
$t_1$	0.32	107.7636	3.3139	18667.785908
	0.34	116.7001	3.5030	25610.945398
	30	139.8663	3.8136	48121.293864
	40	116.2472	3.4252	24364.140586
	50	99.3572	3.1301	13425.289882
$C_o$	60	86.7263	2.8966	7828.695206
	70	76.9535	2.7072	4737.434016
	0.6	99.4060	3.1310	13451.013916
	0.8	99.3816	3.1305	13437.671733
$C_d$	1.0	99.3572	3.1301	13425.289882
	1.2	99.3329	3.1296	13412.000943
	1.4	99.3085	3.1292	13399.635540
	0.1	114.1888	3.3901	22658.762978
	0.3	106.3041	3.2535	17341.523205
$C_h$	0.5	99.3572	3.1301	13425.289882
	0.7	93.1921	3.0178	10492.600953
	0.9	87.6850	2.9150	8264.487153
	1	116.2472	3.4252	24364.140586
	2	107.1497	3.2684	17931.200832
$C_a$	3	99.3572	3.1301	13425.289882
	4	92.6140	3.0070	10193.764954
	5	86.7263	2.8966	7828.695206
	9.0	65.8917	2.4282	2069.430046
	9.5	79.8322	2.7328	5274.082822
$C_p$	10.0	99.3572	3.1301	13425.289882
	10.5	128.0032	3.6633	36281.439718
	11.0	173.0796	4.4271	111917.335461
	0.13	115.9857	3.3888	23185.470925
	0.14	107.1085	3.2520	17526.974559
$I_p$	0.15	99.3572	3.1301	13425.289882
	0.16	92.5318	3.0207	10393.342691
	0.17	86.4771	2.9219	8113.504528

## Continued

	0.8	101.4209	3.1676	14553.559776
	0.9	100.3788	3.1487	13976.252836
$I_c$	0.10	99.3572	3.1301	13425.289882
	0.11	98.3554	3.1118	12899.287997
	0.12	97.3729	3.0938	12397.003606
	0.02739	102.4244	3.1859	15132.486979
	0.03287	101.2505	3.1646	14462.096251
$M$	0.04109	99.3572	3.1301	13425.289882
	0.04657	98.4373	3.1132	12936.607209
	0.05205	97.3575	3.0934	12381.282146
	0.07123	99.3572	3.1301	13425.289882
	0.07671	99.3572	3.1301	13425.289882
$N$	0.08200	99.3572	3.1301	13425.289882
	0.08767	99.3572	3.1301	13425.289882
	0.09315	99.3572	3.1301	13425.289882
	4.50	78.5951	2.7107	4798.331111
	4.75	88.2968	2.9109	8109.469642
$A$	5.00	99.3572	3.1301	13425.289882
	5.25	112.6870	3.3823	22454.874872
	5.50	128.2019	3.6630	37420.137469

## 6.1. Observations from Table 1

1) As  $a$  increases, the selling price  $p$  rises significantly. The total cycle length  $T$  also increases with  $a$ , and the total cost exhibits exponential growth.

2) As  $b$  increases, the selling price  $p^*$  rises consistently. The total cycle length  $T^*$  also increases, and the total cost shows a rapid increase with increasing  $b$ .

3) As  $c$  increases, the selling price  $p^*$  increases steadily, the total cycle length  $T^*$  increases slightly, and the total cost grows significantly.

4) As  $\alpha$  increases, the selling price  $p^*$  shows significant growth, the total cycle length  $T^*$  increases moderately, and the total cost experiences exponential growth.

5) As  $\beta$  increases, the selling price  $p^*$  decreases, the total cycle length  $T^*$  decreases, and the total cost also decreases significantly.

6) As  $\gamma$  increases, the selling price  $p^*$  increases significantly, the total cycle length  $T^*$  grows steadily, and the total cost shows exponential growth.

7) As  $t_1$  increases, the selling price  $p^*$  and the total cycle length  $T^*$  rise consistently, and the total cost grows rapidly.

8) As  $C_o$  increases, the selling price  $P^*$ , the total cycle length  $T^*$  and the total cost decrease.

9) As  $C_d$  increases, there is a slight decrease in the selling price  $P^*$ , the total cycle

length  $T$ , and the total cost.

10) As  $C_h$  increases, the selling price  $P^*$ , the total cycle length  $T^*$ , and the total cost decrease steadily.

11) As  $C_a$  increases, the selling price  $P^*$ , the total cycle length  $T^*$ , and the total cost decrease.

12) As  $C_p$  increases, the selling price  $P^*$ , the total cycle length  $T^*$ , and the total cost grow significantly.

13) As  $I_p$  increases, the selling price  $P^*$ , the total cycle length  $T^*$ , and the total cost decrease consistently.

14) As  $I_e$  increases, the selling price  $P^*$ , the total cycle length  $T^*$ , and the total cost decrease slightly.

15) As  $M$  increases, the selling price  $P^*$ , the total cycle length  $T^*$ , and the total cost decrease gradually.

16) No significant changes in the selling price  $P^*$ , the total cycle length  $T^*$ , and the total cost are observed as  $N$  varies.

17) As  $A$  increases, the selling price  $P^*$ , the total cycle length  $T^*$ , and the total cost grow significantly.

**Table 2.** The feasible solution to the inventory problem.

Cases	$M$	$N$	$t_1$	$T$	$p$	Total Cost ( $TC$ )
Case I: $0 < M < N < t_1 < T$	0.04166	0.08219	0.3	3.1338	99.5609	13526.0000
Case II: $0 < t_1 < M < N < T$	0.3013	0.3150	0.3	3.3643	124.7984	24029.2613
Case III: $0 < N < M < t_1 < T$	0.08219	0.04166	0.3	3.0627	95.7250	11571.5496
Case IV: $0 < t_1 < N < M < T$	0.3150	0.3013	0.3	3.5759	136.3396	34159.16940

## 6.2. Observations from Table 2

### Case I:

- The supplier offers customers a shorter credit period than the retailer does. This implies that the retailer must settle their dues with the supplier earlier than when they collect payments from customers.
- The time at which deterioration starts is significantly later than both  $M$  and  $N$ . This allows sufficient time to sell products before deterioration impacts inventory.
- The total cycle length is shorter than in other cases, helping reduce holding costs. With a moderate selling price, the total cost is relatively low.
- Case I demonstrates a cost-efficient setup where the trade credit periods, cycle length, and selling price are balanced to minimize total costs.

### Case II:

- The retailer's credit period from the supplier is longer than when deterioration starts. Additionally, the retailer offers customers an even longer credit period.

This extended credit arrangement increases financial risks.

- Deterioration starts early in the cycle relative to credit periods, meaning part of the inventory might deteriorate before being sold.
- The total cycle length is relatively long, increasing holding costs. The high selling price leads to a significantly higher total cost, making this configuration the most expensive.
- Case II highlights inefficiencies in aligning credit periods with deterioration timing, leading to the highest total cost.

**Case III:**

- The customer's credit period is shorter than the retailer's credit period from the supplier. This favorable arrangement ensures that the retailer collects payments from customers before having to pay the supplier.
- Deterioration starts well after both credit periods, reducing the risk of holding deteriorated inventory before sales.
- A relatively shorter cycle length and a low selling price result in the second-lowest total cost.
- Case III is the most cost-efficient configuration resulting from its optimal alignment of credit periods, inventory management, and selling price.

**Case IV:**

- The customer's credit period is shorter than the retailer's credit period from the supplier. This alignment is favorable, but the close proximity of  $N$  and  $M$  creates a smaller buffer.
- Deterioration starts early, before the customer credit period ends, increasing the risk of holding deteriorated inventory before payments are collected.
- The total cycle length is the longest among the cases, significantly increasing holding costs. The high selling price contributes to the second-highest total cost.
- Case IV shows inefficiencies due to a longer cycle length and a higher selling price.

## 7. Conclusions

In this paper, we have developed an EOQ model for deteriorating items with a two-level trade credit period and multifactor dependent demand where demand is influenced by price, stock and advertising. We present the optimal strategy to minimize cost. As in this model, a two-level trade credit policy is considered, so this model is more beneficial for retailers to make their inventory strategy to minimize cost and maximize their profit. As a result, the retailer can better manage their financial expenses. Furthermore, by leveraging insights from demand patterns such as boosting advertising during peak seasons or dynamically adjusting prices, retailers can influence purchasing behavior and maximize the turnover of deteriorating items. Implementing an integrated inventory model enables retailers to determine optimal order quantities, pricing, promotional schedules, and stock levels while minimizing losses from spoilage and stock outs. Ultimately, this model

empowers retailers to maximize profitability, maintain product freshness, and respond more effectively to market trends, all within the constraints of supplier agreements and product perishability. The impact of varying parameter values on the solution has been analyzed through sensitivity analysis. It is observed that changes in the value of price sensitivity in demand function ( $b$ ), advertisement cost ( $C_A$ ), ordering cost  $C_0$ , retailer's credit period ( $M$ ) as well as holding cost ( $c_h$ ) and purchase cost ( $C_p$ ) lead to significant result on cycle length, selling price and total cost.

The scope of this work can be expanded in the following directions. One possible extension could be allowing shortages; one could consider time deterioration rate and preservation technology to reduce deterioration in future research.

### Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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