American Journal of Industrial and Business Management, 2025, 15(1), 41-70
https://www.scirp.org/journal/ajibm
ISSN Online: 2164-5175
ISSN Print: 2164-5167

Dynamic Volatility Spillovers among Green Bonds, Green Stocks and Carbon Markets under the COVID-19: Evidence from China

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How to cite this paper: Ye, S. Y., Zhang, H. M., & Lou, Q. (2025). Dynamic Volatility Spillovers among Green Bonds, Green Stocks and Carbon Markets under the COVID-19: Evidence from China. *American Journal of Industrial and Business Management, 15,* 41-70.
https://doi.org/10.4236/ajibm.2025.151004

Received: December 10, 2024
Accepted: January 18, 2025
Published: January 21, 2025

Abstract

The carbon emissions trading markets are strongly linked to the green bond and green stock markets. To gain a deeper understanding of the complex relationships among these three markets, we apply dynamic Copula and VAR-BEKK-GARCH-X models to investigate the dynamic dependence and volatility spillovers between green bond indices (i.e. RGBI), green stock prices (i.e. RGLS), and carbon trading allowances (i.e. RHBC). Additionally, it further investigates the impact of exogenous variables (RGBI/RGLS/RHBC) on market volatility. The empirical results show increased dependence and volatility spillovers during the COVID-19 outbreak. RHBC, as an exogenous variable, demonstrates predictive power in the RGBI-RGLS market spillovers. The findings underscore the strategic importance of green asset allocation for investors and policymakers.

Keywords

Dynamic Copula, VAR-BEKK-GARCH-X, Dynamic Dependence, Volatility Spillover, Exogenous Variable, Green Bonds, Green Stocks, Carbon Markets, Green Finance, Sustainable Development

1. Introduction

Climate change within the framework of the United Nations Framework Convention on Climate Change (UNFCCC) is an essential global public policy issue (Jiang et al., 2023). Global economic growth has led to a rise in carbon dioxide emissions. It is widely believed that global warming and environmental degradation are primarily caused by uncontrolled carbon emissions (Solomon et al., 2009; Shujah-ur-

Rahman et al., 2019). Countries are actively improving their financial policy toolbox, vigorously promoting green financial services that promote ecological enhancement, climate change mitigation, and efficient resource utilization, and establishing a carbon emissions trading system to promote environmental protection and achieve sustainable economic and social development goals.

Green finance provides the necessary project finance for developing an environmentally friendly economy (Sachs et al., 2019). The results of Lv and Li's (2021) study show that the widespread promotion of green finance helps to reduce the emission of environmental contaminants like carbon dioxide. Beyond a decade ago, the green finance market exhibited remarkable development potential. Research on the green financial markets includes aspects such as green bonds (e.g. Banga, 2019; Reboredo, 2018; Tolliver et al., 2020), green stocks (Rezec & Scholtens, 2017; Pham, 2021), and other aspects. Furthermore, the implementation of a carbon emissions trading system is essential for environmental conservation and achieving sustainable development objectives. By defining ownership of carbon emission permits and tying them to economic incentives, businesses can be encouraged to enhance production techniques and innovate new technologies aimed at decreasing carbon emissions. Data show that as of December 31, 2023, the national carbon market has accumulated 442 million tons of carbon emission allowances, with a total turnover of 24.919 billion yuan (Wang & Lyu, 2024).

The potential for shifts in the interconnectedness of variables in financial markets is a core concern in economics and finance. Researchers have gathered extensive evidence on this topic. Deng et al. (2022), by utilizing Diebold and Yilmaz's (2012) spillover index model as well as Baruník and Křehlík's (2018) frequency-domain spillovers approach, and the Dynamic Conditional Correlation (DCC) model, primarily examined the interactions between six green sector stocks, green bonds, and traditional financial markets. They found that the green bond market is least affected by shocks from these six green industry stock markets. Wan et al. (2023) studied the impact direction, spatial spillover effects, and other aspects of green finance development on energy structure transformation. They found a significant spatial spillover effect of green financial development on China's energy structure transformation by constructing several models, including the fixed effects model and the spatial Dubin model. Abakah et al. (2023) studied spillovers and dependencies in the green investment, carbon market, financial market, and commodity market during COVID-19. They discovered that under extreme market conditions, the marketability of green bonds compared to other assets rises with the magnitude of the shock. Wang and Li (2024) proposed threshold time-varying Copula-GARCHSK model that effectively studies the nonlinear time-varying dependence in the U.S. and Chinese green bond markets under different states, revealing the asymmetric bidirectional risk spillover effects between the two markets. Notably, when both markets decline simultaneously, the risk spillover from the Chinese market to the U.S. market significantly increases.

The above analysis shows a close relationship among green bonds, green stocks, and the carbon emission trading markets. How does the dynamic dependency between them change? What is the direction of influence transmission among them? Is it unidirectional or bidirectional? In addition, in discussing the relationship of multiple variables, many researchers use one of the variables as an exogenous variable and study its impact on the relationship between the other variables (e.g. Lin et al., 2019; Chen et al., 2023; Han et al., 2020). An exogenous variable refers to a variable that is determined outside the model under study. It is not influenced by the other variables within the system and instead acts as an independent factor that impacts the dependent variables. Therefore, we want to know, in the context of this study, especially during the COVID-19, the question arises whether a similar effect exists among green bonds, green equities, and the carbon emissions trading market.

This study uses the China Green Bond Index, CSI 300 Green Leading Stock Price, and Carbon Emission Trading Allowance Daily Closing Price data to measure the green bond market, the green stock market, and the carbon emission trading market, respectively. To address the issues mentioned above, this study employs dynamic Copula and VAR-BEKK-GARCH-X models. Specifically, the dynamic Copula model is utilized to explore the dynamic dependence between the three markets, thus revealing the dynamic dependence between the relationships over time. Subsequently, to identify the transmission paths of the effects, the paper investigates the volatility spillovers among the three markets using the VAR-BEKK-GARCH model. Ultimately, by introducing the exogenous variable X, we further examined how these markets, when each is considered as exogenous variable, influence the volatility of the other two markets.

There are three contributions to this paper. Firstly, the index for green bonds in China showcases the performance of environmentally friendly bonds within the Chinese market, the CSI 300 Green Leading Stock Price is the first comprehensive green index in China and serves as a market benchmark for green stocks in China, and the Hubei Carbon Emission Trading Quota can represent a specific range of industrial activities in China, so studying the complex relationship among these three can help promote sustainable development, optimize investment strategies and help policymakers formulate relevant policies. Secondly, the dynamic Copula and VAR-BEKK-GARCH models are used to grasp the dynamic dependencies and spillovers between pairs of these three markets, respectively, especially during the COVID-19 period. Thirdly, we incorporate the exogenous variable X within the framework of the VAR-BEKK-GARCH model. The three markets act as exogenous factors to capture their spillovers to the other two markets in different periods.

The rest of this paper is organized as follows. Section 2 presents the modeling framework employed in this study. Section 3 discusses the selection and rationale behind the data, gives its descriptive statistics, and analyzes them accordingly. Section 4 presents the empirical results analysis and discussion. Finally, Section 5 provides conclusions and recommendations.

2. Methods

2.1. Estimation of Marginal Distribution

We perform ARMA modeling on the obtained data and assess the clustering of volatility in the residuals using GARCH. Furthermore, financial practitioners know that most economic data display various asymmetries (Perez-Quiros & Timmermann, 2001; Babsiri & Zakoian, 2001). To some extent, although classical GARCH models perform well in capturing heavy tails and high peaks in the data and characterize autocorrelation, heteroskedasticity, and volatility aggregation well, they are less effective in describing asymmetric behaviours like leverage effects on asset price fluctuations triggered by adverse or favorable news in financial markets. Consequently, leveraging the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model, we constructed an ARMA-GJR-GARCH(1, 1) model aimed at fitting the marginal distribution (Yeo, 2004). This model not only inherits the strengths of the classical GARCH model but also better captures the asymmetries such as "leverage effect". The marginal distribution expressions for each log return can be shown below:

$$R_{i,t} = \ln\left(\frac{X_{i,t}}{X_{i,t-1}}\right), \tag{1}$$

$$R_{i,t} = \mu_i + \varphi_i R_{i,t-1} + \phi_i \varepsilon_{i,t-1} + \varepsilon_{i,t}, \tag{2}$$

$$\varepsilon_{i,t} = \sqrt{\sigma_{i,t}^2} e_{i,t}. \tag{3}$$

ARMA-GARCH(1, 1)-std model:

$$e_{i,t} \sim \mathrm{std}(v),\ i = 1, \tag{4}$$

$$\sigma_{i,t}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2, \tag{5}$$

$$t(e_{i,t}; v) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)} \left[\pi(v-2)\sigma_{i,t}^2 \left(1 + \frac{e_{i,t}^2}{v-2}\right)^{v+1}\right]^{-\frac{1}{2}}. \tag{6}$$

ARMA-GJR-GARCH(1, 1)-sstd model:

$$e_{i,t} \sim \mathrm{sstd}(v_i, \lambda_i),\ i = 2,3, \tag{7}$$

$$\sigma_{i,t}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + I\left(\gamma_i \varepsilon_{i,t-1}^2\right), \tag{8}$$

$$\text{skewed-t}(e_{i,t}; v, \lambda) = \begin{cases} bc\left(1 + \frac{1}{v-2}\left(\frac{b\varepsilon_{i,t} + a}{1-\lambda}\right)^2\right)^{-\frac{v+1}{2}}, & \varepsilon_{i,t} < -a/b \\ bc\left(1 + \frac{1}{v-2}\left(\frac{b\varepsilon_{i,t} + a}{1+\lambda}\right)^2\right)^{-\frac{v+1}{2}}, & \varepsilon_{i,t} \geq -a/b \end{cases} \tag{9}$$

$$\text{with } a = 4\lambda c \frac{v-2}{v-1}, b = 1 + 3\lambda^2 - a^2, c = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)}},$$

where $X_{i,t}$ denotes the original series data, $i = 1, 2, 3$ (representing the China Green Bond Index, the CSI 300 Green Leading Stock Price, and the Carbon Emission Right Trading Allowance, respectively), the sequence is indexed by t, which ranges from 1 to T, with $T = 2077$. The variables $X_{i,t}$ denote the daily closing prices at times t. $R_{i,t}$ is the log return of the original data, μ_t represents the constant term in the mean equation, θ_t indicates how the current data are influenced by the previous period's residuals, ϕ_t captures how the current data are influenced by historical data from one period ago. $\varepsilon_{i,t}$ denote the sequence of residuals at time t, while σ_t^2 denote the conditional volatility at time t. α_i represent the ARCH coefficients, which reflect the impact of lagged one-period historical residuals on the current fluctuations, β_i represent the GARCH coefficients, which reflect the impact of lagged one-period historical volatility on the current fluctuations. ω_i represent the constant term in the variance equation. γ_i represents an asymmetry parameter that indicates how the impact of past adverse or favorable news affects current volatility, and is employed to assess the leverage effect. To guarantee that conditional volatility remains positive, it is typically required that $\alpha > 0$, $\beta > 0$. For the GARCH model, it is also required that $0 < \alpha + \beta < 1$. For the GJR GARCH model, it is required that $0 < \alpha + \beta + \gamma/2 < 1$. The distribution properties of the sequence $R_{i,t}$ permit the selection of a variety of probability distribution functions $e_{i,t}$, including the normal distribution, t-distribution, extreme value distribution, and so on. $I(\cdot)$ is the indicator function representing $\varepsilon_{i,t-1}$. In the paper, we will investigate the residual terms obeying normal, skewed normal, t and skewed t distribution, respectively. The fitting results are compared, so as to select the best marginal fitting distribution function.

2.2. Dynamic Copula Model

Correlations between financial markets can be nonlinear and volatile with changes in the external conditions, so a dynamic nonlinear model is needed to describe this dynamic correlation structure. The Copula function introduced by Sklar in 1959 can be used to study this nonlinear dependence (Sklar, 1959). Copula functions address certain drawbacks of using Pearson correlation coefficients as the most straightforward measure of dependence (Embrechts et al., 1999). The application of Copula models to analyze temporal dependence is referred to as dynamic Copula modeling. The Time-varying Copula developed by Patton (2001, 2006) and the DCC Copula proposed by Engle (2002) are both used to obtain the dynamic dependence between variables that changes over time fluctuations.

Dependence structures are often chosen from elliptic Copula (Wen et al., 2012), such as the Gaussian Copula (which may be the benchmark Copula in economics) and the Student-t Copula, for which the symmetric tail dependence suggests that the tail dependence varies by the same amount under extremely favorable or extremely unfavorable market conditions, which, however, may not be the case in real life. In general, markets are more likely to experience simultaneous "crashes" in bear markets. In contrast, markets are more likely to rise simultaneously in bull markets, even though they may not "crash" or boom at similar rates. Therefore, it is interesting to consider further models with asymmetric dependence in the upper and lower tails, such as the Clayton Copula model, an important feature of which is its consideration of asymptotic lower tail dependence, and the SJC Copula model, which considers both upper and lower tail dependence.

Therefore, to better characterize the market dependence structure, this paper refers to the research of Wen et al. (2012) and chooses these four Copula types mentioned above, i.e. DCC Gaussian Copula, DCC Student-t Copula, Time-varying Clayton Copula, and Time-varying SJC Copula models.

2.2.1. DCC Copula Model

For the Gaussian Copula and Student-t Copula models, this paper specifies that the linear correlation parameter ρ_t changes over time, as demonstrated in Engle's DCC(1,1) model (Engle, 2002).

$$Q_t = \left(1-\bar{\alpha}-\bar{\beta}\right)\cdot\bar{Q}+\bar{\alpha}\xi_{t-1}\cdot\xi_{t-1}^{\mathrm{T}}+\bar{\beta}\cdot Q_{t-1}, \tag{10}$$

$$\rho_t = Q_t^{*-1} Q_t Q_t^{*-1}, \tag{11}$$

$$\tau_t = \left(2/\pi\right)\sin^{-1}\rho_t, \tag{12}$$

where Q_t is the covariance matrix of the standardized residual vector ξ_t, $\bar{Q}$ is the unconditional covariance. Q^* is a square matrix where the non-diagonal elements are 0, and the diagonal elements are the square roots of Q_t. Kendall τ is utilized to capture nonlinear dependencies that cannot be measured by linear correlation. As mentioned above, Gaussian Copula and Student-t Copula have only the linear dependence parameter ρ_t in their cumulative distribution functions, Equation (12) represents the relational equation between linear dependence ρ_t and nonlinear dependence τ_t between Gaussian Copula and Student-t Copula. In contrast, the Clayton Copula has $\tau_{clayton}$ ($\delta_{Clayton} = 2\tau/(1-\tau)$) while the SJC Copula has upper and lower tail τ_{SJC} ($\kappa_{SJC}^U = 1/\log_2\left(2-\tau^U\right)$, $\gamma_{SJC}^L = -1/\log_2\left(\tau^L\right)$).

2.2.2. Time-Varying Copula Model

The Joe-Clayton Copula (Patton, 2006) is defined as:

$$\begin{gathered} C_{JC}\left(u,v\mid\tau^U,\tau^L\right) = 1-\left(1-\left\{\left[1-(1-u)^k\right]^{-\gamma}+\left[1-(1-v)^k\right]^{-\gamma}-1\right\}^{-\frac{1}{\gamma}}\right)^{\frac{1}{\gamma}}, \\ \gamma = -\frac{1}{\log_2\left(\tau^L\right)}, \\ k = \frac{1}{\log_2\left(2-\tau^U\right)}. \end{gathered} \tag{13}$$

Furthermore, the definition of the Symmetrized Joe-Clayton (SJC) Copula is that:

$$
\begin{aligned}
&C_{SJC}\left(u,v \mid \tau^U,\tau^L\right)\\
&=\frac{1}{2}\left(C_{JC}\left(u,v \mid \tau^U,\tau^L\right)+C_{JC}\left(1-u,1-\nu \mid \tau^U,\tau^L\right)+u+\nu-1\right),\\
&\tau_t^U=\Lambda\left(\omega_U+\beta_U\tau_{t-1}^U+a_U\times\frac{1}{10}\sum_{j=1}^{10}\left|u_{t-j}-\nu_{t-j}\right|\right),\\
&\tau_t^L=\Lambda\left(\omega_L+\beta_L\tau_{t-1}^L+a_L\times\frac{1}{10}\sum_{j=1}^{10}\left|u_{t-j}-\nu_{t-j}\right|\right).
\end{aligned}
\tag{14}
$$

here, $\Lambda(X)=\left(1+\mathrm{e}^{-x}\right)^{-1}$ is a logistic function whose purpose is to ensure that the variable τ_t^U and τ_t^L varies within the range (0, 1).

The probability distribution function of the bivariate time-varying Clayton Copula is defined as:

$$
C(u,v,\delta)=\left(u^{-\delta}+v^{-\delta}-1\right)^{-1/\delta}. \tag{15}
$$

The time-varying correlation parameter is characterized by the following evolution equation:

$$
\rho_t=\Lambda\left(\omega+\beta\rho_{t-1}+\alpha\times\frac{1}{q}\sum_{i=1}^{q}(u-v)^2\right). \tag{16}
$$

The function $\Lambda(x)=\mathrm{e}^x$ is a transformation function whose purpose is to ensure that the variable ρ_t varies within the range (0, 1), generally with a lag order of $q \le 10$. Similar to Ahdika et al. (2021), the initial values of the parameters are selected as shown below: $\omega_0=\Lambda^{-1}(\delta)$, $\alpha_0=0$, $\beta_0=0$, where δ represents the parameter of the static Clayton Copula model and $\rho_0=\delta$.

Using the Dynamic Copula Toolbox code by Vogiatzoglou (2017), the time-varying copula model presented in this paper was developed.

2.2.3. Parameter Estimation Methods

For the sake of argument, consider the joint distribution function of two distinct residual variables with parameters δ_i and δ_j, where $i \ne j$. This function can be expressed as:

$$
F\left(\varepsilon_{i,t},\varepsilon_{j,t};\zeta_{h,t}\right)=C\left(F_i\left(\varepsilon_{i,t};\delta_i\right),F_j\left(\varepsilon_{j,t};\delta_j\right);\zeta_{h,t}\right), \tag{17}
$$

where $C(\cdot)$ is the time-varying Copula and $\zeta_{h,t}$ is the dynamic parameter of the time-varying Copula defined by Equation (17). Below is the log-likelihood function of the model:

$$
\begin{aligned}
\iota\left(\delta_i,\delta_j,\zeta_{h,t}\right) &= \log\left(\prod_{t=1}^{T} f\left(\varepsilon_{i,t},\varepsilon_{j,t};\zeta_{h,t}\right)\right) \\
&= \log\left(\prod_{t=1}^{T} \frac{\partial^2 F\left(\varepsilon_{i,t},\varepsilon_{j,t};\zeta_{h,t}\right)}{\partial\varepsilon_{i,t}\partial\varepsilon_{j,t}}\right) \\
&= \log\left(\prod_{t=1}^{T} \frac{\partial^2 C\left(F_i\left(\varepsilon_{i,t};\delta_i\right),F_j\left(\varepsilon_{j,t};\delta_j\right);\zeta_{h,t}\right)}{\partial\varepsilon_{i,t}\partial\varepsilon_{j,t}}\right) \\
&= \log\left(\prod_{t=1}^{T} f_i\left(\varepsilon_{i,t};\delta_i\right) f_j\left(\varepsilon_{j,t};\delta_j\right) c\left(F_i\left(\varepsilon_{i,t};\delta_i\right),F_j\left(\varepsilon_{j,t};\delta_j\right);\zeta_{h,t}\right)\right) \\
&= \sum_{t=1}^{T}\log f_i\left(\varepsilon_{i,t};\delta_i\right) + \sum_{t=1}^{T}\log f_j\left(\varepsilon_{j,t};\delta_j\right) \\
&\quad + \sum_{t=1}^{T}\log c\left(F_i\left(\varepsilon_{i,t};\delta_i\right),F_j\left(\varepsilon_{j,t};\delta_j\right);\zeta_{h,t}\right).
\end{aligned} \tag{18}
$$

Since it is difficult to achieve maximum likelihood estimation of all parameters simultaneously in multivariate modeling, the two-stage estimation procedure of Inference Function for Marginal (IFM) proposed by Joe and Xu (1996) is used in this paper. Joe demonstrated that the IFM is more accessible and efficient than the traditional maximum likelihood estimation method. First, the parameters $\left(\delta_i,\delta_j\right)$ of the marginal variables are calculated, followed by the estimation of the parameters $\zeta_{h,t}$ of the time-varying copula. Consequently, Equation (18) can be expressed as follows:

$$
\iota\left(\delta_i,\delta_j,\zeta_{h,t}\right) = \iota_{f_i}\left(\delta_i\right) + \iota_{f_j}\left(\delta_j\right) + \iota_{c_k}\left(\zeta_{h,t}\right). \tag{19}
$$

here, $\iota_{f_i}\left(\delta_i\right)$ and $\iota_{f_j}\left(\delta_j\right)$ represent the log-likelihood functions of the marginal variables, while $\iota_{c_h}\left(\zeta_{h,t}\right)$ represent the log-likelihood function of the dynamic Copulas with $h=1,2,3,4$. Where h represents the DCC Gaussian Copula, DCC Student-t Copula, Time-varying Clayton Copula, and Time-varying SJC Copula, respectively.

Finally, $\zeta_{h,t}$ it is estimated by maximizing $\iota_{c_h}\left(\zeta_{h,t}\right)$.

$$
\hat{\zeta}_{k,t} = \arg\max_{\zeta_{h,t}} \iota_{c_k}\left(\zeta_{h,t}\right). \tag{20}
$$

2.3. VAR-BEKK-GARCH-X Model

The dynamic Copula model can capture dependencies between markets but cannot determine the direction in which such dependencies arise between markets. Volatility spillovers are when price fluctuations in one market cause price fluctuations in another market due to the interconnectedness of different markets or the same risk factors. Therefore, to assess the direction of volatility spillovers between markets, this study applies the BEKK-GARCH model established by Engle and Kroner (1995). Additionally, to enhance forecasting accuracy, the VAR model is incorporated into the conditional mean function (Jayasinghe et al., 2014; Mensi et al., 2014), resulting in the VAR-BEKK-GARCH model. Before building the VAR model, the appropriate lag length of the sample was selected using the AIC and BIC criteria. Here is the formulation of the VAR-BEKK-GARCH-X model.

$$R_t = \mu + \sum_{i=1}^{p} \alpha_i R_{t-i} + \varepsilon_t\ , \tag{21}$$

$$\varepsilon_t \sim N(0, H_t), \tag{22}$$

where R_t represents the return matrix, including RGBI, RGLS, and RHBC, p is lag length, μ is a 2×1 vector matrix of constants, ε_t is a 2×1 vector matrix of residuals that conforms to a normal distribution with zero means, and the definition of the conditional variance-covariance matrix is shown in Equation (23).

$$H_t = CC' + A(\varepsilon_{t-1}\varepsilon'_{t-1})A' + BH_{t-1}B' + EE_t'^2 \tag{23}$$

$$C = \begin{bmatrix} C_{11} & 0 \\ C_{21} & C_{22} \end{bmatrix},\ E = \begin{bmatrix} E_{11} & 0 \\ E_{21} & E_{22} \end{bmatrix},\ A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$

$$B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix},\ e_{t-1} = \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \end{bmatrix},\ H_{t-1} = \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{bmatrix},$$

where C is a constant matrix, E is an exogenous variable coefficients matrix. A_{ij} is employed to detect ARCH effects in the residuals of market i and market j. B_{ij} is employed to detect GARCH effects in the conditional variances of market i and market j. When A_{ij} and B_{ij} are zero or insignificant, market i exhibits no volatility spillover impact on market j. According to Lin et al. (2019), RGBI, RGLS, and RHBC serve as exogenous variables in constructing a VAR-BEKK-GARCH-X model (where X represents the exogenous variable). The objective is to assess whether the introduction of exogenous variables can impact the volatility of a market. For example, when RGBI acts an exogenous variable, if parameter E_{11}^2 is negatively significant, it suggests that RGBI exerts a net negative spillover effect on the RGLS market; if parameter E_{21}^2 is negatively significant, it suggests that RGBI exerts a partial negative spillover effect on the RHBC market; if parameter E_{22}^2 is negatively significant, it suggests that RGBI exerts a net negative spillover effect on the RHBC market. A similar understanding can be reached when RGLS or RHBC are used as exogenous variables.

3. Data

3.1. Data Selection

In this study, we select the CBS-China Green Bond Index, CSI 300 Green Leading Stock Prices, and Hubei Carbon Emission Allowance Daily Closing Price data from July 2, 2015 to April 15, 2024 to measure the green bond, green stock, and carbon emission right markets, respectively. This period spans from the COVID-19 outbreak to the end of the post-outbreak phase, allowing us to examine the dynamic dependencies and spillovers between these markets during the outbreak period. The COVID-19 outbreak in Hubei Province at the beginning of 2020 resulted in decreased economic activity and lower carbon emissions, leaving January 24, 2020 to March 22, 2020 without trading data. Therefore, to ensure consistent trading days for each variable, 2077 observations were gathered for the empirical study by omitting holidays and other days when trading does not occur using market trading day data from the past nine years.

Given that the national carbon emissions trading market was established in July 2021, less than three years ago, the information contained in its data is not sufficient. Here, this paper selects the data of the Hubei carbon emissions trading market as the sample data, the main reason is: among all the pilot trading markets in the country, is the Hubei carbon emissions trading market has the largest trading volume, the largest number of transactions days and the most complete volume, and the coverage rate of the enterprises is also maintained at a high level.

To more precisely identify the spillover effects pre- and post-COVID-19, this study segments the sample into three parts and constructs a VAR-BEKK-GARCH-X model for each. Based on the research by Mariana et al. (2021), the three subsample periods are: from July 2, 2015 to December 8, 2019, from December 9, 2019 to December 7, 2022, and from December 8, 2022 to April 15, 2024. The first breakpoint was December 8, 2019, when the Chinese government reported the first case to the World Health Organization (WTO). The second breakpoint is December 7, 2022, when the State Council of China released the "New Ten Articles" to liberalize epidemic prevention and control. For simplicity, the entire sample period was designated as P_1, prior to the onset of the COVID-19 pandemic. This was followed by P_2, which spanned the duration of the pandemic. After the pandemic, the period was designated as P_3, and finally, P_4, which spanned the period following the pandemic.

This paper uses RGBI to denote the China Bond-China Green Bond Index, i.e. the green bond market. RGLS denotes the price of CSI 300 Green Leading Stocks, i.e. the green stock market. RHBC denotes the Hubei Carbon Emission Allowance, i.e. the carbon emission right trading market. All the above data were obtained from the China Wind database. This paper processed data using Rstudio, Matlab 2018b, and WinRATS Pro 8.0 software.

3.2. Descriptive Statistics

Figure 1 shows that the daily yield fluctuation of the RGLS and RHBC markets is more significant than that of the RGBI market, mainly because the stock market is more sensitive to information and the price is more volatile, while the bond market is less risky, and the price is more stable. In addition, the daily return series of these three markets show the phenomenon of volatility aggregation, that is, small fluctuations usually follow small fluctuations, and large fluctuations usually follow large fluctuations, which provides a basis for the subsequent analysis using the GARCH model.

Table 1 displays the descriptive statistics of the log returns for the three markets. The analysis shows that the standard deviations of the RGLS and RHBC return series are similar and both higher than that of RGBI, indicating that RGLS and RHBC are more volatile and susceptible to unexpected information and extreme events, while RGBI has the lowest volatility. The kurtosis of all three markets exceeds 3, demonstrating "high kurtosis and thick tails" characteristics, suggesting a heightened likelihood of extreme risks compared to a normal distribution, particularly in the RGBI market. The Jarque-Bera test results indicate that the return series of all three markets significantly deviate from the normal distribution. The ARCH-LM statistics reveal significant ARCH effects in these markets, a necessary condition for GARCH modeling. The Ljung-Box statistics show that all three time-series exhibit serial correlation, allowing the use of the ARMA model to fit the mean return equations (Yao & Li, 2023).

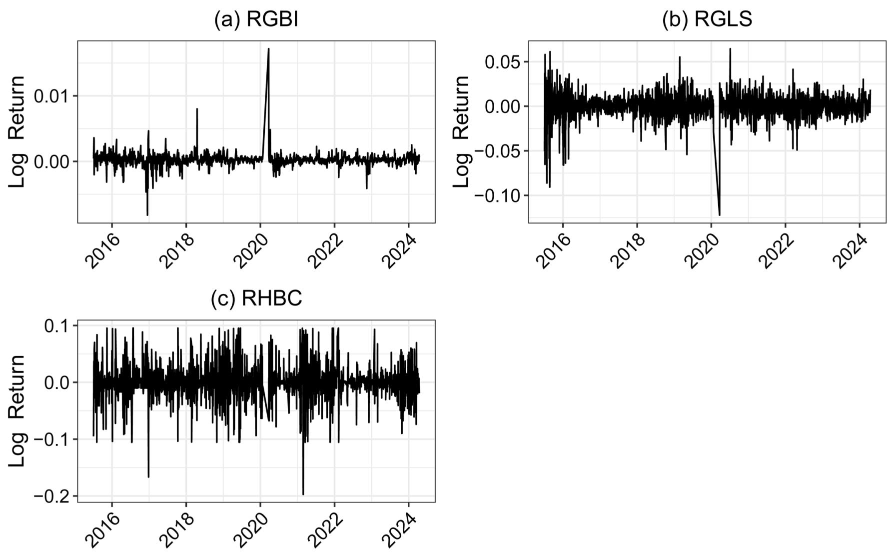

Figure 1. The plot of daily log return from July 2, 2015 until April 15, 2024.

Table 1. Descriptive statistics.

Variables	mean	median	max	min	sd	skew
RGBI	2.058e−04	2.058e−04	0.0172	−8.183e−03	8.650e−04	3.0602
RGLS	−2.117e−05	3.749e−04	0.0645	−0.1222	0.0127	−1.0886
RHBC	2.191e−04	0	0.0956	−0.1972	0.0289	−0.2985

Variables	kurtosis	JB test	LB test(13)	ARCH−LM test(13)
RGBI	84.795	581956*** (<2.2e−16)	618.09*** (<2.2e−16)	31.855*** (0.0025)
RGLS	12.683	8520.9*** (<2.2e−16)	27.793*** (0.0097)	127.3*** (<2.2e−16)
RHBC	7.331	1653*** (<2.2e−16)	39.261*** (0.0002)	339.17*** (<2.2e−16)

Notes: The table displays summary statistics of daily returns over the period from July 2015 to April 2024. ARCH-LM is the Lagrange Multiplier test for autoregressive conditional heteroscedasticity. *, **, and *** indicate statistical significance at the 10%, 5% and 1% level, respectively.

4. Results and Analysis

4.1. Marginal Distribution

According to descriptive statistical analysis in **Table 1**, the distributions of green bond, green stock, and carbon emissions trading markets are characterized by asymmetry, volatility clustering, and "peaks and thick tails". Therefore, ARMA-GARCH(1, 1) and ARMA-GJR-GARCH(1, 1) models estimate the marginal distributions of the green bond, green stock, and carbon emission rights trading markets, respectively, the fitting results shown in **Table 2**. By comparing the Log-Likelihood (LL) and AIC values in **Table 2**, it is evident that to achieve better fitting of the marginal distribution of the time series, the ARMA-GARCH(1, 1)-std model should be chosen for RGBI, while the ARMA-GJR-GARCH(1, 1)-sstd model is more suitable for RGLS and RHBC. **Table 3** is the estimated marginal distribution parameters for the markets.

Table 2. Comparison between GARCH and GJR-GARCH model.

		GARCH(1, 1)				GJR GARCH(1, 1)			
		norm	snorm	std	sstd	norm	snorm	std	sstd
RGBI	LL	12462.78	12375.12	**12730.5**	12727.57	12442.63	12456.31	12720.34	12721.92
	AIC	−12.001	−11.915	**−12.258**	−12.254	−11.980	−11.993	−12.247	−12.248
RGLS	LL	6392.363	6398.138	6478.905	6480.056	6396.639	6401.365	6480.208	**6481.497**
	AIC	−6.1526	−6.1572	−6.235	−6.2351	−6.1557	−6.1593	−6.2353	**−6.2355**
RHBC	LL	4785.23	4786.727	5080.869	5080.926	4785.277	4786.777	5080.889	**5080.956**
	AIC	−4.6043	−4.6047	−4.8881	−4.8872	−4.6033	−4.6038	−4.8863	**−4.8872**

Notes: LL denotes the Log-Likelihood.

Table 3. Result of estimation of the marginal distribution model.

	RGBI	RGLS	RHBC
μ	0.000198*** (0.000)	0.000154 (0.483476)	−0.000171 (0.517021)
ϕ	0.617926*** (0.000)	−0.822098*** (0.000)	0.024791 (0.776732)
θ	−0.224440*** (0.000041)	0.837431*** (0.000)	−0.249775*** (0.003290)
ω	0 (0.982745)	0.000003*** (0.087142)	0.000086*** (0.000031)
α	0.063497*** (0.000017)	0.054959* (0.000361)	0.613257*** (0.000)

Continued

β	0.907452*** (0.000)	0.903362*** (0.000)	0.396994*** (0.000)
γ	-	0.036509 (0.126065)	−0.022386 (0.808635)
$\lambda(skew)$	-	0.953040*** (0.000)	1.008797*** (0.000)
$v(shape)$	4.293542*** (0.000)	5.393195*** (0.000)	3.183708*** (0.000)
$\alpha+\beta$	0.970949	-	-
$\alpha+\beta+\gamma/2$	-	0.976576	0.999058

Notes: *, **, and *** indicate statistical significance at the 10%, 5% and 1% level, respectively. "-" indicates a null value.

From Table 3, it can be observed that the range of α values extends from 0.054959 to 0.613257, while the values of β span from 0.396994 to 0.907452, and each marginal distribution satisfies this necessary precondition required by each of them, i.e. $0<\alpha+\beta<1$, and $0<\alpha+\beta+\gamma/2<1$, respectively. This evidence suggests that the constructed model effectively characterizes the time-series of the log returns of the three markets. In addition, the magnitude of λ characterizes the asymmetry of the time-series. In this context studied, only the RGLS and RHBC markets exhibit varying degrees of asymmetric effects, with RHBC reflecting a relatively more substantial degree. The value v indicates the thickness of the distribution, with the most significant magnitude of v observed for green stocks. This indicates that the tails of green stocks are thicker and more prone to extremes.

The Copula function imposes specific requirements on the standardized residuals, and thus, in this study, the standardized residuals obtained after GARCH modeling are tested for heteroskedasticity. From Table 4, it can be seen that the Ljung-Box statistic and ARCH Lagrange Multiplier test indicate that the null hypothesis should be rejected, suggesting that there is no serial autocorrelation or heteroskedasticity. Consequently, it is valid and reasonable to model the marginal distribution using the corresponding GARCH model. Subsequently, the standardized residual series of the marginal distribution was transformed probabilistically to ensure that the series conformed to the uniform distribution between 0 and 1. The K-S test was employed to ascertain whether the sequence in question obeys U(0, 1). The resulting *p*-value was more significant than 0.05, indicating that the original hypothesis cannot be rejected. This implies that the three standardized residual errors after PIT obey U(0, 1), which provides a basis for the subsequent analysis using the Copula model.

4.2. Dynamic Dependence

To capture the dynamic relationship between the three markets, this paper is divided into three steps. First, the static copula function (4 types) and time-varying copula function (4 types in total) are used to fit the dynamic correlation structure of the three market pairs RGBI-RGLS, RGBI-RHBC, and RGLS-RHBC; second, a better fitting model is selected based on the optimal LL, AIC, and BIC. Finally, the dynamic evolution path of the three market pairs is obtained based on the best-fitting model.

Table 4. Test of standardized residuals on the marginal distribution model.

	RGBI	RGLS	RHBC
L-B test(2)	0.24839 (0.8832)	1.4404 (0.4866)	2.521 (0.2835)
ARCH LM test(2)	0.00049496 (0.9998)	4.2273 (0.1208)	3.767 (0.1521)
K-S test	0.020231 (0.7892)	0.018786 (0.8573)	0.033719 (0.1886)

Note: The values in parentheses are p-values.

The estimation results LL, AIC, and BIC of the static copula model and the dynamic copula model are shown in **Table 5**. For RGBI-RGLS, even though the BIC value of the Student-t Copula model is the largest, the LL value of the DCC Student-t Copula model is the largest and the AIC value is the smallest. For the RGLS-RHBC market pair, the dynamic DCC Gaussian Copula model is a relatively good choice in combination with the results of LL, AIC, and BIC, as well as the goodness-of-fit test (**Table 6**). For RGBI-RHBC, the AIC and BIC in the static Student-t copula are smaller than those in the DCC Student-t copula, and the gap is slightly larger, but the LL values obtained by the two are not much different. Therefore, the static Student-t Copula should be considered when constructing the correlation structure of RGBI-RHBC. At the same time, this paper also constructs the DCC Student-t copula and obtains the evolution path of its correlation coefficient, as shown in **Figure 3**. Its fluctuation range is very small, which further verifies the conclusion that RGBI-RHBC should adopt the static Student-t Copula.

In summary, the four time-varying copula models are compared by LL, AIC, and BIC. From the final fitting effect, to characterize the time-varying characteristics of interdependence within the research interval, especially to study whether the interdependence between markets will change during specific events, such as the global public health security period caused by COVID-19, this paper selects DCC Student t copula to fit the overall correlation structure of RGBI-RGLS; static Student-t copula is selected to fit the overall correlation structure of RGBI-RHBC; DCC Gaussian Copula is selected to fit the overall correlation structure of RGLS-RHBC.

Table 5. The fitting effect test of different types of copula functions.

	Type of copula	LL	AIC	BIC
		RGBI-RGLS		
Constant	Gaussian	0.331	1.338052	7.103197
	Student-t	5.368	−8.7364	−2.9712
	Clayton	−0.117	2.2348	7.9999
	SJC	−2.554	12.00001	45.82919
Dynamic	DCC Gaussian	8.397	−12.7948	−1.5184
	DCC Student-t	**20.993**	**−35.9857**	**−19.0712**
	Time-varying Clayton	1.721	2.5574	19.4720
	Time-varying SJC	−17.457	46.9144	80.7436
		RGBI-RHBC		
Constant	Gaussian	0.001892	1.996216	7.761361
	Student-t	3.151	**−4.3014**	**1.4637**
	Clayton	0.2	1.599	7.3642
	SJC	3.112	5.776965	39.60615
Dynamic	DCC Gaussian	0.049	3.9029	15.1793
	DCC Student-t	**3.316**	−0.6329	16.2817
	Time-varying Clayton	3.301	−0.0628	16.8518
	Time-varying SJC	−4.853	21.7056	55.5348
		RGLS-RHBC		
Constant	Gaussian	0.2934	1.413231	7.178376
	Student-t	0.678	0.6447	**6.4099**
	Clayton	−2.8719e−05	2.0001	7.7652
	SJC	0.12	11.76005	45.58924
Dynamic	**DCC Gaussian**	2.963	**−1.9259**	9.3505
	DCC Student-t	**3.172**	−0.3447	16.5699
	Time-varying Clayton	−1.044	8.0880	25.0026
	Time-varying SJC	−9.610	31.2196	65.0488

Table 6. Goodness of fit test (RGLS-RHBC).

Type of copula	GoF p-value
DCC Gaussian Copula	**0.7847**
DCC Student-t Copula	0.7757
Student-t Copula	0.7557

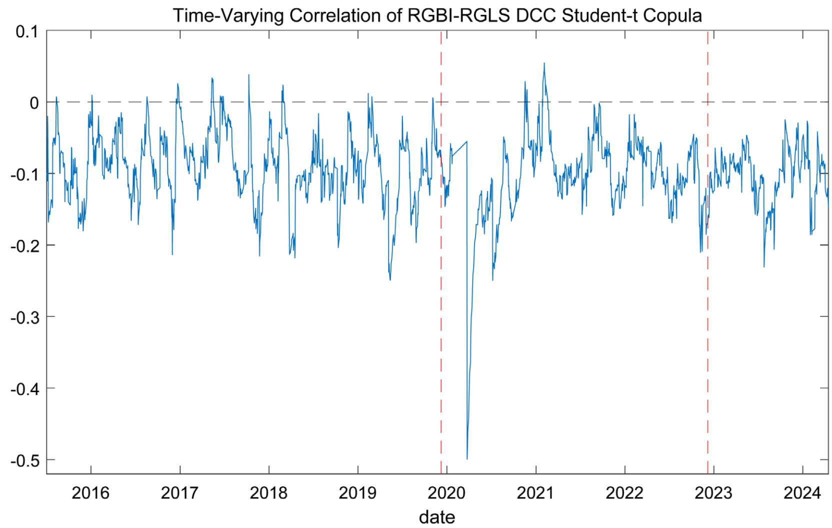

Figure 2. Dynamic correlation plot of RGBI-RGLS based on DCC Student-t.

Finally, the goodness of fit test is performed on the selected copula for each market pair, and the results are shown in Table 7. The p-values of the goodness of fit test are much greater than 0.05, indicating that the selected copula can fully describe the dependency between variables. Once again, the reliability of the selected copula is verified.

Table 7. Goodness of fit test.

Variable	Type of copula	GoF p-value
RGBI-RGLS	DCC Student-t copula	0.2702
RGBI-RHBC	Student-t copula	0.8906
RGLS-RHBC	DCC Gaussian copula	0.7847

Based on Equations (14)-(16), we derived the dynamic evolution paths of the resulting dependence coefficients (see Figures 2-4). The red dashed lines indicate the time breaks of December 8, 2019, and December 7, 2022, respectively, while the black dashed line indicates the dotted line with the y-axis zero. As illustrated in Figure 2 and Figure 4, the correlation coefficients of RGBI-RGLS versus RGLS-RHBC are significantly time-varying. The former fluctuates in the range of −0.5 to 0.1, while the latter fluctuates in the range between −0.23 to 0.24. Both fluctuate around small values of −0.1 and −0.028, respectively. However, the former is mainly below 0. So, the correlation coefficient between these two markets shows a negative correlation. During periods of market turbulence, for example the beginning of the COVID-19 outbreak, the dependence coefficient between these two markets shows a minimum value of −0.5. As can be seen from Figure 3, the time variation of the correlation coefficient between the RGBI-RHBC markets is not strong, fluctuating around a small value of 0.005, with frequent changes, but a small fluctuation range. Similarly, the minimum value appeared in the early stage of COVID-19, which further verifies the above conclusion on the selection of copula, that is, RGBI-RHBC should use the static Student-t copula model to construct its correlation structure, and the correlation coefficient is −0.07505. To put it differently, the overall interdependence between the markets is relatively low, especially between RGBI and RHBC. However, there are special circumstances during certain periods. For example, the economic crisis sparked by COVID-19 became an excellent opportunity to observe a breakthrough, during which the dependence among markets intensified.

4.3. Volatility Spillovers

This section primarily presents estimates derived from the VAR-BEKK-GARCH-X model across P_1, P_2, P_3, P_4. First, Rstuio software is used to obtain the most suitable lag order of the VAR model for each pair of markets. WinRATS Pro 8.0 software is used to establish the BEKK-GARCH-X model, which is analyzed for volatility spillover, and the specific direction of volatility spillover between these markets is obtained. Because of the limited space, these chosen results of the most suitable lag order of the VAR model are shown in Table A1 in **Appendix A**, and only the spillover effect results when RGBI is used as an exogenous variable are shown in the text. The rest of the results are given in **Appendix B**.

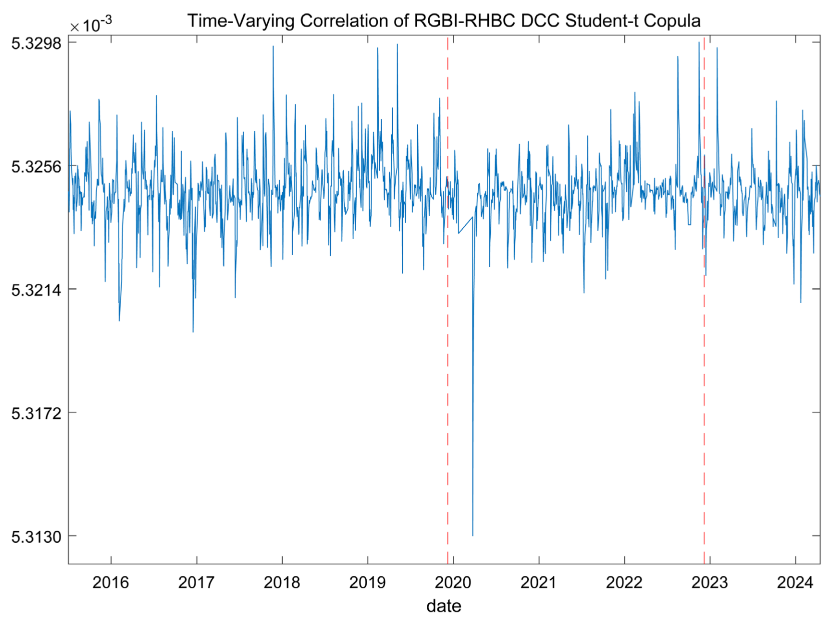

Figure 3. Dynamic correlation plot of RGBI-RHBC based on DCC Student-t Copula model.

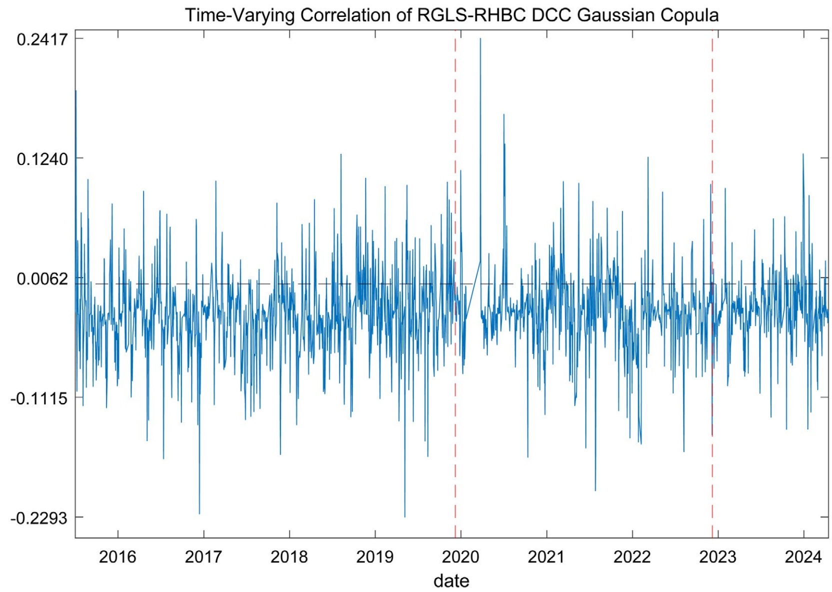

Figure 4. Dynamic correlation plot of RGLS-RHBC based on DCC Gaussian Copula model.

Table 8 shows the parameter estimation results of the VAR-BEKK-GARCH model with RHBC as an exogenous variable, and **Table 9** shows the Wald test results of each period when RHBC is an exogenous variable. This paper interprets the empirical results from three perspectives: Cross-market spillover, spillover after introducing exogenous variables, and individual market spillover effects. First, from the perspective of cross-market spillover, there existed a two-way volatility spillover impact between RGBI and RGLS in all periods except P_2, and this relationship gradually strengthened as the COVID-19 subsided. Next, from the perspective of the effect of introducing the exogenous variable RHBC, significant results occurred in each period. During P_1, E_{11} is markedly negative, and E_{22} is markedly positive, suggesting that RHBC exhibits a negative net spillover impact on RGBI and a positive net spillover impact on RGLS. During P_2 and P_3, E_{11} is significant, suggesting that RHBC exerted a positive net spillover impact on RGBI while adversely affected RGBI during P_3. More interestingly, during P_3 and P_4, E_{22} became significant and positive, indicating that RHBC began to exert a positive net spillover impact on RGLS after the epidemic outbreak, and the effect gradually increased. This can be interpreted as the pandemic's outbreak changing RHBC's role, rendering it no longer an indicator affecting RGBI after the pandemic ended. Moreover, the outbreak altered the direction of its impact on RGBI and caused RGLS to start showing signs of being a predictive indicator at the early stages of COVID-19.

The parameter estimation results of the VAR-BEKK-GARCH model with RGLS as an exogenous variable are presented in **Table B1** in **Appendix B**. **Table B2** in **Appendix B** displays the results of the Wald test for each period when RGLS is utilized as an exogenous variable. First, from the perspective of cross-market spillovers, there is no bidirectional spillover effect between RGBI and RHBC only before and after the COVID-19 (i.e. P_2 and P_4), but there is a unidirectional spillover effect during the rest of the period. Throughout the whole period, there was a two-way spillover effect between these two markets, and the correlation between them gradually increased as the epidemic waned after the outbreak. From the perspective of the effect of introducing the exogenous variable RGLS, E_{11} and E_{22} are significant in all periods except for the P_2 period, indicating that RGLS exerts a net spillover effect on both RGBI and RHBC. During P_3, E_{11} is markedly negative, while the rest of the periods are significantly positive. This spillover effect is gradually enhanced after the epidemic. Only during periods P_2 and P_3, E_{21} is markedly positive, suggesting that RGLS has a partially positive effect on RHBC during periods P_2 and P_3. Furthermore, this impact tends to strengthen with the weakening of COVID-19.

Table 8. Estimated results of VAR-BEKK-GARCH-RHBC model.

Period	P_1		P_2		P_3		P_4	
Variable/ Parameter	Coef.	Sig.	Coef.	Sig.	Coef.	Sig.	Coef.	Sig.
C(1, 1)	0.0002***	0.000	0.0002***	0.000	0.0004***	0.000	0.0002***	0.000
C(2, 1)	0.0004	0.401	0.0003	0.275	0.0014	0.551	−0.0062***	0.000
C(2, 2)	0.0011***	0.000	0.0007***	0.000	0.0044***	0.000	−0.0003	0.913
A(1, 1)	0.5094***	0.000	0.5332***	0.000	0.1854***	0.000	0.7206***	0.000
A(1, 2)	0.0210	0.930	0.5878**	0.037	2.1630***	0.001	−3.0022*	0.069
A(2, 1)	−0.0179***	0.000	−0.0003	0.877	−0.0448***	0.000	−0.0070	0.103
A(2, 2)	0.2421***	0.000	0.2277***	0.000	0.3409***	0.000	−0.0441	0.618
B(1,1)	0.7574***	0.000	0.8288***	0.000	0.2244**	0.022	0.6161***	0.000
B(1, 2)	−0.3128	0.650	−0.2678**	0.030	1.3926	0.379	2.7873	0.105
B(2, 1)	0.0070***	0.004	−0.0002	0.774	0.0089	0.194	0.0182***	0.002
B(2, 2)	−0.9654***	0.000	0.9706***	0.000	−0.8453***	0.000	0.5683***	0.001
E(1, 1)	−0.0080***	0.000	0.0026***	0.004	−0.0043***	0.000	−0.0013	0.323
E(2, 1)	0.0175*	0.058	−0.0065	0.489	−0.0043	0.862	−0.0403	0.339
E(2, 2)	0.0217***	0.006	0.0100	0.373	0.0314*	0.057	0.0767 **	0.048

Note: RHBC is used as an exogenous variable. *, **, and *** indicate statistical significance at the 10%, 5% and 1% level, respectively. "Coef." represents the coefficient value, and "Sig." represents the p-value. "P_1" represents the period from July 2, 2015, to April 15, 2024. "P_2" represents the period from July 2, 2015, to December 6, 2019. "P_3" represents the period from December 6, 2019, to December 7, 2022. "P_4" represents the period from December 9, 2022, to April 15, 2024.

Table 9. Wald test of VAR-BEKK-GARCH-RHBC model.

Period	Spillage scenarios		Chi-Squared	*p*-value
P_1	A&B	=>	0.49	0.782
		<=	82.72	0.000
		<=>	109.00***	0.000
	mean spillover	A11 A22	563.34***	0.000
		B11 B22	33001.91***	0.000
		A11 A22 B11 B22	55153.24***	0.000
P_2	A&B	=>	4.98*	0.083
		<=	0.37	0.832
		<=>	5.69	0.223
	mean spillover	A11 A22	251.22***	0.000
		B11 B22	46558.52***	0.000
		A11 A22 B11 B22	274510.14***	0.000
P_3	A&B	=>	10.79***	0.005
		<=	305.23***	0.000
		<=>	306.15***	0.000
	mean spillover	A11 A22	125.44***	0.000
		B11 B22	228.55***	0.000
		A11 A22 B11 B22	390.95***	0.000
P_4	A&B	=>	4.07	0.130
		<=	15.87***	0.000
		<=>	18.43***	0.001
	mean spillover	A11 A22	46.24***	0.000
		B11 B22	36.16***	0.000
		A11 A22 B11 B22	258.17***	0.000

Note: RHBC is used as an exogenous variable. ***, ** and * indicate 1%, 5% and 10%; if "=>" is significant, then there is spillover from variable 1 to variable 2; if "<=" is significant, then there is spillover from variable 2 to variable 1; if "<=>" is significant, it indicates a two-way spillover between variable 1 and variable 2.

The parameter estimation results of the VAR-BEKK-GARCH model with RGBI as an exogenous variable are presented in **Table B3** in **Appendix B**. **Table B4** in **Appendix B** displays the corresponding Wald test results. First, with regard to cross-market spillovers, the two-way spillover effect is absent only during P_4, and the correlation between the two diminishes with the weakening of the epidemic after the outbreak. From the perspective of the effect of introducing the exogenous variable RGBI, E_{11} is found to be significantly positive in all periods except during P_2, while E_{22} is significantly negative in all periods except during P_2. This indicates that RGBI has a positive net spillover effect on RGLS and a negative net spillover effect on RHBC. However, E_{21} is significantly positive only during P_2, indicating that the exogenous variable RGBI has a partially positive effect on RHBC.

Finally, in terms of individual market spillovers, the Wald test for each market pair of A_{11}, A_{22}, B_{11}, and B_{22} reveals that RGBI, RGLS, and RHBC have significant mean and volatility spillovers, i.e. the volatility itself has significant ARCH and GARCH effects, which means that new information in the market will have an impact on future volatility. At the same time, past fluctuations also affect future fluctuations.

4.4. Discussions

This paper establishes the corresponding dynamic Copula and VAR-BEKK-GARCH-X models to explore the dynamic dependence between RGBI-RGLS, RGBI-RHBC, and RGLS-RHBC and the impact of volatility spillover.

Firstly, for the market pair RGBI-RGLS, a DCC Student-t Copula model was developed, which shows that RGBI-RGLS is negatively correlated in general, and this correlation changes frequently, especially during the COVID-19 period. Regarding the RGBI-RGLS relationship, it can be explained that the performance of green bond and green stock markets highly depends on the macroeconomic environment and policy changes. Green bonds support specific environmental projects with relatively stable returns and robust market performance, while the green stock market is susceptible to market sentiment and macroeconomic and is more volatile. Therefore, the negative correlation between the two reflects investors' risk aversion behavior under different market conditions. As shown in **Figure 2**, at the beginning of 2020, the green stock market experienced a steep decline as a result of the volatility in the global economy and financial markets caused by the outbreak of COVID-19, while the government's fiscal and monetary policies stabilized the green bond market, leading to a prominent negative relationship between the two. The global energy crisis and inflation stemming from the Russia-Ukraine conflict in 2022 also affected the Chinese green finance market.

Secondly, for the RGBI-RHBC market pair, a static Student-t Copula model is established, and the results show that RGBI-RHBC presents a weak negative correlation. This negative correlation fluctuates with different macroeconomic environments and policy changes. In certain periods, such as the global economic recession, energy crisis, and the COVID-19 pandemic, the performance of the green bond market and the carbon emission trading market may differ greatly, further exacerbating this weak negative correlation. By deeply analyzing these factors, especially the role of policy changes and market sentiment, we can better understand the volatility of this relationship.

Thirdly, for the market pair RGLS-RHBC, a DCC Gaussian Copula model is established, and the results show that RGLS-RHBC is negatively correlated in general and close to zero, indicating that the correlation between the two is weak. This can be discussed from the perspectives of supply and demand and corporate costs: as the carbon market develops, firms need to buy more carbon allowances, leading to higher prices and increased operating costs, which negatively affect profits and stock prices. As a result, when RHBC prices rise, RGLS may fall and vice versa, leading to a negative correlation. However, in certain periods, this correlation changes, and a short-term positive correlation occurs. As shown in **Figure 4**, from 2016 to 2019, the correlation between green stocks and carbon trading allowances fluctuates frequently but with small magnitudes, usually close to zero, indicating a more stable relationship. In early 2020, the outbreak of COVID-19 increases market uncertainty, resulting in changes in the correlation. In 2022 and 2023, as China introduces new policies in green finance and carbon emissions, the market again shows significant volatility.

Ultimately, the VAR-BEKK-GARCH-X model was employed to investigate the volatility spillover among markets. Firstly, in terms of market dynamic adjustment, with the development of green financial policies and carbon emissions trading market, enterprises' operating costs and market expectations have changed significantly. For example, the purchase of carbon emission rights by enterprises increases operating costs, leading to fluctuations in green stock prices, and this market dynamic adjustment is reflected in the spillover effects of RGLS and RHBC. Secondly, concerning policy effects, the implementation of green finance and carbon emission policies—including the establishment of the carbon emissions trading market and support policies for green bonds—directly influences the market performance of RGBI, RGLS, and RHBC. The changes in these policies are particularly significant during the COVID-19 period, leading to changes in the spillover effect in different periods. Finally, from the perspective of market uncertainty, macroeconomic events like COVID-19 have heightened market unpredictability, resulting in notable changes in spillover effects. For instance, during the initial phase of COVID-19, RHBC showed an increased positive spillover effect on RGLS, reflecting increased market confidence in green stocks, while its impact on RGBI weakened. By analyzing the spillover effects of RGBI, RGLS, and RHBC, one can observe how the green finance market dynamics respond to policy changes and macroeconomic events.

5. Conclusion

In this study, a dynamic Copula model and a VAR-BEKK-GARCH-X model (when X is an exogenous variable) are constructed using data of RGBI, RGLS, and RHBC from 2015 to 2024, so as to explore the dynamic correlation and volatility spillover effect situation between the markets. Its findings can be summarized from two perspectives: individual investors and policymakers.

Individual investors can use the negative correlation between green bonds (RGBI) and green stocks (RGLS) to hedge their risk, particularly in volatile market conditions like during COVID-19, where RGLS declines, and RGBI is more robust. Specific events like COVID-19 and the Russia-Ukraine conflict have affected the correlation of RGBI and RGLS, and investors should pay attention to these events and adjust their portfolios. Although the dependence between the RGBI and the carbon trading market (RHBC) is faint, there may be significant fluctuations during specific periods, necessitating attention to policy changes and market expectations. The negative correlation between RGLS and RHBC provides hedging tools for risk management by allocating these two types of assets.

For policymakers, green finance and carbon emissions policies directly impact the market performance of RGBI, RGLS, and RHBC. Policymakers should pay attention to the timing and effect of policy implementation, such as policy support during COVID-19 stabilizing the green bond market. However, the impact on green stocks is notably complex. In 2022, China introduced several policies to support green finance, fostering the growth of green bonds and stocks, and prompting adjustments in market expectations. Macroeconomic events such as COVID-19 have increased market uncertainty, and policymakers need to take regulatory measures to stabilize markets, such as fiscal and monetary policies, to stabilize the green bond market during the initial phases of COVID-19, despite its effects on the carbon market. Long-term green finance and carbon reduction policies, such as CERF, can help respond to market changes and enhance market stability and development.

Overall, the green finance market generally has a complex dynamic relationship under macroeconomic events and policy changes. Investors should pay attention to market fluctuations and policy changes and adjust investment strategies to realize risk hedging and maximize returns. Policymakers must ensure market stability and sustainable development through reasonable policy support and regulation measures. In the context of supporting green development, the rational allocation of green bonds, green stocks, and carbon emission trading assets is strategically vital to both sides.

Future research could explore the potential of integrating machine learning or advanced econometric techniques into predictive modeling to more accurately forecast the dynamic dependencies and volatility spillover effects under complex macroeconomic conditions. Secondly, when studying the dynamic dependence of green bonds and carbon emission trading markets, there is a very weak negative correlation. In the future, the DCC model can be used to further refine the changes in the correlation between the two during this period, especially considering the different impacts of policies and market events, to reveal a more complex dynamic relationship.

Data Availability Statements

The datasets that support the findings of this study are openly available in wind database at https://www.wind.com.cn/portal/en/WDS/database.html.

Declaration

This article uses ChatGPT to revise some code and Grammarly to correct grammar, ensuring smooth sentence flow.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix A

Table A1. Optimal choice of lag order of VAR model.

MODEL		AIC	HQ	SC	FPE
RGBI-RGLS	k	**3**	2	1	**3**
	LL	18009.406	18005.563	17996.034	18009.406
RGBI-RHBC	k	**3**	2	1	**3**
	LL	16288.263	16285.267	16278.048	16288.263
RGLS-RHBC	k	**1**	**1**	**1**	**1**
	LL	10539.653	10539.653	10539.653	10539.653

Note: Bolded in the table is the optimal VAR model order. "k" represents the order. "LL" represents the log-likelihood function.

Appendix B

Table B1. Estimated results of VAR-BEKK-GARCH-RGLS model.

Period	P_1		P_2		P_3		P_4	
Variable/ Parameter	Coef.	Sig.	Coef.	Sig.	Coef.	Sig.	Coef.	Sig.
C(1, 1)	0.0000	0.714	0.0001***	0.000	−0.0001***	0.000	0.0000	0.39
C(2, 1)	0.0002	0.849	0.0138***	0.000	0.0013	0.155	−0.0011	0.57
C(2, 2)	0.0103***	0.000	0.0082*	0.094	0.0045***	0.000	0.0068***	0.00
A(1, 1)	0.4345***	0.000	0.3557***	0.000	0.3285***	0.000	0.3835***	0.00
A(1, 2)	3.0279***	0.000	5.5488***	0.000	2.2702	0.101	−8.5877**	0.02
A(2, 1)	−0.0003	0.415	−0.0004	0.604	−0.0010**	0.044	−0.0002	0.88
A(2, 2)	0.6683***	0.000	0.7549***	0.000	0.7053***	0.000	0.4529***	0.00
B(1, 1)	0.8553***	0.000	0.9302***	0.000	0.8143***	0.000	0.8228***	0.00
B(1, 2)	−7.9088***	0.000	−4.121***	0.000	−0.3544	0.583	−4.6804	0.46
B(2, 1)	−0.0050***	0.000	0.0000	0.964	0.0009***	0.001	−0.0022	0.48
B(2, 2)	−0.6825***	0.000	0.4599***	0.000	0.7843***	0.000	−0.7865***	0.00
E(1, 1)	0.0150***	0.000	−0.0003	0.889	−0.0208***	0.000	0.0222***	0.00
E(2, 1)	−0.0461	0.111	0.1825**	0.045	0.0764*	0.066	−0.0812	0.61
E(2, 2)	0.2705***	0.000	−0.0877	0.379	0.3218***	0.000	0.6207***	0.01

Note: RGLS is used as an exogenous variable. *, **, and *** indicate statistical significance at the 10%, 5% and 1% level, respectively. "Coef." represents the coefficient value, and "Sig." represents the p-value. "P_1" represents the period from July 2, 2015, to April 15, 2024. "P_2" represents the period from July 2, 2015, to December 6, 2019. "P_3" represents the period from December 6, 2019, to December 7, 2022. "P_4" represents the period from December 9, 2022, to April 15, 2024.

Table B2. Wald test of VAR-BEKK-GARCH-RGLS model.

Period		Spillage scenarios	Chi-Squared	*p*-value
P_1	A&B	=>	74.56***	0.000
		<=	44.84***	0.000
		<=>	114.38***	0.000
	mean spillover	A11 A22	830.28***	0.000
		B11 B22	3821.57***	0.000
		A11 A22 B11 B22	10291.38***	0.000
P_2	A&B	=>	50.37***	0.000
		<=	0.70	0.703
		<=>	50.55***	0.000
	mean spillover	A11 A22	464.84***	0.000
		B11 B22	7797.21***	0.000
		A11 A22 B11 B22	22004.50***	0.000
P_3	A&B	=>	2.80	0.247
		<=	10.90***	0.004
		<=>	13.80***	0.008
	mean spillover	A11 A22	284.93***	0.000
		B11 B22	4741.75***	0.000
		A11 A22 B11 B22	11756.75***	0.000
P_4	A&B	=>	5.31*	0.070
		<=	0.53	0.767
		<=>	5.34	0.255
	mean spillover	A11 A22	57.12***	0.000
		B11 B22	443.30***	0.000
		A11 A22 B11 B22	1108.59***	0.000

Note: RGLS is used as an exogenous variable. ***, ** and * indicate 1%, 5% and 10%; if "=>" is significant, then there is spillover from variable 1 to variable 2; if "<=" is significant, then there is spillover from variable 2 to variable 1; if "<=>" is significant, it indicates a two-way spillover between variable 1 and variable 2.

Table B3. Estimated results of VAR-BEKK-GARCH-RGBI model.

Period	P_1		P_2		P_3		P_4	
Variable/ Parameter	Coef.	Sig.	Coef.	Sig.	Coef.	Sig.	Coef.	Sig.
C(1, 1)	0.0012***	0.000	0.0008***	0.001	0.0059***	0.000	0.0015	0.188
C(2, 1)	0.0033**	0.035	−0.0062	0.111	−0.0026	0.170	−0.0069*	0.095
C(2, 2)	0.0092***	0.000	0.0125***	0.000	0.0040*	0.072	0.0100***	0.000
A(1, 1)	0.2554***	0.000	0.2313***	0.000	0.2243***	0.000	−0.0107	0.893
A(1, 2)	0.0744*	0.068	0.2269***	0.001	−0.1731***	0.008	0.2360	0.276
A(2, 1)	0.0164**	0.040	−0.0146	0.259	0.0387*	0.062	−0.0008	0.974
A(2, 2)	0.6531***	0.000	0.7380***	0.000	0.7535***	0.000	0.6133***	0.000
B(1, 1)	0.9575***	0.000	0.9709***	0.000	0.8265***	0.000	0.9740***	0.000
B(1, 2)	−0.0529***	0.009	−0.0264	0.452	0.1308	0.282	0.1396	0.228
B(2, 1)	−0.0133***	0.002	0.0109	0.291	−0.0087	0.392	0.0039	0.812
B(2, 2)	0.7354***	0.000	0.5390***	0.000	0.7775***	0.000	0.5450***	0.000
E(1, 1)	0.9929***	0.000	−0.2695	0.216	1.8030***	0.000	1.0171**	0.020
E(2, 1)	2.5481***	0.005	7.8220***	0.000	0.7510	0.311	3.5190	0.718
E(2, 2)	−5.2318***	0.000	1.7763	0.490	−2.3432*	0.057	−25.854***	0.000

Note: RGBI is used as an exogenous variable. *, **, and *** indicate statistical significance at the 10%, 5% and 1% level, respectively. "Coef." represents the coefficient value. "Sig." represents the p-value.

Table B4. Wald test of VAR-BEKK-GARCH-RGBI model.

period		Spillage scenarios	Chi-Squared	p-value
P_1	A&B	=>	7.46**	0.024
		<=	9.91***	0.007
		<=>	20.21***	0.000
	mean spillover	A11 A22	637.33***	0.000
		B11 B22	25762.99***	0.000
		A11 A22 B11 B22	94592.46***	0.000
P_2	A&B	=>	11.20***	0.004
		<=	1.31	0.520
		<=>	13.40***	0.009
	mean spillover	A11 A22	329.36***	0.000
		B11 B22	46898.82***	0.000
		A11 A22 B11 B22	290759.66***	0.000

Continued

P_3	A&B	=>	8.06**	0.018
		<=	4.03	0.133
		<=>	12.17**	0.016
	mean spillover	A11 A22	263.94***	0.000
		B11 B22	2159.23***	0.000
		A11 A22 B11 B22	6743.67***	0.000
P_4	A&B	=>	2.99	0.224
		<=	0.09	0.955
		<=>	3.16	0.531
	mean spillover	A11 A22	53.77***	0.000
		B11 B22	1071.96***	0.000
		A11 A22 B11 B22	1258.38***	0.000

Note: RGBI is used as an exogenous variable. ***, ** and * indicate 1%, 5% and 10%; if "=>" is significant, then there is spillover from variable 1 to variable 2; if "<=" is significant, then there is spillover from variable 2 to variable 1; if "<=>" is significant, it indicates a two-way spillover between variable 1 and variable 2.